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# Large-scale experiments on wave-induced shallow turbulent coherent structures

Experimental observations and interpretation

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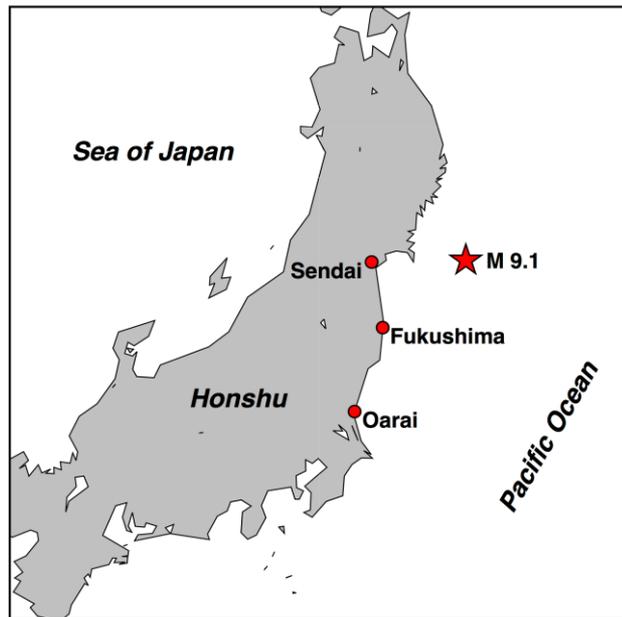
**USC** University of  
Southern California



# Observation of a tsunami-induced eddy

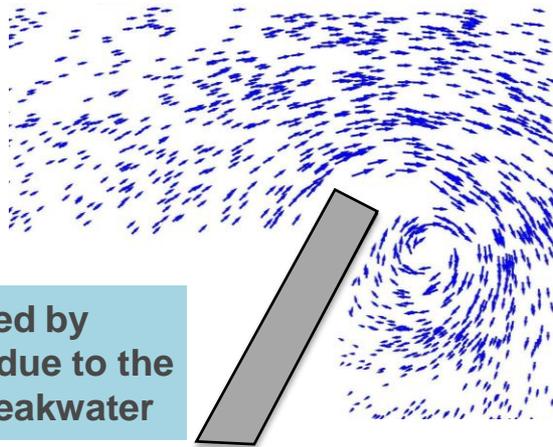
## Port Oarai, Japan

- Interaction of the 2011 tsunami currents with the coastline created special effects.
- Formation of a gigantic rotational flow structure seen in a helicopter video.



# Turbulent Coherent Structures (TCS)

- 2D TCS definition:
  - “Two-dimensional, connected, large-scale turbulent fluid masses that extend uniformly over the full water depth and contain a phase-correlated vorticity, with the exception of a thin near-bottom boundary layer”. (Hussain 1983; Jirka 2001)
- TCSs form in unidirectional and fully turbulent ( $Re \gg 10^3$ ) shallow flows ( $L/H \gg 1$ ).
  - $L$  is a characteristic length-scale,  $H$  is the flow depth.
- Kinetic energy decay is dominated by bottom friction.



TCS generated by transverse shear due to the presence of a breakwater



# Large-scale experiments

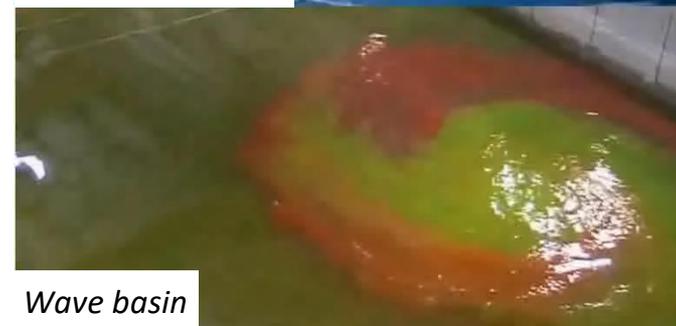
- Lack on quantitative data on wave-induced TCS: setup a laboratory experiment.
- Re-create a shallow TCS in a well controlled environment.
- Study flow field during the TCS spatial growth and spin-down.
- Develop tools to estimate time-scales of TCS development and decay.



The wave basin used for the experiments.

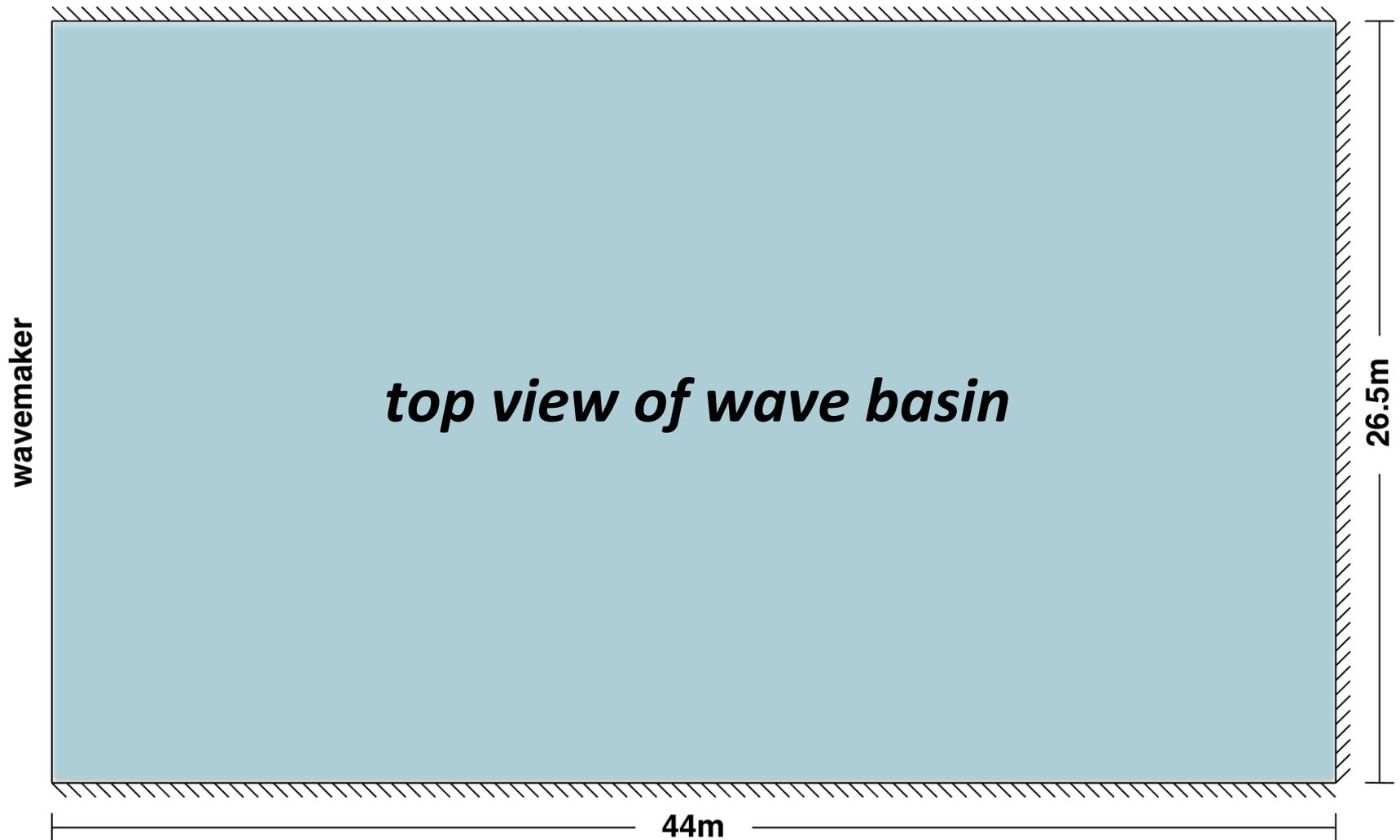


Oarai, Japan 2011

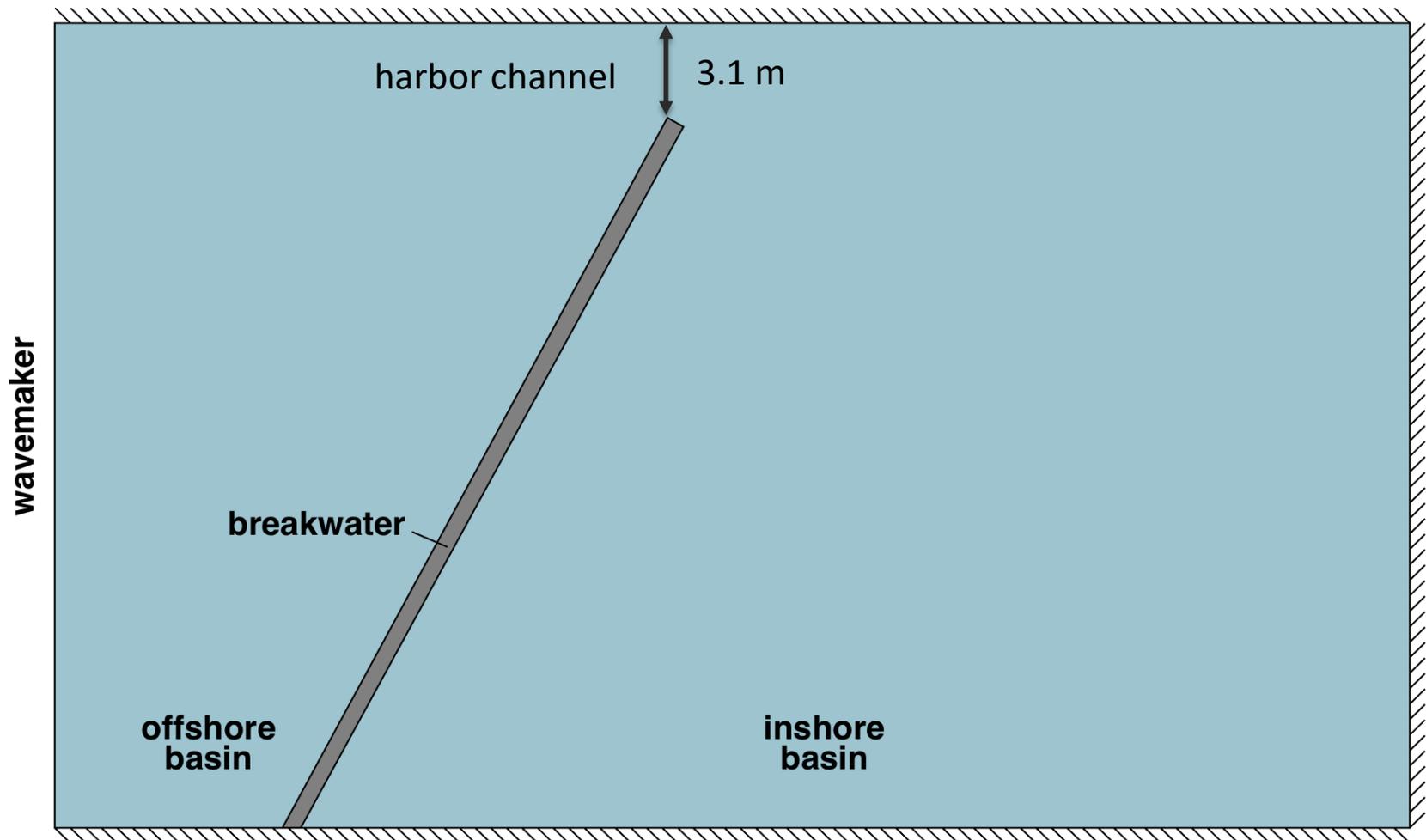


Wave basin

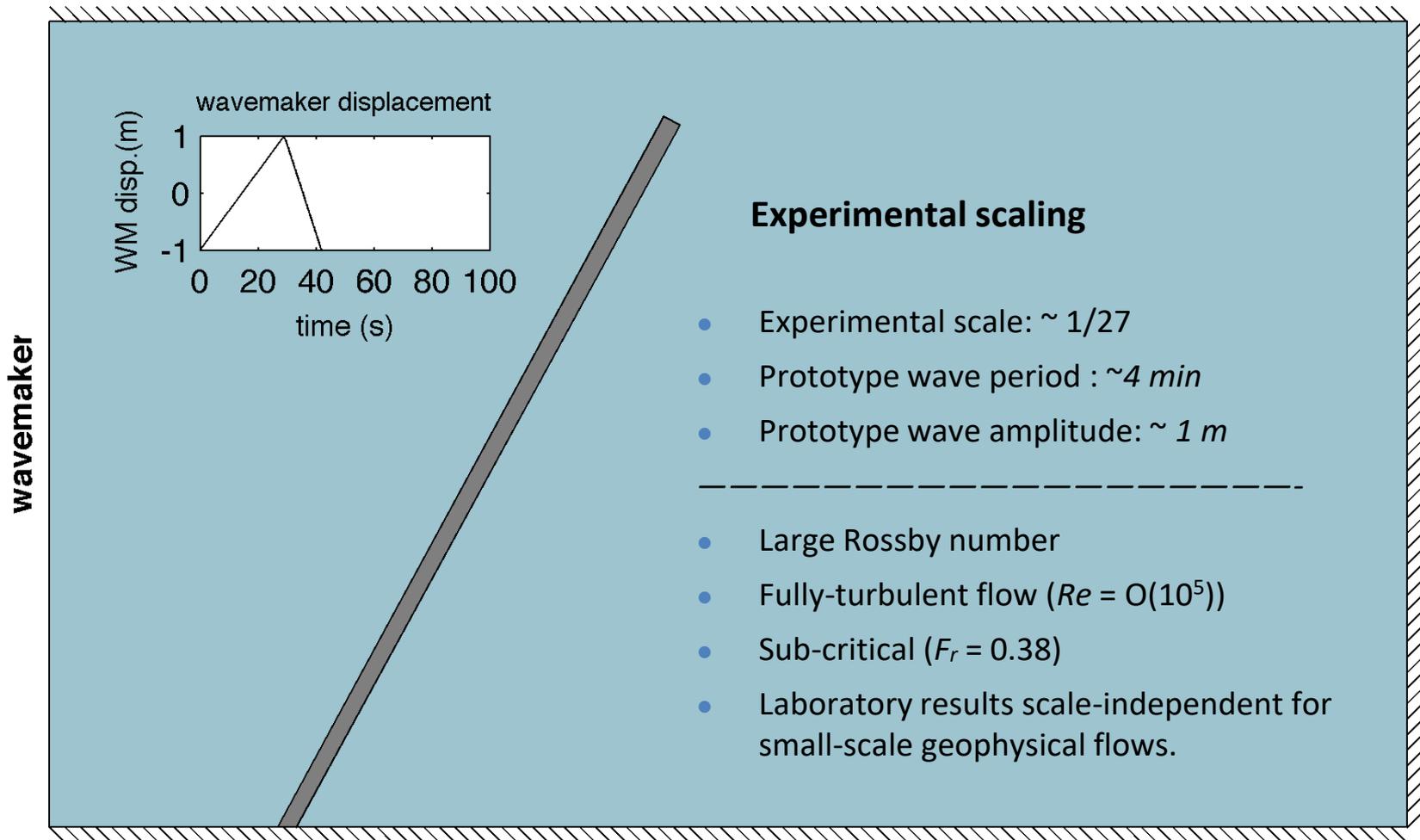
# Experimental setup



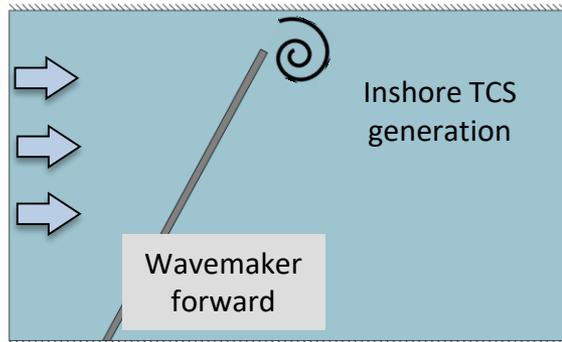
# Experimental setup



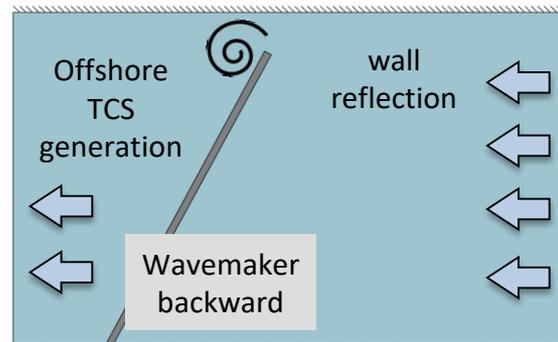
# Experimental setup



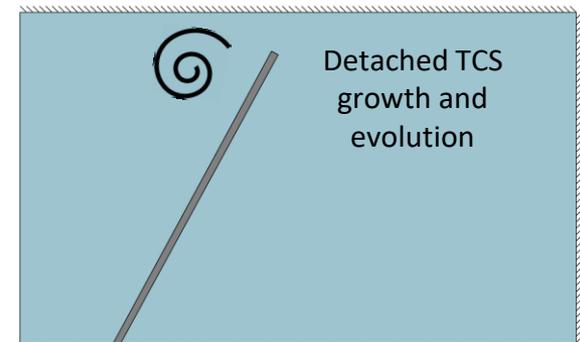
# The three flow phases in the laboratory



*Phase 1*

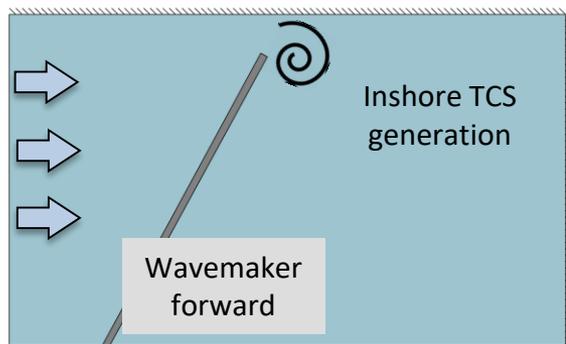


*Phase 2*

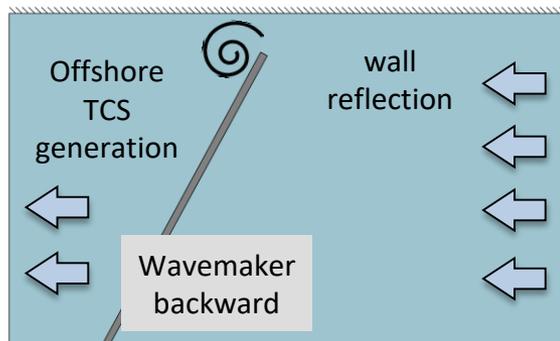


*Phase 3*

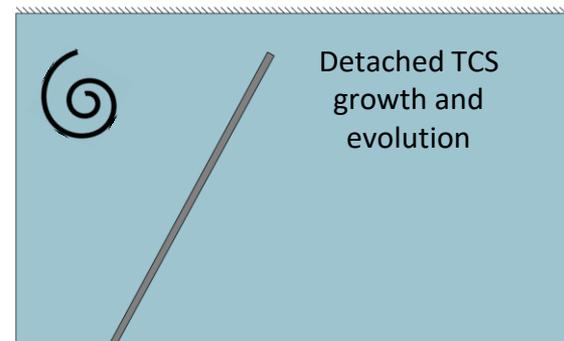
# The three flow phases in the laboratory



Phase 1

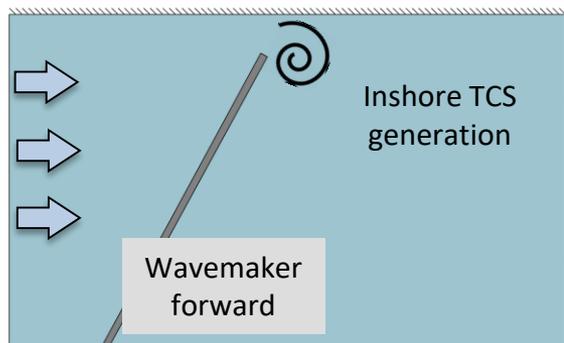


Phase 2

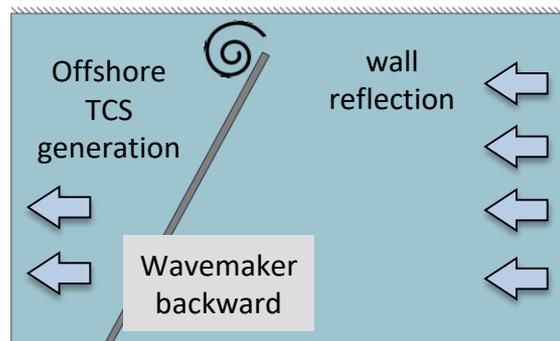


Phase 3

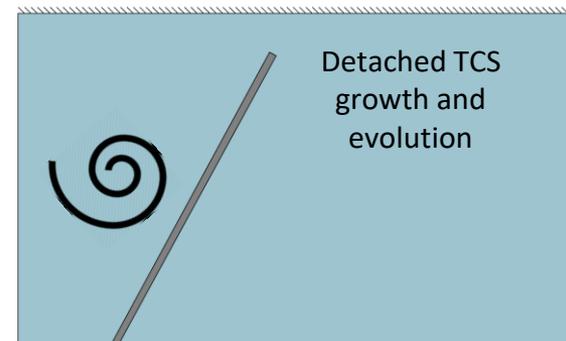
# The three flow phases in the laboratory



Phase 1

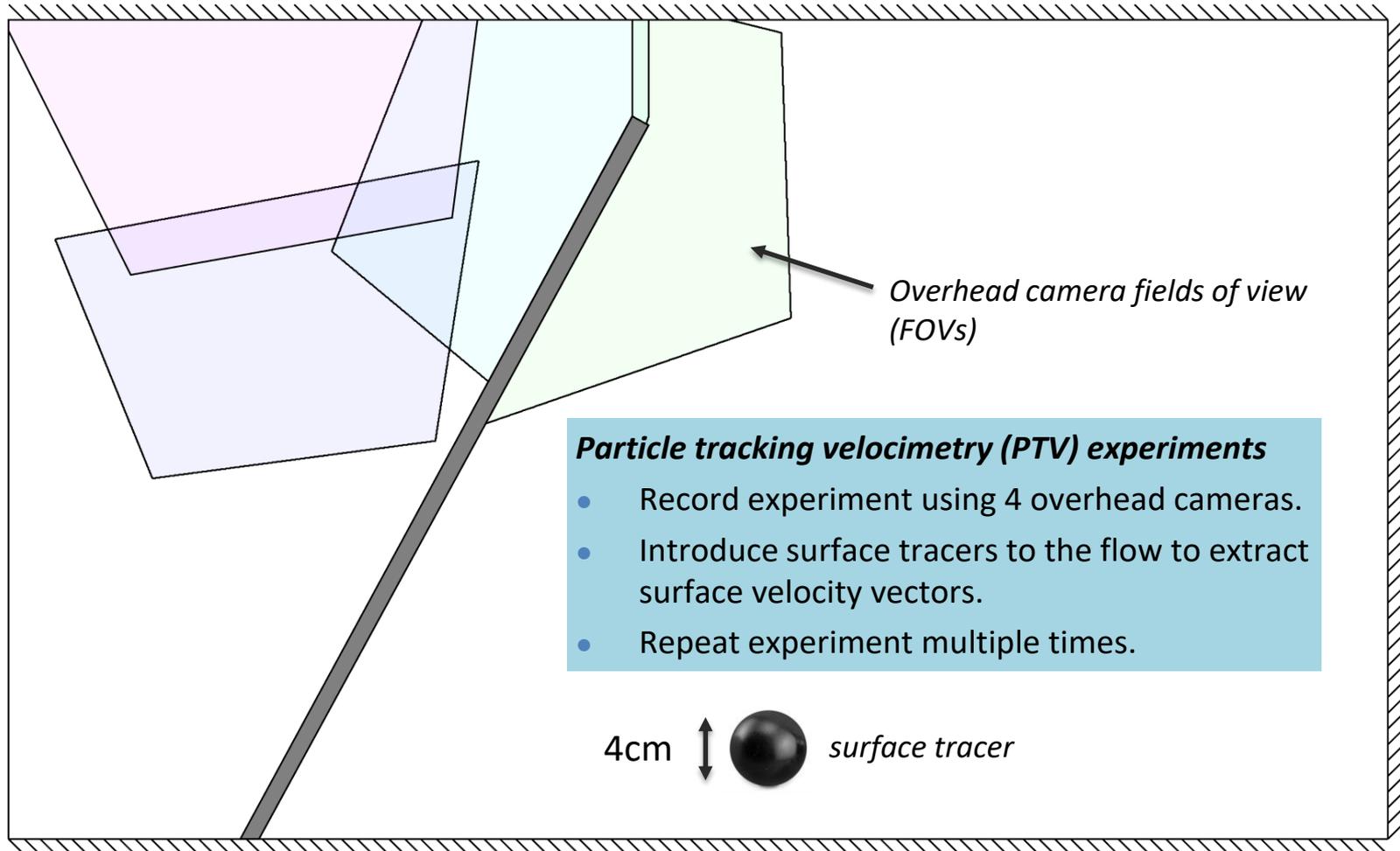


Phase 2

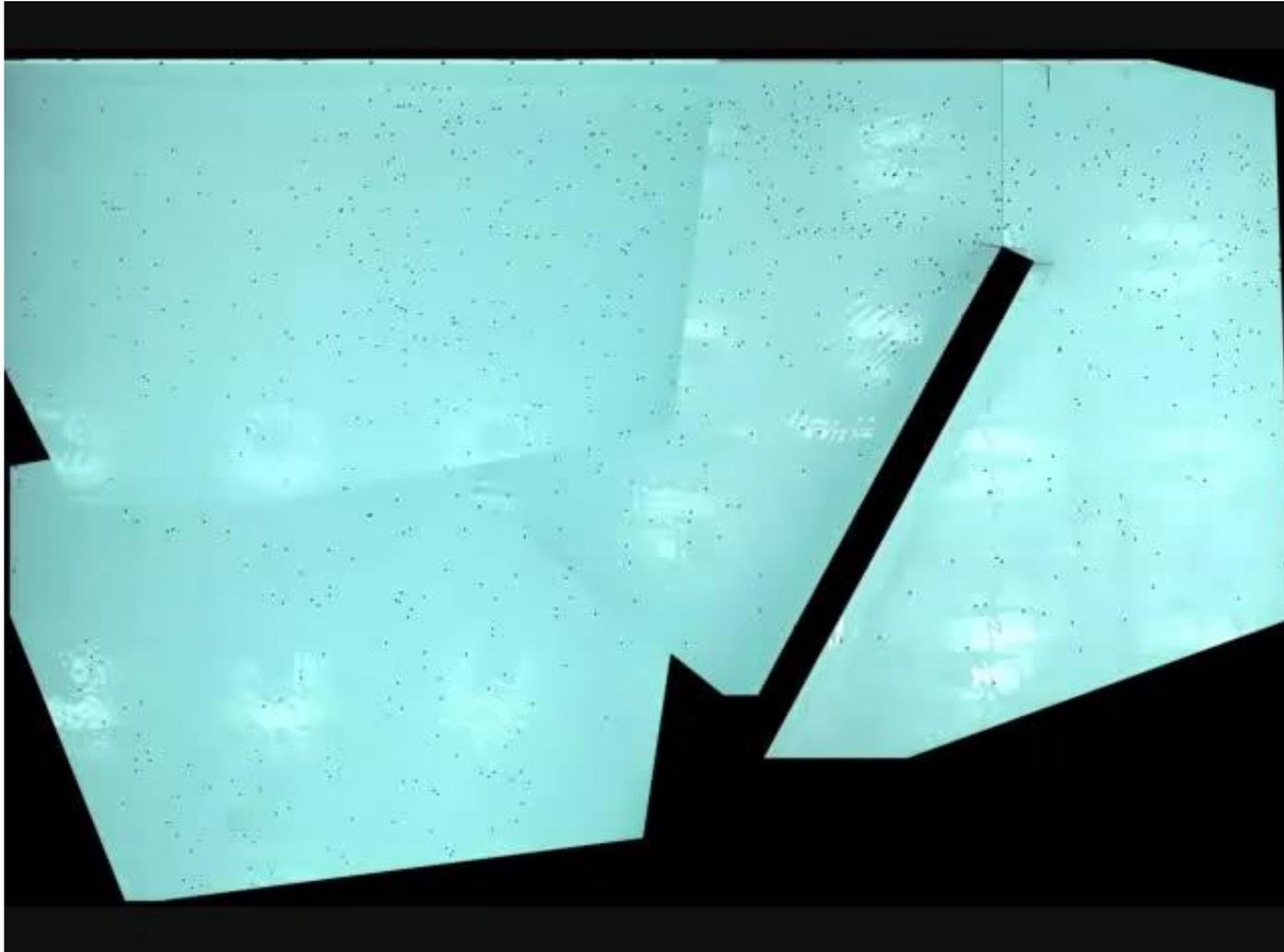


Phase 3

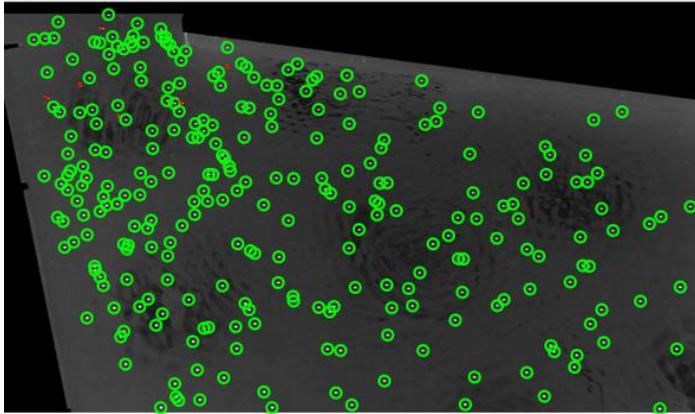
# 2D PTV experiments



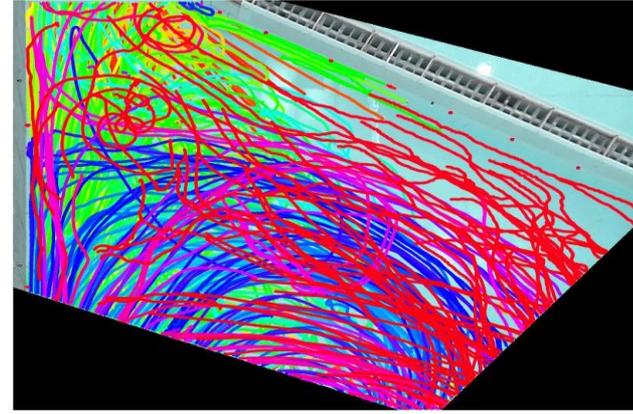
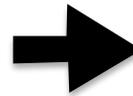
# A single 2D PTV experiment



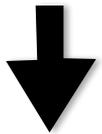
# 2D PTV steps to extract surface velocity vectors



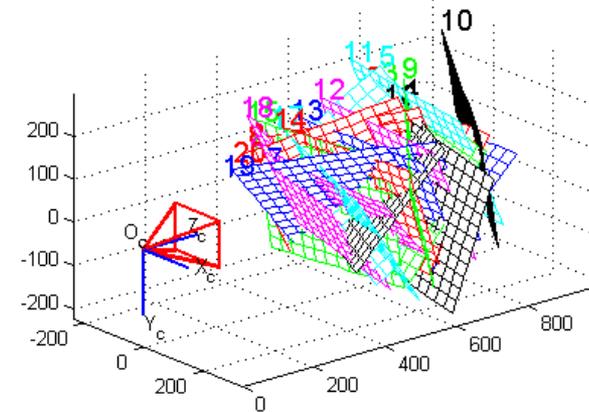
(1) Tracer center detection



(2) Particle tracking



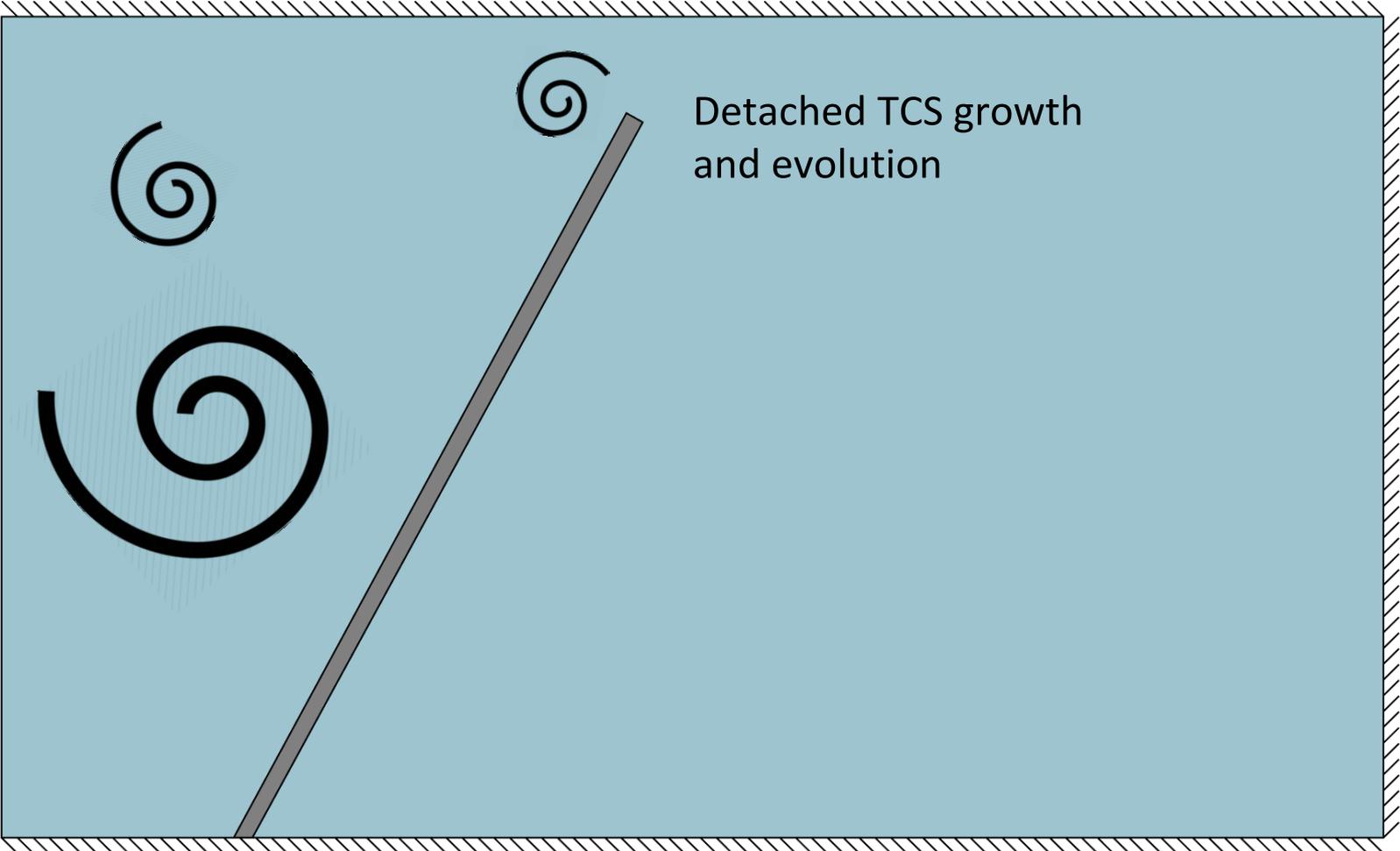
(4) Extraction of surface velocity vectors



(3) Image- to world-coordinate transformation

Kalligeris and Lynett (2017), *Exp Fluids* (in preparation)

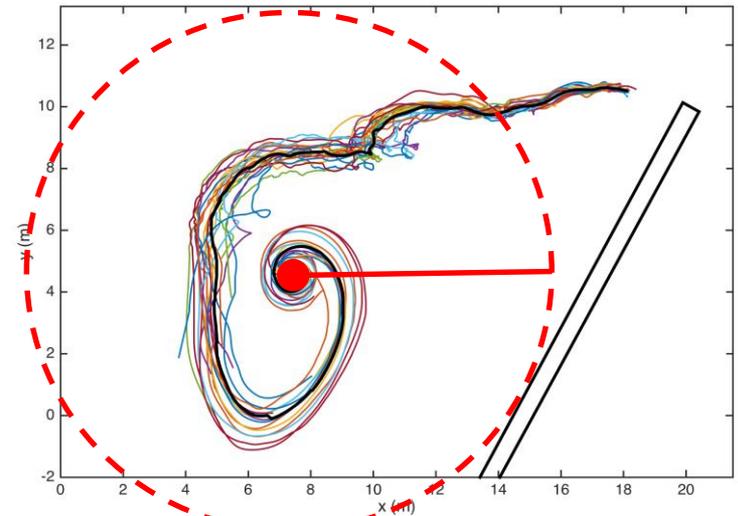
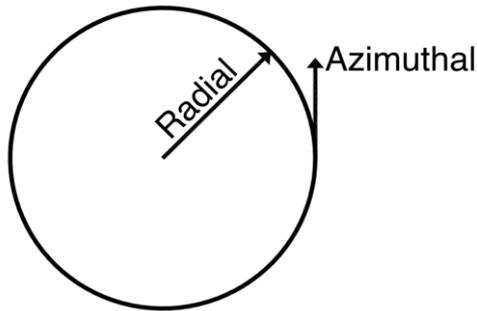
# Experimental analysis – phase 3



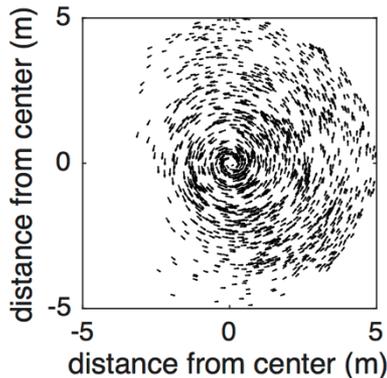
# TCS-centered ensemble

## Obtain azimuthal-averaged TCS velocity profiles

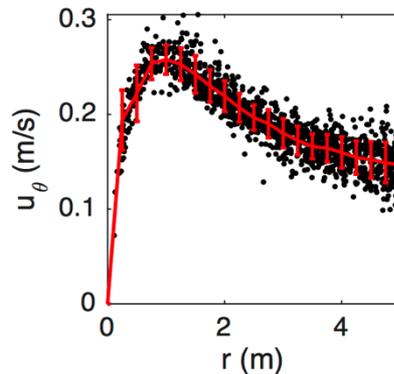
- Construct a TCS-centered ensemble.
- Get azimuthal-averaged velocity profiles.



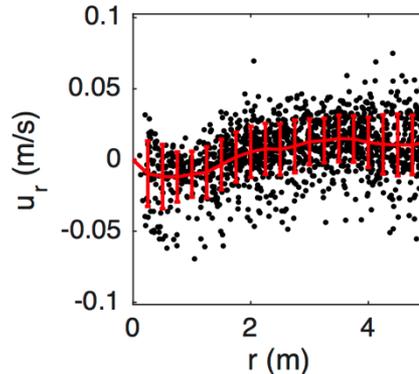
TCS paths from all experimental trials



Top view of TCS-centered vector ensemble



Azimuthal velocity



Radial velocity

- azimuthal average (mean)
- standard deviation
- scattered points

# Governing equations of motion

- Depth-averaged, incompressible equations of motion: hydrostatic, axisymmetric

$$\frac{\partial \eta}{\partial t} + \frac{1}{r} \frac{\partial (r d \bar{u}_r)}{\partial r} = 0, \quad \text{continuity equation}$$

$$\left. \begin{aligned} \frac{\partial \bar{u}_r}{\partial t} + \bar{u}_r \frac{\partial \bar{u}_r}{\partial r} - \frac{\bar{u}_\theta^2}{r} &= -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu_{eff} \left[ \frac{1}{d} \frac{\partial \eta}{\partial r} \frac{\partial \bar{u}_r}{\partial r} + \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial (r \bar{u}_r)}{\partial r} \right) \right] - \frac{\tau_{br}}{\rho d}, \\ \frac{\partial \bar{u}_\theta}{\partial t} + \bar{u}_r \frac{\partial \bar{u}_\theta}{\partial r} + \frac{\bar{u}_\theta \bar{u}_r}{r} &= \nu_{eff} \left[ \frac{1}{d} \frac{\partial \eta}{\partial r} \frac{\partial \bar{u}_\theta}{\partial r} + \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial (r \bar{u}_\theta)}{\partial r} \right) \right] - \frac{\tau_{b\theta}}{\rho d}. \end{aligned} \right\} \text{Momentum equations}$$

*convective terms*                      *turbulent diffusion terms*                      *bottom shear stress term*

- $h$  is the still water depth.
- $\eta$  is the surface elevation.
- $d$  is the total water depth ( $d=h+\eta$ ).
- $u_\theta$  is the azimuthal velocity.
- $u_r$  is the radial velocity.
- $\nu_{eff}$  is the added turbulent viscosity.

# Governing equations of motion

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- Assume purely azimuthal flow ( $u_r = 0$ )

$$\frac{u_\theta^2}{r} = \frac{1}{\rho} \frac{\partial p}{\partial r} \quad \text{cyclotrophic balance equation}$$

- Turbulent viscosity  $\nu_{eff} = \text{Sqrt}(c_f/2)Uh$
- Bottom shear stress  $\tau_{b\theta} = c_f \rho u_\theta^2/2$ 
  - $c_f$  is the quadratic law friction coefficient

$$\frac{\partial u_\theta}{\partial t} = \sqrt{\frac{c_f}{2}} u_\theta h \left[ \frac{1}{h} \frac{\partial \eta}{\partial r} \frac{\partial u_\theta}{\partial r} + \frac{1}{r} \frac{1}{\partial r} \left( r \frac{\partial u_\theta}{\partial r} \right) - \frac{u_\theta}{r^2} \right] - \frac{c_f u_\theta^2}{2h} \quad \text{radial diffusion equation}$$

# TCS kinetic energy decay

- Radial diffusion equation

$$\frac{\partial u_\theta}{\partial t} = \sqrt{\frac{c_f}{2}} u_\theta h \left[ \frac{1}{h} \frac{\partial \eta}{\partial r} \frac{\partial u_\theta}{\partial r} + \frac{1}{r} \frac{1}{\partial r} \left( r \frac{\partial u_\theta}{\partial r} \right) - \frac{u_\theta}{r^2} \right] - \frac{c_f u_\theta^2}{2h}$$

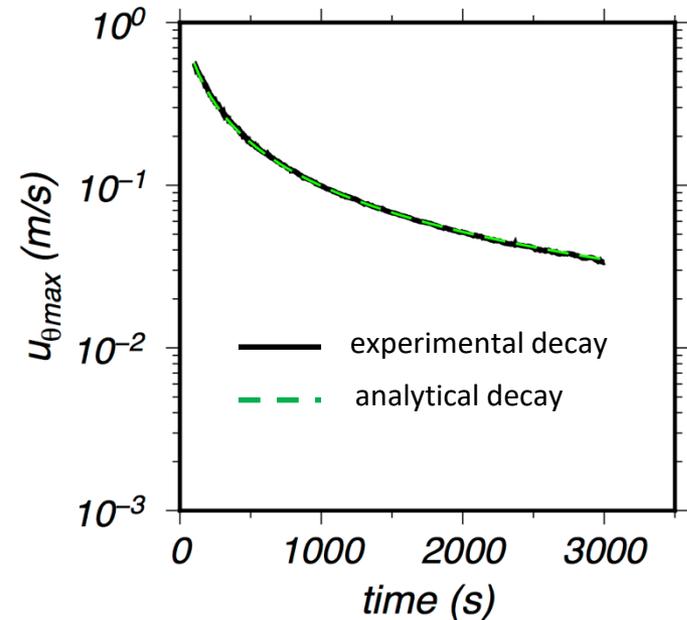
*bottom friction term*

- Bottom friction dominates kinetic decay

$$\frac{\partial u_\theta}{\partial t} = -\frac{c_f u_\theta^2}{2h}$$

- Separation of variables provides temporal azimuthal-velocity dependence

$$u_{\theta, max}(t) = \frac{1}{\frac{1}{u_{\theta, max, 0}} + \frac{c_f}{2h} t}$$



Decay of maximum  $u_\theta$  with time

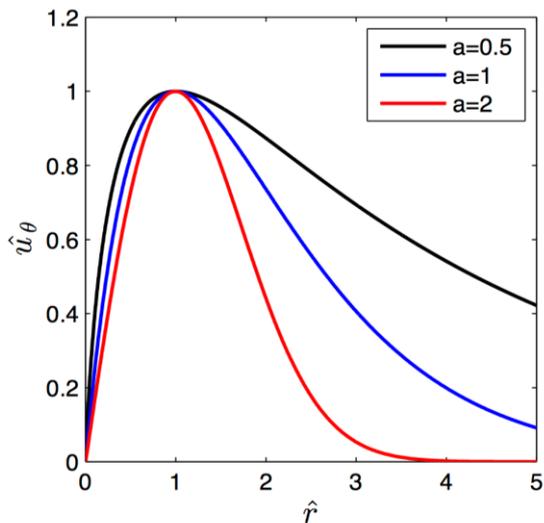
# TCS primary flow radial profile

- Compare azimuthal velocity profiles with the stirring vortex profile (or  $\alpha$ -profile)

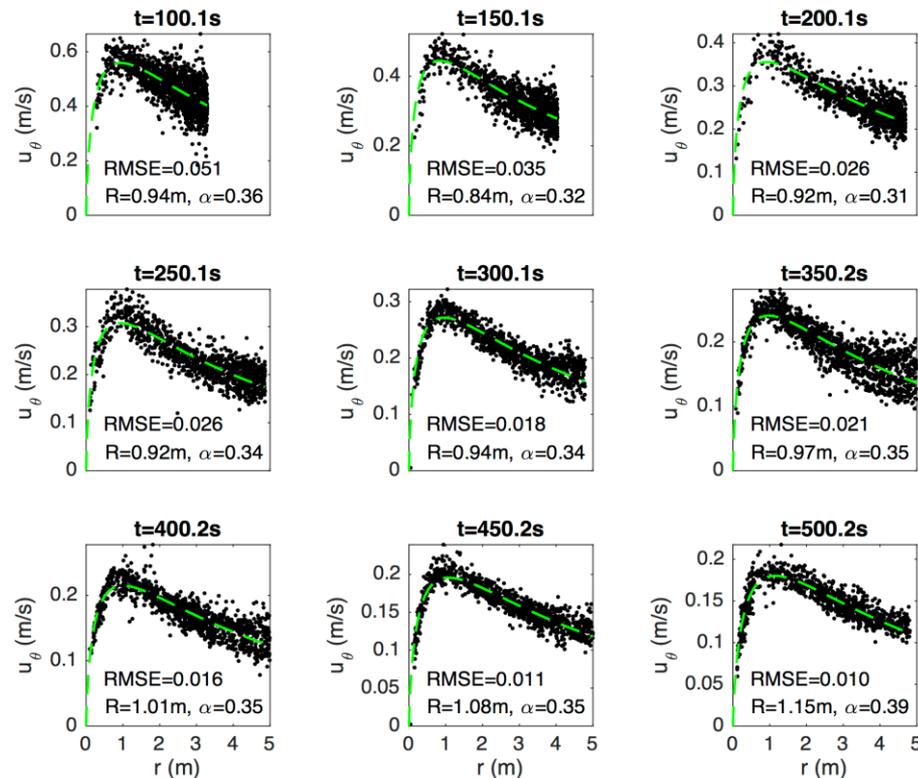
$$\frac{r}{R} \exp\left(\frac{1 - (r/R)^\alpha}{a}\right)$$

-  $R(t)$  is the radius of  $u_{\theta, \max}(t)$

-  $a$  is a free parameter



Vortex  $\alpha$ -profile



Fit  $u_\theta$  scattered data to stirring vortex profile

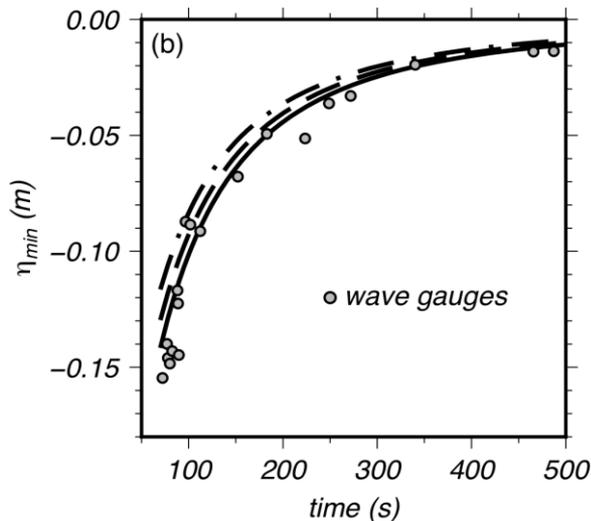
# TCS free surface elevation (FSE) profile

- Full azimuthal-velocity profile (primary flow)

$$u_{\theta}(r, t) = u_{\theta, max}(t) \frac{r}{R} \exp\left(\frac{1 - (r/R)^a}{a}\right), \quad u_{\theta, max}(t) = \frac{1}{\frac{1}{u_{\theta, max, 0}} + \frac{c_f}{2h} t}$$

- Use cyclostrophic balance equation to infer the experimental TCS FSE profile

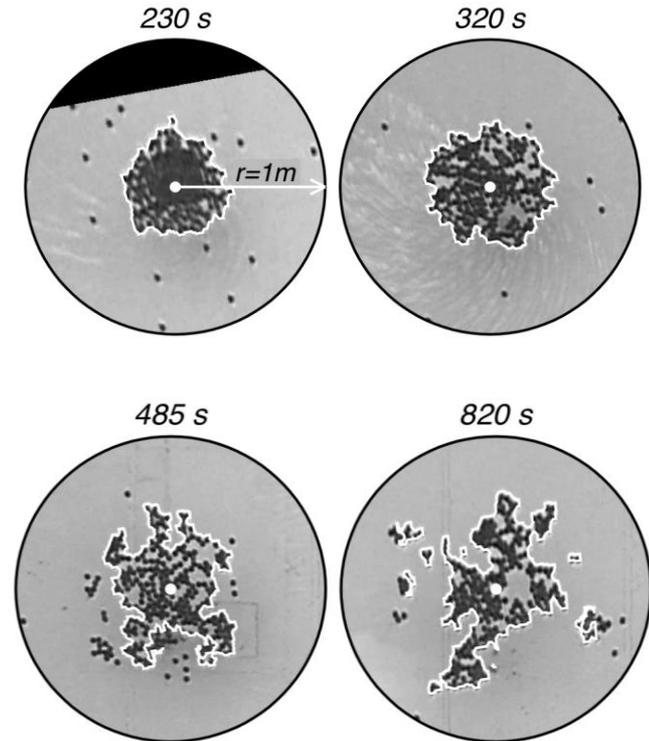
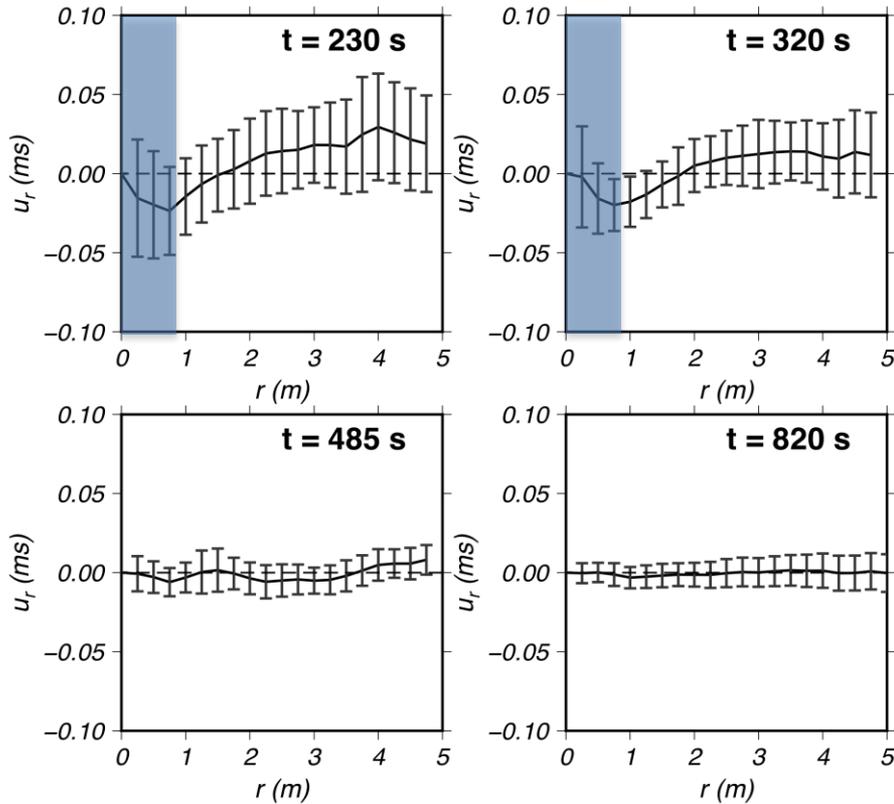
$$\frac{\partial \eta}{\partial r} = \frac{1}{g} \frac{u_{\theta}^2}{r} \quad \longrightarrow \quad \eta(r, t) = \frac{u_{\theta, max}^2(t)}{g} \int \frac{r}{R_{v_{max}}^2(t)} \exp\left(2 \frac{1 - (r/R_{v_{max}}(t))^a}{a}\right) dr$$



—  $a=1/3$   
 - -  $a=2/5$   
 - ·  $a=1/2$

Predicted  $\eta_{min}$  decay at  
 TCS center

# Secondary flow components - radial velocity

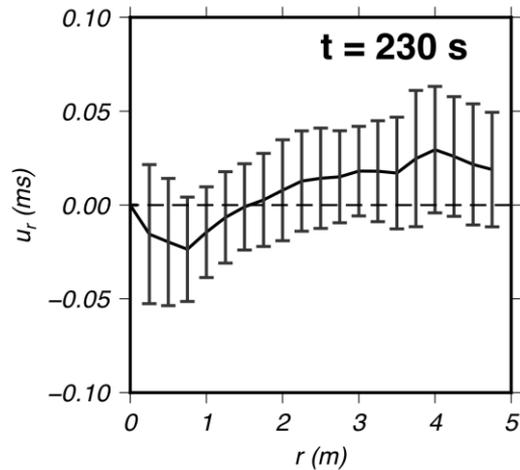


# Secondary flow components - vertical velocity

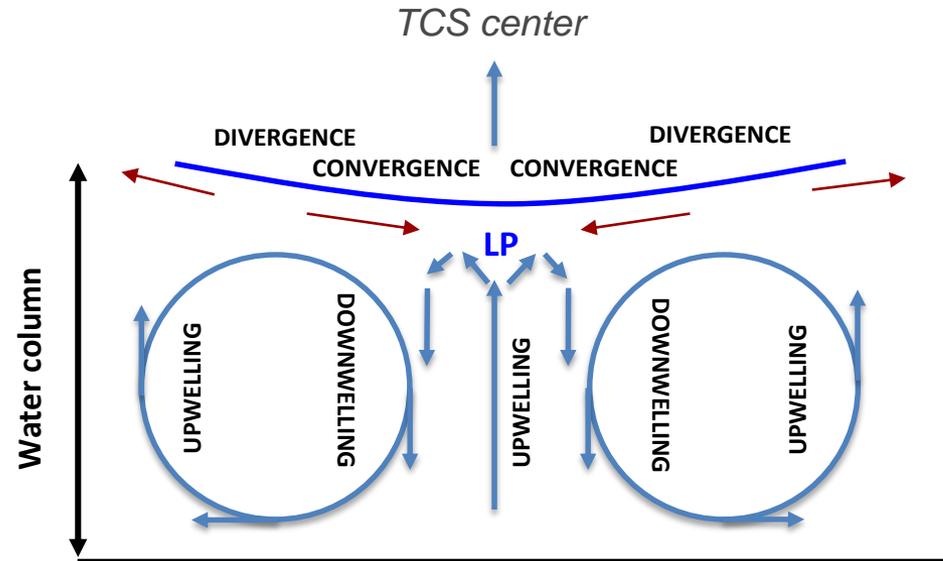
- Use kinematic free surface boundary condition for axisymmetric flow

$$w(r, \eta) = \frac{\partial \eta}{\partial t} + \frac{u_r u_\theta^2}{gr}, \text{ where } w(r, \eta) \text{ is the vertical velocity at the free surface (+ upwards)}$$

positive quantity



Radial velocity profile



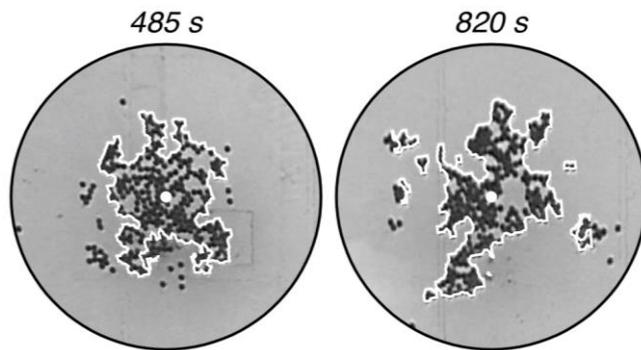
TCS re-circulation pattern around center

# Flow transition to Q-2D

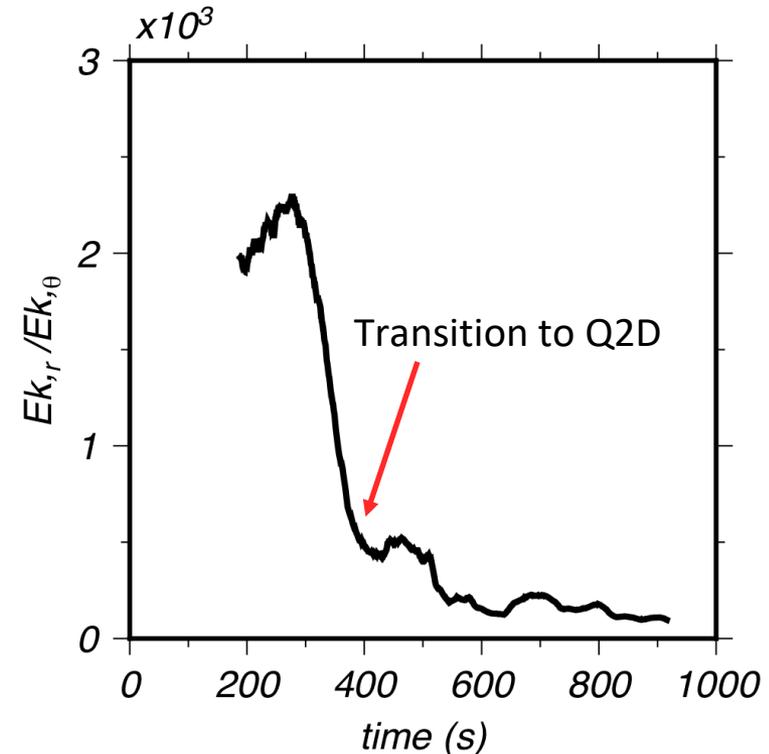
- Study the decay of the kinetic energy of the radial and azimuthal flow components

- The kinetic energy of velocity components ( $u_r$ ,  $u_\theta$ ) in the free surface are evaluated from

$$E_{k,i} = \pi\rho \int_0^R u_i^2(r) r dr,$$



Tracer-conglomerate compactness  
at TCS center



Ratio of radial and azimuthal kinetic energy

## Summary

- The flow structure of a wave-induced TCS was studied through a series of large-scale experiments in a wave basin
- First-order models were derived to describe TCS flow field, spatial growth, kinetic decay, and the FSE around the TCS-center
- The secondary flow components suggest a flow recirculation pattern along the water depth
- The secondary flow components decay faster than the primary flow component, leading to a more Q-2D flow at later stages.

## Funding

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- Myronis Foundation fellowship.



## Acknowledgements

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- Many thanks to Aykut Ayca (USC) and Adam Ryan (OSU) for their help during the experiments at OSU.