

STOCHASTIC EXTREME WAVES GENERATOR FOR THE MID RIO DE LA PLATA ESTUARY NORTHERN COASTS

Sebastián Solari¹, Luis Teixeira¹ and Ismael Piedra-Cueva¹

Whenever the failure of a maritime work has important economic, social or environmental consequences, its design and verification should be performed using probabilistic techniques (e.g. Losada 2001). For breakwaters the dominant agents in hydraulic failure modes are sea-level and waves, so in order to perform Level III probabilistic verifications it is necessary to develop simulation models of these agents. This paper presents the development of a methodology for the simulation of time series of extreme agents (waves and sea levels) and its application to the northern coast of the mid Río de la Plata estuary. This development poses particular challenges related to the own characteristics of the estuary and to the available information.

Keywords: extremes; waves; winds; storm surge; simulation; Monte Carlo, SWAN model

INTRODUCTION

Verification equations for breakwaters and other coastal structures that are based on Ultimate Limit States (ULS) requires wave state parameters and sea level as input. As a consequence it is required to have joint distribution functions of extreme wave parameters and extreme sea levels for defining the working conditions that should be used on the design and verification these coastal structures. In turn, for performing Level III verification of coastal structures it is convenient to have a simulation methodology for combined extreme wave and sea level conditions (Monte Carlo simulation methodologies).

Montevideo harbor area is located on the northern cost of the mid Río de la Plata, a shallow estuary, where extreme wave conditions are locally generated by winds and strongly conditioned by the water depth, that is affected by the local winds as well.

OBJECTIVE

The objective of this work is to develop and implement a methodology for the simulation of extreme wave and sea level conditions for the Montevideo harbor area (mid Río de la Plata northern coast) that:

- Gives the joint distribution of extreme wave state parameters and extreme sea level
- Could be used for applying Level III verification technics based on the use of ULS formulas.
- Gives the distributions of severest conditions that a given structure should withstand during its lifetime.
- Allows to include hypothesis of sea level rise on the simulations.

METHODOLOGY

In the northern coast of the mid and inner Río de la Plata extreme waves are of local generation, being conditioned by wind speed and direction, as well as by sea level, which influences friction dissipation and depth induced breaking. At the same time storm surges in the estuary, which predominate over tides during storm conditions, is also related to the wind (Santoro et al. 2013).

Based on the above a methodology is developed for the simulation of extreme wave conditions based on the random generation of extreme wind events and the sea levels associated with such events. The simulation method, outlined in Figure 1, is as follows:

- (A.1) The number of wind extreme events per year and per direction is simulated using a Poisson distribution.
- (A.2) Maximum wind speed of each event is simulated conditional to the direction of the event, using generalized Pareto distributions (GPD), i.e. one distribution per direction.
- (B) Maximum sea level of the event, conditional to the wind speed and direction of the event is simulated, using copulas for defining the dependence between sea level and wind speed and direction.
- (C) Once the wind-sea level event series is simulated, the hybrid downscaling method proposed by Camus et al. (2011) is used for estimation of wave condition at several location of the coast.

¹ IMFIA-Universidad de la República, Julio Herrera y Reissig 565, 11300, Montevideo, Uruguay

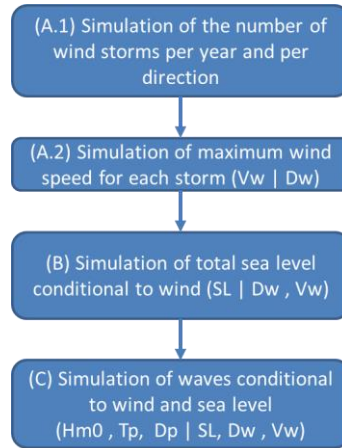


Figure 1. Outline of the proposed methodology.

For steps (A) a 3-hourly wind data measured in the Rio de la Plata are available, from which the peak over threshold (POT) methodology is applied, clustering the data in 8 directions. For step (B) a joint distribution of extreme winds (speed and direction) and sea level conditioned to wind is required. To obtaining this distribution two issues deserve special attention:

- Defining a joint distribution for each of the wind directions (i.e. directional methodology) significantly reduces the size of the sample used for each distribution. Also, not clear dependence structure was obtained between the wind speed and the sea-levels following the directional methodology.
- For sea-levels analysis a series of 110 years of annual maximum (AM) and a series of approx. 30 years of hourly levels are available. Extreme distribution obtained with AM series results in a more severe extreme climate than is obtained from the hourly series. So, although it is not possible to determine wind and sea-level joint events from the AM series, it is desirable to include the information that emerges from it in the simulation process.

Step (A).

The way these issues were tackled is discussed below.

For step (C) the methodology proposed by Camus et al. (2011) is implemented, using the SWAN wave model. An advantage of the proposed methodology over the direct stochastic simulation of wave parameters is that it allows for the inclusion of trends in mean sea-level before applying the methodology of Camus et al. (2011), without making the methodology more complex.

Sea level distribution conditional to wind speed and direction

To tackle the first issues raised above in regards to the simulation of sea levels, it is proposed to construct a new variable for defining the dependence structure between wind and sea level, by finding the direction that maximizes the linear correlation between sea-levels and the wind intensity in that direction:

$$\theta_p = \arg \max \{ \rho(SL, W(\theta)) \} \quad (1)$$

where $W(\theta)$ is the projection of the wind on the direction θ , ρ is linear correlation function and θ_p is the direction that gives the maximum linear correlation between projected wind and sea level.

With regards to the second issue, hourly data is used for fitting the dependence structure between extreme sea levels and extreme winds, while annual maxima data is used for defining the upper tail of the distribution of sea levels.

Thus, step B is divided in five sub-steps, as follows:

- (B.1) Identify, using the hourly data, the maximum sea level reached during each wind storm.
- (B.2) Find which projection of the wind gives the maximum linear correlation with the maximum sea level reached during the storm.
- (B.3) Define the probability distribution of the projected wind speeds.
- (B.4) Define the probability distribution of the extreme sea levels, such that the upper tail reproduces the behavior of the annual maxima distribution.

(B.5) Fit a Copula for modelling the dependence structure between the extreme winds (projected) and the extreme sea levels.

Steps B.1, B.2 and B.5 are straightforward. However, in order to apply a copula model on step B.5, marginal distributions for the projected wind speed and for the extreme sea-level are required. How these marginal distributions are defined in steps B.3 and B.4 is described in more detail below.

Defining the probability distribution of the projected wind speeds

A probability distribution function for the projected wind speed is required, such that is able to accommodate new, more extreme conditions, than that used for fitting the model. Thus, the use of an empirical distribution for the projected wind speed is discarded. A second option would be to fit a parametric distribution using the projected wind speeds, but there is no guarantee that the tail of this new distribution would be in agreement with the long-term upper tail obtained by directional simulation.

Here, we define the probability distribution function of the projected wind speeds using the GPD fitted to the directional series and the empirical distribution of the directions, as (Eq. 2):

$$F(W_p) = \sum_{i=1}^{N_D} F\left(\frac{W_p}{\cos(\theta_i - \theta_p)} \middle| \theta_i\right) \Pr(\theta_i) \quad (2)$$

where W_p is the wind speed projected on direction θ_p , θ_i is the wind direction discretized in N_D bins, $\Pr(\theta_i)$ is the probability of the i bin, $F(X/\theta_i)$ is the probability distribution of wind speeds with direction θ_i (see Figure 2).

Since both the GPD fitted to the directional series and the empirical distribution of the wind directions will be used for simulation, the use of the distribution given by Eq. 2 assures that there will be agreement between the upper tail of distribution of the wind speeds and the upper tail of the distribution of the projected wind speeds, and also that the distribution is able to accommodate all the extreme values arising during the simulation.

Define the probability distribution of the extreme sea levels, with the upper tail defined by the annual maxima distribution

Determining the marginal distribution of the maximum sea levels reached during the storms should take into account the second issue raised above, which involves incorporating the information coming from the Generalized Extreme Value (GEV) distribution fitted to the AM series.

According to Salvadori et al. (2007), given a Poisson process of parameter ν , whose exceedances over the threshold follow a GPD with parameters (a, b, γ) , the AM distribution of the process follows a GEV distribution whose parameters (a', b', γ') are given by:

$$a' = \frac{\nu^\gamma - 1}{\gamma} b + a \quad ; \quad b' = b \nu^\gamma \quad ; \quad \gamma' = \gamma \quad (3)$$

where (a, a') are position parameters (the threshold for the case of the GPD), (b, b') are scale parameters and (γ, γ') are shape parameters. Then, defining the Poisson parameter ν as a function of the threshold a , the GPD parameters (a, b) are calculated using the GEV parameters (a', b') .

Thus, under the assumption that the highest sea-levels occur associated with extreme wind events, it is imposed that the upper tail of the marginal distribution of the maximum sea levels reached during wind storms follows a GPD whose parameters (a, b, γ) are calculated from the AM distribution. Then, a mixture distribution is constructed for the marginal distribution of sea levels (SL), using the empirical distribution when $SL < a$ and the $GPD(x|a, b, \gamma)$ when $SL > a$.

To calculate the parameters (a, b, γ) the following relation between the Poisson parameter ν and the threshold a is defined

$$\nu(a) = \nu \Pr(SL > a) \quad (4)$$

where ν is the mean number of wind storms per year and $\Pr(SL > a)$ is the empirical probability of maximum sea levels reached during a wind storm event, estimated from the sample of maximum wind speed and maximum sea level during a wind storm event. Then, parameter a is obtained by solving Eq.

$$(a'-a)\gamma + \frac{1-v(a)^\gamma}{\gamma} = 0 \quad (5)$$

and having a parameter b is obtained from Eq. 6

$$b = \frac{b'}{v(a)^\gamma} \quad (6)$$

APPLICATION

Number of storms and maximum wind speed and direction

There are 40 years of wind data measured in situ available for the analysis.

After quality control, only years with more of 90% of data are retained, and the Peak Over Threshold (POT) methodology is applied. The number of storms per year is fitted with a Poisson distribution, and the mean number of storms per direction is modeled empirically using 8 direction bins.

Each directional POT series is fitted by means of a Generalized Pareto Distributions (GPD), except for the northern directions that are pooled together before fitting (i.e. NE, N and NW storms are considered together, not affecting the wave modeling in the northern coast of the estuary).

Figure 2 shows the frequency of the wind direction of the POT series and the GPD fitted to the S and SW POT series.

For verifying that the directional simulation methodology is adequately reproducing the extreme wind conditions, a simulation of 10^6 years of wind storms is performed and the result are compared with the extreme distribution obtained by fitting a GPD to the omnidirectional POT series. Figure 3 shows both distributions along with the empirical omnidirectional POT series. It is noted that the results of the directional simulation methodology are in agreement with the GPD fitted to the omnidirectional POT series, but with a significant reduction in the width of the confidence intervals. This reduction may be related with the homogeneity of the POT series that is achieved by using the directional approach, something that is not taken into account when using the omnidirectional approach.

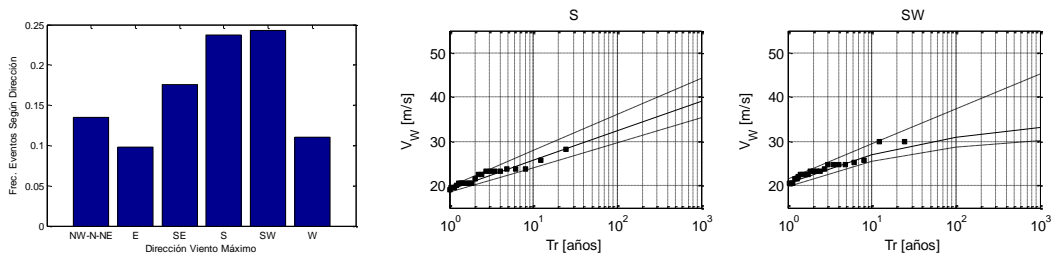


Figure 2. Left: frequency of the directional bins. Center: GPD fitted to the south POT wind speeds. Right: GPD fitted to the southwest POT wind speeds.

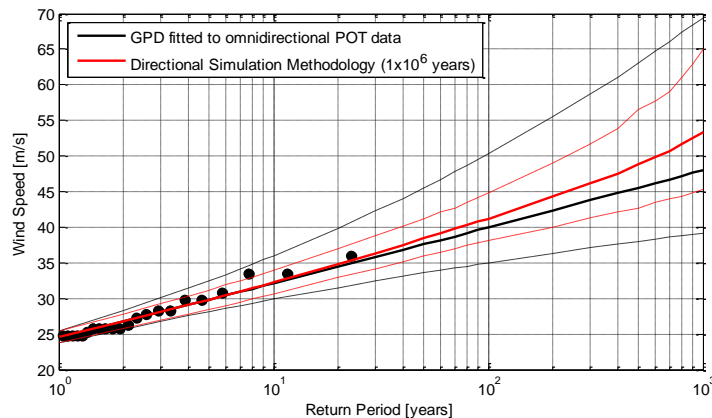


Figure 3. Comparison of the GPD fitted to the omnidirectional POT wind speeds and the extreme distribution obtained from directional simulation of 10^6 years of wind storms.

Sea level conditional to wind

There are two sea level series available at Montevideo: hourly data (approx. 40 years) and annual maxima data (approx. 105 years).

Relation between maximum sea level and maximum wind speed during storms can only be obtained using of the hourly sea level data, however extreme value distribution obtained with the annual maxima series gives higher sea levels than those obtained with the hourly series through a POT analysis, mainly due to extreme condition registered during first quarter of 20th century, as shown in Figure 4. Then, steps B.1 to B.5 are followed in order to define the joint distribution of maximum sea level and maximum wind speed reached during a wind storm.

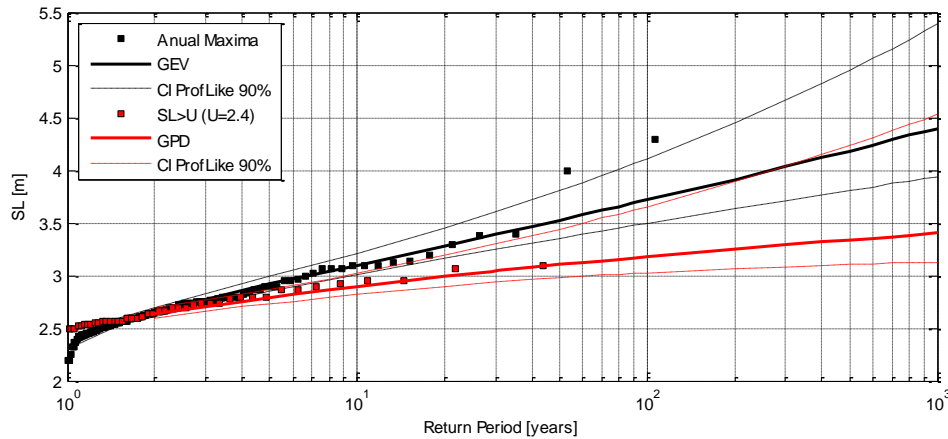


Figure 4. Comparison of the AM and the POT sea level data, and the GEV and GPD fitted to them.

First hourly data is used to identify the maximum sea level reached during each wind storm, obtaining approximately 10 events per years for 20 years of good quality wind and sea level data. Figure 5 (top) shows the obtained data. It is noted that no clear relation can be inferred between wind speed and sea level.

Then, the direction that gives the maximum linear correlation between the projected wind speed and the sea level is looked for (Eq. 1). In this case it is found to be the SW direction. Figure 5 (bottom) shows the obtained projected wind speed versus sea level. In this case a clear relation between both variables is identify.

Next, the marginal distribution of the sea levels and the projected wind speeds is constructed. For the former the GEV distribution fitted to the annual maxima data (Figure 4) is used for determining the parameters of the GPD used for the upper tail of the distribution. For the latter the empirical distribution of the directions and the GPD fitted to the directional POT data (see Figure 2) are used, along with Eq. 2. Figure 6 and 7 shows the obtained distributions. It is noted that both of them gives a good fit to the data.

After fitting the marginal models, a Clayton copula is fitted to the marginal probabilities in order to model the dependence between sea level and projected wind speed (for an introduction to Copulas applied to extreme value analysis the reader is referred to Salvadori et al. 2007).

Finally, the marginal distributions and the copula fitted in this section are used for the simulation of a maximum sea level for each one of the 10^6 years of wind storms simulated in the previous section. Figure 8 shows a comparison of the sea levels extreme value distribution obtained from the simulations and the one fitted to the original annual maxima data. It is noted that there is good agreement between both.

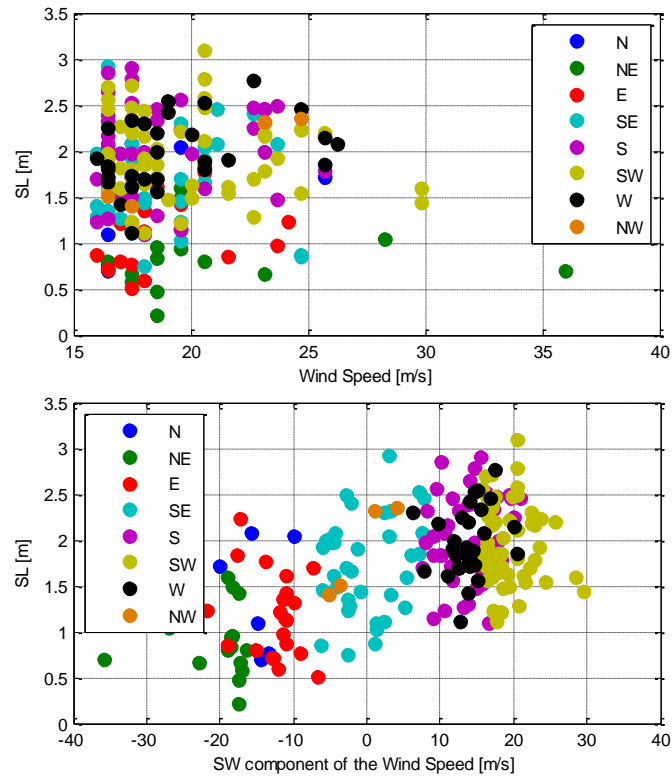


Figure 5. Top: Wind speed versus sea level. Bottom: Projected wind speed versus sea level. Wind directions are given in colors.

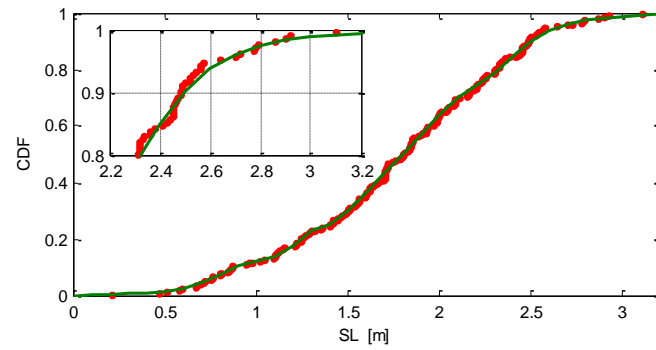


Figure 6. Empirical (red dots) and mixture (green line) distributions of the maximum sea levels reached during wind storms.

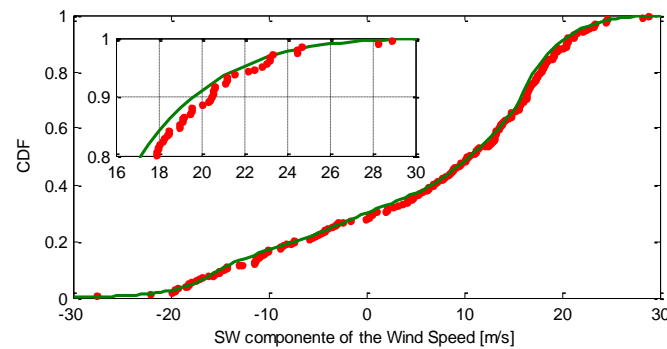


Figure 7. Empirical (red dots) and parametric (green line, eq. 2) distributions of the projected wind speed.

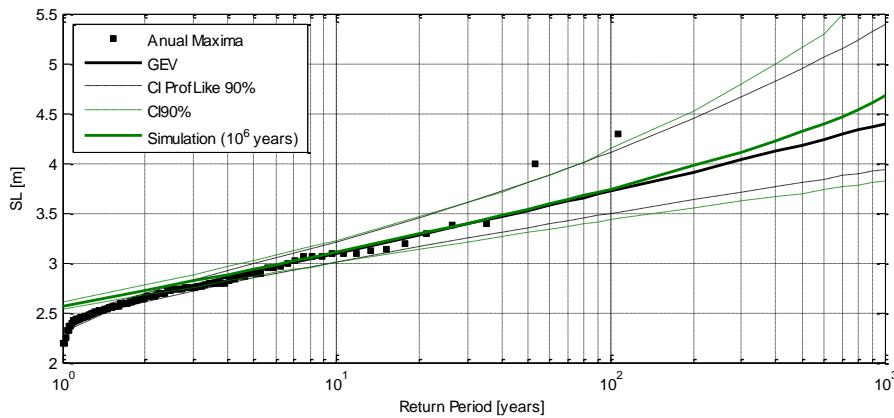


Figure 8. Comparison of the GEV distribution fitted to the annual maxima sea level data and the extreme distribution obtained from directional simulation of 10^6 years of wind storms.

Waves conditional to wind and sea level

The last step of the methodology is to construct a data base of SWAN simulations that allows to estimate wave conditions given wind and sea level condition.

For this a set of three nested grids are used, as shown in figure 9, and the SWAN simulations are run under stationary and uniform wind and sea level conditions, without considering incoming waves in the outer boundary. For the calibration of the model 15 severe events measured at Montevideo coast are available (less than two years of wave measurement), but no extreme event is available for calibration (return period of the available events is approx. 1 year).

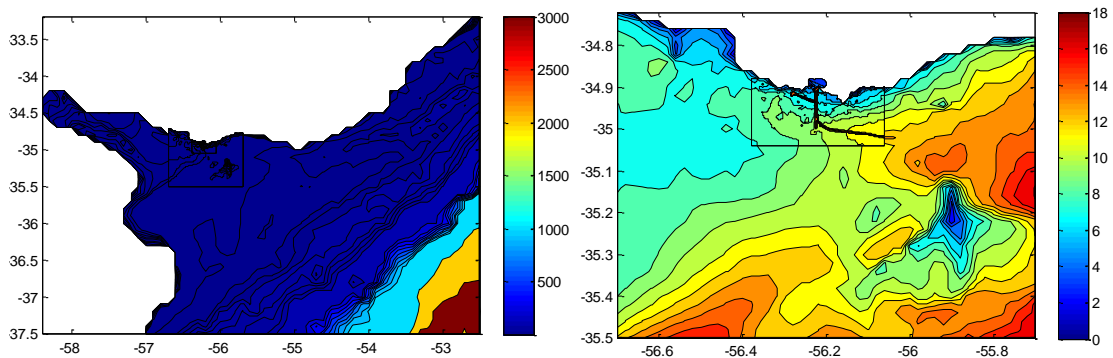


Figure 9. Bathymetry of the three nested grids used for SWAN simulation.

RESULTS

Using the proposed methodology several thousand years of wind, sea level and waves are simulated.

Figure 10 shows the joint distribution of significant wave height and simultaneous sea level obtained for a given point of Montevideo coast, obtained without considering an increment in mean sea level and considering a linear increment of 30 cm in the next 50 years. The joint distribution is constructed using 10,000 simulation of 50 years each. The curves represent combinations of constant joint return period of $(H_{m0} > X \cap SL > Y)$.

The simulation could be used also for the estimation of the more severe conditions that will reach a given maritime structure during its lifetime. For example, Figure 11 shows the joint distribution function of the maximum significant wave height obtained for each 50 years long simulation and its simultaneous sea level, thus the maximum expected significant wave height reaching a structure of 50 years lifetime and its confidence intervals can be estimated, as well as the expected sea level and its confidence intervals.

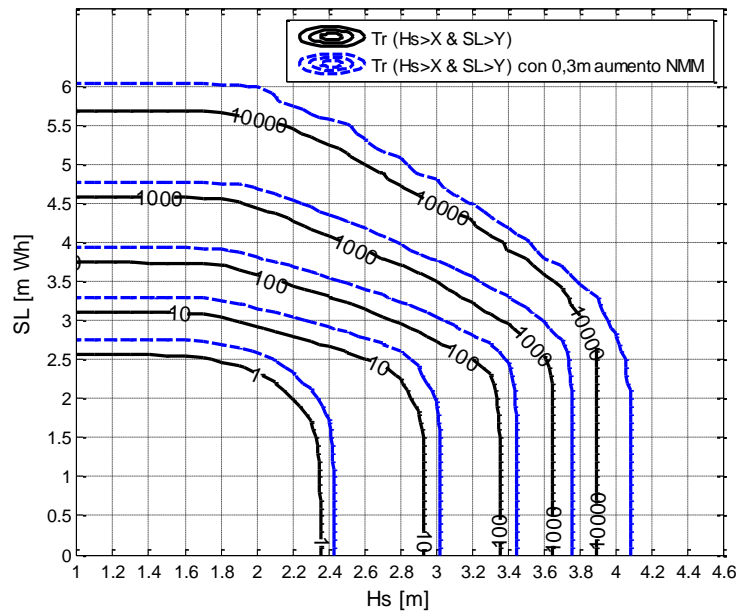


Figure 10. Joint distribution of extreme significant wave height and sea level, with (blue) and without (black) sea level rise.

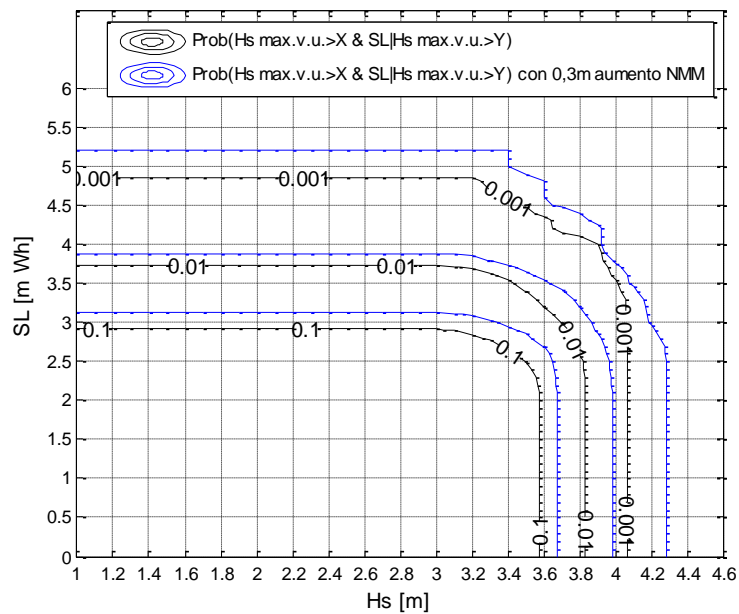


Figure 11. Joint distribution of maximum significant wave height and corresponding sea level expected for a 50 years lifetime, with (blue) and without (black) sea level rise.

DISCUSSION AND CONCLUSIONS

An ad hoc methodology was developed for the simulation of joint extreme wave and sea levels at Montevideo coastal area, in the northern coast of the mid Rio de la Plata estuary. Main highlights of the methodology are: (a) annual maxima information is included on the simulation of storm peak sea levels, and (b) a single dependence model between extreme wind (speed and direction) and sea level was developed based on the definition of a projected wind speed and its marginal probability distribution function.

Main limitations of the methodology comes from the simplifying assumptions and from the limited availability of wave data.

Regarding the hypothesis of the model, it is noted that maximum values of wind, wave and sea level were considered simultaneous and stationary. This is on the safe side as long as all variables add to the load terms of the verification equation, although there is no quantification of the level of overestimation. Also, base hypothesis of the model is that extreme sea level and wave condition only happened under extreme wind conditions. So far this hypothesis has proved to be reasonable for the mid Río de la Plata, but it should be verify every time that new information become available.

On the other hand, the limited availability of wave information in the area results in that some relevant parameters of the numerical models can not be calibrated. As a consequence numerical modelling uncertainties are not quantify by the methodology; e.g. SWAN breaker parameter defining H_{max} of the Battjes and Janssen (1978) model can not be calibrated with the available data (several tens of years would be required for this), however it significantly affects the high return period values of H_{m0} (see Figure 12).

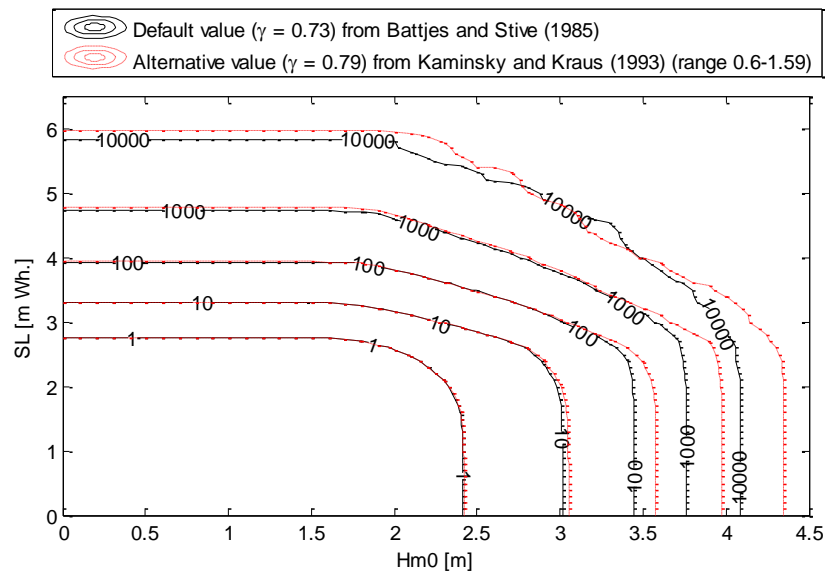


Figure 12. Joint distribution of extreme significant wave height and sea level, obtained for different values of the H_{max} parameter of the Battjes and Janssen (1978) model.

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