ESTIMATION OF MAXIMUM WAVE HEIGHTS BY WAVE SPECTRAL MODEL AND NONLINEAR WAVE THEORY

Nobuhito Mori[1], Yusuke Takagi[2], Koji Kawaguchi[3], Hiroaki Kashima[3], Hajime Mase[1], Tomohiro Yasuda[1] and Hiroaki Shimada[2]

The strong winter storm crossed from the west to east of the Sea of Japan developing rapidly in early April, 2012. The minimum pressure of the winter storm reached 964 hPa at 21:00 on 3rd April. The storm generated wind waves more than 10 m in the significant wave height and gave severe damages to coastal structures around Japan. This study analyzed observed wave records at 12 stations along the Sea of Japan. A series of numerical analysis was performed to understand the characteristics of extreme wave sea condition. The maximum wave heights were estimated based on the spectral wave model and nonlinear short wave statistical theory. The estimated maximum wave heights by the nonlinear theory show better agreement with the observed peak of maximum wave height than that by the linear theory.

Keywords: extreme wave, spectral wave model, maximum wave height, freak wave

INTRODUCTION

A third generation spectral wave model has been developed to estimate wave spectral energy evolution of wind waves and swells the last few decades. The wave model shows reasonable accuracy of characteristic wave properties against observed wave records. Although, the wave model only estimate integral of spectral energy such as significant wave height H_s, the maximum wave height H_{max} is also important to assess coastal hazards (e.g. offshore wind turbine, vessel). The mathematical model to estimate a maximum wave height in a wave train has been developed as a part of freak wave prediction the last decade. It has been demonstrated that the quasi nonlinear interactions plays important roles to enhance the extreme wave probability.

The maximum wave height theory has been developed in the context of freak wave prediction (Yasuda et al. 1992; Onorato et al. 2001). The occurrence probability of extreme wave is increased if the fourth order moment of surface elevation, kurtosis, becomes large. Janssen (2003) found that the ratio between the steepness, a measure of the nonlinearity, and the spectral bandwidth, a measure of the dispersion, is an important parameter for determining the probability of finding a large wave. After Janssen, this ratio between nonlinearity and dispersion was named the Benjamin–Feir index (BFI); its relation to the kurtosis has been found in Janssen (2003) in the limit of large times and for narrow banded spectra neglecting directional dispersion. The role of the skewness in the wave height distribution is less important with respect to kurtosis. The skewness comes usually as a result of second-order corrections (bound modes) and is weakly affected by the dynamics of free waves. In Mori and Janssen (2006) the formal relation between kurtosis, the maximum wave height and occurrence probability of a freak wave has been discussed. The kurtosis enters in the maximum wave height distribution function as a nonlinear correction to the Rayleigh distribution. Waseda (2006) and Onorato et al. (2009a) investigated the monotonic decrease of kurtosis as directional effects become significant and the comparison of two different directional wave experiments was summarized in Onorato et al. (2009b). These results show that the directional dispersion effects suppress the kurtosis enhancement in directional sea states. Mori et al. (2011) proposed new parameterization of kurtosis from spectra considering directional dispersion effects. Once the kurtosis is estimated from wave information, the probability of maximum wave height in a train can be estimated by the nonlinear wave height distribution (e.g. Mori and Janssen, 2006). The theoretical frame work has been validated by wave flume experiments and numerical experiments, respectively. However, the validation of theoretical frame work does not well examined in the real ocean yet.

The state of art of nonlinear theories can estimate probability of maximum wave height from given directional spectra for given initial conditions in wave tank or numerical experiments. This study examines the maximum wave height prediction in cooperating with wave spectral model for the severe stormy condition in field. The validation of nonlinear theory of maximum wave height from wave spectra is conducted with observed buoy data along the storm track.

[1] Disaster Prevention Research Institute, Kyoto University, Uji, Kyoto 611-0011, Japan
[2] Kansai University, 3-3-35 Yamate-chou, Suita, Osaka 564-8680, Japan
[3] The Port and Airport Research Institute, 3-1-1 Nagase, Yokosuka 239-0826, Japan

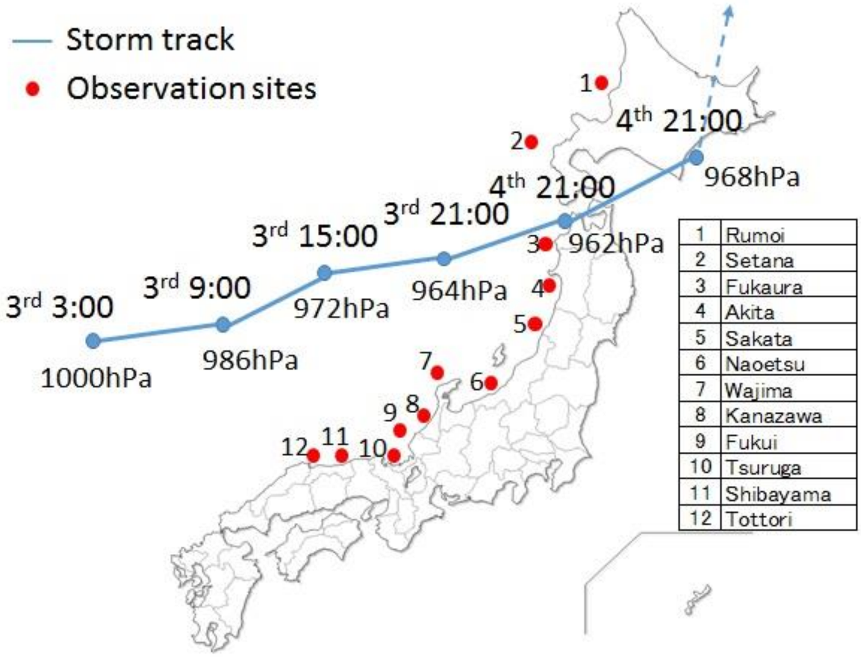


Figure 1. Track of winter storm (-) during April 3th-4th in 2012 and locations of wave observation (●) (line: center of storm track, circle: observed locations)

METHODOLOGY

The estimation of wave field was conducted spectral wave model and SWAN model was used to simulate wave spectral evolution. The extreme weather event is selected as a case study because the enhancement of maximum wave height by quasi-resonant nonlinear interactions is expected for severe weather wave field. The selected weather event is the strong winter storm crossed the Sea of Japan in early April of 2012 (see Figure 1). A series of numerical experiments was performed to understand the characteristics of extreme wave sea condition for 2012 event.

An estimation of maximum wave height, H_{max}, is important issue for disaster reduction and designing structures, although significant wave height $H_{1/3}$ (calculated by wave profile) or H_s (calculated by spectra) plays major factor for engineering design due to statistical uncertainty. However, recent progress of research on extreme wave modeling found that H_{max} and related freak wave is associated with short-time scale four wave interactions (quasi-resonant interactions) and nonlinear short term statistical properties. The developed framework of theory and parameterization has been verified by physical wave modeling and numerical experiments by nonlinear Schrodinger equations and Zakharov equation and the others (e.g. Mori et al., 2007).

The maximum wave heights in this study are estimated by the framework combining the spectral wave model and nonlinear wave theory formulated by the authors (Mori et al., 2006 and 2011; denotes MaxWaveNL hereafter). The measure of quasi-resonant interactions for H_{max} in deep-water directional wave filed is kurtosis of surface elevation and Benjamin-Feir Index in wave directional wave spectra (BFI-2D). There are several parameterization of relation between kurtosis and BFI, we adopt following parameterization proposed by Mori et al. (2011).

$$\mu_4 = \frac{\pi}{\sqrt{3}} \frac{BFI}{1+\alpha R} \tag{1}$$

where μ_4 is kurtosis of surface elevation, *BFI* is defined by $\sqrt{2}ak/\delta_\omega$, $R=\delta_\theta^2/2\delta_\omega^2$ is ration of directional and frequency spectrum bandwidth, ak the characteristic wave steepness, δ_ω and δ_θ are directional and frequency spectrum bandwidth, α the empirical constant. Three parameters, ak, δ_ω and

Table 1. Numerical setup of spectral wave mode (Δx: spatial resolution, $\Delta\theta$: directional resolution, Δf: frequency resolution, GSE: threshold period of garden sprinkler effects)

No.	Δx [deg.]	$\Delta\theta$ [deg.]	Δf [bin]	GSE [s]
Case 1	0.10	10.00	24	
Case 2	0.10	3.33	24	
Case 3	0.10	10.00	24	30
Case 4	0.10	10.00	24	60
Case 5	0.05	10.00	24	
Case 6	0.05	10.00	24	60
Case 7	0.05	3.33	24	

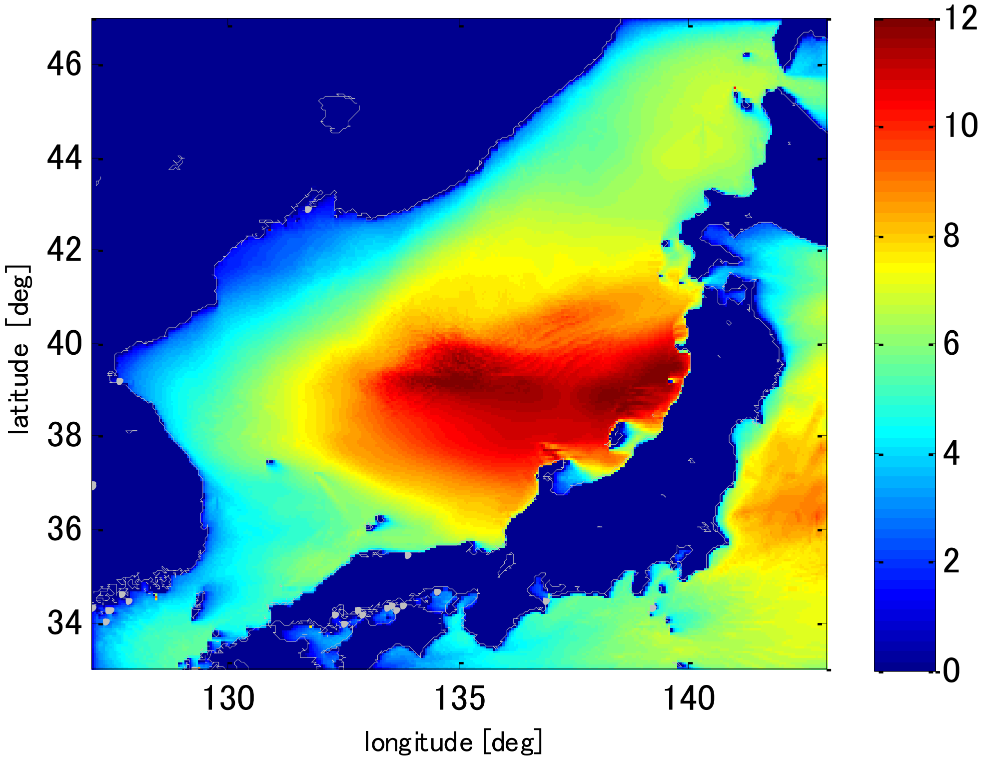


Figure 2 Estimated maximum H_s by wave model with JMA analysis forcing U_{10} over the period (contour: H_s in meter)

δ_θ can be calculated from the directional spectra. Once kurtosis is estimated, the probability density function of H_{max} can be estimated by the nonlinear statistical theory (Mori and Janssen, 2006).

$$p_m(H_{\max}) = \frac{N}{4} H_{\max} e^{-\frac{H_{\max}^2}{8}} \left[1 + (\mu_4 - 3) A(H_{\max})\right] \times \exp\left\{ -N e^{-\frac{H_{\max}^2}{8}} \left[1 + (\mu_4 - 3) B(H_{\max})\right] \right\} \qquad (2)$$

where p_m the probability density function of H_{max}, N the number of waves in the wave train, A and B are polynomials. The probability of H_{max} depends on wave nonlinearity and length of storm, respectively. In addition Eq.(2) is identical to the Rayleigh based H_{max} theory if the wave field follows the linear condition (μ_4=3). The combining Eq.(1) and (2) using the directional spectrum information, it is possible estimate the statistical properties of H_{max} (e.g. expected value of H_{max}).

A series of numerical experiments was performed to understand the characteristics of extreme wave sea condition for 2012 event. The estimation of wave field was conducted spectral wave model and SWAN model was used to simulate wave spectral evolution. The configuration of SWAN model was examined changing the both spatial and directional resolutions and nonlinear wave interaction term of S_{nl} as DIA or exact interactions to check the sensitivity of spectral evolution (see configuration in Table 1). The source terms were given by the standard setup for S_{in} as Janssen's linear growth proposed and S_{ds} as Komen's dissipation, respectively.

Forcing of SWAN was the wind speed at 10 m height, $U_{10,}$ atmospheric reanalysis data by MSM model by Japan Metrological Agency. The bathymetry was given by GEBCO_08 Grid (a global 30 arc-second grid) with spatial linear interpolation. The spatial resolution Δx and directional resolution $\Delta\theta$ were given by 0.05 or 0.10 degree and 3.33 or 10.0 degree, respectively. The garden sprinkler effect was also examined to reduce the radial energy propagation from storm center to outside as shown in Table 1. As explain later part of this manuscript, the most accurate results is case 2 in Table 1, and the results by case 2 is going to discuss hereafter.

EVENT OF TARGET AND OBSERVATION

A target of storm for analysis is winter depression passed over the Sea of Japan from 3rd to 4th in April, 2012 (Figure 1). The storm passed from west to east and was grew up to 964 hPa at 21:00 in April 3rd, although it was 1006 hPa at 21:00 in April 2nd (JMA 2012). The growth of the storm was 42 hPa within 24 hours and minimum pressure, 964 hPa, was the most severe winter storm in the historical record in this area. The storm generated wind waves more than 10 m in H_s and gave several damages along the west part of Japanese coast as sliding caisson, overtopping and etc. (Figure 2). The winter storm at the Sea of Japan is generally severe similar to the North Sea but the Sea of Japan is semi enclose sea. Therefore no swell effects can be considered for wave field. The extreme wave study at the Sea of Japan can be regarded as fetch limited experiments as some field experiments in a lake, although the size of Sea of Japan is quiet large. Therefore, the Sea of Japan is ideal area for the field experiments due to enclosed lateral boundary condition.

The validation of numerical model and theory was conducted for observed wave records at 12 stations along the Sea of Japan (Figure 1). The wave data was taken by NOWPHAS system (Japanese Nationwide Ocean Wave information network for Ports and HArbourS) operating by Ministry of Land, Infrastructure, Transport and Tourism (MLIT) of Japan. The raw data was independently to analyze focusing on the maximum wave height for this study. The surface elevation was filtered by the post-processing with despiking and etc. The short wave statistics including, the maximum wave height, wave steepness, frequency bandwidth, directional bandwidth and the others were computed in detail.

RESULTS AND DISCUSSION

The storm generated more than 10 m in H_s along the western coast of Japan (Figure 2). The observed $H_{1/3}$ and H_{max} were extremely large at the Sea of Japan. The maximum significant wave height at Wajima (point 7 in Figure 1) was 9.3 m and the maximum $H_{1/3}$ was 11.0 m and H_{max} was 20.3m at Sakata (point 2 in Figure 2). The extreme wave is distributed southern side of storm track as shown in Figure 1 and 2.

The observed H_{max} and nonlinear wave statistics was clear. There is weak correlation between H_{max} and kurtosis as same as previous studies. The kurtosis was slightly large at the beginning of storm and became smaller near peak of storm generally. The observed kurtosis distributed in the range of 2.7 to 3.4, thus the wave field had weak nonlinear properties.

The hindcast results gives satisfactory correlation between observed H_s and simulated H_s as giving the correlation coefficient R=0.94 (SI: 11%, Bias: 0.41 m). The errors of maximum H_s of each location are listed in Table 2. The estimated maximum H_s is slightly overestimated at the most of location in the range of 0.2 to 0.6 m which agrees with previous studies of wave hindcast. The bias is not clearly observed but the negative bias can be found in the south part of the Sea of Japan (right side of storm track) and more positive bias can be found in the north part of it. The overestimation of H_s, positive bias, can be found the most of severe storm area from point 1 to 9 in Figure 1 and Table 2. The numerical setup gave significant impact on the accuracy of hindcast as shown in Table 2. Although the nonlinear energy transfer term S_{nl} and GSE do not play significant change on the accuracy of H_s, the both spatial and directional resolution changes the results greatly. The mean difference and absolute difference of H_s at the peak of storm are smaller than 0.2 m and 0.8 m, respectively. Overall the

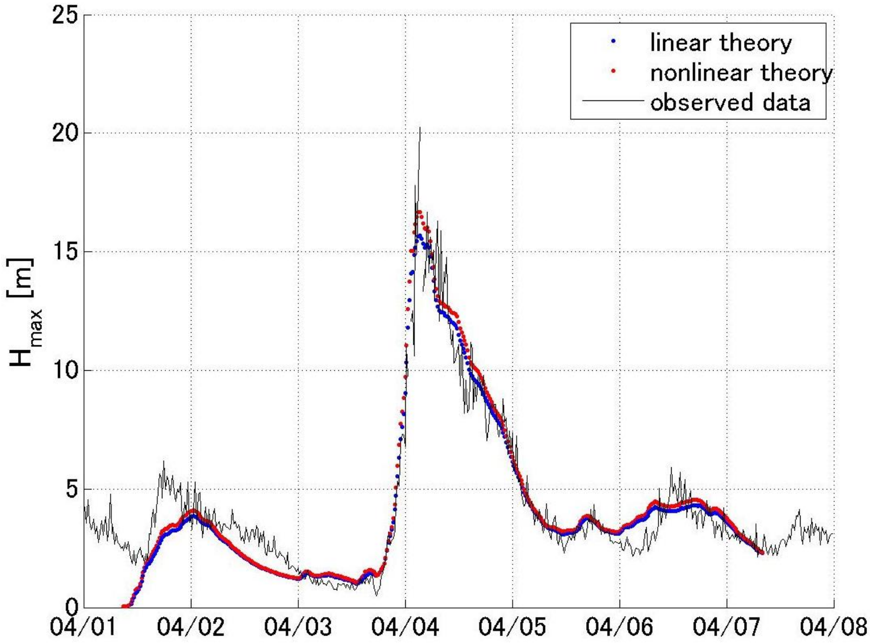
Table 2 Accuracy of maximum H_s at the peak of storm at each location: model minus observation (location #1-12 from top to bottom, case #: different model configuration, unit: m)

Case	1	2	3	4	5	6	7
	Δx=0.1	Δx=0.1	Δx=0.1	Δx=0.1	Δx=0.05	Δx=0.05	Δx=0.05
	Δθ=10	Δθ=3.3	Δθ=10	Δθ=10	Δθ=10	Δθ=10	Δθ=3.3
Location			GSE=30s	GSE=60s		GSE=60s	
Tottori	1.00	0.37	0.99	1.01	1.32	1.31	0.64
Shibayama	-0.37	-1.02	-0.36	-0.37	0.05	0.06	-0.76
Tsuruga	-0.20	-0.27	-0.21	-0.20	0.53	0.53	0.36
Fukui	-1.10	-1.37	-1.10	-1.10	-1.00	-1.00	-1.32
Kanazawa	0.73	0.57	0.73	0.73	1.04	1.04	0.87
Wajima	0.90	0.55	0.90	0.90	0.86	0.86	0.36
Naoetsu	1.74	1.14	1.72	1.76	2.68	2.67	1.50
Sakata	0.43	-0.49	0.41	0.40	0.25	0.24	-0.68
Fukaura	2.02	1.40	2.03	2.04	0.24	0.24	-0.72
Setana	1.04	0.86	1.04	1.04	1.03	1.03	0.79
Rumoi	0.62	0.32	0.63	0.63	-0.08	-0.09	-0.42
Error: diff	0.62	0.19	0.62	0.62	0.63	0.63	0.06
Error: abs	0.83	0.73	0.83	0.83	0.71	0.71	0.77

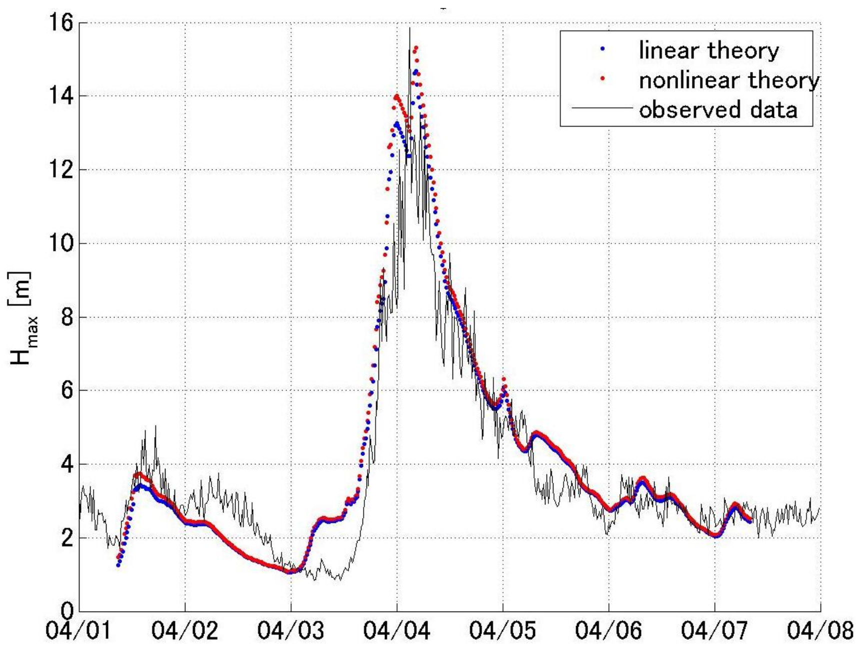
Table 3 Accuracy of estimated maximum H_{max} at the peak of storm at each location : model versus observation (only Case 2 in Table 1)

Case 2	Difference [m]		Ratio [%]	
	Linear	Nonlinear	Linear	Nonlinear
Tottori	-1.27	-0.65	-12.2	-6.2
Shibayama	-3.39	-2.89	-26.5	-22.6
Tsuruga	-1.23	-0.97	-11.8	-9.3
Fukui	-3.22	-2.62	-21.4	-17.4
Kanazawa	0.94	1.5	8.6	13.8
Wajima	-1.13	-0.51	-7.1	-3.2
Naoetsu	-0.48	-0.03	-4.1	-0.2
Sakata	-4.25	-3.19	-21.0	-15.8
Fukaura	2.13	2.63	18.2	22.6
Setana	-1.55	-1.15	-15.6	-11.6
Rumoi	-0.88	-0.43	-8.2	-4.0
Error: diff	-1.30	-0.76	-9.18	-4.91
Error: abs	1.86	1.51	14.07	11.51

directional resolution gives the most significant changes which can improve the accuracy of H_s about 20-50% of difference of maximum value at the peak of storm depends on the locations. The best hindcast results were obtained for the cases of 2 and 7 which conditions were $\Delta\theta$=3.33 degree and Δx=0.05 degree (about 5 km), respectively. The finer directional resolution gives better energy propagation from storm center to outside. The results of maximum wave height estimation will be shown following section.



(a)Sakata point (#5 in Figure 1)



(b)Wajima point (#7 in Figure 1)

Figure 3 Time history of observed and estimated H_{max}
(●: line theory, ●: nonlinear theory, line: observed data)

The maximum wave height for each time segment of 60 min window were computed based on Eq.(1) and (2) giving by the computed wave directional information and duration of storm by wave period information. The difference and ratio of maximum wave height H_{max} at the peak of storm for the case 2 are shown in Table 3. The underestimation of H_{max} can be found both the linear and nonlinear estimations, although the hindcast of H_s is slightly underestimated. Comparing the linear and nonlinear estimations, the observed data shows the nonlinear enhancement of H_{max} significantly. Due to nonlinear

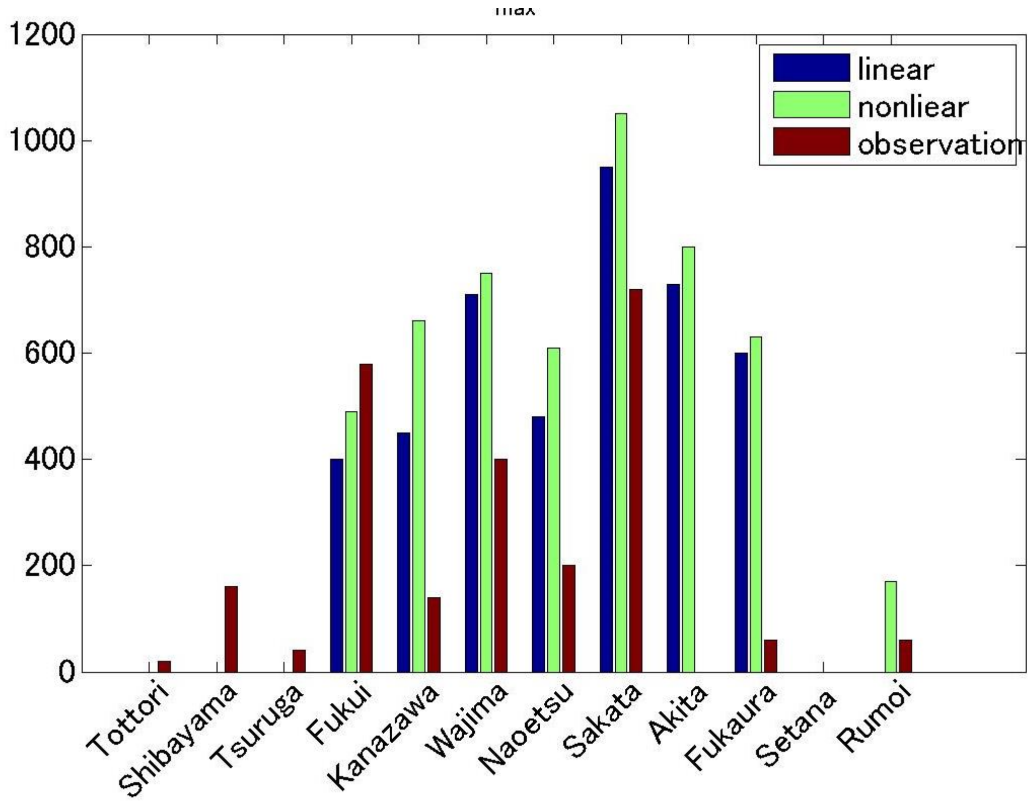


Figure 4 Duration period of maximum wave height H_{max} exceeds 10 m [unit: min]
(■: linear theory, ■: nonlinear theory, ■: observed data)

effects depends on wave steepness and shape of wave spectra, the observed data shows the significant difference at the peak of storm. The differences of estimated maximum H_{max} between theories and data are -1.3 m by the linear theory and -0.76 m by the nonlinear theory on average, respectively. The nonlinear theory reduced the bias error of H_{max} estimation from -9.2% to -4.9% which gives reasonable improvement of prediction of H_{max} at the peak of storm. The MaxWaveNL requires detail shape of directional wave spectra, therefore, the higher directional resolution is important to estimate the maximum wave height in the train. It is found that the higher directional resolution gives better performance of H_s and H_{max} hindcast due to improved energy propagation and better estimation of directional wave spectra.

Figure 3 shows time series of H_{max} both the linear and nonlinear theories with observed data at Sakata (#5) and Wajima (#7), respectively. The maximum observed H_{max} was recorded 20.3m at Sakata point in this event. The solid black line indicates observed data, the red and blue dots indicate H_{max} estimated by the linear and nonlinear theories (Eq.(1) and (2)). The value of H_{max} is statistically unstable generally. Therefore, there are large fluctuation of observed H_{max} in the time series. On the other hand, the theoretically estimated H_{max} is expected value of pdf of H_{max} as shown in Eq.(1). We need to keep in mind for comparison. The averaged relative value (e.g. mean ratio of difference) is appropriate for comparison as shown in Table 2. The estimated H_{max} by the nonlinear theory show better agreement with the observed peak of H_{max} than the linear theory (Figure 3 and Table 2). The estimated values of H_{max} by the linear and nonlinear theories are identical if wave field is moderate. This is because there is enhancement of kurtosis for moderate wave condition and nonlinear effects to pdf of H_{max} only corrects positively. Therefore, the nonlinear theory always increases the H_{max} than the linear theory.

The peak of storm duration is another important factor for wave hindcast. Figure 4 shows the duration period of maximum wave height H_{max} exceeds 10 m. The different bars indicate ■ linear theory, ■ nonlinear theory, and ■ observed data, respectively. The storm was continues over 24 hours and thus the duration was relatively short as a tropical cyclone. The observed duration of peak of storm H_{max} exceeds 10 m is shorter than the theories. The estimation of duration is important factor for caisson sliding distance. Therefore it is important to improve the accuracy and validation of peak storm duration near future.

CONCLUSION

The state of art of nonlinear theories can estimate probability of maximum wave height from given directional spectra for given initial conditions in wave tank or numerical experiments. This study examined the maximum wave height prediction in cooperating with wave spectral model for the severe stormy condition in field. The selected weather event was the strong winter storm crossed the Sea of Japan in early April of 2012. A series of numerical experiments was performed to understand the characteristics of extreme wave sea condition for 2012 event. The validation of nonlinear theory of maximum wave height from wave spectra is conducted with observed buoy data network along the storm track. The maximum observed significant wave height and maximum wave height were 11.0 m and 20.3m at Sakata in Yamagata Prefecture, Japan. It is found that the higher directional resolution gives better performance of H_s and H_{max} hindcast due to improved energy propagation and better estimation of directional wave spectra. The nonlinear theory improves the bias error of H_{max} estimation from -9.2% to -4.9% in comparison with the linear theory.

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