A Numerical Study on Surface Wave Evolution over Viscoelastic Mud

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Abstract
A numerical modeling approach is applied to investigate the combined effect of wave-current-mud on the evolution of nonlinear waves. A frequency-domain phase-resolving wave-current model that solves nonlinear wave-wave interactions is used to solve wave evolution. A comparison between the results of numerical wave model and the laboratory experiments confirms the accuracy of the numerical model. The model is then applied to consider the effect of mud properties on nonlinear surface wave evolution. It is shown that resonance effect in viscoelastic mud creates a complex frequency-dependent dissipation pattern. In fact, due to the resonance effect, higher surface wave frequencies can experience higher damping rates over viscoelastic mud compared to viscous mud in both permanent form solution and random wave scenarios. Thus, neglecting mud elasticity can result in inaccuracies in estimating total wave energy and wave shape.

Introduction

The interaction among surface waves, currents, and muddy sea beds in coastal waters is complex; it is well known that surface waves are dissipated while propagating over a muddy seabed as a result of wave energy dissipation within the bed (e.g. Gade,1958). Besides, wave-induced and ambient currents can transport muds along and across the coasts. Therefore, accurate characterization of mud behavior is critical in assessing wave energy and wave-induced coastal erosion.

Wave energy dissipation over muddy seafloors has been observed in both long and short waves. Sheremet and Stone (2003) found that wave dissipation over muddy seabed is important not only for the long waves, but also for short waves which do not penetrate deep into the water column to interact directly with the muddy seafloor. Their observations suggest that nonlinear wave-wave interactions may result in wave energy dissipation in short-wave bands. Kaihatu et al. (2007) incorporated the Ng (2000) mechanism in a parabolic phase-resolving frequency-domain nonlinear wave model. Numerical tests show that damping of high frequency waves occurs through sub harmonic interactions. Kaihatu and Tahvildari (2012) extended the wave-current interaction model of Kaihatu (2009) to include mud-induced dissipation mechanism of Ng (2000). The model shows that co-propagating currents act to reduce mud-induced wave dissipation while counter propagating flows increase the dissipation.

Previous studies have made various assumptions for mud rheology. The most common assumptions are viscous fluid (e.g. Dalrymple and Liu, 1978b; Ng, 2000), viscoelastic medium (e.g. Macpherson, 1980; Mei et al., 2010), Bingham plastic (e.g. Mei and Liu, 1987; Chan and Liu, 2009), and other non-Newtonian medium (e.g. Zhao et al., 2006). Jain and Mehta (2009) discuss that among mud rheological behaviors, viscoelastic fluid assumption can predict wave attenuation over the widest range wave of wave action and sediment properties. An accurate model for wave evolution over muddy seabed requires an accurate characterization of mud behavior. Liu and Chan (2006) studied the small amplitude long wave propagation over a thin viscoelastic mud layer theoretically. They showed that the coupling between mud viscosity and elasticity results in complexities in predicting wave dissipation rate. They found that the wave attenuation rate does not always diminish as the elastic shear modulus increases, and that there is a possible resonance effect when the magnitude of wave frequency becomes closer to the natural frequency of the viscoelastic mud layer. They concluded that wave attenuation can increase remarkably in the range of small shear modulus.

In this research, we extend the model of Kaihatu and Tahvildari (2012) which captures wave-current interaction in the presences of viscous mud, by accounting for viscoelastic mud. The damping mechanism is that of Liu (2006) in which the mud layer is assumed to be thin and behave as a viscoelastic medium. This model provides a more versatile predictive tool for wave behavior in coastal waters.

Numerical Model

The wave-current interaction model is based on the spectral model developed by Kaihatu (2009). Kaihatu (2009) model is based on the wave-current interaction model of Kaihatu and Kirby (1998) but includes second order terms that improve energy transfer calculations in high frequencies. Model derivation is outlined in Kaihatu and Kirby (1995), and Kaihatu (2009) in a comprehensive manner, hence we limit the model description here to essential details. The wave-current interaction model is based on the mild-slope equations Smith and Sprinks (1975), and it solves the nonlinear interactions among resonant triads (e.g. Phillips, 1981). The formulation assumes irrotational flow, mildly varying depth in the horizontal plane $h(x, y)$, and invariant current with respect to the vertical coordinate, z. The velocity potential function is given by:

$$\emptyset = \emptyset_0 + f_n(k_n, h, z)\emptyset_n(k_n, \omega_n, x, y, t) \tag{1}$$

In which:

$$f_n(z) = \frac{\cosh k_n(h+z)}{\cosh kh} \quad \emptyset_n = -\frac{igA_n}{2\sigma_n}e^{i\int k_n dx - \omega_n t} + c.c. \tag{2}$$

Where A_n is the amplitude of $n - th$ frequency component in the free surface elevation η, and $c.c.$ denotes the complex conjugate. Limiting the model to one horizontal dimension results in:

$$\frac{\partial A_n}{\partial x} + \frac{\sigma_n}{2(C_{gn}+U)}\left[\frac{\partial}{\partial x}\left(\frac{C_{gn}+U}{\sigma_n}\right)\right]A_n + D_nA_n$$

$$= \frac{-i}{8(C_{gn}+U)\sigma_n}\left(\sum_{l=1}^{n-1}RA_lA_{n-l}e^{i\theta_{l,n-l}} + 2\sum_{l=1}^{N-n}SA_l^*A_{n+l}e^{i\theta_{n+l,-l}}\right) \tag{3}$$

Where the nonlinear interaction coefficients are:

$$R = \left(\frac{g}{\sigma_l\sigma_{n-l}}\right)[\sigma_n^2 k_l k_{n-l} + (k_l+k_{n+l})(\sigma_{n-l}k_l+\sigma_l k_{n-l})] - \frac{\sigma_n^2}{g}(\sigma_l^2 + \sigma_l\sigma_{n-l} + \sigma_{n-l}^2) \tag{4}$$

$$S = \left(\frac{g}{\sigma_l \sigma_{n+l}}\right)[\sigma_n{}^2 k_l k_{n+l} + (k_{n+l} - k_l)(\sigma_{n+l} k_l + \sigma_l k_{n+l})] - \frac{\sigma_n{}^2}{g}(\sigma_l{}^2 - \sigma_l \sigma_{n+l} + \sigma_{n+l}{}^2) \quad [5]$$

In these equation, $\theta_{l,n-l} = \int k_l + k_{n-l} - k_n dx$, and $\theta_{n+l,-l} = \int k_{n+l} - k_l - k_n dx$. In the equation (3), energy dissipation is represented by the term $D_n A_n$. The energy dissipation can be due to wave breaking, mud-induced damping by mud, or any other mechanism. In this study, we improve the models of Kaihatu et al. (2007) and Kaihatu and Tahvildari (2012) by incorporating the effect of viscoelastic mud. The viscoelastic mud model is based on Liu and Chan (2006). Liu and Chan (2006) studied the effect of a thin visco-elastic mud layer with the depth d on small amplitude wave propagation. Model derivation is outlined in Liu and Chan (2006) in a comprehensive manner, hence we limit the model description here to essential equations. Their approximate solutions for wave number k_r and mud-induced dissipation D_{mw} are:

$$k_r h = k_1 h - \frac{(k_1 h)^2 \gamma \left(\frac{\delta_{me}}{h}\right)}{\sinh 2k_1 h + 2k_1 h}\left[2\lambda - \frac{\Omega_M \sinh(2\lambda\Omega_M) + \Omega_P \sin(2\lambda\Omega_P)}{\cosh(2\lambda\Omega_M) + \cos(2\lambda\Omega_P)}\right] \quad [6]$$

$$D_m \omega = \frac{(k_1 h)^2 \gamma(\frac{\delta_{me}}{h})}{\sinh 2k_1 h + 2k_1 h}\left[\frac{\Omega_P \sinh(2\lambda\Omega_M) - \Omega_M \sin(2\lambda\Omega_P)}{\cosh(2\lambda\Omega_M) + \cos(2\lambda\Omega_P)}\right] \quad [7]$$

The parameters in thiese equation are: $\vartheta_{me} = \vartheta_m + \frac{iG_m}{\omega\rho_m}$, $\delta_{me} = \sqrt{\frac{2|\vartheta_{me}|}{\omega}}$, $\Omega_P = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}$, $\Omega_M = \cos\frac{\theta}{2} - \sin\frac{\theta}{2}$, $\tan\theta = \frac{G_m}{\omega\rho_m\vartheta_m}$, $\gamma = \frac{\rho_w}{\rho_m}$, $\lambda = \frac{d}{\delta_{me}}$, and $\frac{\omega^2 h}{g} = k_1 h \tanh k_1 h$.

In which ϑ_{me} is the effective kinematic viscosity of the mud, ρ_m is the mud density, ω the angular frequency, G_m the mud elasticity modulus, and $i = \sqrt{-1}$.

Results

The model is used to study the effect of viscoelasticity on surface wave evolution. A comparison between the results of numerical wave model and the laboratory experiments of Soltanpour and Samsami (2011) confirms the accuracy of the numerical model. Figure (1) shows the variation of damping rate with surface wave frequency for various values of mud shear modulus. In the case of purely viscous mud, the damping rate is smallest in the low and high end of the frequency range and varies relatively mildly in between. The variation of damping rate for viscoelastic muds is more pronounced around a certain frequency due to the resonance effect. Resonance occurs when the surface wave frequency approaches the natural frequency of oscillation in the mud layer and results in amplification of interfacial waves. As a result, a high shear stress is developed within the mud layer and the surface wave is dissipated at a high rate due to mud viscosity.
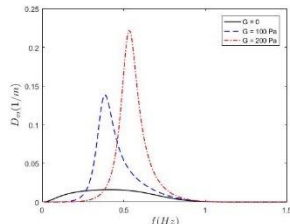

Figure 1 - Variation of surface wave damping rate with frequency for different values of mud shear modulus $\xi = 100, h = 1\,m, d_m = 0.20\,m, and\ \rho_m = 1111\,kg/m^3$.

We considered both Cnodial and randome waves solutions. Fifteen harmonic are used for the Permanent

form solution. The figure 2 shows The figure shows the distinct non-monotonicity with frequency and there is a a significant loss of amplitude at the high frequencies over the mud patch. Subharmonic interaction with the lower, actively-damped frequencies is probabaley causing this reduction in amplitude as the other scientists mentions before.
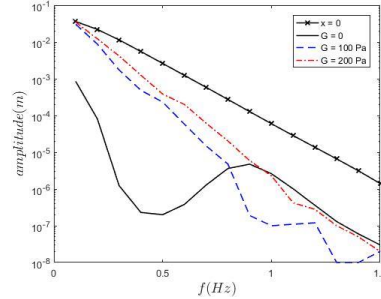

Figure 2 - Evolution of frequency amplitudes in a cnoidal wave spectrum over muds with di erence shear moduli. $\xi = 100, h = 1\,m, d_m = 0.20\,m, and\ \rho_m = 1111\,kg/m^3$.

Figure (3) shows the permanent form solution in the presence of current.

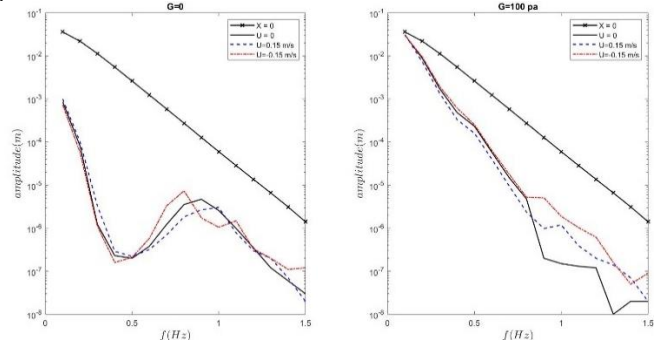
Figure 3 - Evolution of frequency amplitudes in a cnoidal wave spectrum over muds with current for G=0 and G=100 Pa, $\xi = 100, h = 2\,m, d_m = 0.20\,m$, and $\rho_m = 1111\,kg/m^3$.

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