MODELLING LONG-TERM COASTAL MORPHOLOGY USING EOF METHOD

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Coastline is constantly changing due to the action of wind, waves, tides and sea level variations. Coastal erosion and coastal flooding become increasingly concerned for coastal engineers and coastal zone managers. With global warming due to the climate change which leads to the sea level rise, the frequency and severity of storms are increasing, so that coastal defence becomes even more challenging. In the past decades, various coastal defence structures have been built worldwide to protect our coasts and coastal environment. These structures include sea wall, breakwaters, groynes, and other forms, which often are the combinations of those mentioned, in addition to the soft engineering approaches, such as beach nourishment. To ensure the coastal defence structures to be effectively functional for their design life, it requires the designers to fully understand the effects of the structures on the hydrodynamics and morphodynamics in the surrounding areas, and the response of the coastline in long term. In recent years, the data-driven models have been widely used for long-term modelling of shoreline changes, as such Empirical Orthogonal Functions (EOF) method or one-line model, but require extensively the field data. This paper is to present the details of the EOF analysis using the results obtained from a process-based model COAST2D and the parameterisation procedure for predicting longer term morphological changes from the extrapolated spatial and temporal EOF components. The results presented in this paper illustrate the novelty and effectiveness of the approach as a practical tool for long-term morphological predictions.

Keywords: EOF method; parameterisation, shore-parallel breakwaters; morphological modelling

INTRODUTION

Coastal erosion is a concerning issue for the United Kingdom and other countries around the world. As reported by the UK DEFRA (DEFRA,2010), around 44% of the English and Welsh coastline is defended and the 6% in Scotland. Even if the situation requires an important investment nowadays, this is expected to increase in the near future. Over the period of ten years previous to 2007, the investment in coastal defences had doubled. These costs on coastal defences are likely to increase due to the sea level rise and the change in the frequency and directions of the storms. More recently, during December 2013 and February 2014, there have been a number of severe storms which caused major damages to some stretches of the UK coastline, including the severe erosion of a main railway line at Dawlish, Southwest of England, resulting in line closure for several months.

In order to defend the shoreline against storms and coastal erosion, different actions can be taken. These actions include "hard engineering" solutions such as building seawalls, groynes or breakwaters, and "soft engineering" approaches like beach nourishment or shoreline realignment. Either of them has advantages and disadvantages, and some may be more suitable for specific locations.

With the hard engineering solutions, shore-parallel breakwaters have been proved to be effective in reducing coastal erosion and protecting the coastlines worldwide. In the past decades, examples in the UK include schemes built at Sea Palling, Elmer, Sidmouth and Jaywick in the forms of shore-parallel breakwaters, groynes or the combinations of both. Due to the complex interactions between waves, tides, morphological changes and the presence of nearshore structures, to fully understand the hydrodynamics and morphodynamics becomes extremely difficult, and to accurately predict long-term impacts of the coastal defences on beach morphology is even more challenging. Therefore, laboratory experiments have been extensively used in assessing the performance and effectiveness of the structures, see Ilic et al. (1999). Whilst the laboratory experiments provide valuable data for designing the structures, they are limited by the excessive costs and time required. With the rapid development of computer power and computing techniques in recent decades, numerical modelling plays a key role in assisting the designs. Together with laboratory experiments and field monitoring programmes, the computer modelling has been significantly advanced for both short and medium term predictions of coastline changes under combined wave and tide conditions. Taking the shore-parallel breakwater schemes at Sea Palling (Norfolk, the UK) as an example, the earlier modelling work was done by (Fleming and Hamer 2000). Since the construction of the breakwaters, extensive research has also been carried out at this site, with combination of extensive field measurements for storm events (Dolphin et al. 2009), longer-term remote sensing monitoring of the coastline (Fairley et al. 2009), process-based modelling (Pan et al. 2005; Du et al. 2010), as well as probabilistic modelling (Wang and Reeve 2010). The research has also led to the current design guidelines to be improved (Johnson et al. 2010). Further information of the relevant research projects can also be found in Pan et al. (2010). Those research

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exercises are important in gaining detailed insights into the complex interactive hydro-morpho-dynamics in the areas for the existing schemes, and to identify the potential deficiency in designing the structures, such as the tide impacts, as the most UK coastal waters have meso- or macro-tides, which have been ignored largely in the current designs.

As more field monitoring data becomes available, the data-driven modelling has also been extensively developed in the recent decades. For example, the Empirical Orthogonal Functions (EOF) technique has been commonly used to understand the trends of shoreline changes or the behaviours of morphological changes (Reeve et al. 2001; Li 2007). However, the results of the data-driven modelling, as its name suggests, depend heavily on the quantity and quality of the data available. Linking the model results to the hydrodynamic conditions is also difficult in most cases due to the non-inclusion of physics.

This paper presents a novel method to apply the EOF technique to the results of a process-based model for a longer term morphological predictions. This approach combines the advantages of the physical processes included the process-based models and the statistical aspects from the data-driven technique (such as EOF) to improve the accuracy of the morphological predictions without needing large amount of field data, but with all necessary physical processes being included. In this study, a 2D process-based model COAST2D (Pan et al. 2005; Du et al. 2010) is used to calculate the morphological changes at an idealised site with the presence of a shore-parallel breakwater scheme under combined wave and tide conditions. Then the output of the model results are analysed using the EOF technique and the time varying spatial, as well as the temporal EOF components are examined and parameterised. The parameterised EOF components are finally extrapolated for longer term morphological predictions. The morphological changes predicted by both the parameterised EOF components and the COAST2D model are compared to assess the accuracy and applicability of the proposed approach. In this study, the EOF analysis is applied the unit volumetric changes in order to overcome the difficulties arose from the rapid changes of the shoreline during the simulations as the relatively energetic wave conditions are used.

EMPIRICAL ORTHOGONAL FUNCTIONS (EOF) METHOD

The Empirical Orthogonal Functions (EOF) method is a mathematical tool which can be used to analyse temporal and spatial variations of a set of data. As pointed out by Medina et al. (1990), the EOF technique has been used widely in different disciplines. For example, Lorenz (1959) used the EOF method for atmospheric prediction and David (1976) to predict the temperature in the sea surface.

In the coastal engineering, the EOF method has also been commonly used. Different approaches had been used in the last decades to study the changes of coastal morphology. Inman and Nordstom (1975) used the EOF method to characterize beach profiles. Medina et al. (1990) analysed the variability of the cross shore profile to a mean profile, the ‘equilibrium profile’. Medina et al. (1994) studied the sediment size variation along the beach profile. Other authors have also studied the alongshore changes. Munoz-Perez et al. (2001) studied the shoreline variability during three years in a three kilometres beach in Spain. Miller and Dean (2007) applied the EOF method to three different beach stretches in USA and Australia. All the studies mentioned above used data obtained from bathymetric surveys in the field. However more recently, data obtained from video monitoring has been also used, which represents lower economic costs and greater spatial resolution. Fairley et al. (2009) applied the EOF method by using data obtained from video site, explaining the behaviour of the tombolo and salient generated by the offshore breakwater scheme at Sea Palling, Norfolk, UK. Those studies are all site-specific and focused on individual schemes, and mostly little information on hydrodynamic conditions is included.

The EOF method calculates a set of orthogonal functions (eigen vectors), or temporal and spatial components, that can be used to reconstruct the original data set at any point during the studied period. It is a similar process than Fourier decomposition series but without imposing the solution to be sines and cosines.

Therefore, by using the EOF method it is possible to reproduce the coastline changes in the surveyed period by using a reduced number of orthogonal functions. The set of functions provided is sorted, so the first couple of functions, both temporal and spatial components, explain the most part of the data variability. The EOF method also guarantees that number of functions is the lowest possible compared with other methods. In addition, it provides separately the temporal variations and the spatial variations. These features make the EOF method a simple and objective method to analyse the shoreline variations.

Whilst detailed descriptions of the method can be found in Munoz-Perez et al. (2001), a brief description will be provided here for illustration purposes. Let F(n,m) be a ‘n’ by ‘m‘ matrix containing the data of 'n' points of the variable of interest in space during ‘m’ surveys in time. The EOF method provides the functions X(m) and T(n) that made:

$$d(x=n,t=m)=\sum_i X(n,i)T(m,i)a(i) \qquad (1)$$

where, d(x=n,t=m) is the reconstructed variable, a(i) is a scale factor and i represents the number of components/functions taken into account. To obtain the X(m) and T(n) functions, the eigen values and vectors have to be solved using:

$$\begin{aligned}[A-\lambda I]X &= 0 \quad where: A = F\times F^T \\ [B-\lambda I]T &= 0 \quad where: B = F^T\times F\end{aligned} \qquad (2)$$

where, dimensions of A=[n×n] and B=[m×m]. λ is the eigen value matrix, which provides information about the weight of each eigen vector on the total variance of the data. Therefore it is possible to decide how many components to be used in reconstructing to certain level of accuracy. In general, using 1st 3-4 EOF components would be sufficient to represent 99% of the total variance. As the EOF method separates the spatial variations from the temporal variations it is possible to clearly determine the point where the bigger erosion will occurs or where the salient will be closer to the breakwater. Also it can explain when major changes are occurring.

PORCESS-BASED MODEL COAST2D

In this study, the EOF method is applied to the results of a process-based model COAST2D. The COAST2D model is a 2D depth-averaged hydrodynamic and morphodynamic model, which has been developed, used and well validated in a number of research projects, see Pan et al. (2005, 2007) and Du et al. (2010). The model consists of a number of fully interactive modules, mainly: a wave module to determine wave-period averaged wave energy or wave height and wave direction for the wave transformation from offshore to nearshore; and a current module to compute the depth-integrated current velocity and water surface elevation under both tide and wave actions; and a morphological module to compute the sediment transport rates using equilibrium formulae, as well as the resulting bed level changes. The model also includes full wave-current and hydrodynamic-morphological interactions.

Computational Domain & Test Conditions

The model is set up on a computational domain consisting in a 6025 m alongshore distance and 1665 m across-shore distance, as shown in Figure 1. The offshore water depth is set to be 15 m and the initial beach slope is set to 1:50. The breakwater scheme, which is similar to that at Sea Palling, consists of four shore-parallel breakwaters, located 200 m from the initial shoreline. Each breakwater is 200 m in length with a gap of 250 m between. The crest of the breakwaters is set to be 3 m above the mean sea level, being surface piercing in all cases. The sediment size of the bed material is assumed to be 250µm. A mesh articulated by 241x111 nodes with sizes of 25 m and 15 m in alongshore and cross-shore directions respectively is used.

The incident waves of 2 m in height and 6 sec in period is imposed along the offshore boundary and the incident wave direction is 30 deg from shore-normal. Stationary M2 tide with tidal range of 3 m is used as shown in Figure 1, in the simulations.

Simulations are carried out over 2000 h (hours) with the single sea state described above. The hydrodynamic and morphological parameters, such as wave height, wave level, current velocity, sediment transport rate, bathymetry and bed level changes are output hourly. In the present study, the analysis is focused on the shoreline changes which are derived from the bathymetric contours. Figure 2 shows the final bathymetry after 1000 h simulation. It clearly shows the formation of the embayments and salients/tombolos in the leeside of the breakwaters. Bed level changes (ie the difference between the final and initial bathymetries) are shown in Figure 3. It can be seen that during the 1000 h simulation, the deposition occurred in the updrift area of the scheme and erosion in the downdrift. The results also indicate that considerable volume of the sediment is deposited in the front of the breakwater and sediment is shifted from updrift to downdrift inside the embayment. The magnitudes of the deposition/erosion are in a range of ±4 m, which are regarded as reasonable under the relatively energetic sea state (2 m incident waves).

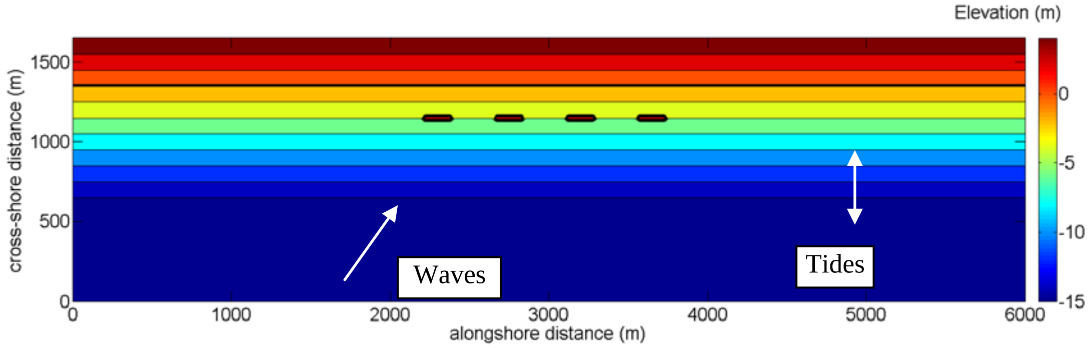

Figure 1. Computational domain and initial bathymetry for COAST2D model.

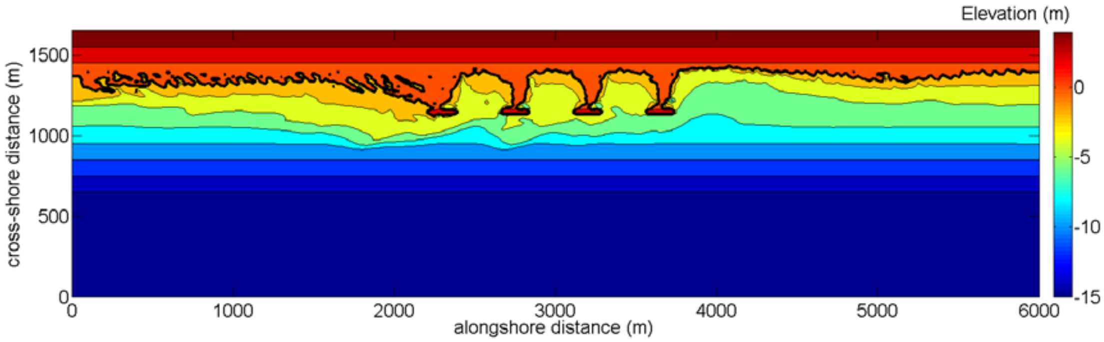

Figure 2. Final bathymetry after 1000 h simulation.

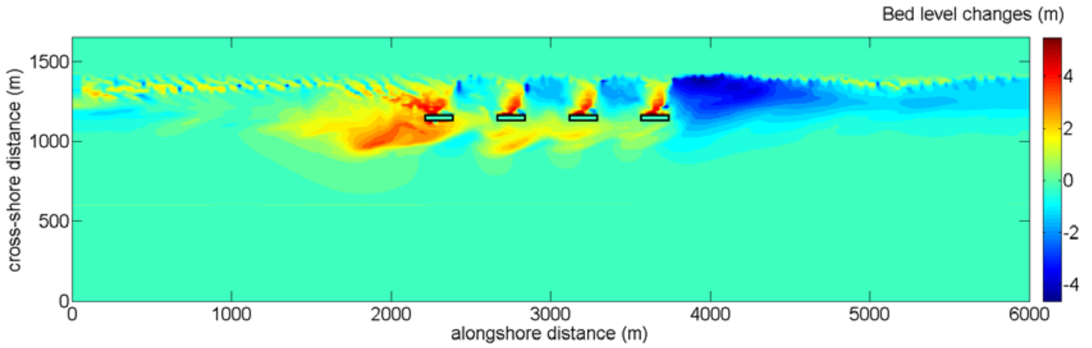

Figure 3. Bed level changes after 1000 h simulation.

EOF ANALYSIS

In most of previous studies on coastal morphology, the EOF method has been usually applied the shoreline positions to understand geomorphological behaviour. As shown in Figure 2, the shoreline position produced by the COAST2D model illustrates some rapid changes under the single and relatively energetic sea state, which makes identifying the exact shoreline more difficult and in some due to overlapping. Having examined the results closely, it was found that using the unit volumetric changes can make the process simpler and more reliable. The unit volumetric change is the total volumetric change integrated in the cross-shore direction at each longshore computational section as:

$$VC_i = \sum_{j=1}^{NJ} BLC_{i,j} \cdot \Delta x \cdot \Delta y \tag{3}$$

where, $BLC_{i,j}$ is the bed level change at node point (i,j); NJ is the total node points in the cross-shore direction; and Δx & Δy are the spacings in x and y directions respectively. Once the EOF analysis is done on the unit volumetric changes (VC_i), the unit volumetric changes is converted back to the shoreline position using the one-model concept, with a assumed depth of closure. From the results of the COAST2D model, unit volumetric changes can be extracted hourly for each longshore section with the given hydrodynamic forcing. Due to the high density of the model results both in time and space, sufficient EOF spatial and temporal components can be calculated, examined and analyzed in detail. In order to demonstrate the process of parameterization of the EOF components, the unit volumetric changes obtained from the COAST2D model over 2000 h are divided into two parts. The results from the first 1000 h are used for parameterization, and the parameterized EOF components are extrapolated to predict the unit volumetric changes for further 1000 h (to 2000 h), and thus the unit volumetric changes and shoreline position at 2000h can be reconstructed. The reconstructed shoreline can finally be compared with that predicted by the COAST2D model directly to examine the performance and accuracy of the proposed methodology.

To proceed with what was described previously, unit volumetric changes are calculated from the bed level changes (BLCs) at hourly intervals. Within the computational domain shown in Figure 1, there are 241 longshore sections, therefore an array of 241 unit volumetric changes can be obtained. Using those values as F in Eq. 1 with the one-dimensional EOF analysis, spatial and temporal EOF components can be obtained from a 241 by 1000 matrix. Figure 4 shows the 1st spatial EOF component in the longshore direction (top panel), as well as the 1st temporal EOF component (bottom panel). From Figure 4, it can be seen that the 1st spatial EOF component at 1000 h follows closely the pattern of embayment and salients shown in Figure 3, with the positive values in the updirift area (x=0-2000 m) representing the deposition and the negative values in the downdrift area (x=4000-6000 m) representing the erosion, and the alternative positive and negative values in between. Other EOF components can also be obtained similarly (not shown here). The temporal EOF component shows an exponential behaviour, which means that the variable of interest: volumetric changes, is growing fast at the early stage of the morphological changes and then it keeps growing at a decreasing rate.

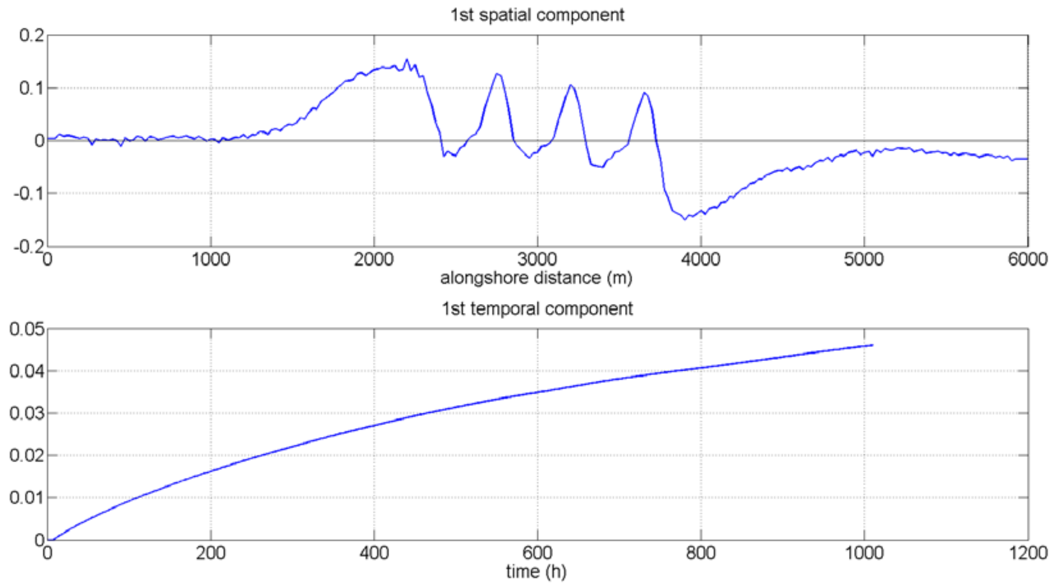

Figure 4. 1st spatial (top panel) and temporal (bottom panel) EOF components from the analysis over 1000 h.

In using the EOF technique, for a system is already close to an equilibrium state, the mean value of the parameter of interest can be removed before applying the EOF, see Muñoz-Perez (2001). If the mean value is not removed in the EOF analysis, the 1st EOF component will show the mean behaviour of the volumetric changes and the next components will be changes over the mean value. In this study, as the bathymetry evolves from its initial (flat) state to an equilibrium state, it is considered to be more suitable not to remove the mean values for the EOF analysis. The results of the EOF analysis on the the unit volumetric changes shows that the 1st spatial EOF component in this study accounts for the 96.3% of the total variance, which represents the main volumetric changes during the 1000 h simulation. The 1st temporal EOF component also shows an equilibrium tendency of the unit volumetric changes during the simulation.

To justify the use of the unit volumetric changes for the EOF analysis, the same approach has also applied to shoreline position changes and sediment transport rates. The results of their EOF components, however, indicated the first EOF component to be between the 50% and 90%, showing a large variability. Therefore, it is thought that applying the EOF analysis to the unit volumetric changes are the most suitable.

EOF COMPONENT PARAMETERISATION & EXTRAPOLATION

From the EOF analysis preformed in the previous section, the EOF components offer insights into the complex hydro/morpho-dynamic processes and shoreline evolutional behaviour in the nearshore area. The results also show that the 1st spatial and temporal EOF components represent 96.3% of the total variance. Therefore, with the 1st spatial and temporal EOF components obtained, the unit volumetric changes at 1000 h can be reconstructed near-perfectly. However, the information obtained is insufficient to enable the prediction of future shoreline changes beyond 1000 h. In order to achieve this, parameterisation is proposed here so that the spatial and temporal EOF components can be extrapolated accurately for longer term predictions. In this study, the aim is to predict the shoreline position at 2000 h using the parameterised results of 1000 h simulation, and the parameterisation is performed on the 1st EOF components only as they represent 96.3% of the total variance.

Since the hourly bed level changes are available from the COAST2D model, the unit volumetric changes at given time intervals can be readily calculated. In this study, the unit volumetric changes at 40 hourly intervals are calculated. When the EOF analysis is applied progressively, it yields a series of spatial and temporal EOF components with the first 1000 h simulation. This operation results in 25 sets of spatial and temporal components to be generated. Figure 5 shows the 25 first temporal EOF components obtained.

From Figure 5, it can be seen that the 1st temporal EOF components are time varying, but follows an exponential behaviour. It is therefore assumed that the following general equation can be used for parameterisation or curve fitting:

$$C = a_t\left(e^{-b_t t} - 1\right) \qquad (4)$$

where, C is the parameterized EOF component; a_t and b_t are two arbitrary constants to be determined by the curve fitting technique such as "Least Square Method"; and t is time. Once the constants a and b in Eq. (4) are determined, the time varying EOF components (spatial and temporal) can be estimated and extrapolated further beyond the original period of parameterization.

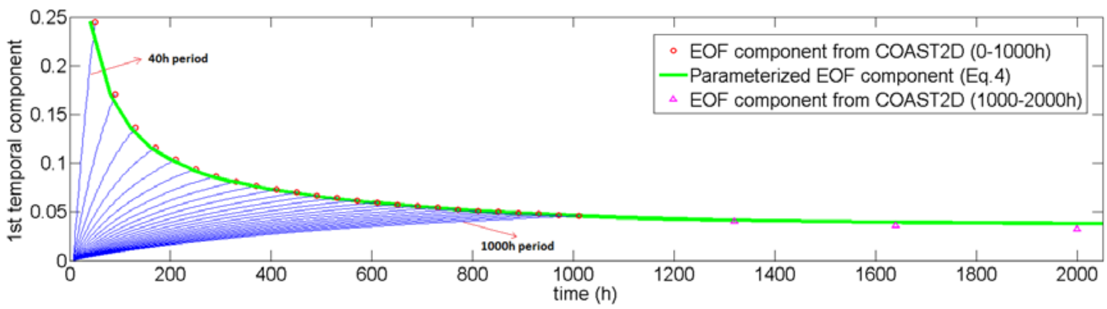

Figure 5. 1st temporal EOF components obtained from COAST2D and parameterized (Eq. 4).

Having done the parameterisation, the constants a and b in Eq. 4 is obtained and Eq. 4 can be used to calculate the 1st temporal EOF components beyond 1000 h, which are shown in Figure 5. The values calculated by Eq. 4 are also compared with the EOF components directly calculated from the results of COAST2D model at selected times, as also shown in Figure 5, showing a good agreement. Therefore, the proposed parameterisation (Eq. 4) can be used confidently to extrapolate the EOF components.

Similarly, the same approach is attempted to apply to the 1st spatial EOF components. However, as the spatial EOF components vary in time differently at different locations, the approach will need to be applied to each longshore location. Figure 6 shows 25 spatial EOF components at all longshore locations. Taking the time-varying spatial EOF components at a particular longshore location, say Section 100 (x=2500 m) as an example, 25 spatial EOF components are shown in Figure 7. Parameterising these 25 spatial EOF components using a similar formula expressed in Eq. 4, the spatial EOF components can be extended, the results of which are also shown in Figure 7, in comparison with those directly calculated from the results of COAST2D. It is evident that the proposed parameterisation works well at this particular location. For other longshore locations, the same can be applied, and the results of the parameterised spatial EOF components at 1000 h for all locations are shown in Figure 8.

Comparison with those calculated from the results of the COAST2D model shows a good agreement as expected.

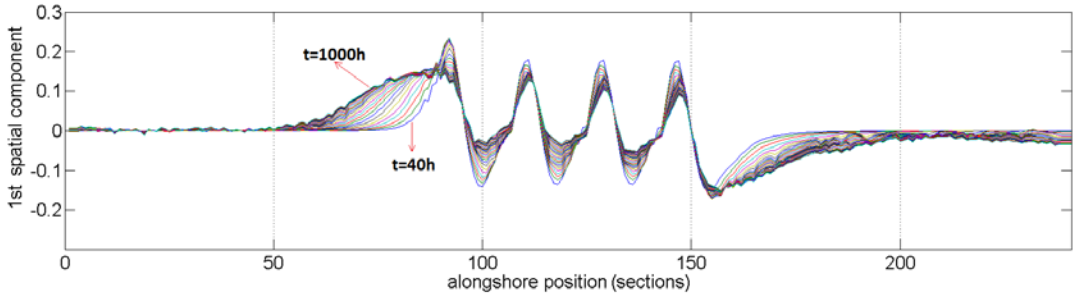

Figure 6. 25 1st spatial EOF components from the analysis over 1000 h.

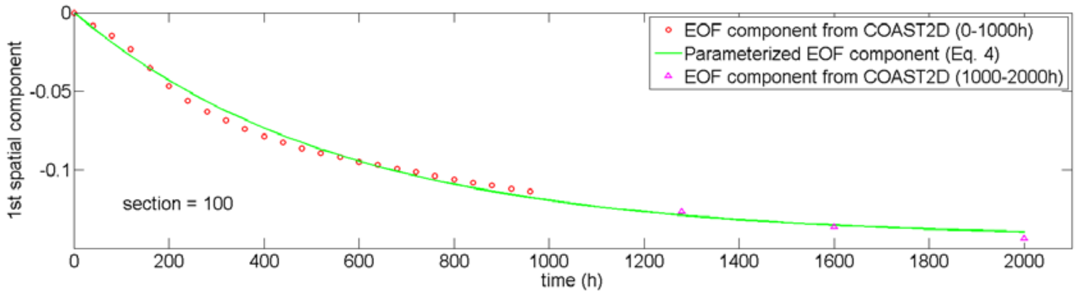

Figure 7. 1st spatial EOF component for section 100 (x=2500m) at different times, parameterized spatial component and spatial EOF component from COAST2D for 1000-2000 h.

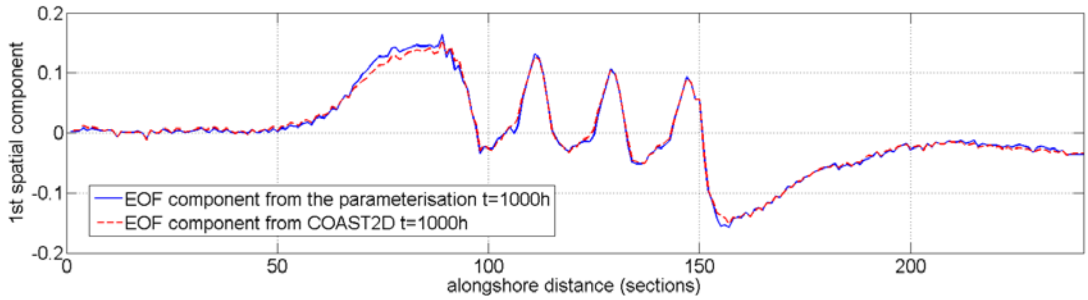

Figure 8. 1st spatial EOF components from COAST2D and parameterized spatial component for t=1000h.

Now, further extrapolating the parameterised spatial EOF components to 2000 h yields the results as shown in Figure 9. The comparison with those calculated from the COAST2D model indicate a good agreement in general, although discrepancies can be found in area updrift of the breakwaters, where considerable sediment transport and bed level changes are taking place, see Figure 3.

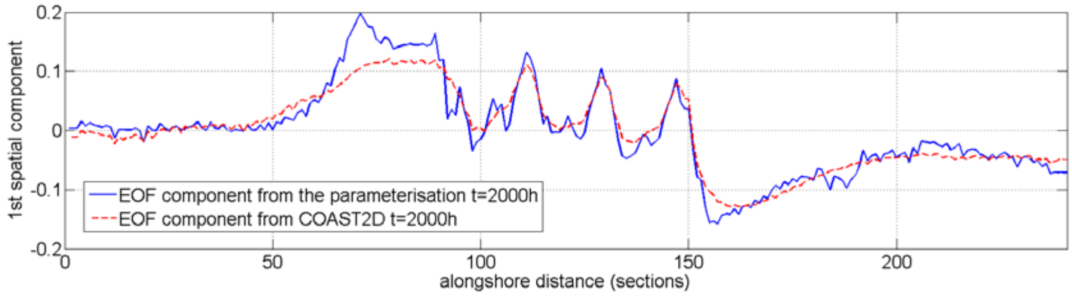

Figure 9. 1st spatial EOF components from COAST2D and parameterized spatial component for t=2000h.

SHORELINE CHANGES

With the EOF analysis on the unit volumetric changes, the parameterisation process proposed has been seen working well. Both the 1st spatial and temporal EOF components can be extended well from 1000h to 2000h, which is significant in predicting the longer term morphological changes. However the purpose of this work is to predict the shoreline position. Therefore volumetric changes have to be converted into shoreline positions. In order to do so, a conversion similar to the one-line model principle (Reeve et al., 2011) is used. According to this concept, the advance or recess, of the shoreline can be explained as a function of the volumetric changes, the dimension of the berm and the depth of closure as shown in equation 6;

$$\Delta S_j = \frac{VC_j}{(B+d^*)\Delta x} \qquad (6)$$

where, VC_j is the unit volume change at Section j; B is the height of berm; and d* the depth of closure. The depth of closure is defined as the maximum water depth where the sediment is moved by the currents. In this study, different values for the depth of closure are considered at different alongshore locations as the breakwaters modify the nearshore hydrodynamics, and the depth of closure can no longer be assumed to be constant for the whole domain. Therefore three different values will be considered: 1) for the open sea areas, where the hydrodynamics are not affected by the presence of the breakwaters; 2) behind the breakwaters, where this represent a physical limitation to the shoreline advance; and 3) in the embayments, where the currents generated in between the breakwaters affect the sediments movement. These three values are determined by observation of bathymetry variation.

Figure 10 shows the reconstructed shoreline position at 2000 h using the aforementioned principle, ie Eq. 6, from the reconstructed unit volumetric changes based on the extrapolated spatial and temporal EOF components from the results at 1000 h. In Figure 10, the reconstructed shoreline position is also compared with that directly computed by the COAST2D model, showing a good agreement. As described previously, during the reconstruction of shoreline position, the depth of closure is set differently according the longshore location. As indicated in Figure 10, the values of the depth of closure range from 15 m for the open coasts, to 2.5 m behind the breakwaters (BW). Since the 1st EOF components represent over the 95% of the total variance, using the 1st components can sufficiently re-produce the shoreline position reasonably accurately. This is indicated by the good agreement in Figure 10 when the reconstructed shoreline position are compared with the results obtained from COAST2D. There are some minor discrepancies toward the lateral boundaries of the computational domain, likely due to the influence of the boundary conditions.

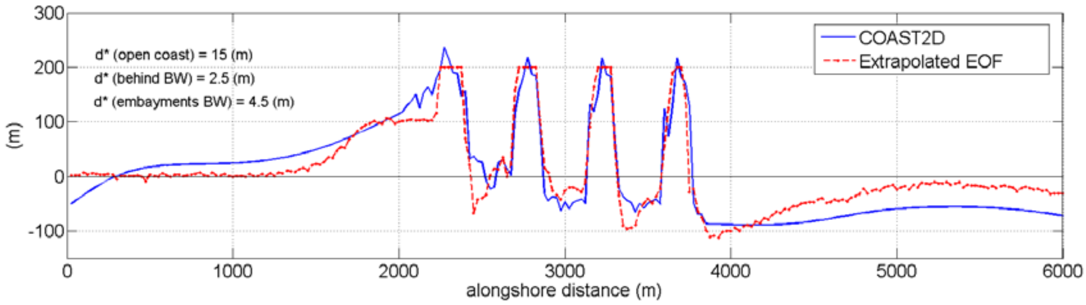

Figure 10. Shorelines reconstructed using the extrapolated EOF components and obtained from COAST2D model at t=2000h

CONCLUSIONS

A novel methodology for predicting the long-term morphological changes has been proposed and demonstrated in this paper. This approach combines a process-based model and a data-driven approach by applying the EOF method to the results of a process-based model COAST2D to generate time series of the EOF components, which are consequently parameterised for extrapolation. The extrapolated EOF components are then used to predict the longer term morphological changes and shoreline evolution. In this study, it is found that the unit volumetric changes are the most suitable parameter to be analysed using the EOF technique, which can then be converted to the shoreline position using appropriately chosen depth of closure.

For an energetic sea state used in the study, it is found that the proposed approach is capable of predicting the shoreline position at 2000 h based on the results of a process-based model at 1000 h. This is significant in terms of forecasting the long-term shoreline evolution, as it makes the process-based modelling much more efficient. The novelty of the proposed approach is also reflected by the fact that the data-driven modelling can now include more physical processes by using the results of the process-based model, and the reliance of the data-driven models on the field measurements is significantly reduced. This is particularly useful at the design stage of the coastal defence structures, where field measurements may not yet be available.

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REFERENCES

DEFRA. 2010. Charting Progress 2. *Feeder Report: Productive Seas.*

Dolphin, T.J., C.E. Vincent, J.C. Bacon, E. Dumont, E, Terentjeva, A. 2009. Decadal-scale impacts of a segmented, shore-parallel breakwater system, *Coastal Engineering, 66, 24-34.*

Du, Y., S. Pan and Y. Chen. 2010. Modelling the effect of wave overtopping on nearshore hydrodynamics and morphodynamics around shore-parallel breakwaters, *Coastal Engineering,* 57, 812-826.

Fairley, I., M. Davidson, K. Kingston, T. J. Dolphin and R. Phillips. 2009. Empirical orthogonal function analysis of shoreline changes behind two different designs of detached breakwaters, *Coastal Engineering, 56, 1097-1108.*

Fleming, C. and B. Hamer. 2000. Successful implementation of an offshore reef scheme, *Coastal Engineering 2000, 1813-1820.*

Ilic, S., S. Pan, B. Chapman, B., A.J. Chadwick, B.A. O'Connor and N.J. MacDonald. 1999. Laboratory measurements of flow around a detached breakwater, *Coastal Structures '99, (ed.) I.J. Losada, ASCE, New York, pp. 813-821.*

Li, Y. 2005. Investigation of the multi-scale variability of beach profiles, *PHD thesis, University of Plymouth.*

Johnson, H., J. Wilkens, A. Parsons, and T. Chesher. 2010. Guidance for outline design of nearshore detached breakwaters on sandymacro-tidal coasts, *Environment Agency Report: SCHO0210BRYO-E-P, Environment Agency.*

Medina, R., M. A. Losada and R. A. Dalrymple. 1990. Análisis de perfiles de playa por medio de funciones ortogonales empíricas (método FOE), *Revistas de obras públicas, Junio 1990, 9 - 17.*

Medina, R., M. A. Losada and C. Vidal. 1994. Temporal and spatial relationship between sediment grain size and beach profile, *Marine Geology 118, 195-206.*

Miller, J. K, Dean, G, R. 2007. Shoreline variability via empirical orthogonal function analysis: Part I temporal and spatial characteristics, *Coastal Engineering, 54, 111-131.*

Miller, J. K, Dean, G, R. 2007. Shoreline variability via empirical orthogonal function analysis: Part II relationship to nearshore conditions, *Coastal Engineering, 54, 133-150.*

Muñoz-Perez, J. J., R. Medina and B. Tejedor. 2001. Evolution of longshore beach contour lines determined by E.O.F. method, *Scientia Marina 65(4): 393-402.*

Munoz-Perez, J. J., L. Tejedor and R. Medina. 2001. Las funciones empiricas ortogonales y los cambios en el perfil de playa a corto, medio y largo plazo, *Física de la Tierra, 13, 139-166.*

Pan, S., Reeve, D., Davidson, M., O'Connor, B., Vincent, C., Dolphin, T., Wolf, J., Thorne, P., Bell, P., Souza, A., Chesher, T., Johnson, H. and Leadbetter, A. (2010), "Larger-scale morphodynamic impacts of segmented shore-parallel breakwaters on coasts and beaches: An overview of the LEACOAST2 project", Shore & Beach, 78(4) /79 (1), 35-43

Pan, S., M. Li, Y. Zhu, B. A. O´connor, C. E. Vincent, J. A. Taylor, T. J. Dolphin and J. C. Bacon. 2005. Effect of shore-parallel breakwaters on coastal morphology under storm conditions, *Coastlines, Structures and Breakwaters 2005* (ed.) N. W. H. Allsop, Thomas Telford Ltd, London, pp. 64-73

Pan, S., D. Reeve, M. Davidson, B. O'Connor, C. Vincent, T. Dolphin, J. Wolf, P. Thorne, P. Bell, A. Souza, T. Chesher, H. Johnson and A. Leadbetter. 2010. Larger-scale morphodynamic impacts of segmented shore-parallel breakwaters on coasts and beaches: An overview of the LEACOAST2 project, *Shore & Beach*, 78(4) /79 (1), 35-43
Pan, S., J. Wolf, Y. Chen, P. Bell, Y. Du, P. Fernando and M. Li. 2007. Modelling nearshore waves with presence of shore-parallel breakwaters, *Coastal Structures 2007*, (eds.) L. Franco, G. R. Tomasicchio and A. Lamberti, World Scientific Publishing Co. Pte. Ltd, pp. 1125-1134
Reeve, D. E., A. J. Chadwick and C. A. Fleming. 2011. Coastal Engineering: Processes, Theory and Design Practice, *SPON Press, London, 2nd Edition.*
Reeve, D.E., B. Li, and N. Thurston. 2001. Eigenfunction analysis of decadal fluctuations in sandbank morphology, *Journal of Coastal Research*, 17(2), 371-382.
Wang, B. and D.E. Reeve. 2010. Probabilistic modelling of long-term beach evolution near segmented shore-parallel breakwaters, *Coastal Engineering*, 57, 732-744.