THE OCCURRENCE PROBABILITIES OF ROGUE WAVES IN DIFFERENT NONLINEAR STAGES

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The occurrence probabilities of Rogue Waves in different nonlinear states are investigated based on high-order spectral method (HOSM), which is a direct phase-resolved numerical method. The focus is given to the occurrence probability of Rogue Waves in the nonlinear evolution stage where the Benjamin-Feir Instability is not the dominate mechanism due to the quartet resonance interactions for wave fields with a broad range in frequencies. The initial wave trains are generated from Stokes waves and two sidebands. Based on the simulation, we find that the Kurtosis evolves distinctly at three nonlinear stages and shows a weak relation to the probabilities of the Rouge Waves. We also introduce a simple Entropy formula, turning out to close a stable value.

Keywords: Rouge Waves, HOS, Nonlinear mechanism, probability, Kurtosis, Entropy

1. Introduction

The concept of Rouge Wave was first put forward by Draper (1965) to describe exceptionally high waves surprisingly appearing on the sea surface, which may cause disastrous damages to offshore structures and surface vessels. One typical and common operational definition of Rouge Wave defines that its wave height should exceed the significant wave height in 2 times. In the recent two decades, a large number of Rogue Wave studies have been conducted theoretically, numerically, experimentally and based on field data, which have significantly advanced our knowledge of this kind special and notorious disastrous wave. Some new results including the discussion and debate for the Rouge Waves have been summarized by Akhmediev & Pelinovsky (2010) in the special topics of the European Physical Journal. Due to the potential risk, the Rogue Waves have attracted the attention of the shipping and offshore industry. So that some big international research projects have been initiated as mentioned by Bithner-Gregersen & Toffoli (2012). However, there is still not a unifying concept towards the Rogue Waves can be concluded. The occurrence probability of Rogue Waves is one of the most interesting topics since it related to the engineering design practice directly. In the last decades, a number of works haves been done is this field, such as Mori (2006), Akhmediev (2011), Tomas (2012), Xiao (2013), Wang(2014) based on milestone work conducted by Janssen (2003), where the Benjamin-Feir-Index(BFI) was introduced and explained theoretically. However, physically, BFI is useful because it is a parameter showing the conditions of the Benjamin-Feir Instability. If a wave train can propagate in a large scale distance, or a sea state can evolve for a long time, such as the ocean in the monsoon seasons, the nonlinearity may grow to a stage beyond the Benjamin-Feir Instability can control, since the quartet resonance interactions for wave fields with a broad range in frequencies and directions. The effectiveness of BFI characterizing the occurrence probability of Rogue Waves seems to be a problem.

One of The latest studies for the probability of Rogue Waves has been performed by Bithner-Gregersen & Toffoli (2012), focusing on the design practice. One of their results is that 'the extreme crest considerably increases for prolonged sea state durations'. Considering the concept, which indicates that there are different kinds of Rogue Waves according to their intrinsic nonlinear mechanisms, shown by Liu (2006) based on field data and Tao(2011, 2012a, 2012b) via numerical simulations. It does need carry out some studies to understand the characteristics of the Rouge Waves in a long time evolved wave state, including the occurrence of them.

2. Approach

After a systemically verification with experimental data provided by Shemer & Sergeeva(2009), the high-order spectral method(HOSM) developed by Dommermuth & Yue(1987) is applied for direct phase-resolved simulation of nonlinear random wave fields evolution. HOSM resolves the phase of a large number (N) of wave modes and accounts for their nonlinear interactions up to an arbitrary high order (M) including broadband non-resonant and resonant interactions up to any specified order. Meanwhile viscous dissipation and wave breaking dissipation are modelled in domain. Due to its high efficiency and accuracy, the HOSM is an effective approach for long-time and large-space simulation of nonlinear wave-field evolutions.

3. Modulated Stokes wave train long time evolution

In this section, we will investigate the characteristics of long time evolution processes generated from modulated Stokes wave train. First we studied the evolution process of narrow band modulated wave train, constructed by a Stokes carrier wave and imposed sideband according to most unstable modulation instability condition. Then the characteristics of extreme waves will be discussed to introduce the evolutions of different parameters.

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3.1 Initial condition with weakly modulated wave train

For the narrow band modulated wave train, the initial conditions we used are as following.

$$\left.\begin{aligned}\eta(x,0) &= \eta[\varepsilon_0, k_0] + r_1 a_0 \cos(k_- x - \theta_-) + r_2 a_0 \cos(k_+ x - \theta_+) \\ \phi^s(x,0) &= -\phi^s[\varepsilon_0, k_0] + \frac{r_1 a_0}{\sqrt{k_-}} e^{k_- \eta} \sin(k_- x - \theta_-) + \frac{r_2 a_0}{\sqrt{k_+}} e^{k_+ \eta} \sin(k_+ x - \theta_+)\end{aligned}\right\}$$

where $\eta(x,0)$ and $\phi^s(x,0)$ are respectively the free-surface elevation and potential of a right-going Stokes wave of steepness ε_0 and wave number k_0. Here $\varepsilon_0 = k_0 a_0$ and a_0 is the surface elevation of carrier wave. $k_\pm = k_0 \pm \Delta k$ and $\theta_\pm$ are wave numbers and phases of sideband respectively. Those parameters are chosen as the most unstable conditions in the initial period. The initial wave relative amplitude spectrum and wave surface for $\varepsilon_0 = 0.05$ are shown in Fig.1. For all the cases, the calculation domain is $200 l_0$. The parameters for HOS are M=7, N=8192, $T_0/dt = 128$. Here dt is the time step for simulation.

Tab.1. Parameters for modulated wave train					
Cases	M	$\Delta k / k_0$	$\theta_\pm$	$r_{1,2}$	Evolution time (t/T0)
I	7	0.1	$\pi / 4$	0.1	30000

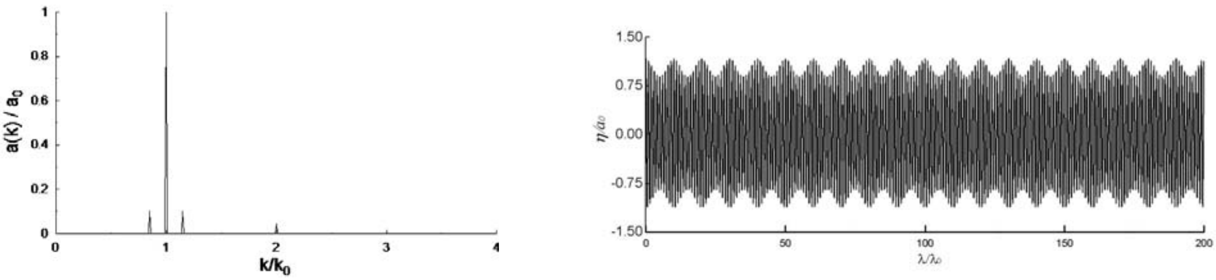

Fig.1. Initial wave relative amplitude spectrum (left) and wave surface (right) for $\varepsilon_0 = 0.05$

Using HOS method, calculation accuracy would significantly enhances as wave modes (N) being incredibly close to nonlinear wave order (M). Higher the nonlinear wave order (M), smaller the round-off errors. However, higher precision requiring higher demand of computing makes it necessary for us to determine the suitable M for this research. The variable E/E_0 shown in Fig.2 indicates the conservation of the wave train with minute energy dissipation (only less than 0.01%) over evolution time. As shown in the picture, it has smooth evolution process, assuring the numerical results.

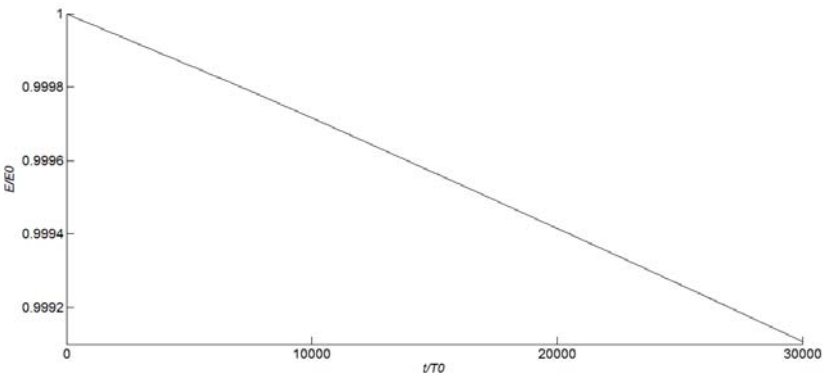

Fig.2 Energy evolution of Strokes wave train for $\varepsilon_0 = 0.05$**, r1=0.1**

3.2 Characteristics of Freak waves and the related mechanisms

3.2.1 Kurtosis

Characteristic parameters such as Kurtosis, occurrence probability, etc. change as waves propagate and the Kurtosis is calculated as

$$Kurtosis(t_i) \equiv \frac{1}{N_x} \sum_{j,t=t_i} \eta_j^4 / \eta_{rms}^4 \;, \qquad \eta_{rms} = \sqrt{\frac{1}{N_x} \sum_i \eta_i^2}$$

where η is the sea level displacement with zero mean level, η_{rms} is root-mean-square value of η, and N_x is the number of values. Kurtosis describes the degree that actual probability distribution deviates from Gaussian distribution with the value of 3. In this paper, the Kurtosis is calculated based on the wave surface elevation of whole calculation domain for each carrier wave period T_0. From Fig.3, it can be seen clearly that the Kurtosis presents distinct different properties in different nonlinear stages. The Kurtosis tends to increase gradually and the average value is greater than 3 beyond the B-F instability dominated stage.

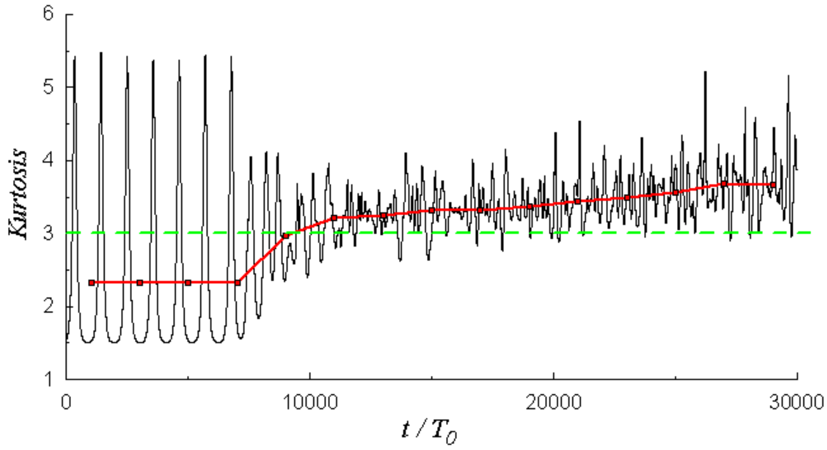

Fig.3.The evolution of Kurtosis and the average trend

Fig.4 is given to find out the relationships between Kurtosis and extreme waves in each time scale. Interestingly, there appears to have distinct trends at different stages of evolution. In Fig.5, the relationship between the Kurtosis and the extreme waves at the first stage (0<t/T0<=7100) can be given as a fitting equation ($y = 0.5052e^{0.9582x}$ with correlation coefficient of R=0.9875) which is close to exponential manner while the relationship at the third stage (9000<t/T0<=30000) shows a linear trend with much lower correlation. There is also an intermediate stage (7100<t/T0<=9000) between the two physical nonlinear ones. It can be deduced that the Kurtosis is close related to the extreme waves in different nonlinear stages.

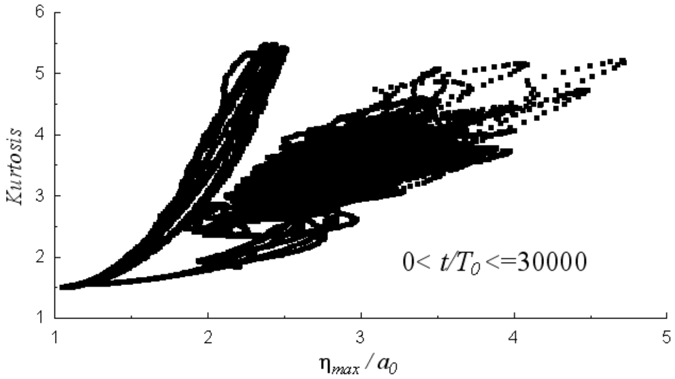

Fig.4.Relationship between the Kurtosis and the extreme waves

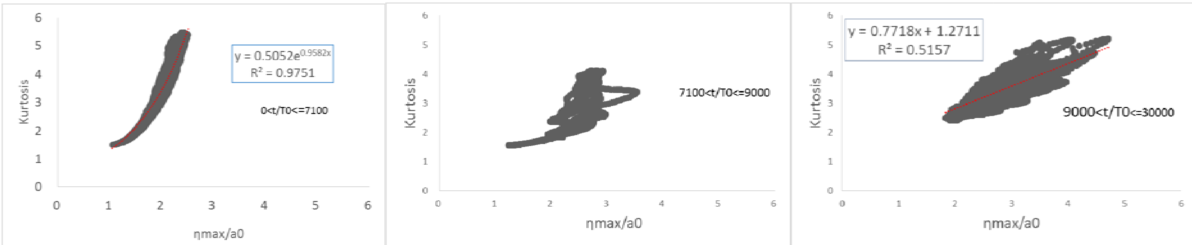

Fig.5.Relationships between the Kurtosis and the extreme waves at three nonlinear stages

3.2.2 Occurrence probability

Occurrence probabilities of the Rouge Waves, which are calculated based on the wave surface elevation of whole calculation domain at each period, evolve with time as left picture of Fig.6 shows. It could be perceived that the probabilities of Rouge waves are always large at the peak of modulation. In order to catch the trend, the probabilities are averaged for each 3000 T_0 shown in the right picutre, which become stable and present increase trend beyond the B-F instability dominated stage.

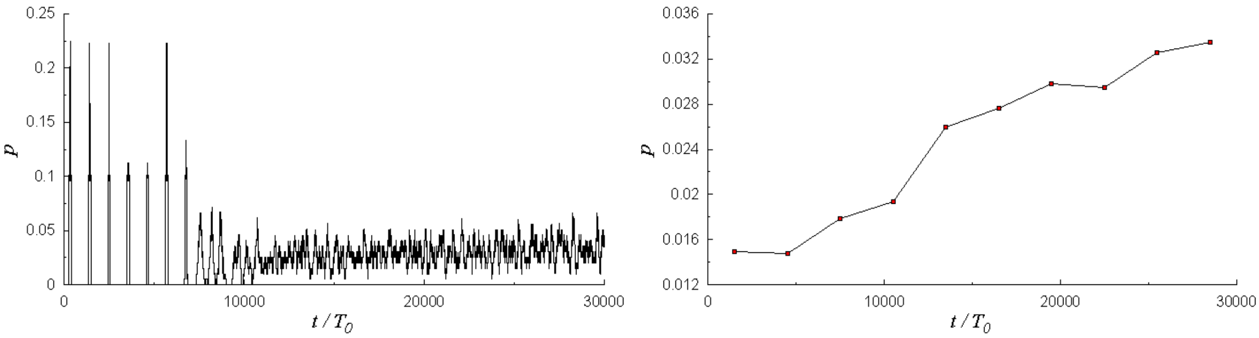

Fig.6.The evolution of occurrence probabilities of Rouge Waves and its average trend

We try to find parameters which related to the probability, the results are shown in Fig.7. The relationship between probability of Rouge Waves and extreme wave heights is obscure, which means higher Rouge Waves and high probabilities may not occur together, while the probabilities weakly related to the Kurtosis, which means the probabilities have a trend to increase with high nonlinearity.

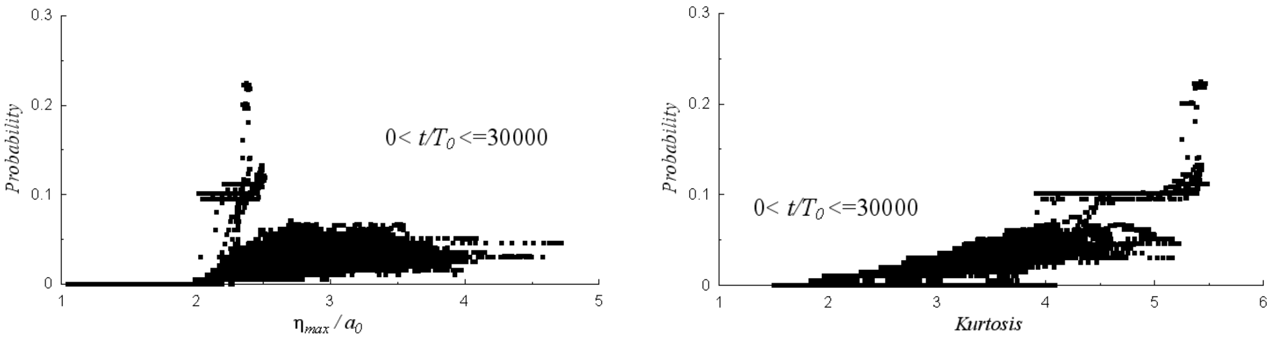

Fig.7.Relationships between occurrence probability and other parameters (extreme waves and the Kurtosis respectively, from left to right)

To further study the varying pattern of probabilities of Rouge Waves based on fix point wave surface, we set four fix points working as anchoredbuoys (shown in Fig.8).We calculated the probabilities based on the wave surface time series on spot1 for each 128 T_0 and the average values are calculated from each 16 probabilities. Clearly to see from Fig.8, the probabilities from fix points present a similar trend with those from whole calculation domain.

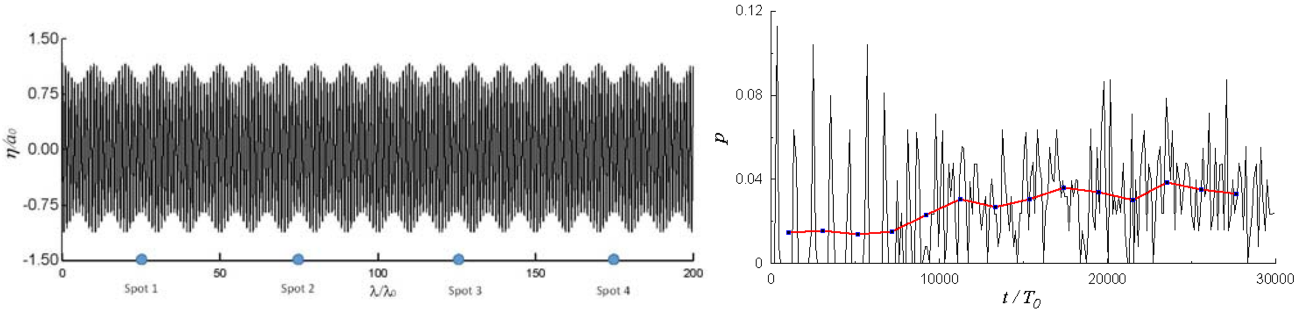

Fig.8.Fix points setting schematic and the evolution of probabilities based on fix point wave surface (from left to right)

3.2.3 Entropy

After initial wave train evolves over enough long periods, it processes into random waves. In order to investigate the fundamental changes for the whole wave field, a simple Entropy formula is introduced to express the generated random waves. Here, Entropy is calculated as

$$E = \frac{-\sum p \ln(p)}{\ln(N)}$$

Wave heights are counted appling the zero-up (or zero-down) crossing method and divided into several intervals. In the expression above, p presents the probability that wave heights lie in a certain interval and N is the number of intervals. Entropy ranges from 0 to 1 while the value corresponding to the actual ocean waves falls in between. The Entrophy of Gauss spectrum and JONSWAP spectrum are 0.29 and 0.39 respectively. The Entrophy of our case is increasing along with the evolution process from imposed wave train to random waves as shows in Fig.9. Its tendency, being to close a stable value, may relate a particular wave spectrum.

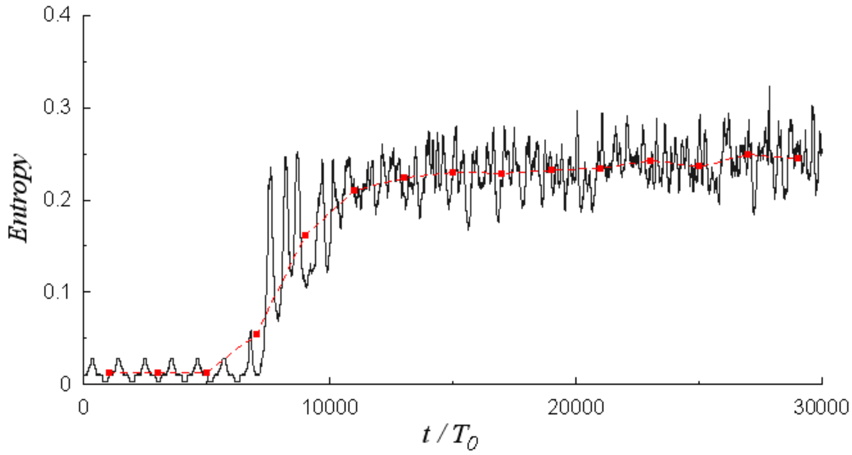

Fig.9.Evolution of the Entropy of wave heights

4 Conclusion

We consider a very long-time evolution of modulated Stokes wave trains, which is obtained using nonlinear HOS simulation (with order M =7). On the basis of simulated results, the Kurtosis is close related with extreme waves in the whole long time evolution processes while the relationships are different distinctly for three nonlinear stages. For the probabilities of Rouge Waves, it presents a weak trend to increase with evolution time going on. Finally, by introducing a simple Entropy, we find the wave field displays a trend to be random waves and the wave spectrums tend to be a stable formula.

Acknowledgments

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