FREQUENCY DOWNSHIFTING OF WAVE SPECTRA AND FORMATION OF FREAK WAVES ON NON-UNIFORM OPPOSING CURRENT

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Physical and numerical modeling of initially monochromatic wave propagation on stepwise opposite current in conditions of deep water was carrying out. The physical experiments demonstrated two steps of downshifting of spectral maximum in the regions of increasing of opposite current. Freak waves arise at the moments of downshifting, when several peaks of spectrum exist, due to superposition of waves, provided by each of spectral peaks. To explain the phenomena of downshifting the dynamical evolution set of equations are constructed. Their solutions perfectly described the main qualitative features of wave transformation during the physical experiments.

Keywords: waves on opposite current; frequency downshifting; freak waves

INTRODUCTION

The time-space evolution of storm wave spectrum is still incompletely solved problem. It is proved by existing of numerous nowadays wave models.

The downshifting of wave spectra during the wave propagation is well known empirical fact, but details of this process are the subject of the investigation still now. In depends on initial and external condition the downshifting could be continuous or discreet in frequency space. The samples of discreet downshifting (DD) with frequency step like the step of modulation instability of Benjamin-Feir was discovered in field and laboratory conditions as reported by few authors (for example, Chawla, Kirby, 2002; Kuznetsov et al., 2006; Kuznetsov, Saprykina, 2008, Ma et al., 2010).

All of the reports highlight the fact that the frequency step of DD coincides with frequency step of modulation instability – after some distance of wave run the maximum of wave spectra is shifted to the frequency of lower sideband. The modulation instability occurs in enough steep waves at the moments of waves steepening under the influence of wind increasing or opposite current increasing or immediately after the generation of steep waves by wavemaker.

Explanations of mechanism of DD are various: some authors declare that higher sidebands are dissipated during wave breaking due to frequency selectivity of dissipation that saves the lowest sideband only, others explain DD on opposite current as that only lower sidebands pass through the blocking point. But the DD had taken a place in case of absence of breaking and in absence of current for mechanically generated waves (Kuznetsov, Saprykina, 2010). So the physical explanation of DD effect was not clear up to now.

The influence of opposite current on waves is practically important problem in the coastal zones at tidal inlets and river mouths. The DD strongly affects on wave blocking or unblocking and can accompanied with abnormally high wave arising. For example, recent investigation had shown that interaction of wave with currents can leads to formation of extremely high (or freak) waves that is very important to be taken in account in many applied tasks of coastal engineering and of navigation. As was shown by many researches, abnormal waves can be formed due to evolution of narrow banded spectra of steep waves (for example, Kuznetsov, Saprykina, 2006, 2008; Saprykina, Kuznetsov, 2009). Opposing currents can be considered as "natural source" of wave steepening.

The main aim of this work is to investigate the details of frequency downshifting process, the same as details and mechanism of arising of freak waves. For the last purpose we use the space varying opposing current that can be considered as easiest way for realization "natural" steepening of waves in the laboratory conditions.

EXPERIMENT

Experiment was carried out in middle size flume of Tainan Hydraulics Laboratory: 200 m long, 2 m wide and 2 m deep. Setup of experiment is shown on Fig. 1. A current was generated by pump. A space variety of current was provided by decreasing of water depth above underwater bar, placed in

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the middle of the flume. Mechanically generated wave propagate initially on still water, then on slow opposite current 0.21 m/s and then on opposite current 0.45 m/s. Waves were synchronously registered by 35 capacity type gauges at sampling rate 50 Hz. The current was measured by three components ADV and two components EMC gauges. In total an evolution of 33 series of initially monochromatic, bichromatic and irregular JONSWAP spectrum waves was investigated.

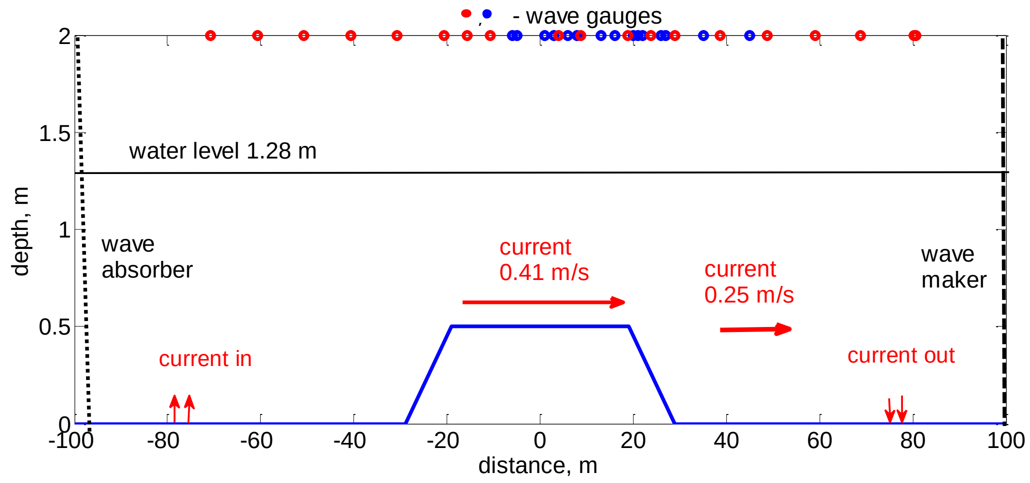

Figure 1. Setup of experiment in THL middle size flume.

DISCUSSION

Results of experiment

It was revealed that according the stepwise increasing of opposite current the steepness of waves increase also (Fig.2).

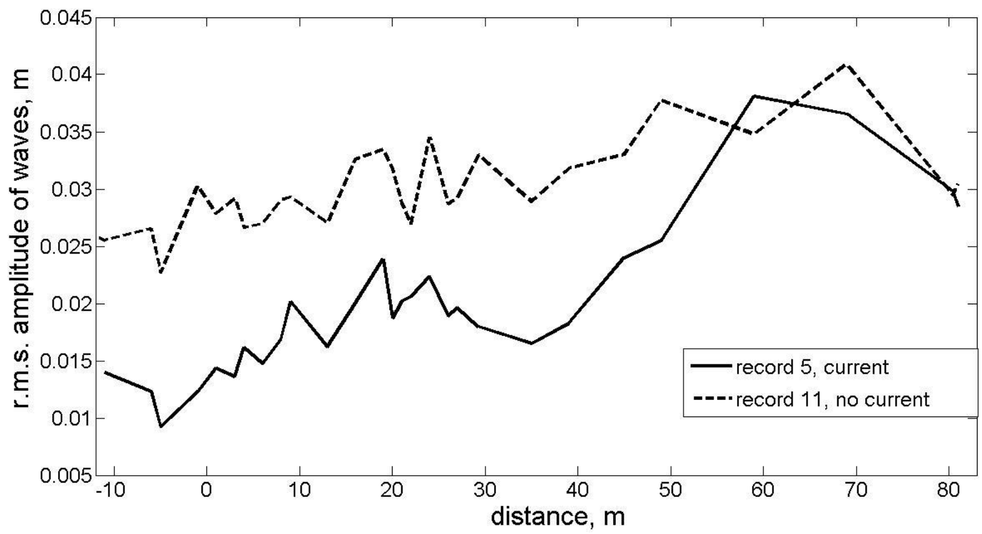

Figure 2. Evolution of r.m.s. amplitudes of initially monochromatic waves H=0.1 m, T=1 s, without current (dashed line) and with opposing current (solid line).

At each step of current/steepness increasing the same scenarios was realized: a growth of side bands of Benjamin-Feir modulation instability and then downshift of frequency of maximum of wave spectrum to low-frequency band (Fig.3, lower). This process does not strongly depend from intensity of wave breaking. For more steep initially waves this scenario is developed faster.

For the comparison the evolution of wave spectra and wave amplitude (root mean squared) in case of absence of current are shown on Fig.3 (upper) and on Fig. 2. The evolution of amplitudes with distance in both cases is similar, wave breaking affects only on total decreasing of amplitude but not on its space variability.

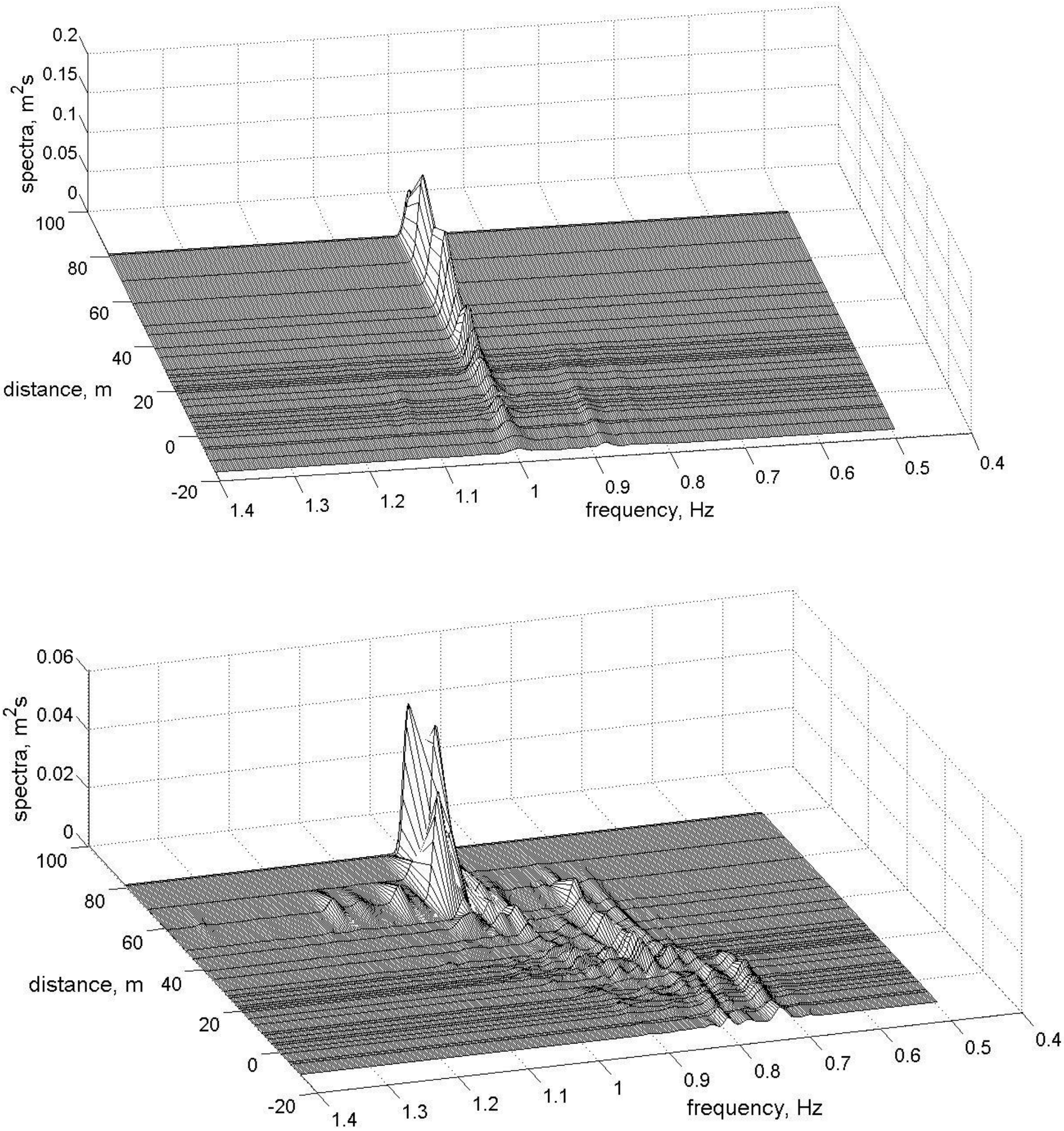

Figure 3. Evolution of spectra of initially monochromatic waves H=0.1 m, T=1 s, without current (upper) and with opposing current (lower). Wavemaker was at 100 m, waves propagate on still water at distances 100-75 m, on current 0.25 m/s at distances 75-19 m and on current 0.41 m/s at distances 19 m and further.

Freak waves arise at the moments of DD, when several peaks of spectrum exist. A superposition of waves, provided by each of spectral peaks, forms the extreme waves, exceeding of significant wave height in twice (Fig.4).

On Fig. 5 the wavelets transform of wave chronogram that contained the freak waves, shown on Fig. 4, demonstrate the of intermittence the frequency structure. Around time sample 1600 the “high frequency” packet exists. Around 2100 – the low frequency packet exist. At the coincidence in time the high and low frequency packets at 3240 time sample the freak waves occur due to superposition of waves, provided by each of this frequency scales .

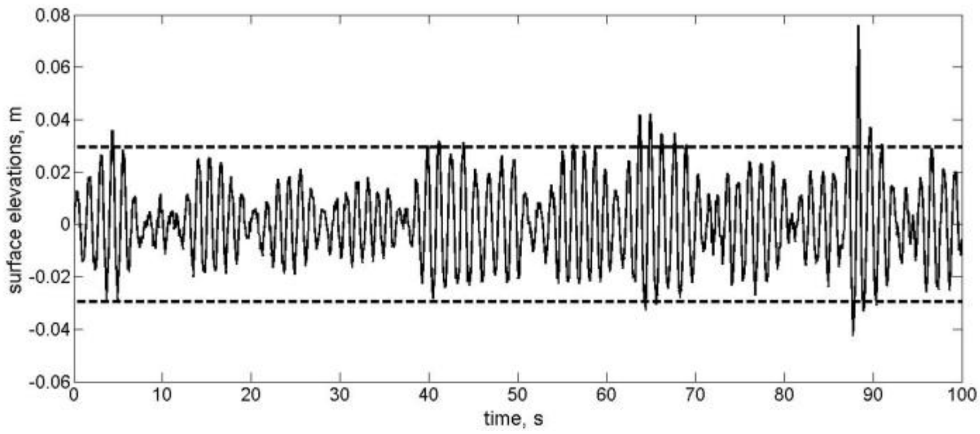

Figure 4. Example of freak wave, arising at frequency downshifting. Record 5, distance 6 m from the center of flume, initially monochromatic waves H=0.1 m, T=1 s. The dashed lines are the significant wave height.

The coincidence in time their crests at 3240 time sample provide the freak wave formation as shown on Fig.6. The slope of local maxima lines demonstrate that coincidence in time the crest and through of waves of different scales: the vertical local maxima lines –the freak waves; the inclined – no freak waves. The reasons of space-time variability of relative crest positions of waves of different scale are the difference in their wave lengths and in their celerities due to the dispersion.

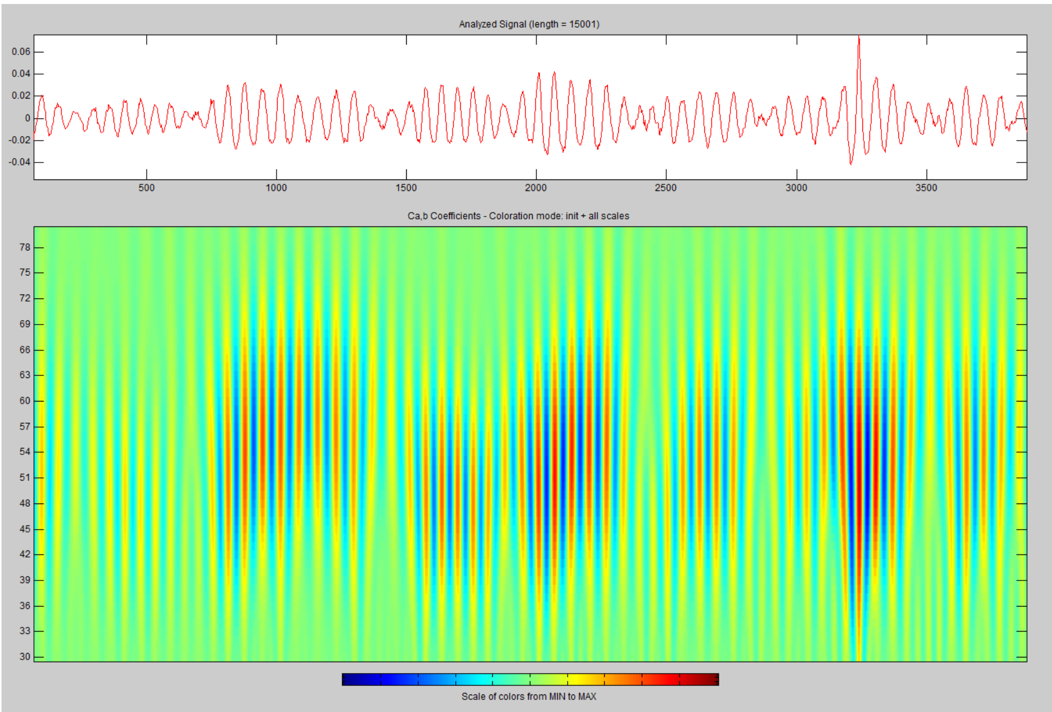

Figure 5. Wavelet transform of chronogram shown on Figure 4.

After the two cascades of downshifting the waves becomes almost linear and their propagation well described by the linear theory that confirmed by coincidence the measured and calculated celerities, as demonstrated at Fig.7.

So on the base of physical experiments results we can conclude that non-uniform opposing current significantly increases steepness of waves and evolution of a spectrum of steep waves on current, in general, does not strongly depend from breaking processes and occurs approximately at the same scenario: a growth of side bands of Benjamin-Feir modulation instability and then downshift of frequency of maximum of wave spectrum in low-frequency band. With increasing of wave steepness this scenario is developed faster and several cascades of downshift of frequency can be observed. Considered downshifting is discrete both on space, and in time. Freak waves arise at the moments of downshifting, when several peaks of spectrum exist.

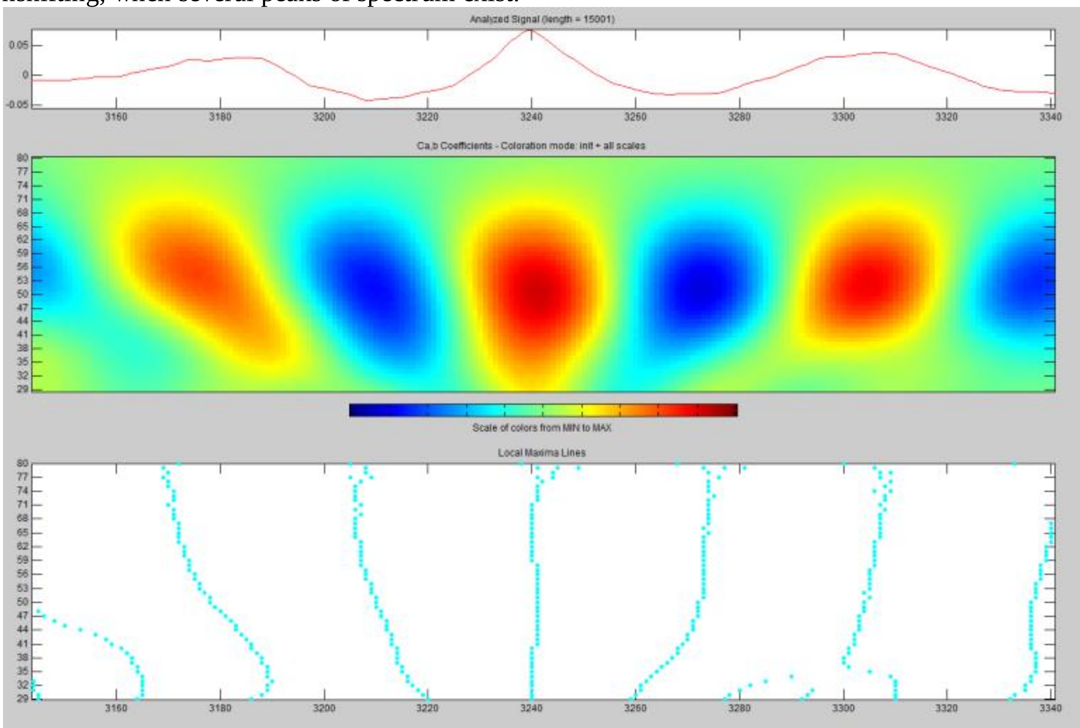

Figure 6. The wavelet transform and it maxima lines around 3240 time sample provide the freak wave.

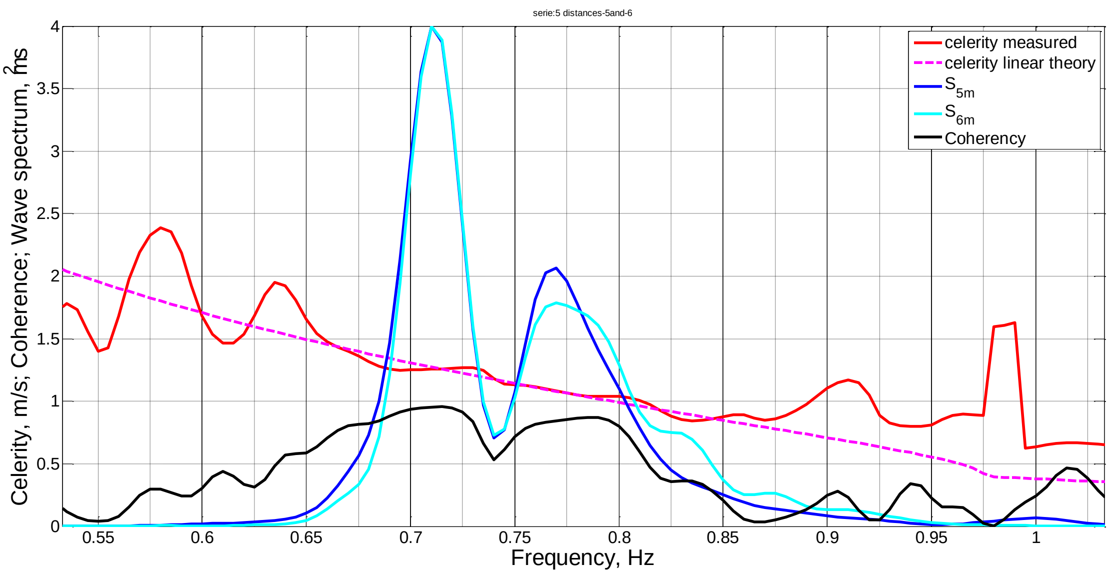

Figure 7. Celerities measured and linear theory at distances 6 and 5 m, wave spectra and coherence function.

Theoretical model of the near-neighbour wave resonances

To explain the phenomena of downshifting, the dynamical evolution set of equations are constructed. Initially constant nonlinear Stokes waves with the amplitude, wave number and frequency (a_0, k_0, σ_0) is unstable to the perturbation in the form of pair small waves with the closed frequencies and wave numbers: superharmonic ($a_1, k_1 = k_0 + \Delta k, \sigma_1 = \sigma_0 + \Delta\sigma$) and subharmonic ($a_{-1}, k_{-1} = k_0 - \Delta k, \sigma_{-1} = \sigma_0 - \Delta\sigma$). Most unstable modes are the following $\Delta\sigma / \sigma_0 = \varepsilon; \Delta k / k_0 = 2\varepsilon$, where $\varepsilon = a_0 k_0$ - initial steepness of the Stokes wave (Benjamin and Feir 1967). That is the Benjamin-Feir or modulation instability of Stokes wave. For surface gravity waves, resonant interactions occur at the third order between four wave components. Three monochromatic wave trains form a nearly resonant "quartet" for one particular configuration, which occurs when two of waves coincide (one has to "count" the carrier wave twice) (Phillips 1967; Shemer et. al. 2002):

$$\begin{cases} \frac{1}{\omega_{-1}}(\partial_T A_{-1} + \mathrm{C}_{g(-1)}\partial_X A_{-1}) = i A_{-1}\sum \gamma_{-1j}\left|A_j\right|^2 + iFA_0^2 A_1^* / 2 \\ \frac{1}{\omega_0}(\partial_T A_0 + \mathrm{C}_{g(0)}\partial_X A_0) = i A_0 \sum \gamma_{0j}\left|A_j\right|^2 + iFA_{-1}A_1A_0^* \\ \frac{1}{\omega_1}(\partial_T A_1 + \mathrm{C}_{g1}\partial_X A_1) = i A_1 \sum \gamma_{1j}\left|A_j\right|^2 + iFA_0^2 A_{-1}^* / 2 \end{cases} \qquad (1)$$

where $2k_c\varepsilon^2$ is a slight mismatch of the wave number from Phillips' four wave resonance conditions for infinitesimal waves. Due to the cancellation of this resonant de-tuning in the presence of Stokes amplitude dispersion, sideband waves grow exponentially (Phillips, 1967).

Initially exponential growth of the sidebands corresponds to the linear phase of the instability. The space scale for development of the modulation instability is l / ε^2 , where the characteristic surface wavelength is l .

The dynamical equations for these almost resonance waves look like follows:

$$\begin{aligned} &2\sigma_0 = \sigma_{-1} + \sigma_{+1}, \\ &2k_0 = k_{-1} + k_{+1} - 2k_0\varepsilon^2, \quad , \\ &k_1 = k_0(1+\varepsilon)^2,\ k_{-1} = k_0(1-\varepsilon)^2, \end{aligned} \qquad (2)$$

where $A_i = A_i(X,T)$ are slowly varying complex amplitudes of interacting waves, C_{gi} - constant group velocities of waves, F and γ_{ij} are real - valued interaction coefficients, $(X = \varepsilon x, T = \varepsilon t)$ are slow space and time variables, respectively.

First terms in the right side of these equations describe the Stocks amplitude dispersion and next terms – almost resonance interactions of waves.

Dynamical equations [2] describe instability of the Stokes wave. Initially exponential growth of the sidebands corresponds to the linear phase of the instability and the energy transfer from the primary wave to the sidebands. On the next stage solutions [2] describe the energy recollection back into the primary wave mode. These modulation-demodulation cycles involved into the evolution of an unstable wave train in the absence of dissipation is known as the Fermi-Pasta-Ulam recurrence phenomenon (FPU). It is theoretically and experimentally confirmed in a numerous number of works (see Lake et al., 1978, Tulin & Waseda, 1999).

Equations [2] describe the interaction of the fundamental three modes, but on the later stages together with growing sidebands additional wave resonances begin to play the significant role. The energy is redistributing to higher and lower harmonics for free and bound waves in the most effective way.

Consequently we will consider multi N wave resonance system including the most effective resonances which are characterized by the minimum detuning from the exact resonance conditions. It is easy to show that the minimum of resonance mismatch in the one-dimensional space case have near-neighbor resonance triads:

$$\begin{cases} 2\sigma_{-1} = \sigma_0 + \sigma_{-2}, \\ 2k_{-1} = k_0 + k_{-2} - 2k_{-1}\varepsilon^2, \end{cases}$$
$$\begin{cases} 2\sigma_1 = \sigma_0 + \sigma_{+2}, \\ 2k_1 = k_0 + k_{+2} - 2k_1\varepsilon^2, \end{cases}$$
....
$$\begin{cases} 2\sigma_N = \sigma_{N-1} + \sigma_{N+1}, \\ 2k_N = k_{N-1} + k_{N+1} - 2k_N\varepsilon^2, \end{cases}$$

where all set of frequencies and wave numbers have the equal difference step: $\sigma_{\pm i} = \sigma_0 \pm i\Delta\sigma, k_{\pm i} = k_0 \pm i\Delta k$.

Correspondingly, the dynamical set of equations for the five main resonance waves, for example, takes a form:

$$\begin{cases} \frac{1}{\omega_{-2}}(\partial_T + C_{g(-2)}\partial_X)A_{-2} = i\,A_{-2}\sum\gamma_{-2j}\left|A_j\right|^2 + iGA_{-1}^2A_0^*/2 \\ \frac{1}{\omega_{-1}}(\partial_T + C_{g(-1)}\partial_X)A_{-1} = i\,A_{-1}\sum\gamma_{-1j}\left|A_j\right|^2 + iGA_{-2}A_0A_{-1}^* + iFA_0^2A_1^*/2 \\ \frac{1}{\omega_0}(\partial_T + C_{g(0)}\partial_X)A_0 = i\,A_0\sum\gamma_{0j}\left|A_j\right|^2 + iGA_{-1}^2A_{-2}^*/2 + iFA_{-1}A_1A_0^* + iHA_1^2A_2^*/2 \\ \frac{1}{\omega_1}(\partial_T + C_{g(1)}\partial_X)A_1 = i\,A_1\sum\gamma_{1j}\left|A_j\right|^2 + iFA_0^2A_{-1}^*/2 + iHA_0A_1A_2^* \\ \frac{1}{\omega_2}(\partial_T + C_{g(2)}\partial_X)A_2 = i\,A_2\sum\gamma_{2j}\left|A_j\right|^2 + iHA_1^2A_0^*/2 \end{cases} \tag{3}$$

where $A_i = A_i(X,T)$ are slowly varying complex amplitudes of interacting waves, C_{gi} - constant group velocities of waves, G, F, H and γ_{ij} are real - valued interaction coefficients.

All waves interact with each other consequently in a chain manner, through the connection with the neighbor waves.

Interaction with current and wave breaking dissipation

How does the modulation instability develop in non-conservative media were waves beside interchange also exchange energy with the slowly changed horizontal current? We have to analyze the corresponding set of wave modulations evolution equations in stationary form in the presence of space variable current.

We will analyze the problem assuming the wave motion phase $\theta_i = \theta_i(x,t)$ exists for each of resonance waves in the presence of a slowly varying current $U(\varepsilon^2 x)$, and we define the local wave number k_i and observed frequency ω_i for each of the wave as:

$$k_i = (\theta_i)_x, \omega_i = \sigma_i + k_i U = -(\theta_i)_t, i = 0, \pm1, \pm2.... \tag{4}$$

It is evident that, for the stationary modulation, the intrinsic frequency σ_i and wave number k_i for each of the wave is no more constant. They all slowly change in the presence of variable current $U(\varepsilon^2 x)$, but the resonance conditions:

$$2\omega_i \approx \omega_{i-1} + \omega_{i+1}$$

will remain valid in the entire region of waves propagation due to the stationary value of the observed frequency for each of the harmonics.

The main kinematics wave parameters (σ_i, k_i) are considered as slowly varying functions with the typical scale, $O(\varepsilon^{-1})$, longer than the primary wavelength and period (Whitham, 1974):

$$k_i = k_i(\varepsilon x, \varepsilon t), \sigma_i = \sigma_i(\varepsilon x, \varepsilon t) \ .$$

On this basis, we will recover the effects of long-scaled current and nonlinear wave dispersion additional (having the same order) to the Stokes' term with the wave steepness squared.

The dynamical equations [3] will have the same form, but represent "intrinsic" dispersion relations for each of the resonant harmonics in the moving system of coordinate and wave action law with the wave action flux speed shifted by the current velocity $U(X)$. Modulation equations [3] are closed by the equations of waves phase conservation [4] for each of the waves.

The Stokes waves with high enough initial steepness ε or under influence of the strong adverse current will inevitably reach the breaking threshold for steepness of the water waves. So we have also to include the breaking dissipation effects into the consideration for this case. We will engage the adjusted dissipative model of Tulin (1996), Huang, Yang, Chiang and Shugan (2011) to describe the impact of breaking on water wave's dynamics. An analysis of fetch laws parameterized by Tulin leads to the conclusion that the energy losses rate due to breaking is of the fourth order in the wave amplitude:

$$D_a / e = \omega D a^2 k^2 \quad ,$$

where e is the wave energy density and $D = O(10^{-1})$ is the small empirical constant.

Sink of energy and momentum due to wave breaking process leads to the additional terms in the right side of the wave action and dispersive equations following from [3] for each of the wave. We will use the wave dissipation function for the adjusted model (Huang et al., 2011) includes also the wave steepness threshold function

$$H\left[\frac{|A_X|}{A_S} - 1\right] \qquad ,$$

where H is the Heaviside unit step function, and A_S is the threshold value of the characteristic steepness $A_X = \sum a_i k_i$ to take into account the dissipation.

Model simulations and comparison with experiment

Let us consider some results of the model simulations, corresponding to experiments on wave-current interaction described above. For the demonstration of capability of the model we choose the five main resonance harmonics, shown of Fig.8. The system of 5 evaluation ordinary differential equations was solved.

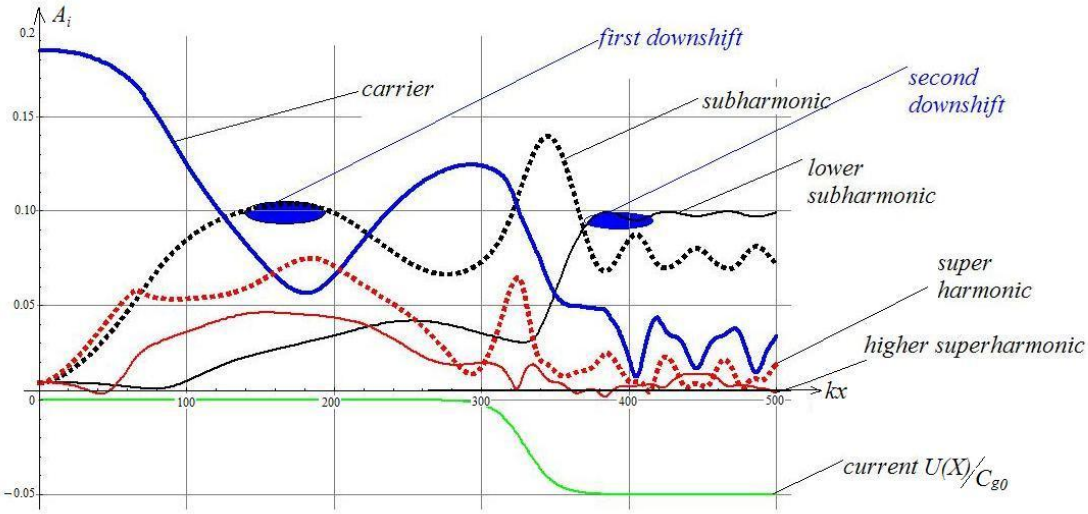

Figure 8. Wave amplitudes along the wave tank in stationary regime of wave propagation, (kx) is the non dimensional coordinate along the wave tank, *k* is the wave number of the initial carrier wave, initial steepness ε=ak=0.18.

Results of simulations for the initially uniform Stokes wave propagation with the steepness $\varepsilon \approx 0.18$ along the wave tank is presented in Fig. 8.

We note the following distinctive features of the wave modeling reasonably corresponding to the results of experiments:

-initial symmetrical grows of the main sidebands with the frequencies difference of the order of wave steepness at the distances up to $kx \sim 50$;

-asymmetrical growth of sidebands ($kx > 70$) and first downshifting of energy to the lower sideband at the distances $kx \sim 120-180$;

-depressing of higher frequency modes ($kx > 180$);

-almost permanent increasing of the lowest sub harmonic along the tank;

-sharp accumulating of energy by the lowest sub harmonic during interaction with increasing opposite current $kx \sim 320-360$;

-final permanent second downshifting of the wave energy;

-permanent second downshifting and total prevailing of the lower frequency modes in the wave spectrum $kx > 360$.

ACKNOWLEDGMENTS

This work was supported by Russian Foundation for Basic Research (project 12-05-92004-HHC_a) and by International Wave Dynamic Research Center (NSC 102-2911-I-006 -302).

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