EOF ANALYSIS OF SHORELINE CHANGES FOLLOWING AN ALTERNATIVE BEACHFILL WITHIN A GROIN FIELD

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The evolution of a beach nourishment project constructed in Long Branch, NJ was investigated using the method of empirical orthogonal functions (EOF). Most applications of EOFs on beach fill projects have focused on traditional linear fills on relatively long, straight, uninterrupted coastlines. The Long Branch project was somewhat unique in that it was designed as a feeder beach and was constructed within a groin field. EOFs were used to analyze shoreline positions and nearshore beach slopes at the site. The first three modes, determined from the shoreline position data set, explain more than ninety percent of the variation from the mean. Mode 1 and Mode 3 illustrate variations of the fill's spreading as material moved in the direction of the net littoral drift, where several shore-perpendicular structures intercepted it. One of these structures was a large outfall pipe, which was shown to have a dominant influence over the shoreline evolution. The second mode was related to seasonal or storm impacts. The EOF analysis of the beach slope data also identified modes related to the spreading of the fill (Mode 2) and seasonal impacts (Mode 1). Overall, the eigenfunctions determined from both data sets reflect the morphological changes which were observed during the field surveys.

Keywords: EOF analysis, beach nourishment, groin

INTRODUCTION

In February 2009, a 700,000 cy, $9 million beach nourishment project was completed in Long Branch, New Jersey. This project was part of the New York District of the US Army Corps of Engineers (USACE), Sandy Hook to Barnegat Inlet, Beach Erosion Control Project. Since the completion of the project, the Davidson Laboratory at Stevens Institute of Technology has been conducting combined topographic and bathymetric surveys of the site using their DUCKS system (Miller & Herrington, 2008). While beach nourishment projects have been extensively studied, much of this research has focused on traditional linear fills located on relatively straight and uninterrupted coastlines. The typical behavior of these fills has been well established. The beach fill template used at Long Branch however, was non-traditional in that it included a feeder beach design, and was constructed within an existing groin field. While the influence of groins and other shore-perpendicular structures on coastline evolution has been studied, the research has not focused on how they influence the spreading of a beachfill over time.

A key goal of this research was to analyze how the interacting groin field and material spreading influenced the morphology at the beach nourishment site. To achieve this goal, empirical orthogonal function (EOF) analysis was used to help identify the patterns associated with the evolution of this project and the relative importance of these patterns. By taking the derivative of the temporal coefficients determined during the EOF analysis, a rate of change was calculated for each of the observed spatial patterns. This rate of change was found to be useful for understanding how these patterns changed with time and how individual events affected beach morphology.

SITE DESCRIPTION

The beach nourishment project was constructed between November 2008 and February 2009 by Weeks Marine, Inc. The USACE project design included a trapezoidal feeder beach placed at the center of the project, which was intended to nourish adjacent areas as it eroded. The feeder beach was designed so that the berm extended 500 ft from the seawall, with the toe of the fill in this section extending 900 ft. In the area outside the feeder beach, the design included a berm extended 300 ft from the seawall as a traditional linear fill. A plan view of the project design is shown in Figure 1. Problems arose when constructing the fill in water depths exceeding 25 ft; as a result, the final beach nourishment's toe extended offshore 300 ft less than originally designed. During the construction of the project, existing structures within the fill template were buried; however, two long outfall pipes and several existing notched groins updrift of the project site remained exposed.

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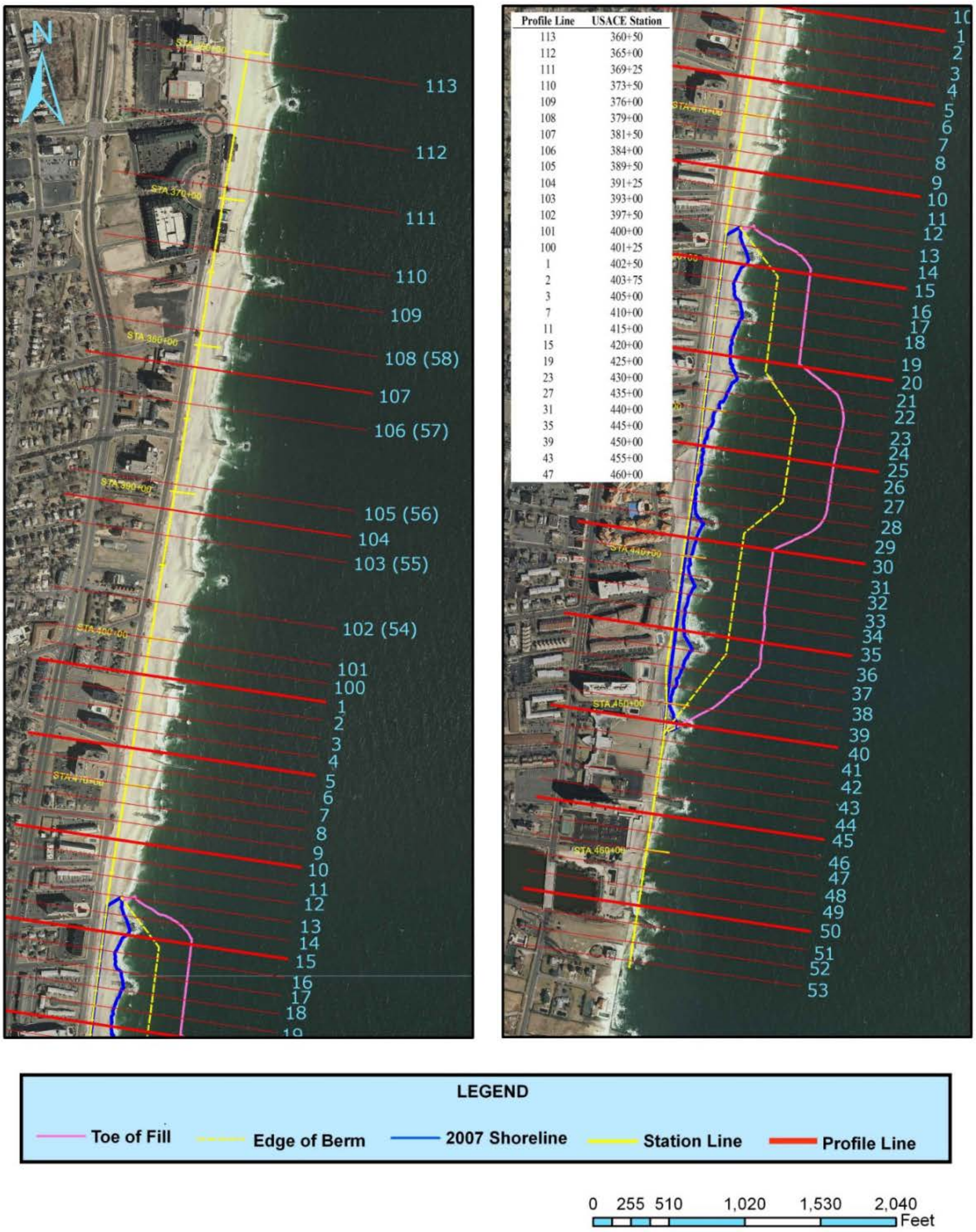

Figure 1: Long Branch nourishment project plan and monitoring lines

At the request of the State of New Jersey, the coastal engineering group at Stevens Institute of Technology developed a monitoring plan designed to capture the morphologic changes of the beach as the fill spread. The original monitoring plan (Miller and Herrington, 2008) consisted of 54 profiles which were spaced 125 ft apart. These profiles were to be surveyed weekly during the first month, biweekly during the second month, and monthly thereafter. Within the first month of survey work it became necessary to add profile lines to the north and remove some to the south, as the initial data indicated that the fill material was being transported faster and farther to the north than originally anticipated. The additional lines added to the north were spaced up to 400 ft apart, and resulted in a final monitoring plan consisting of 64 profile lines, covering a shoreline length of 1.75 miles (Figure 1). Due to the rapid pace of the changes that were observed during the initial monitoring, surveys were performed weekly for the first two months, biweekly for the third month, monthly for the remainder of the first year, and less frequently thereafter. During the period considered in this analysis, several major storms impacted the project site, including Hurricane Bill (October 22, 2009), the October 15, 2009 Nor'easter, the Veteran's Day Storm (November 14, 2009) and the March 12, 2010 Nor'easter.

Surveys were conducted using the Dynamic Underwater and Coastal Kinematic Surveying (DUCKS) system (Miller et al., 2009), and consisted of topographic and bathymetric surveys, in addition to a high resolution shoreline survey. The shoreline data was obtained by walking the Mean High Water (MHW) line separately from the profile lines to improve the resolution of the shoreline data, particularly in the vicinity of the structures. Survey results from the site indicated that the feeder beach eroded as expected with the majority of the material being transported north in the direction of the net littoral drift. Between Postfill Surveys 1 (2/10/2009) and 31 (6/28/2011), the project site experienced a net loss of 366,000 cy, with 238,000 cy transported north to downdrift beaches. During the surveys, qualitative observations were made which suggested that the morphological changes occurred in several stages. Stage One involved the initial spreading of the fill. Within the feeder section of the beach, the slopes remained stable while the shoreline eroded up to 200 ft; changes north of a large outfall pipe near Profile 6 were minimal. During Stage Two, the fill continued to spread and material began bypassing the outfall pipe, increasing the elevation and widening the beaches to the north. Stage Three corresponded to the reemergence of the groins which were buried during the construction of the project. Stage Four included the changes experienced during the first full winter. During this time, major changes occurred, including a reduction in berm height and a flattening of the beach slopes, as the beach transitioned to a more dissipative state. In addition, offshore bars developed and became connected to the exposed groins. In total the shoreline within the feeder feature eroded up to 390 ft from its original post construction position.

METHODOLOGY

Empirical orthogonal function (EOF) analysis was used as a way to objectively validate some of the patterns identified during the surveys and to identify more subtle patterns which may not have been noticed. EOF analysis, also termed principle component analysis (PCA), is a data reduction method used to describe a data set by the least number of independent functions. These functions are also referred to as modes of variability. Each of these modes is comprised of a spatial and a temporal component, where the first mode explains the greatest percentage of variation within the data set, with each successive mode representing less and less of the variance. The EOF technique was originally developed in the early 1900s, by Pearson (1901) and Hotelling (1933), independently, to determine the underlying patterns in seemingly random data. Although strictly a data analysis tool with no inherent physical meaning, it is often possible to relate the results of an EOF analysis with physical phenomena. For this reason, EOFs have been widely used for coastal studies.

Initial applications of EOF analysis in coastal engineering were concentrated on cross-shore variability. In the 1970s, beach profile data collected at Torrey Pines Beach, California was analyzed using EOFs by Winant et al. (1975). In their results, most of the variation in the beach profiles was explained by the lowest three modes. The first mode was determined to be the mean beach function, explaining the majority of the variation contained within the data set. Once this mean was removed, subsequent functions represented variations about the mean beach. The first two were related to physical occurrences; one being the shift from the summer berm to winter bar, and the other the low-tide terrace. Aubrey (1979) built upon this research and identified pivot points for cross-shore transport associated with seasonal changes.

Since its introduction to the coastal engineering field in the 1970's, EOF analysis has been used extensively to extract patterns in the morphologic changes of the beach. Although initially focused on beach profiles and cross-shore variability (Winant, et al., 1975), applications have extended to the study of long-shore variability, Clarke, et al. (1982) being among the first. Since then, applications have included the study of post-nourished beaches in terms of short, medium, and long-term responses of profiles (Larson, et al., 1999) and contour lines (Munoz-Perez, et al., 2001), as well as impacts to dune systems (Bochev-van der Burgh, et al., 2009). Additionally, Munoz-Perez, et al. (2001) illustrated the use of other related statistical measures in the study of the evolution of a nourished beach. More recently, applications have been extended to beaches with offshore coastal structures such as reef flats (Munoz-Perez, et al., 2010) and breakwaters (Fairley, et al., 2009).

In this study, EOF analysis was used to examine specific characteristics of the beach including shoreline position and beach slope. Time series of shoreline positions and slopes, based on data collected during seventeen surveys ranging from February 10, 2009 to June 28, 2011 were analyzed. A complete list of the surveys that were utilized is presented in Table 1. Shoreline positions were measured every 100 ft from a landward baseline parallel to the shoreline. Additional measurements were made near shore-perpendicular structures to capture the shape of each shoreline. Shoreline slopes were calculated by estimating the average shore-face beach slope about the 0 ft NGVD29 (National Geodetic Vertical Datum of 1929) contour line.

Table 1: Long Branch topographical surveys					
Survey	Date	Survey	Date	Survey	Date
1	02/10/2009	13	06/24/2009	21	12/09/2009
3	02/24/2009	14	07/20/2009	23	03/18/2010
8	04/09/2009	16	08/26/2009	25	07/22/2010
9	04/23/2009	17	09/15/2009	27	09/08/2010
10	05/13/2009	18	10/06/2009	31	06/28/2010
11	06/04/2009	20	11/17/2009		

EOF analyses can be performed on either the raw data set or on demeaned data. When performed on the raw data, the first mode typically represents a time varying version of the mean beach (Winant 1975). In the present work, the analysis was first performed on the raw data set, and then on the demeaned data to highlight the variability about the mean (temporal mean for each location was subtracted from the location's measurements).

EOF analysis involves decomposing a set of data into independent functions of space and time,

$$y(x,t) = \sum_{k=1}^{n} a_k c_k(t) e_k(x) \quad (1)$$

where $e_k(x)$ are the spatial eigenfunctions, and $c_k(t)$ are the temporal coefficients or temporal eigenfunctions. The number of independent functions, n, which are summed together, is the lesser of n_x or n_t, where n_x is the number of samples in space and n_t is the number of samples in time. The normalizing factor, a_k is calculated by $a_k = \sqrt{\lambda_k n_x n_t}$, where λ_k is the eigenvalue associated with the k[th] eigenfunction. These values are calculated through solving the eigenvalue problem

$$AE = \Lambda E, \quad (2)$$

where E is a matrix of spatial eigenfunctions, Λ is a diagonal matrix of eigenvalues, and A is the covariance matrix for the data set y(x,t) defined by

$$A = \frac{1}{n_x n_t}(YY^T). \quad (3)$$

where *Y* is a matrix containing the data set (x,t) and n_x and n_t are as defined earlier. *A*, is a measure of the spatial variability in the data set and is a matrix with the dimensions [n_x, n_x]. A similar approach is used to find the temporal coefficients. Each eigenfunction is associated with a single eigenvalue. The sum of the eigenvalues for each eigenfunction is equal to the total variance. Therefore, the percentage of variance explained by each respective eigenfunction, p_k can be determined as follows,

$$p_k = \left(\frac{\lambda_k}{\sum_{k=1}^{K} \lambda_k} \right) \times 100 \quad (4)$$

This property allows for a ranking of eigenfunctions by their contribution to the total variability in the data set.

When using the raw data, it was found that the dominance of the mean beach function obscured many of the underlying patterns. Therefore, the majority of the discussion is focused on the analysis of the demeaned data, where the spatial patterns represent deviations from the mean. In an effort to better understand the rates at which the various patterns identified by the EOF analysis were changing through time, the derivatives of the temporal coefficients were also analyzed. The same approach was used to evaluate both the shoreline position and shoreline slope data sets.

RESULTS AND DISCUSSION

Shoreline Position

When the analysis was performed on the raw time series of shoreline positions, the lowest mode was found to represent the mean beach. The spatial eigenfunction and corresponding time coefficients for the mean beach are displayed (Figure 2). Green vertical lines are used to denote the location of the groins and two long outfall pipes within the monitoring area. In the temporal coefficient plot, red lines are used to identify the major storms which impacted the project site. Both the red and the green lines are used consistently throughout the analysis to help focus the discussion.

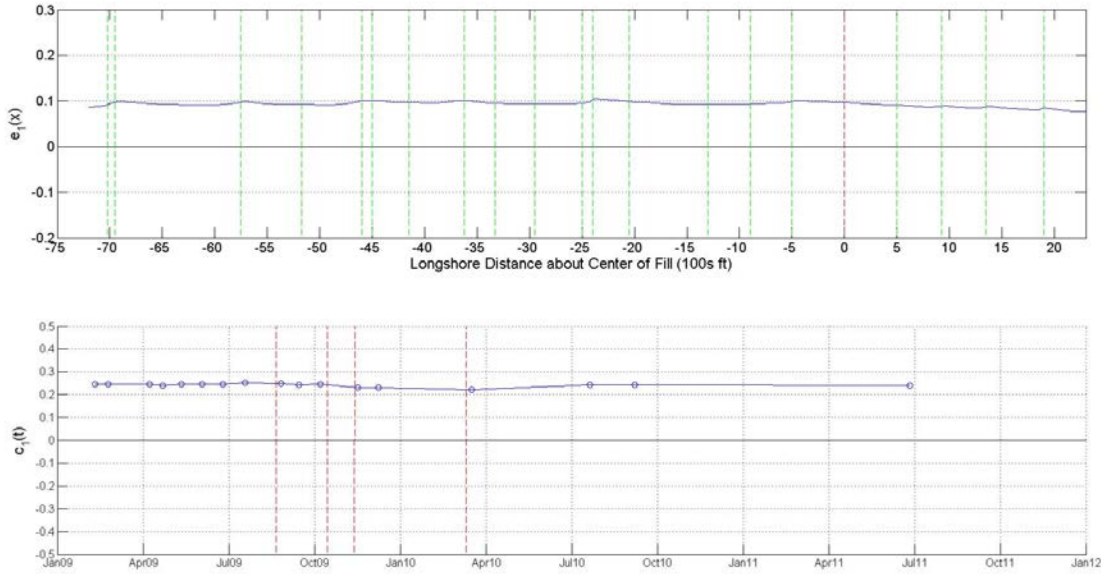

Figure 2: Mean beach spatial and temporal eigenfunction following analysis of shoreline position

As expected, the temporal coefficients associated with the first mode show very little variability, reflecting the fact that the spatial variation depicted represents the mean beach. Higher modes describing variations about the mean are obscured because the first mode describes such a significant portion (99%) of the total variance. To highlight the underlying patterns associated with the spreading of the beachfill and the influence of the groins, the analysis was performed a second time on the demeaned data. The first four spatial and temporal eigenfunctions calculated following the removal of the mean are displayed in Figure 3 and are summarized in Table 2.

The first mode, which explains 58.1% of the total variation in the demeaned data set oscillates about a nodal point 2,500 ft north of the center of the fill. This nodal point coincides with the location of the large outfall pipe located between profiles 5 and 6 (Figure 1). During the field surveys, this structure was noted to have a significant influence on the sediment transport by restricting the spread of the fill to the northern portion of the survey area. The temporal coefficients associated with the first mode consistently decrease with the exception of one disturbance, associated with the Veteran's Day Storm.

Table 2: Shoreline variation explained by eigenfunctions

Eigenfunction	$e_1(x)$	$e_2(x)$	$e_3(x)$	$e_4(x)$	Remainder
Total variance explained (%) Raw Data	99.7	0.22	0.04	0.02	.03
Total variance explained (%) Demeaned Data	58.1	23.4	10.2	3.49	4.78

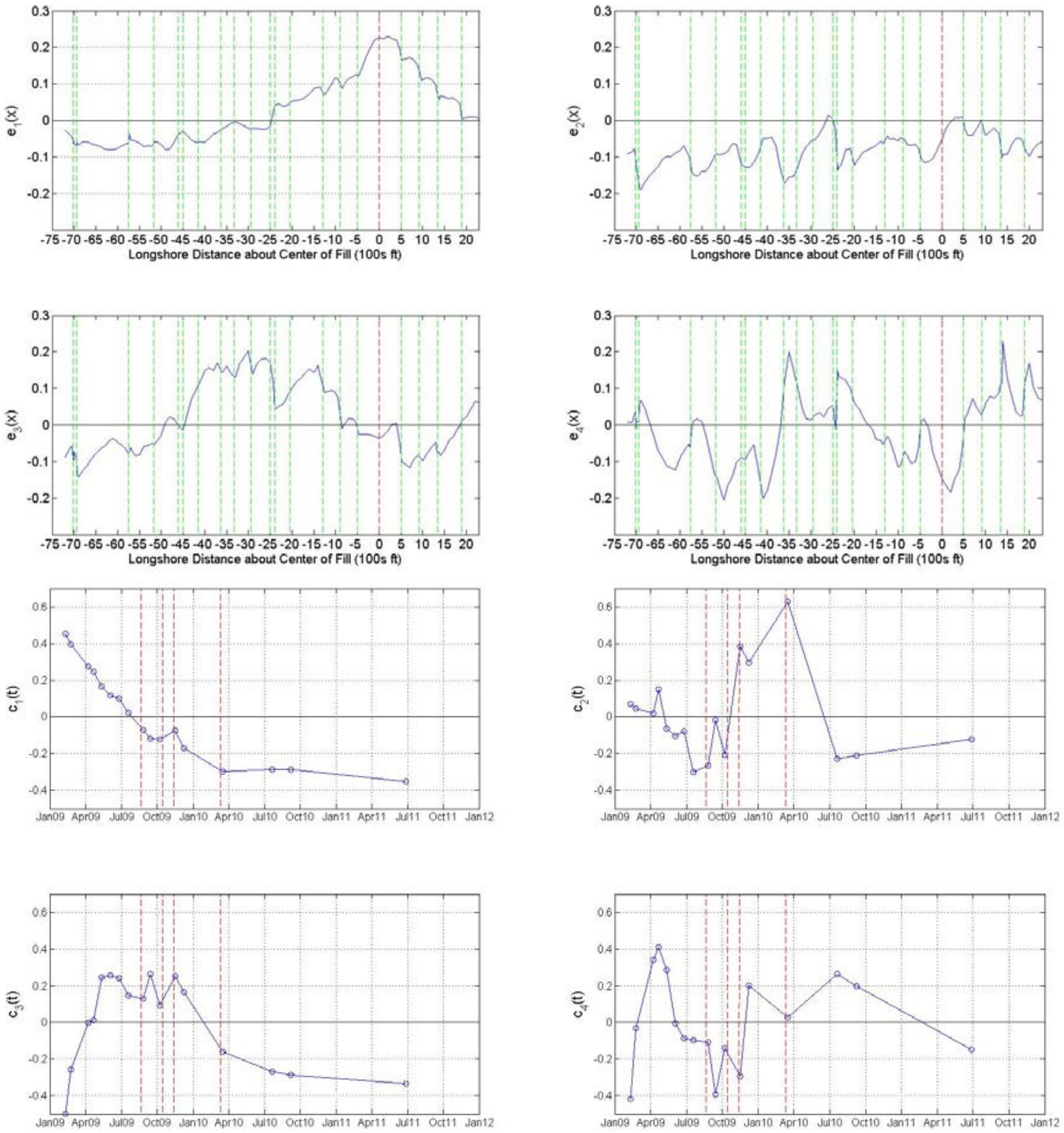

Figure 3: First four spatial and temporal eigenfunctions following analysis of shoreline position

To more clearly illustrate the behavior that Mode 1 represents, the data was reconstructed using the first spatial eigenfunction and selected temporal coefficients in Figure 4. As shown in the figure, the first mode represents consistent shoreline recession within the area of initial fill placement and south of the large outfall pipe. The recession is particularly dramatic in the center of the fill as the fill material spreads to the north and south. While the shoreline south of the pipe recedes, beaches north of the large outfall pipe actually accrete. Between the first (02/10/2009) and most recent survey (06/28/2011), 389 ft of shoreline recession was measured at the center of the fill. The behavior described by Mode 1 explains a significant portion (338 ft) of this total shoreline recession. A secondary feature observable in the first mode is a series of small irregularities related to the presence of the shore-perpendicular structures.

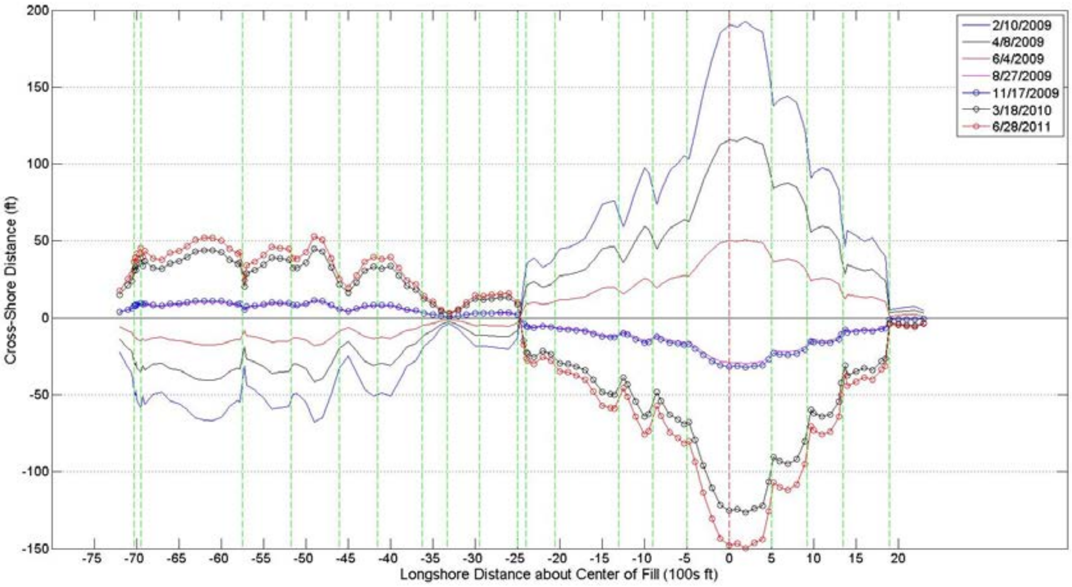

Figure 4: Reconstruction of shoreline position data from the first eigenfunction

The influence of the shore perpendicular structures is even more pronounced in the second mode. Near most of the structures there is a significant discontinuity in $e_2(x)$ across the structure. One of the largest occurs 2,500 ft north of the center of the fill at the large outfall pipe. Apart from the small section of beach between these nodes, the second spatial eigenfunction is consistently negative. This indicates that Mode 2 represents the entire beach (with the exception of the small section between the nodal points discussed above) eroding or widening as a whole depending on the sign of $c_2(t)$. Looking more closely at $c_2(t)$, a clear seasonal pattern is evident. This suggests that Mode 2 describes winter shoreline recession and summer advancement. Also identifiable in $c_2(t)$, are several extremes associated with coastal storms passing offshore of the project site during the analysis. These storms included a powerful nor'easter in March 2010 and the Veteran's Day Storm in November 2009. These two winter storms represent the most significant erosional events described by Mode 2. The fact that the magnitude of $c_2(t)$ increases significantly during the monitoring period matches the observation that major changes in the position did not occur until the onset of the first full winter storm season (~October 2009).

Mode 3 represents shoreline behavior that is similar to that described by Mode 1. The influence of shore-perpendicular structures on the longshore sediment transport is particularly pronounced in Mode 3, where clear nodes separate areas of accretion and erosion. One of these nodes occurs 4,500 ft north of the center of the fill, between profile lines 105 and 106, adjacent to the smaller of the two outfall pipes. The second node occurs at a groin 1,000 ft north of the center of the fill within the area of the initial fill placement, between profile lines 17 and 18 (Figure 1). The shoreline changes represented by Mode 3 were reconstructed in the same manner as those for Mode1. The resulting combined Mode 3 eighenfunction clearly illustrates that in the area between the nodes, the beach was initially shorter than the mean beach; however, immediately following the fill, the area just north of the fill up to the smaller outfall pipe widened until a stable position was reached. After the initial response, the shoreline in this area generally eroded, while areas to the north accreted. The shoreline behavior represented by Mode 3 is consistent with Stages 1 and 2 of the observations made during the surveys, where significant accumulation was noted downdrift of the pipe until bypassing began.

For each temporal eigenfunction, the first derivative was calculated to identify the rate at which the changes represented by each mode were occurring. The results of the first three are shown below in Figure 5. The results can be separated into those that relate to the spreading of the fill, and those that relate to seasonal and storm variations. Modes 1 and 3, which were found to relate to the spreading of the fill have rates that decrease over time. This corresponds to the rate of spreading decreasing as the volume of sand at the fill's center decreased and the beach returned to its equilibrium state. To better visualize how these rates and spatial eigenfunctions work together to describe the spatial and temporal variations, each mode was reconstructed by multiplying the spatial eigenfunction and the first derivative of the corresponding temporal coefficient (Figure 4 and Figure 5).

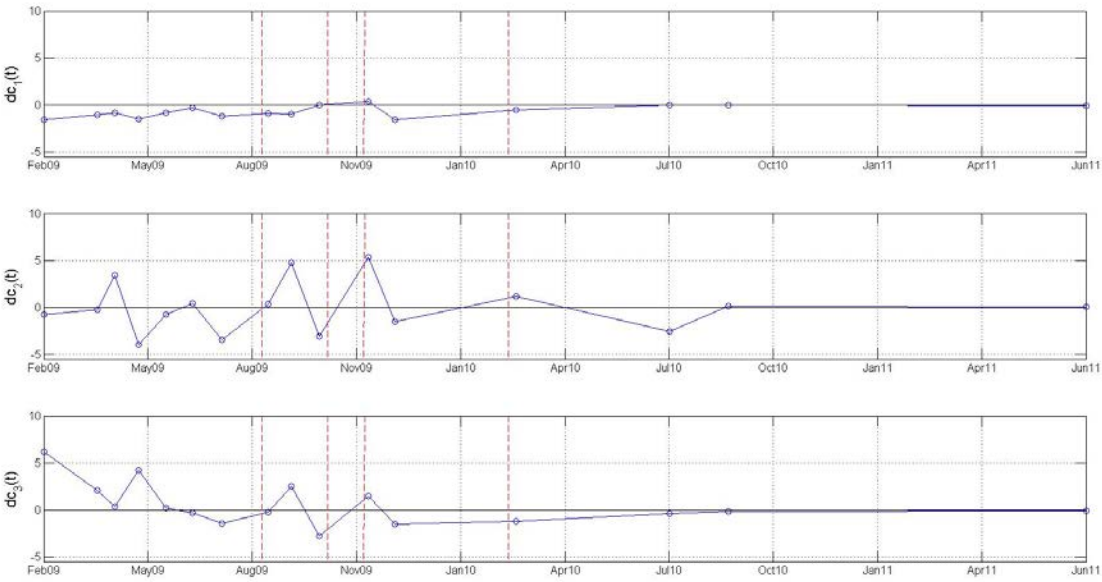

Figure 5: First derivative of first three temporal eigenfunctions following analysis of shoreline position

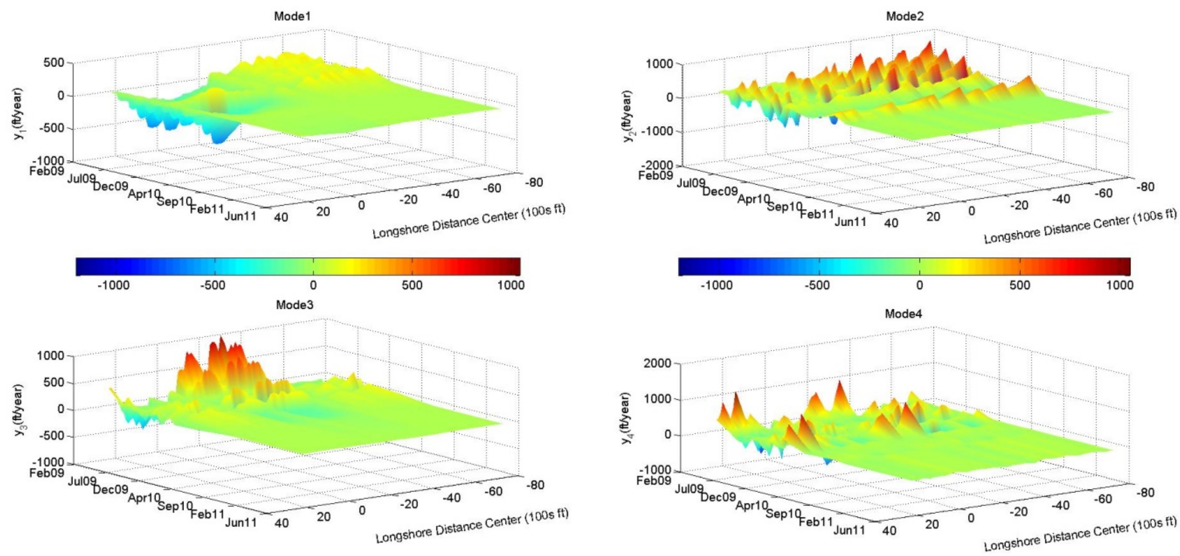

Figure 6: Shoreline position rates of change

Mode 1 depicts that initially the area at the center of the fill experienced high erosional rates while downdrift of the outfall pipe the rates were positive, indicating accretion. The magnitude of each of these rates decreases over time. Mode 3 illustrates a variation on this spreading. During the first six months of the project's life, material continued to collect on the updrift side of the large outfall pipe while bypassing was delayed. The rate at which the shoreline continued to accrete in this region decreased until the location remained relatively stable for several months. Following this point in time, rates were generally negative for this portion of the beach indicating a transport of sand from this location, downdrift to the northern beaches, where the rates became positive.

Oscillations present for Mode 2 represent changes in the direction of shoreline change as the beach alternated between typical summer and winter cross-shore profiles (Figure 3). Although the temporal eigenfunction for the second mode generally depicted how the beach became wider than the mean beach during the summer and narrower in the winter (Figure 3), there were periods of erosion and accretion in both seasons (Figure 5). This function highlights the influence of storms on the resulting shoreline position. After each storm, the value of the temporal coefficient increases from negative to positive. When the spatial eigenfunction is negative, the rate of change is negative as well, indicating that as a result of each storm, the shoreline eroded and became narrower.

Beach Slope

A similar approach to that described above for the shoreline position was followed to analyze the beach slope data. An EOF analysis was initially performed on the raw beach slope data, without the mean removed. As with the shoreline, the first mode was found to describe the majority of the variability in the data set and represent the mean beach condition. When the analysis was rerun with the demeaned data, the first four modes were found to explain more than 70% of the total variation in the data set. When working with the demeaned beach slope data, negative values represent slopes milder than the mean, while positive values represent steeper slopes. For comparison, the first four modes were found to explain more than 95% of the variation in the demeaned shoreline data. This implies that there are more complex variations occurring within the slope data set than in the shoreline data set. The first four modes (Figure 7) and a summary of the results (Table 3) of the beach slope analyses are provided.

Table 3: Beach slope variation explained by eigenfunctions

Eigenfunction	$e_1(x)$	$e_2(x)$	$e_3(x)$	$e_4(x)$	Remainder
Total variance explained (%) Raw Data	92.4	1.52	1.44	0.93	3.7
Total variance explained (%) Demeaned Data	43.1	11.7	9.9	7.2	28.0

Mode 1 of the demeaned data individually, explains 43.1% of the total variation in the data set. Overall for this mode, the spatial coeffiecients are negative, indicating the beach either becomes steeper or milder as a whole, depending on the sign of the temporal coefficient. The temporal coefficients tend to be positive during the winter months, indicating that the beach slope is milder than that of the mean beach. During the spring months, they tend to be negative, indicating that the beach is steeper than the mean beach. The seasonal trend described by Mode 1 is associated with the development of the winter profile which was observed by the survey team in the field as the fourth stage of morphologic change. The temporal coefficient also contains several extremes corresponding to the flattening of the beach slope during the Veteran’s Day storm and the March, 2010 Nor’easter.

The spatial eigenfucntion associated with Mode 2 contains a node approximately 175 ft south of the large outfall pipe, which separates areas where the beach slope is steepening from those where it is flattening. The spatial coefficients are negative north of the node and positive south of it (in the center of the fill). The temporal coefficients were initially positive, reflecting the fact that just after the fill the beach was steeper than the mean beach within the fill template and to the south of the outfall pipe, while the beach was milder than the mean beach north of the outfall. Over time, Mode 2 reflects the beach becoming milder within the fill template and steeper at the downdrift beaches. This pattern is consistent with the coarser fill material moving from the placement site to the beaches in the north. To more clearly illustrate this trend and others, surface plots were created by recombining the spatial eigenfunctions with their respective sets of temporal coefficients (Figure 7). The values resulting from this recombination represent the slope of the beach, relative to that of the mean beach.

Similar to what was done for the shoreline position data, the first derivative of the temporal coefficients associated with the beach slopes was calculated. The first three are shown in Figure 9. The results can be separated into those describing the spreading of the fill material and those describing seasonal or storm variations. Figure 10 depicts the reconstructed rates for the first four modes, where several characteristics stand out. Over time, the magnitude of the first derivative of Mode 2 decreases (Figure 9) as the rate at which the fill material moves away from the center of the fill and the rate at which the beach slope equilibrates slows over time. The rate of change temporal coefficients also illustrate that during each season (represented by Mode 1) there are periods when the beach steepens and flattens (Figure 9). The importance of storms is highlighted by the fact that there is a spike in the magnitude of the rate of change following each storm. As the corresponding spatial coefficients are negative, the increase in rate corresponds to the beach becoming milder after each storm. This agrees with the expected beach profile storm event response.

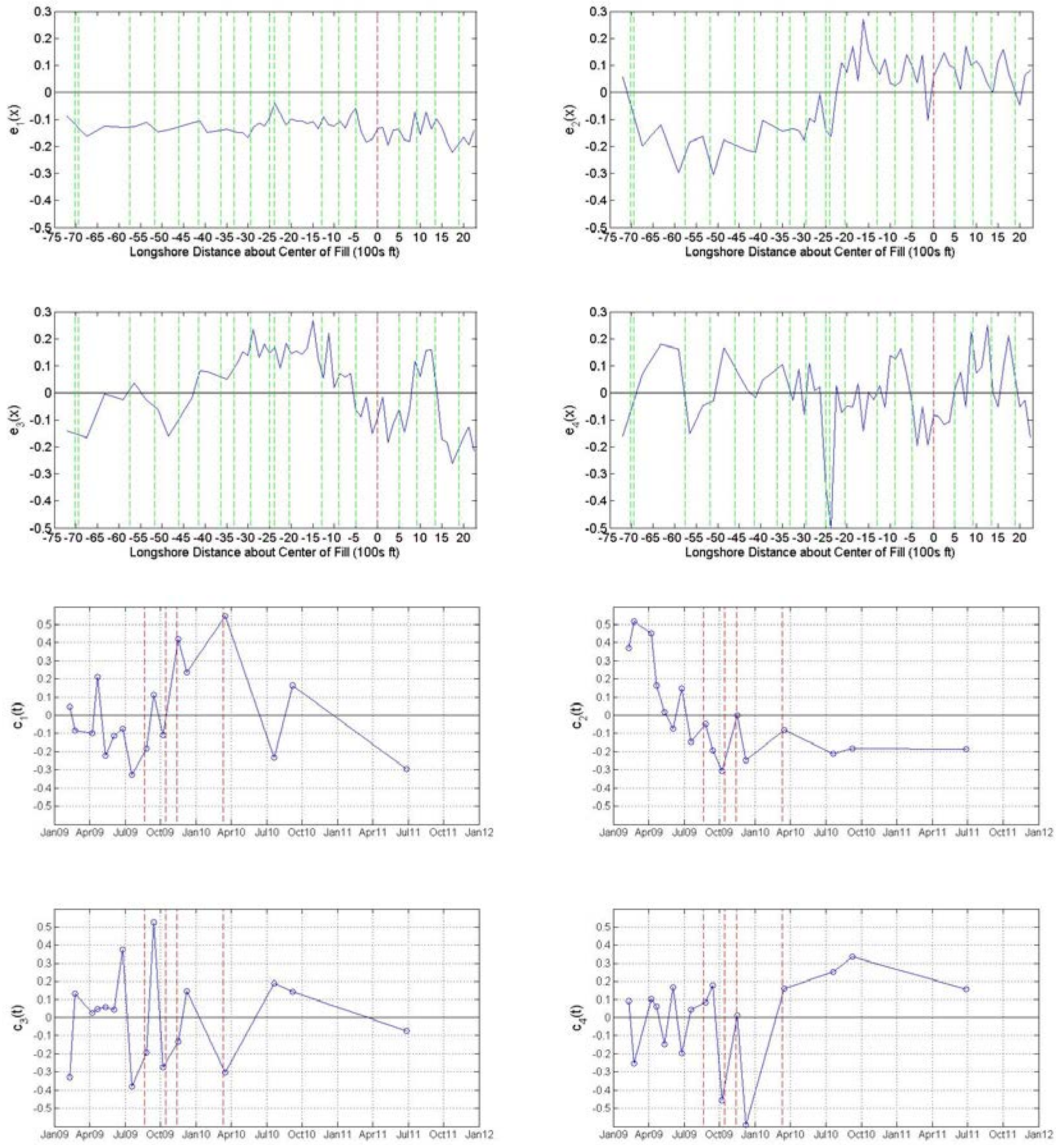

Figure 7: First four spatial and temporal eigenfunctions following analysis of beach slope

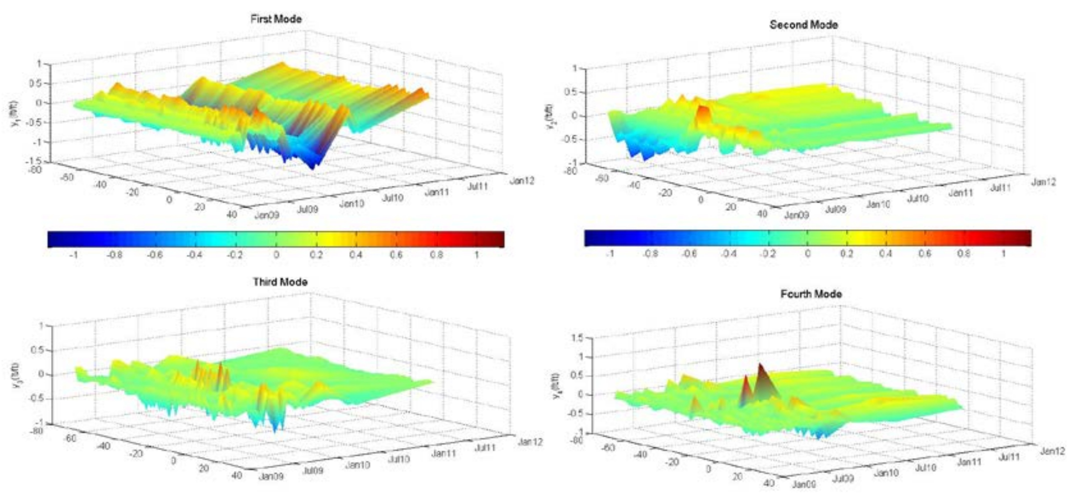

Figure 8: Reconstructed slopes for the first four eigenfunctions

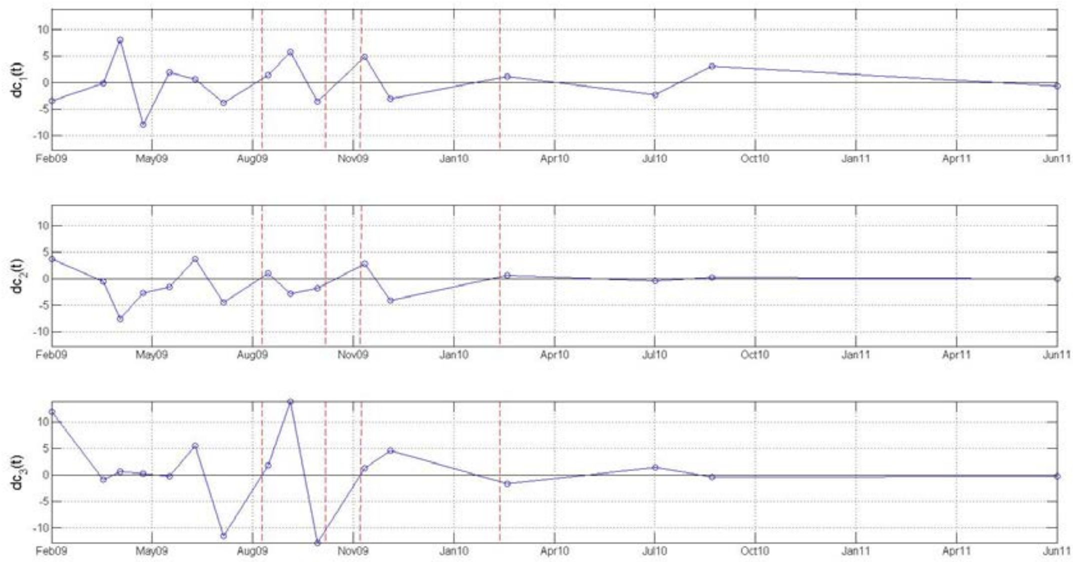

Figure 9: First derivative of first three temporal eigenfunctions following analysis of beach slope

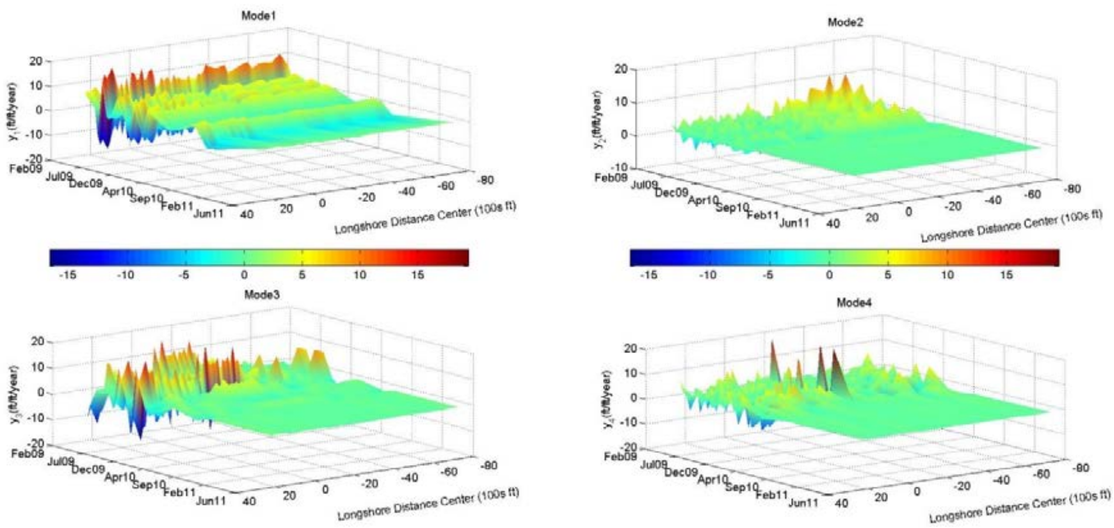

Figure 10: Beach slopes rates of change

CONCLUSION

EOF analysis is a non-physical data reduction technique that has been shown to provide physically meaningful insight into complex coastal systems in the past. In the present work, it was applied to identify patterns in the morphological changes resulting from a non-traditional beachfill at Long Branch, NJ. The results highlight important processes occurring at the site and confirmed patterns observed in the field. The beach fill project at Long Branch was unique in that a non-traditional template including a linear, rectangular fill and a feeder beach, was placed on a coastline within a groin field. EOF analyses of both shoreline position and beach slope were used to help identify patterns within the datasets related to the impact of the feeder beach and the groin field. The first mode identified from the shoreline position data and the second mode from the analysis of the beach slope data reflect the process of the beach fill spreading as the material moved from the center of the fill to areas further north. This is consistent with field observations, as the beach became narrower and milder at the center of the fill and wider and steeper in the north. The rate at which these changes occurred over time decreased, as reflected in the first derivative of the temporal coefficients. This result corresponds to the beach approaching an equilibrium state after the placement of the fill. Other variations that were identified were the result of background processes.

The EOF analysis also identified significant variability related to seasonal variations in shoreline position and slope. This was captured in Mode 2 of the shoreline position EOF analysis and Mode 1 of the beach slope EOF analysis. Each mode represents the beach changing as a whole at any given point in time. The seasonal modes reflect that during the summer months the beach was wider and steeper than the mean beach, while during the winter months it was narrower and milder. This is consistent with typical summer berm and winter bar profiles, and is similar to observations made in the field.

Within each spatial eigenfunction, the influence of the groins on both shoreline slope and position is apparent. Sharp variations in the spatial coefficients are typically observed in the vicinity of the groins. Additionally, some of the larger, more influential structures are observed to separate areas of accretion and erosion. For Mode 1 of the shoreline position results, and Mode 2 of the beach slope results, a large outfall pipe north of the fill area had a significant impact on the morphology of the site, acting as a long impermeable groin. It was observed to divide the portion of the beach that became wider and steeper over time from the portion that became narrower and milder as the fill spread. Other shore-perpendicular structures were also observed to delay the spreading of the fill. Mode 3 derived from the analysis of shoreline position data is related to material being transported from the center of the fill and building up on the updrift side of the smaller outfall pipe, until bypassing began approximately nine months following the project's construction.

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