MODELLING THE FLOW AROUND AN ISLAND AND A HEADLAND: APPLICATION OF A TWO MIXING LENGTH MODEL WITH TELEMAC3D

Nicolas Chini[1] and Peter K. Stansby[2]

Numerical modelling of the circulation around islands and headlands is a challenging task with bottom friction and eddy viscosity controlling the formation of wakes and eddies. The eddy viscosity terms in such configurations can become predominant and an implicit solver is desirable to maintain reasonable computational time. An approach based on coupling the horizontal mixing length to the vertical one following Stansby (2003) is implemented into TELEMAC3D (Hervouet, 2007). This development is then tested against two different datasets of laboratory experiments, the one representing the flow around an island and another one the oscillatory tidal flow in the vicinity of a headland. Comparisons with conventional eddy viscosity models are presented.

Keywords: rubble-mound breakwater; porous concrete; turbulence modeling

INTRODUCION

Representing free surface flows around features such as headlands or islands with numerical models is necessary to predict the dispersion of dilute solutions, sediment transport, bed evolutions. Attempts to model the formation of flow characteristics in the vicinity of those features can be challenging as eddy viscosity terms become predominant. Coastal modelers benefit from a large variety of turbulence closure in order to estimate the eddy viscosity. One can think of classical zero equation models such as Prandlt vertical eddy viscosity model, Smagorinsky horizontal or vertical model, or more complex 2D equation model such k-ε or k-ω models.

A two length scales formulation is proposed by Stansby (2003), for which the horizontal mixing length is a multiple of the vertical one. The standard Prandlt vertical mixing length is assumed. Stansby (2003) tested this formulation against a dataset of experiments dedicated at representing the flow around an island for different inflow conditions and island geometry (Lloyd and Stansby, 1997). The proposed modelling for eddy viscosity was able to represent either eddy formation or a stable wake. The structure of the wake is related on a stability parameter depending on the inflow velocity, the bottom coefficient and the water depth.

Here, it is proposed to test the open source TELEMAC3D, the Navier-Stokes solver from the hydro-informatics suite TELEMAC (Hervouet, 2007), based on finite element methods, on the dataset by Lloyd and Stansby (1997), hereafter referred as LS97. Therefore the eddy viscosity proposed by Stansby (2003) is included into TELEMAC3D, and a model of the experimental dataset of LS97 is built. Given the availability of another dataset of flows around a headland (Lloyd et al., 1998), a second model is constructed to test the numerical modelling against the data collected in laboratory conditions.

After presenting the eddy viscosity model proposed by Stansby (2003), the experimental datasets are introduced. Then the parameterisation of the numerical models is detailed. Results are then presented with prior analyses and conclusion.

TWO MIXING LENGTH MODEL FOR EDDY VISCOSITY

Mixing length models are usually employed for 1D modelling. Based on LES consideration by Rodi (1984), Stansby (2003) considered extending the mixing length approach to the 3 dimensions by using the vertical mixing length to estimate a horizontal mixing length. Thus, Stansby (2003) proposed the following formulation for the eddy viscosity:

$$\nu_t = \left(l_h^4 \left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 \right] + l_v^4 \left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 \right] \right)^{1/2}$$

where u and v represent the horizontal components of the velocity; l_v and l_h are respectively the vertical and horizontal mixing lengths. l_v is estimated according the classical Prandtl mixing length, assuming a mixing layer thickness representing 20% of the total water depth. In order to return conditions for the viscous sublayer with a smooth bed the vertical mixing length is damped according to van Driest formula. To represent the viscous sublayer in the model mesh compression is required. The methodology presented by Stansby (1997) is applied. TELEMAC3D offers the possibility to compute

---

[1] HOCER, 1 Place de Strasbourg, 29200, Brest, France
[2] School of MACE, University of Manchester, M13 9PL, UK

the friction velocity according to the Reichardt law for smooth regime friction. The linear friction velocity for viscous layer is added to TELEMAC3D. TELEMAC3D also includes an option to compute the vertical derivative according to a logarithmic derivation, a suitable method for near wall velocity profiles. That latter option is used when imposing a bottom roughness thickness.

The horizontal mixing length is assumed to linearly depend on the vertical mixing length. Stansby (2003) finds that a ratio of 6 between the two mixing lengths represents the transition between vortex shading and stable wake as presented in LS97 experiments.

ISLAND TEST CASE PRESENTATION

LS97 conducted laboratory experiments in order to quantify flow around a surface-piercing island. The dataset contains several simulations for different inflow conditions leading to wake formation downstream of the island.

The experimental set up consisted of a conical island inserted into a 1.52m wide and 4.52m long flume. Although LS97 tested different island configuration, the island having 8° slope is here considered. The inflow conditions are characterised by a flow velocity of 0.1m/s. Three different outflow water depth are tested (0.019m, 0.0145m and 0.01325m).

Using particle tracking velocimetry technique, LS97 were able to analyse the surface velocity field and they showed that the unsteadiness in the wake was related to a parameter, called the stability parameter, $St = C_f D/h$, where C_f is the free stream bottom friction coefficient, D is the mean island diameter and h is the water depth. Wake stability was observed for critical stability parameter value of 0.40. Lower stability parameter leads to vortex shading. The three different water depth conditions lead to a stability parameter of 0.26, 0.36 and 0.405, respectively.

ISLAND TEST CASE NUMERICAL SET UP

To simulate the island experiments, a numerical model based on the TELEMAC hydro-informatics suite is set up. The domain extent is as considered by LS97. The numerical grid consists of regular triangular elements with a spatial resolution of 0.015m. The vertical is divided into 10 planes. The time step is set to 0.01s.

TELEMAC3D contains a wide variety of numerical options to solve the equations. The model is used with wave equation option for the water depth advection while the advection of the velocity is treated with the Streamline upwind/Petrov-Galerkin formulation. A semi implicit formulation for u and h are considered. The solver for the linear system is GMRES.

The correction for horizontal gradients for gentle slopes proposed by Stansby (2003) is not considered here. The TELEMAC3D option treating the hydrostatic inconsistency is alternatively used.

The boundary conditions are the following. At the inflow boundary, the velocity is imposed to 0.1m/s and water level is left free. Asymmetry is set up by imposing a cross velocity $v = 0.5\sin(\pi t)$ for $t<1$s. At the outflow boundary, only the water depth is imposed. Along the lateral boundary, a slip condition is assumed. To represent the viscous layer the mesh compression proposed by Stansby (1997) is used and a non-slip condition is set at the bottom. The friction velocity is estimated assuming the velocity profile near the wall to be linear.

Different eddy viscosity models are available in TELEMAC3D. Here we consider

1. Smagorinsky model for horizontal eddy viscosity coupled with vertical eddy viscosity based on Prandtl mixing length,
2. The two mixing length model as presented in Stansby (2003).

ISLAND TEST CASE RESULTS

Results presented for the island test case are the surface velocity obtained for three stability parameters and the two considered eddy viscosity models. They are qualitatively compared with the ones obtained by Stansby (2003).

Result for a St=0.26

Fig. 1 shows the free surface velocity for a stability parameter that is related to strong vortex shedding. Both eddy viscosity models present the formation of vortices downstream the island. The use of Smagorinsky model for the horizontal eddy viscosity leads to stronger velocity in the wake of the island.

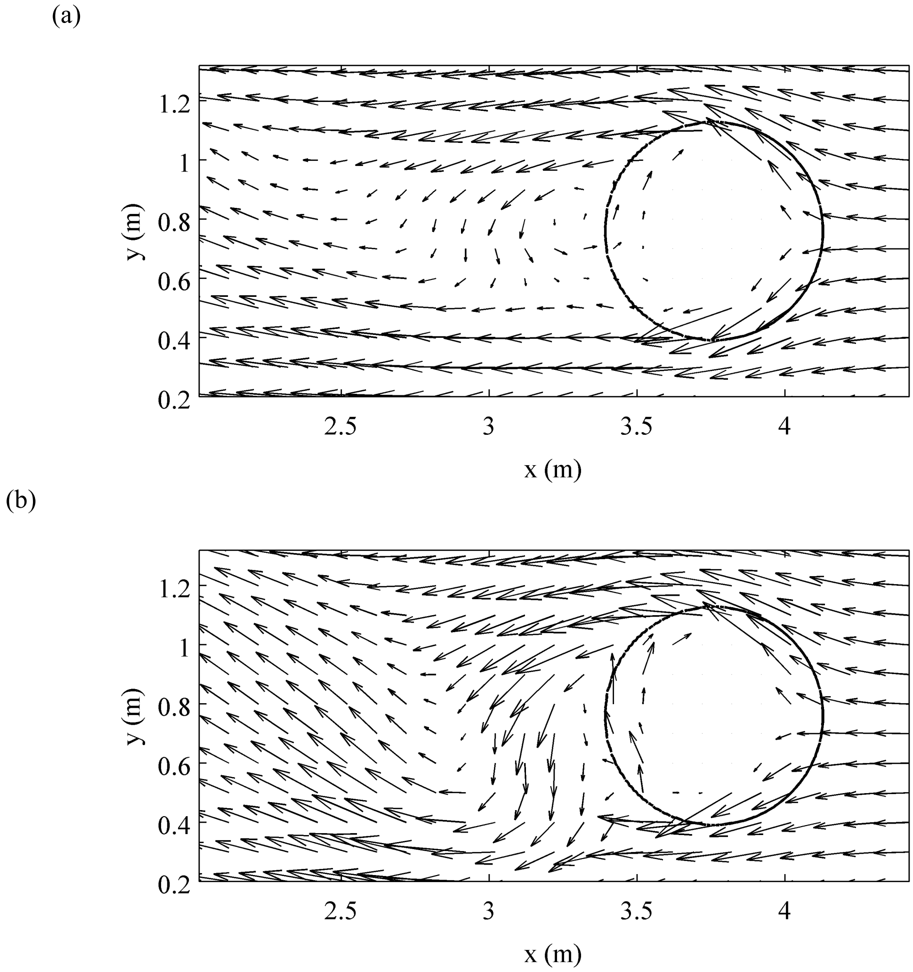


Figure 1. Surface velocity for stability parameter of 0.26.; (a) two mixing length model and (b) Smagorinsky model

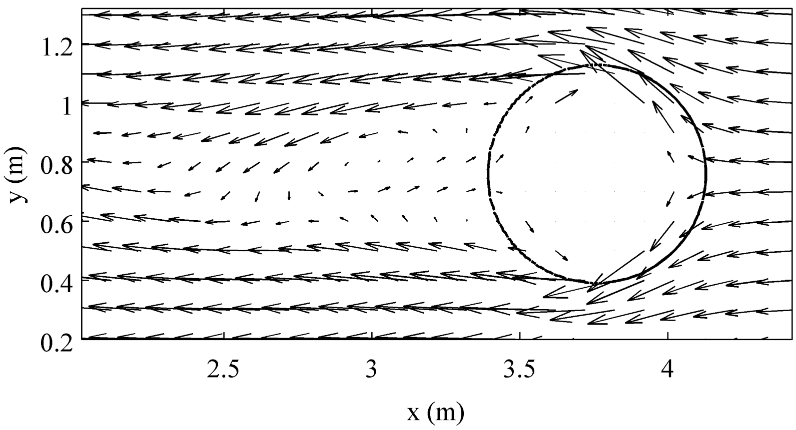
Result for a St=0.36

Results for a stability parameter of 0.36 are presented. For that parameter, LS97 found the wake to be more stable with the formation downstream the island of an unstable bubble. The two mixing length model as formerly presented by Stansby (2003) is able to reproduce the observation by LS97. Results for Smagorinsky model show a strong vortex shedding (Fig. 2).

Result for a St=0.405

This case corresponds to the critical stability parameter for which the wake becomes stable, as illustrated by LS97. Results obtained with the two mixing length model show that the wake is stable. A stagnation point is located at distance of 1.45m downstream the island centre, for the simulations presented here. Stansby (2003) found the stagnation point to be further away from the island, closer to the one observed experimentally by LS97. Results obtained with use of Smagorinsky model do not show a stable wake. Results are presented in Fig. 3.

(a)



(b)

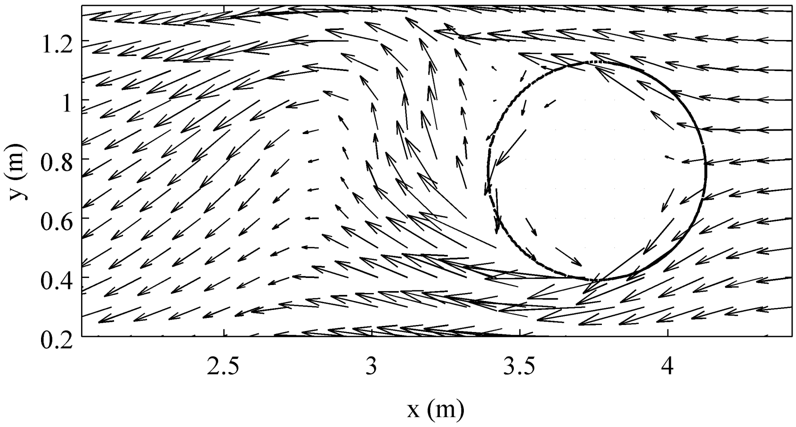
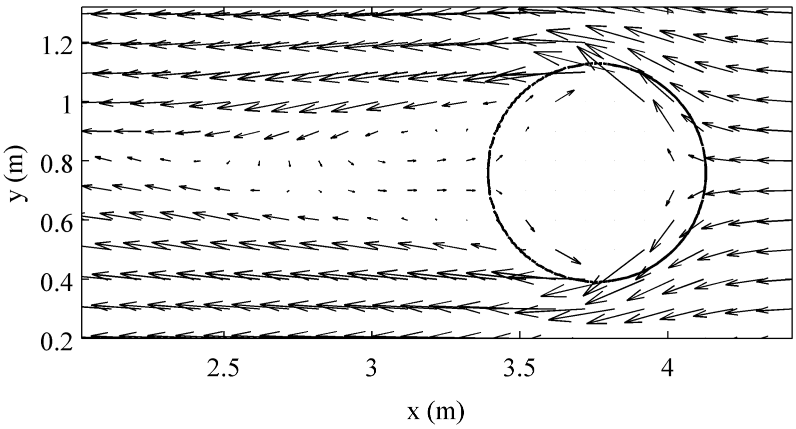


Figure 2. Surface velocity for stability parameter of 0.36.; (a) two mixing length model and (b) Smagorinsky model

(a)



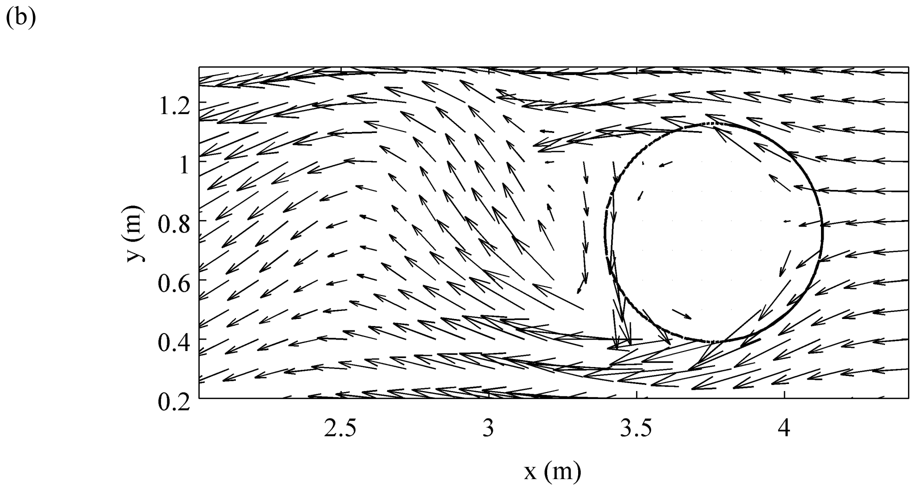


Figure 3. Surface velocity for stability parameter of 0.405.; (a) two mixing length model and (b) Smagorinsky model

HEADLAND TEST CASE PRESENTATION

Following the steady current case of an island, another test is performed for tidal flow around a headland. Experiments were conducted by Lloyd et al. (1998) at the UK Coastal Research Facility CRF. The model consisted in a 36m long and 20m wide rectangular basin. An oscillatory flow is created at each end of the basin to simulate tidal flows.

The cross sectional topography of the basin consists of a beach followed by flat bed offshore. The slope of the beach is 1 in 20. For the flat part of the basin, the experimental water depth is 0.5m. For the headland test case, a conical structure as been created. It is 8m long with a 1 in 5 slope.

Three different experiments are performed for different tidal periods and inflow velocity, with interest in quantifying flows for different Keulegan-Carpenter number. Those conditions lead to different eddy distribution downstream the headland. Table 1 presents the flow parameters tests here extracted from the data collected by Lloyd et al. (1998).

Table 1. Flow parameters for CFR oscillating flow tests.

h (m)	T (s)	U_0 (m)	KC	Re ($x10^5$)
0.48	120	0.10	5.10	1.91
0.48	240	0.15	15.3	8.59

Results from the experiments are ADV temporal series and PTV snapshot of surface current at two moments during the tidal cycle: one at the quarter of period and the other one at the half period.

HEADLAND TEST CASE NUMERICAL SET UP

To simulate the headland experiments, TELEMAC3D is used. The domain extent is similar to the one considered by Lloyd et al. (1998). The numerical grid consists of regular triangular elements with a spatial resolution of 0.1m. The vertical is divided into 10 planes. The time step is set to 0.1s.

The numerical parameters are similar to one used for the island test case.

The model is used for the following eddy viscosity model:

1. Smagorinsky model for horizontal eddy viscosity coupled with vertical eddy viscosity based on Prandtl mixing length,
2. The classical k-ε model
3. The two mixing length model as presented in Stansby (2003).

When a vertical mixing length is used, the mixing layer thickness is 20% of the total water depth.

The boundary conditions are the following. Zero water levels gradient and velocity components are imposed along the open boundary. A slip condition is assumed along the lateral boundary.

The bottom friction is defined in this test case by imposing a uniform Nikuradse parameter based on the roughness of the bed. A value of 0.02m is imposed as defined by Lloyd et al. (1998).

HEADLAND TEST CASE RESULTS

Results are presented for the two following tidal periods: T=120s and T=240s.

In both case, two sets of results are shown. First the results for the surface velocity obtained with the two mixing length are presented. Secondly a quantitative comparison between times series obtained for the different eddy viscosity models are considered.

Result for a T=120s

Fig 4 shows the surface velocity for two different time steps during the tidal period. Results for a tidal period of 120s show the formation of a strong eddy downstream the headland during slack water. Wake formation during the maximum of flood is less obvious. For that tidal period flow separation does not occur downstream the headland.

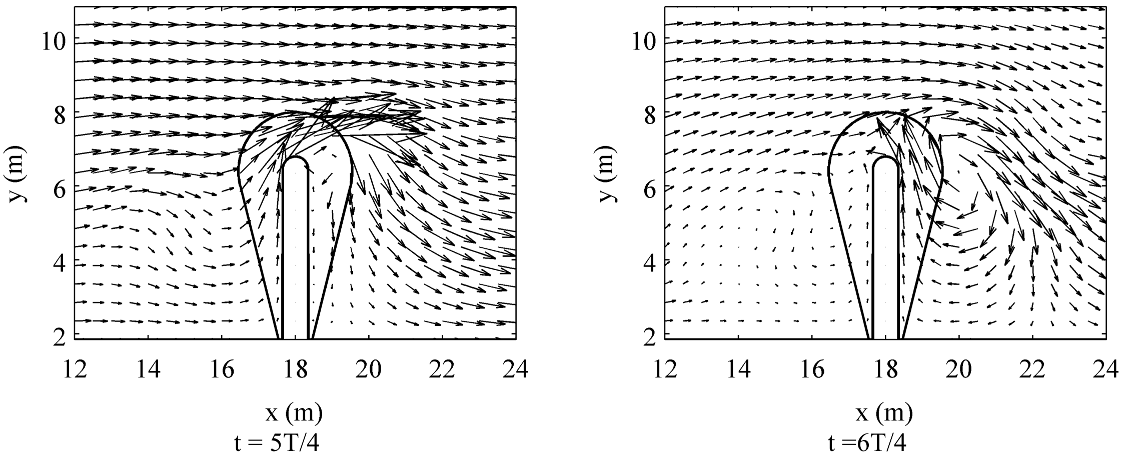
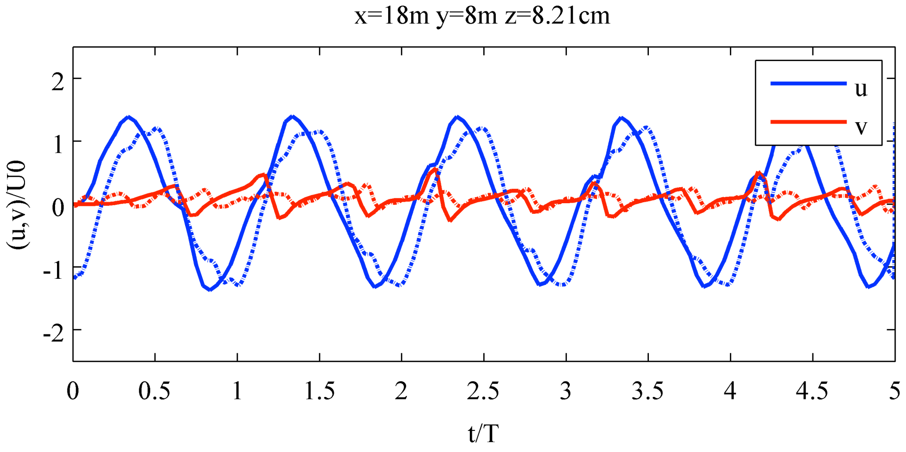


Figure 4. Surface velocity for T=120s

Fig. 5 presents the temporal series of velocity components at x=18m and y=8m, location of headland apex. This figure also shows a comparison with ADV measurement at z=8.21cm. Numerical results are linearly interpolated to estimate the velocity at that water depth. The overall results tend to show that for that particular location, each eddy viscosity model reproduces well the velocity u. For velocity v, which is much smaller, peaks appearing just after slack water, according to the measurements, are represented by both Smagorinsky model and the two mixing length model. Simulations based on k-ε model leads to smoother velocity and those peaks are no longer simulated.

(a)



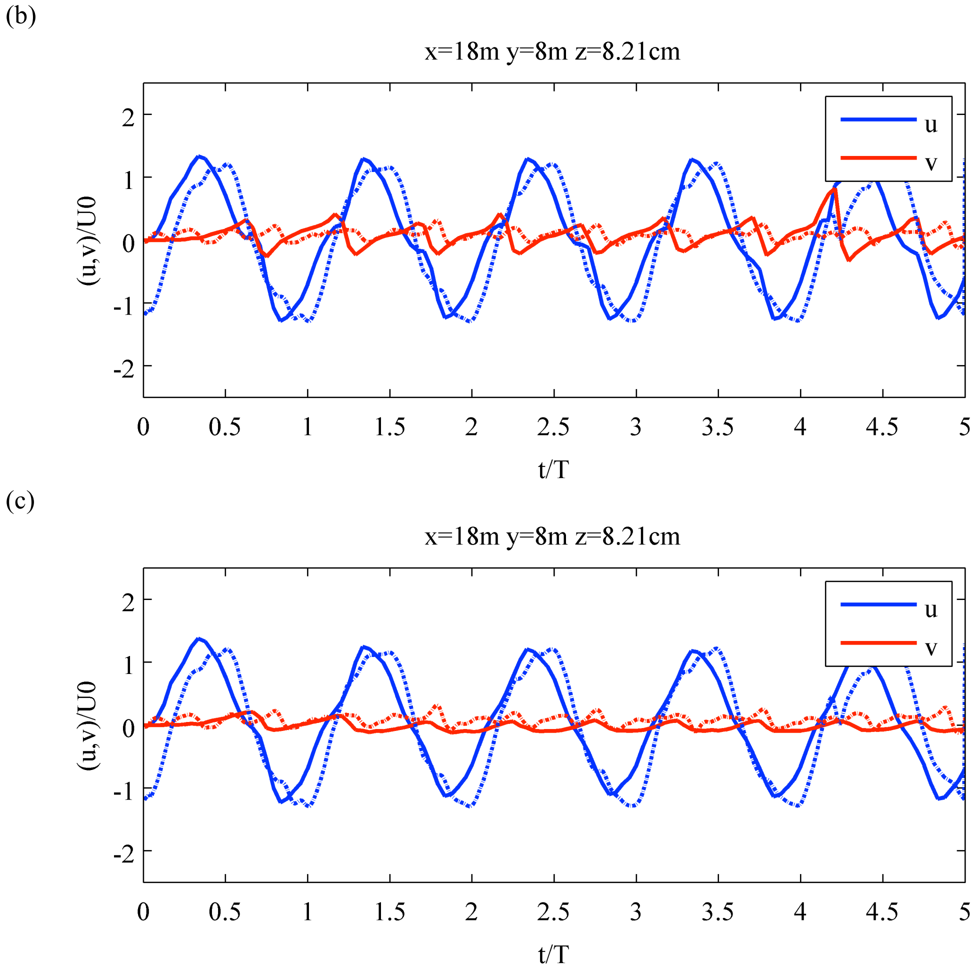
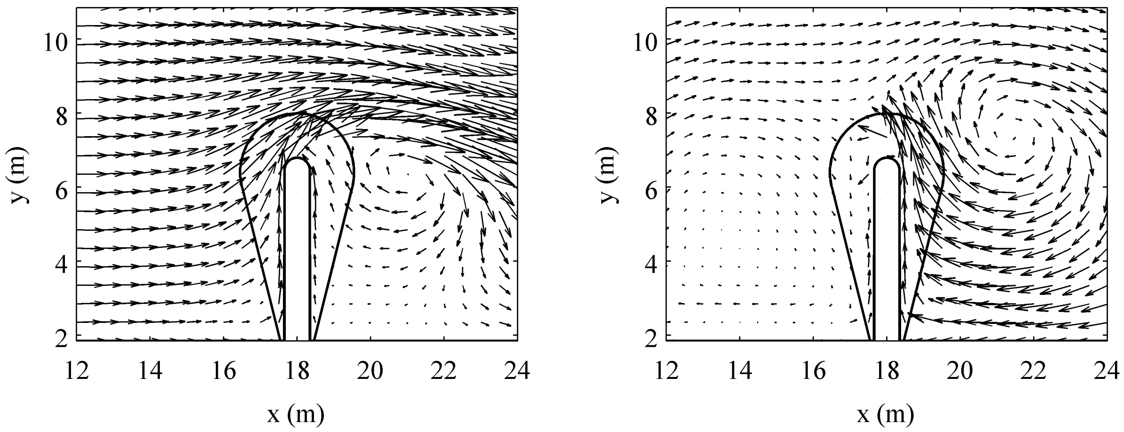


Figure 5. Surface velocity for T=120s. Temporal series of velocity component (a : two mixing lengths model, b: Smagorinsky model, c : k-ε model)

Result for a T=240s

For the tidal period of 240s, flow separation occurs both at slack water and at the maximum of flood, as shown on Fig. 6. The vortex is particularly intense along the edge of the headland at slack water. During the maximum of flood, a return current is noticeable along the downstream edge of the headland. The location of the eddy is not stable during the tidal cycle. It moves in the direction of headland apex between the maximum flood and slack waters.



t = 5T/4 t =6T/4

Figure 6. Surface velocity for T=240s

Quantitative comparisons are made at different locations around the headland apex. Lloyd et al. (1998) monitored the velocity at the 5 locations represented on fig. 7.

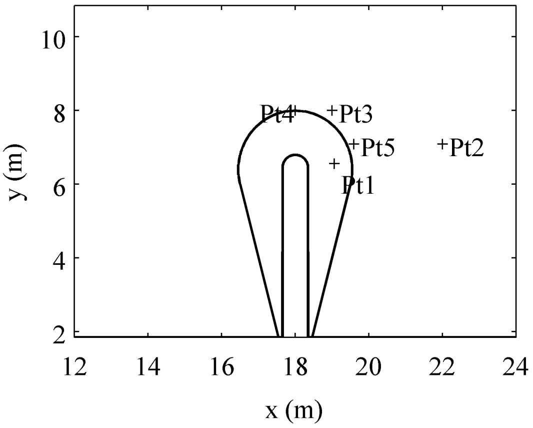
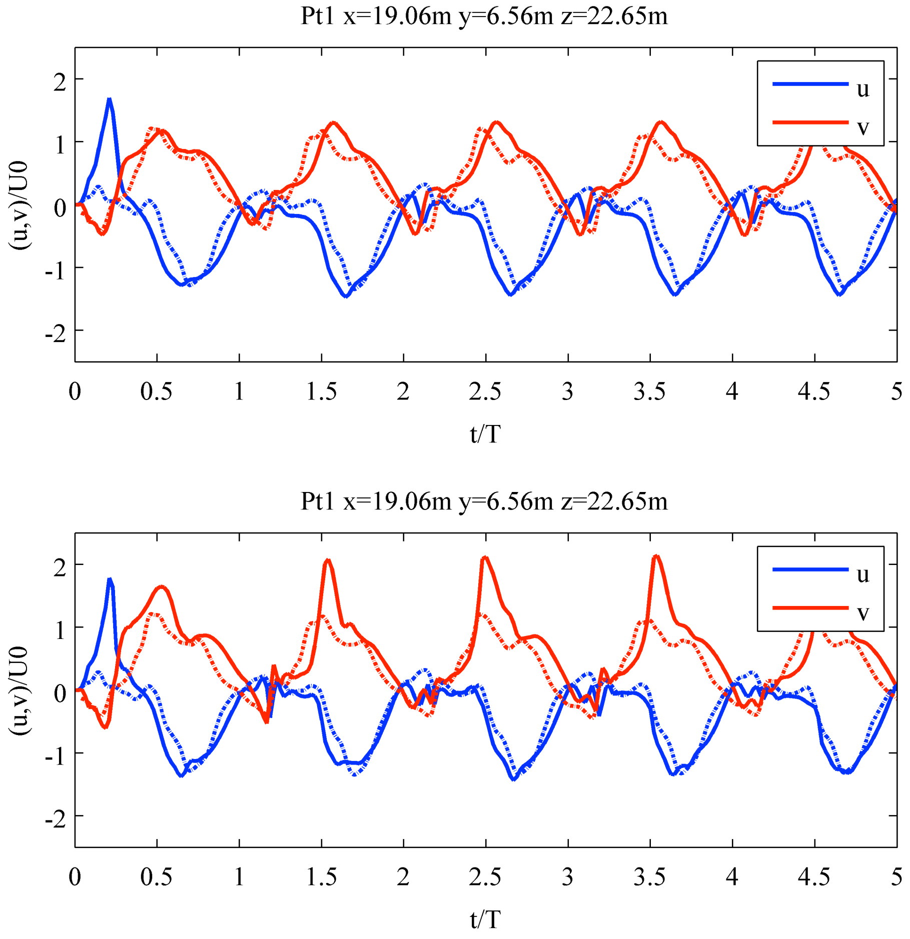


Figure 7. Surface velocity for T=240s

Fig. 8 presents the results at point 1 for velocity measured near the surface at z= 22.65m. Results obtained with the two mixing length model is able to represent the magnitude of each velocity components. Results obtained with Smagorinsky model exhibit comparable validation for cross velocity. However, the longitudinal velocity is overestimated during slack waters. Similar observations can be made for results obtained when the k-ε model is taken into account in the parameterisation of the simulation.

(a)



(c)

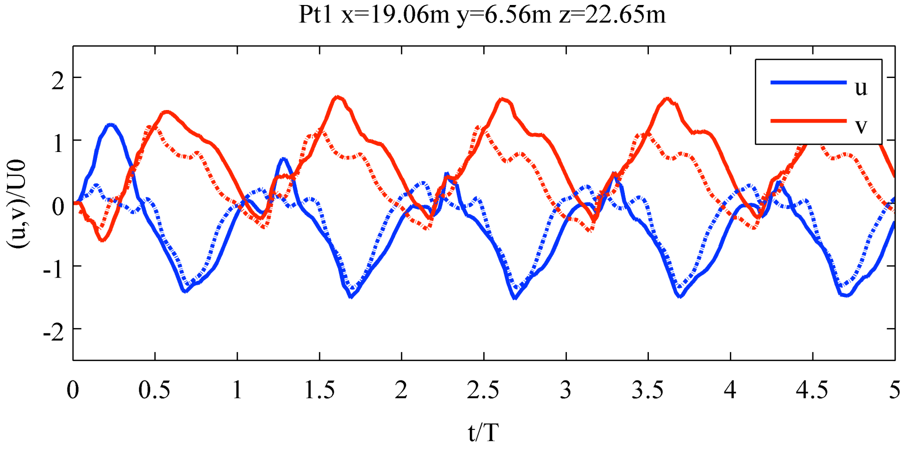
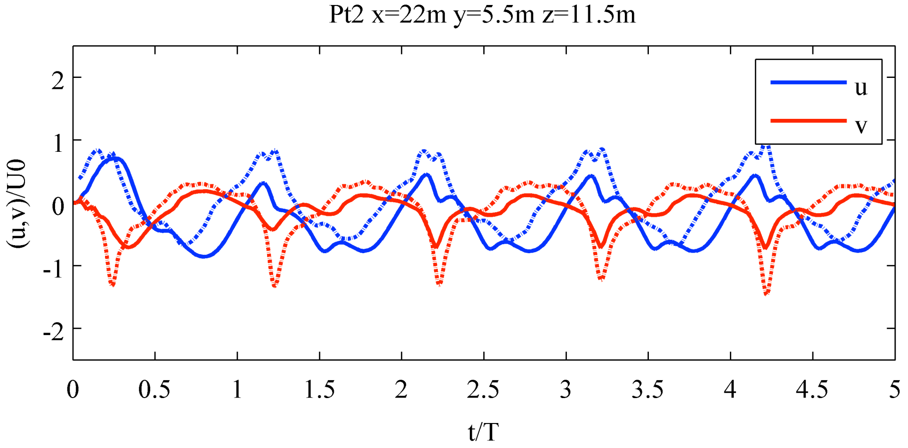


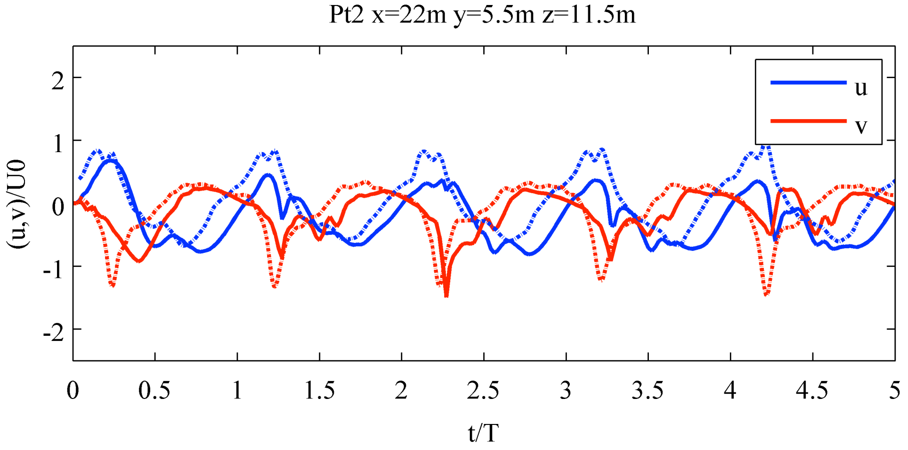
Figure 8. Surface velocity for T=240s at pt1. Temporal series of velocity component (a : two mixing lengths model, b: Smagorinsky model, c : k-epsilon model)

Velocity comparisons at Pt 2, which is the most distant point to the headland, are presented on fig. 9. Measurements indicate that velocities are not symmetrical for the ebb and flood event. This observation is well predicted by the two mixing length model and Smagorinsky models. Results with k-ε are more diffusive and the negative peaks for v-velocity are not represented.

(a)



(b)



(c)

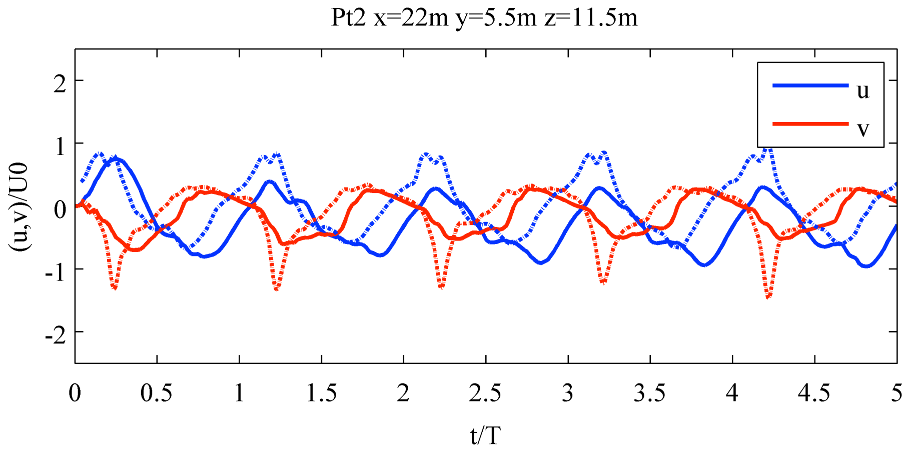
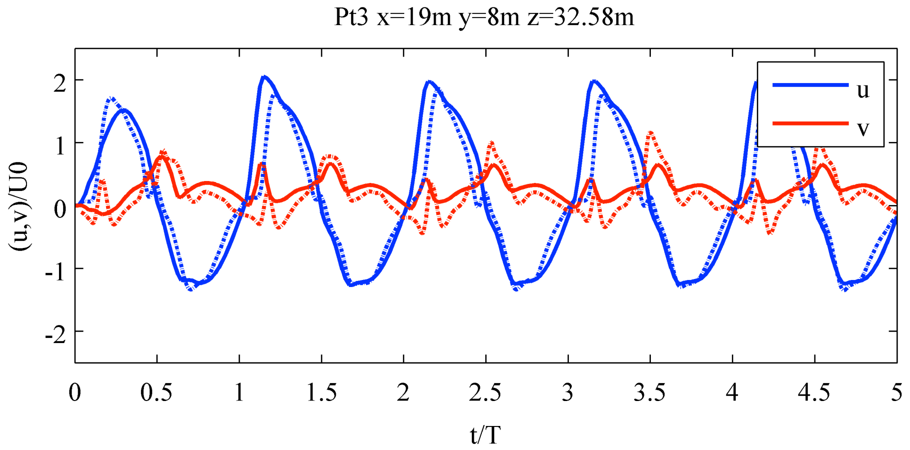


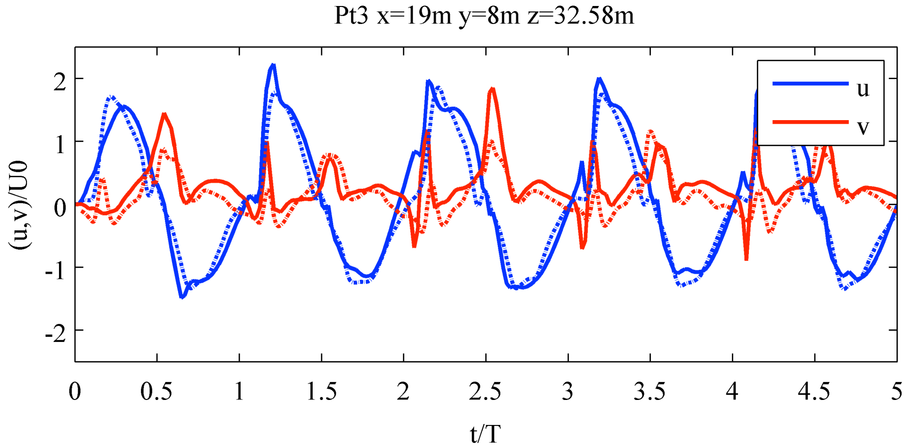
Figure 9. same as fig. 8 but for pt2.

Results are Pt3 are presented on Fig. 10. The point Pt3 is located to the right of the headland apex. Measurements are considered near the water surface. All the eddy viscosity models give satisfactory results at Pt3.

(a)



(b)



(c)

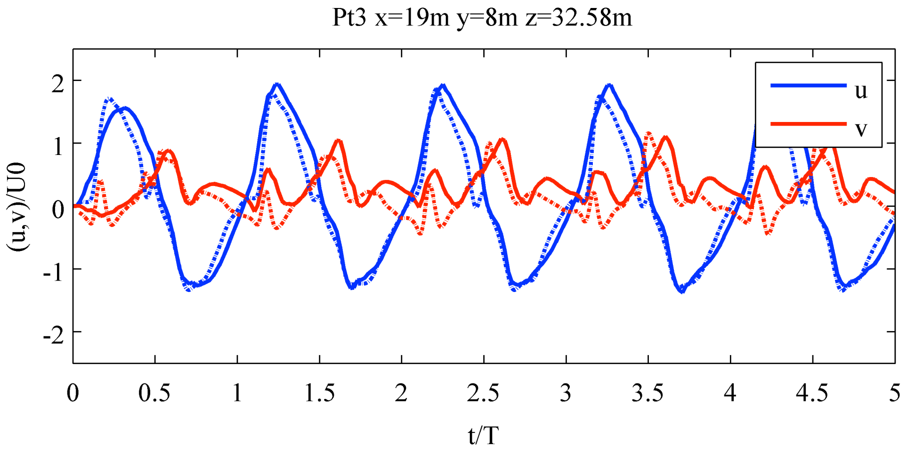
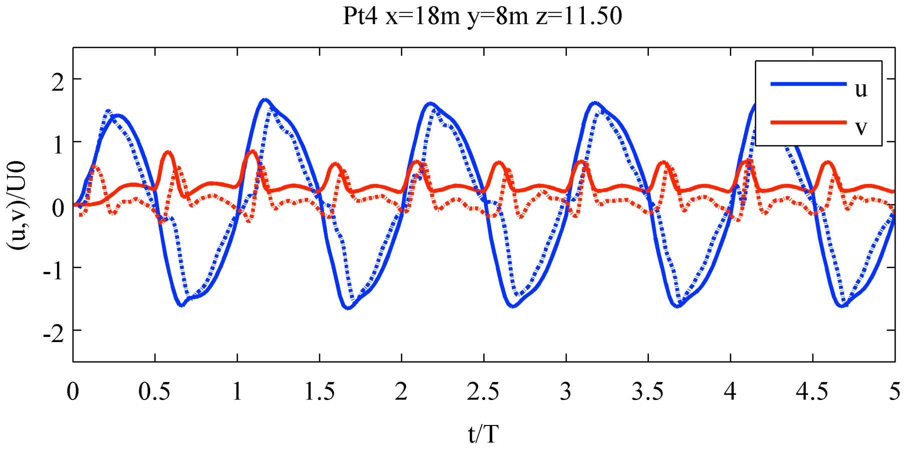


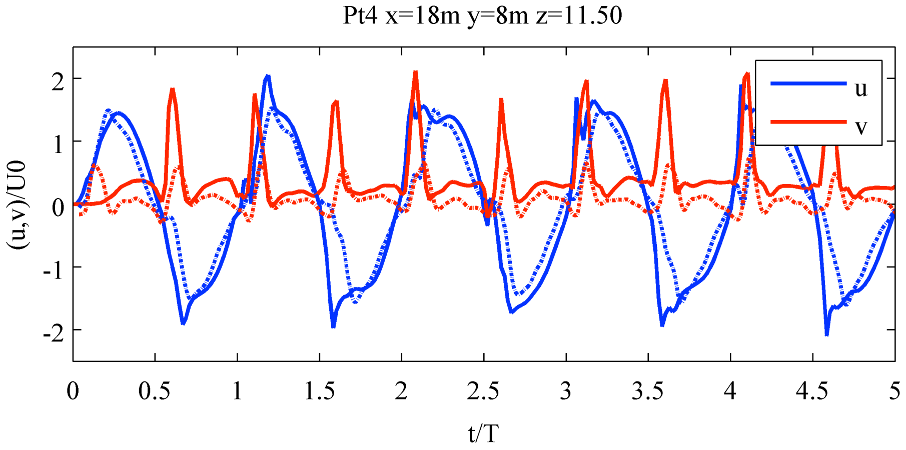
Figure 10. same as fig. 8 but for pt3.

Results at the headland apex are presented on Fig. 11. Results with Smagorinsky model for horizontal eddy viscosity overestimate the peaks of the cross velocity at slack water, although u-velocity is well captured. Results obtained with k-ε model and the tow mixing length model provide similar results, which are in agreement with laboratory measurements.

(a)



(b)



(c)

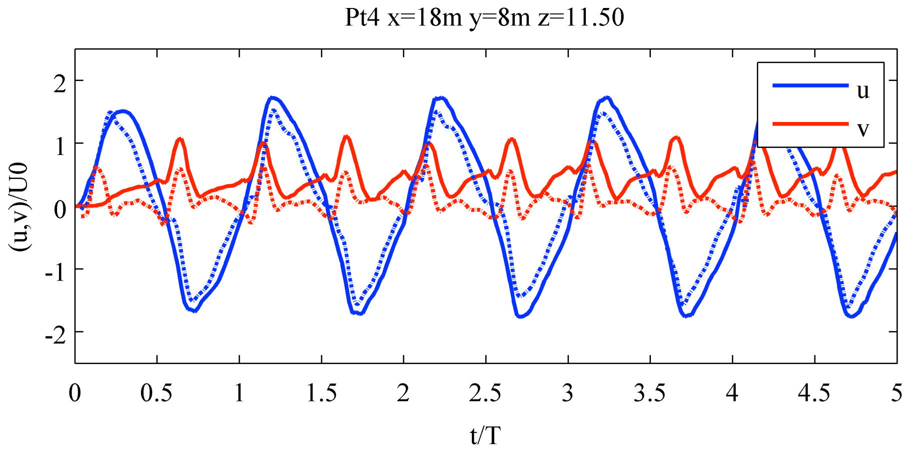
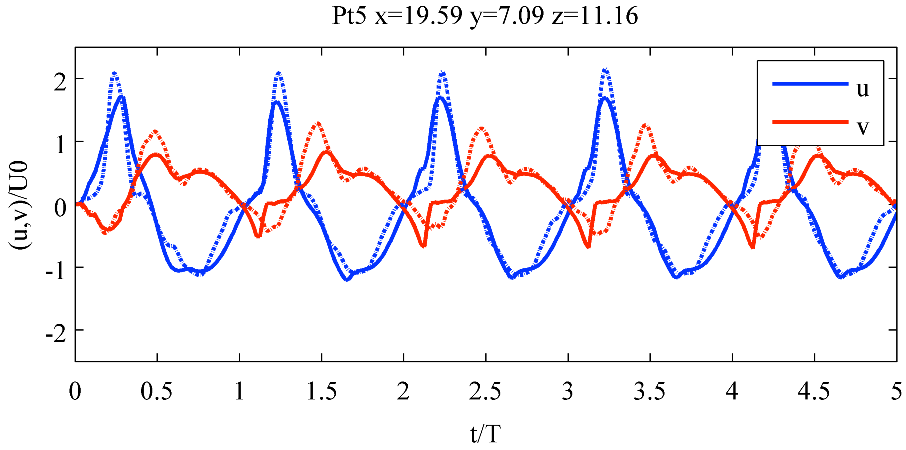


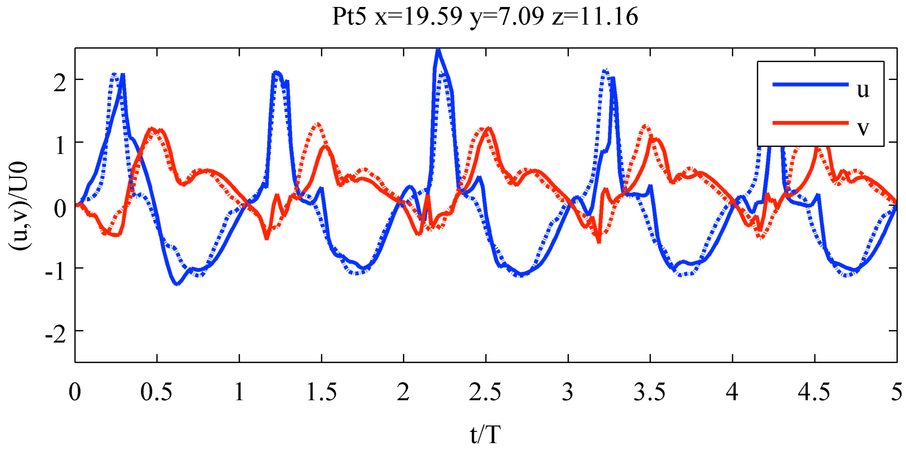
Figure 11. same as fig. 8 but for pt4.

The last point to be considered is Pt5. This one is located near the centre of the vortex observed on Fig. 6. The results for that particular point are presented on Fig. 12. Results for the k-ε model look diffusive and the maximums are under predicted. The simulations based on Smagorinsky model and the two mixing length model leads to similar results, in agreement with the measurements.

(a)



(b)



(c)

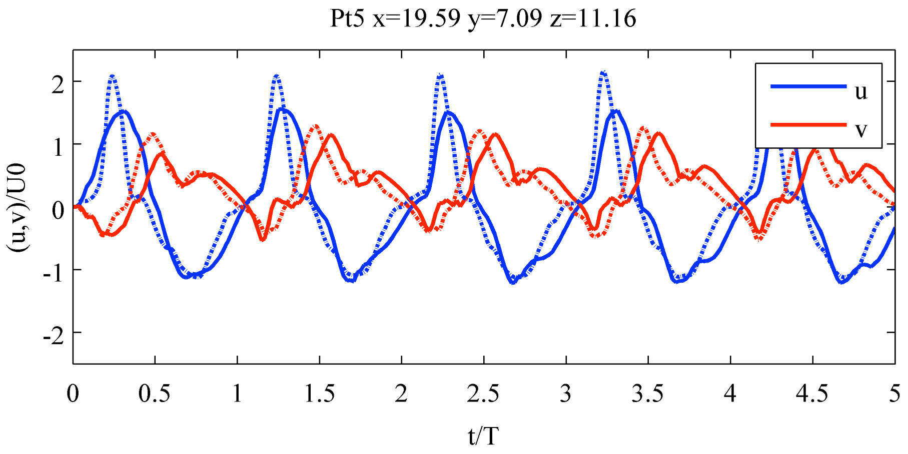


Figure 12. same as fig. 8 but for pt5.

CONCLUSION

The model for eddy viscosity modelling proposed by Stansby (2003) is here implemented in TELEMAC3D and tested against two datasets measured in the Coastal Research Facility.

The first one is the case of an island subjected to different steady flows leading to the formation of noticeable wake patterns (Lloyd and Stansby, 1997). The numerical model was able to represent the flow distribution downstream of the island. For each test case, the model captures the structure of the wake from strong vortex shading to stability.

The second datasets are for the test case of tidal flows around a headland. Once again the model proposed by Stansby (2003) leads to satisfactory results, generally similar to the classical k-ε model or better than the Smagorinsky horizontal eddy viscosity model.

REFERENCES

Hervouet (2007) Hydrodynamics of free surface flows modelling with the finite element method. Wiley. 360pp.

Lloyd and Stansby (1997) Shallow-water flow around model conical islands of small side slope. I: Surface-piercing. J. Hydraul. Engng ASCE 123, 1057–1068.

Lloyd, Stansby and Chen (1998) Flow, mixing and solute transport experiments in the UK Coastal Research Facility. University of Manchester Report.

Rodi, W. 1984 Turbulence models and their applications in hydraulics. IAHR Monograph, 2nd edn.

Stansby, (2003) A mixing-length model for shallow turbulent wakes. Journal of Fluid Mechanics, 495, pp 369-384

Stansby (1997) Semi-implicit finite-volume shallow-water flow and solute transport solver with k − ε turbulence model. Intl J. Numer. Meth. Fluids 25 285–313.