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The State of the Art and Science of Coastal Engineering

A Numerical Study on Surface Wave Evolution Over Viscoelastic Mud

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Introduction

- Muddy coast [1]:

"*a sedimentary-morphodynamic type characterized primarily by fine-grained sedimentary deposits – predominantly silts and clays – within a coastal sedimentary environment*".

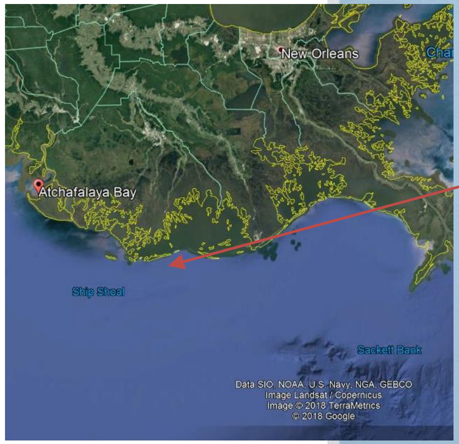
The interaction among surface waves, currents, and muddy sea beds in coastal waters is complex:

- surface waves are dissipated while propagating over a muddy seabed
- wave-induced and ambient currents can transport muds along and across the coasts

Accurate characterization of mud behavior is critical in assessing wave energy and wave-induced coastal erosion.

Examples of muddy coasts around the world:

Atchafalaya Shelf
(Gulf of Mexico)



Cassino Beach
(Brazil)

Handijan muddy coast
(Persian Gulf)

Damping of waves over muddy bed (Cassino Beach, Brazil)

[14] “1998 Airborne Data Acquisition and Registration (ADAR) image showing differential wave breaking due to shallow water mud deposits at the landward location of the field experiment.”, (from K.T.Hollandetal./Continental Shelf Research 29 (2009) 503–514

Previous Studies

Various assumptions for mud rheology:

- Viscous (denser fluid at bottom) ⟶ Gade (1958); Dalrymple & Liu (1978); Ng (2000)
- Viscoelastic (non-rigid bed) ⟶ MacPherson (1980); Ma & Mehta (1990); Liu and Chan (2007)
- Bingham plastic (Solid at low stress and viscous in high stress) ⟶ Mei and Liu (1987)

Waves models:

- Gade (1958); Dalrymple & Liu (1978): Long Waves
- Sheremet and Stone (2003): Short Waves, nonlinear wave-wave interactions may result in wave energy dissipation in short-wave bands

Current Study

In this study, we incorporate the effect of viscoelastic mud layer in a nonlinear frequency domain model for wave-current interaction

- Wave-current interaction model of Kaihatu (2009)
- The effect of mud layer on surface wave is described by Viscoelastic models of Liu and Chan (2007) and MacPherson (1980)

Extension of Study of Kaihatu and Tahvildari (2012) to capture the effect of mud elasticity on surface wave evolution

Nonlinear Wave-current interaction model

- Nonlinear wave-current interaction model of Kaihatu and Kirby (1995), with the second-order correction described by Kaihatu (2009).

- The ambient current filed: constant in vertical (z) coordinate and varies in the horizontal (x,y) directions

- Velocity potential function: $\phi = \phi_0 + f_n(k_n, h, z)\phi_n(k_n, \omega_n, x, y, t)$

$$f_n(z) = \frac{\cosh k_n(h+z)}{\cosh kh}$$

$$\phi_n = -\frac{igA_n}{2\sigma_n} e^{i\int k_n dx - \omega_n t} + c.c.$$

Governing Equations (Numerical Modeling)

$$\frac{\partial A_n}{\partial x} + \frac{\sigma_n}{2(C_{gn}+U)}\left[\frac{\partial}{\partial x}\left(\frac{C_{gn}+U}{\sigma_n}\right)\right]A_n + D_n A_n = \frac{-i}{8\,(C_{gn}+U)\,\sigma_n}\left(\sum_{1}^{n-1} R A_l A_{n-1} e^{i\Theta_{l,n-1}} + 2\sum_{1}^{N-n} S A_l^* A_{n+1} e^{i\Theta_{n+l,-1}}\right)$$

$D_n A_n$: Effect of energy dissipation (Mud-induced damping)

R and S: Nonlinear interaction coefficient:

Kaihatu (2009): Correction of A_n to account for second-order effects due to the dynamic free surface boundary condition for the wave-current interaction problem:

$$B_n = A_n + \frac{1}{4g}\left[\sum_{1}^{n-1} I A_l A_{n-l} e^{\Theta_{l,n-l}} + 2\sum_{1}^{N-n} J A_l^* A_{n+l} e^{\Theta_{n+l,l}}\right]$$

$$I = \frac{\sigma_{n-l}^3\omega_l + \sigma_l^3\omega_{n-l} - g^2 k_l k_{n-l} + \sigma_l^2\sigma_{n-l}^2}{\sigma_l\sigma_{n-l}}$$

$$J = \frac{\sigma_{n+l}^3\omega_l + \sigma_l^3\omega_{n+l} - g^2 k_l k_{n+l} - \sigma_l^2\sigma_{n+l}^2}{\sigma_l\sigma_{n+l}}$$

The first-order effect of ambient currents on waves:

modification of the wave frequency through Doppler shift: $\omega = \sigma + kU \quad \omega^2 = gk\tanh(kh)$

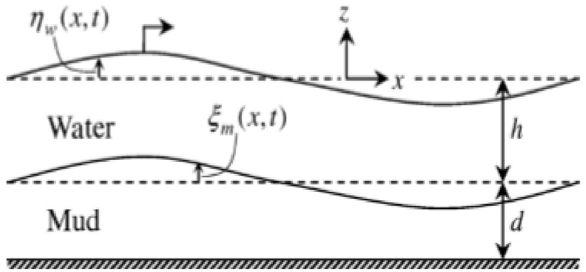
- Intrinsic frequency, σ: is measured with respect to *the coordinate system that is moving with the ambient current*
- Absolute frequency, ω: is measured with respect to *a fixed coordinate system*

Viscoelastic model of Liu and Chan (2007)

$$\eta_w = a\, e^{i(kx - wt)} \qquad k = k_r + iD_{mw}$$

Wave propagation and modulation

Wave dissipation



Assumptions:

- Mud layer is thin with respect to water depth
- Mud layer thickness is the same order of magnitude as the mud boundary layer

The Voigt model is used to represent mud elasticity

$$k_r h = k_1 h - \frac{(k_1 h)^2 \gamma \left(\frac{\delta_{me}}{h}\right)}{\sinh 2k_1 h + 2k_1 h}\left[2\lambda - \frac{\Omega_M \sinh 2\lambda\Omega_M + \Omega_P \sin(2\lambda\Omega_P)}{\cosh(2\lambda\Omega_M) + \cos(2\lambda\Omega_P)}\right]$$

$$D_m w = \frac{(k_1 h)^2 \gamma \left(\frac{\delta_{me}}{h}\right)}{\sinh 2k_1 h + 2k_1 h}\left[\frac{\Omega_P \sinh 2\lambda\Omega_M - \Omega_M \sin(2\lambda\Omega_P)}{\cosh(2\lambda\Omega_M) + \cos(2\lambda\Omega_P)}\right]$$

$$\left\{\begin{aligned}
\omega^2 &= g k_1 \tanh k_1 h, \\
\lambda &= \frac{d}{\delta_{me}}, \quad \gamma = \frac{\rho_w}{\rho_m} \\
\Omega_P &= \cos\frac{\theta}{2} + \sin\frac{\theta}{2} \\
\Omega_M &= \cos\frac{\theta}{2} - \sin\frac{\theta}{2} \\
\tan\theta &= \frac{G_m}{\omega \rho_m \nu_m} \\
\delta_{me} &= \sqrt{\frac{2|\vartheta_{me}|}{\omega}} \qquad \vartheta_{me} = \vartheta_m + \frac{iG_m}{w\rho_m}
\end{aligned}\right.$$

Viscoelastic model of MacPherson (1980)

- Dispersion relation

$$\frac{\rho_w\left(\omega^4-g^2k^2\right)\tanh\left(kH\right)}{gk\tanh\left(kH\right)-\omega^2}+\rho_m gk+T^{'}k^3\;+\rho_m\left(2k^2\nu_{me}-i\omega\right)^2\left[\frac{\left(2k^2-\frac{iw}{\nu_{me}}\right)\left[lC_mC_l-kS_mS_l\right]-2k^2l}{\left(2k^2-\frac{iw}{\nu_{me}}\right)\left[lS_mC_l-kC_mS_l\right]}\right]$$

$$-4\rho_m k^3{\nu_{me}}^2 l\left[\frac{\left(2k^2-\frac{iw}{\nu_{me}}\right)-2k\left[kC_mC_l-lS_mS_l\right]}{2k\left[lS_mC_l-kC_mS_l\right]}\right]=0$$

Parameters: $C_m=\cosh(kd)$ $S_m=\sinh(kd)$ $C_l=\cosh(ld)$ $S_l=\sinh(ld)$ $l=(k^2-\frac{iw}{\nu_{me}})^{0.5}$

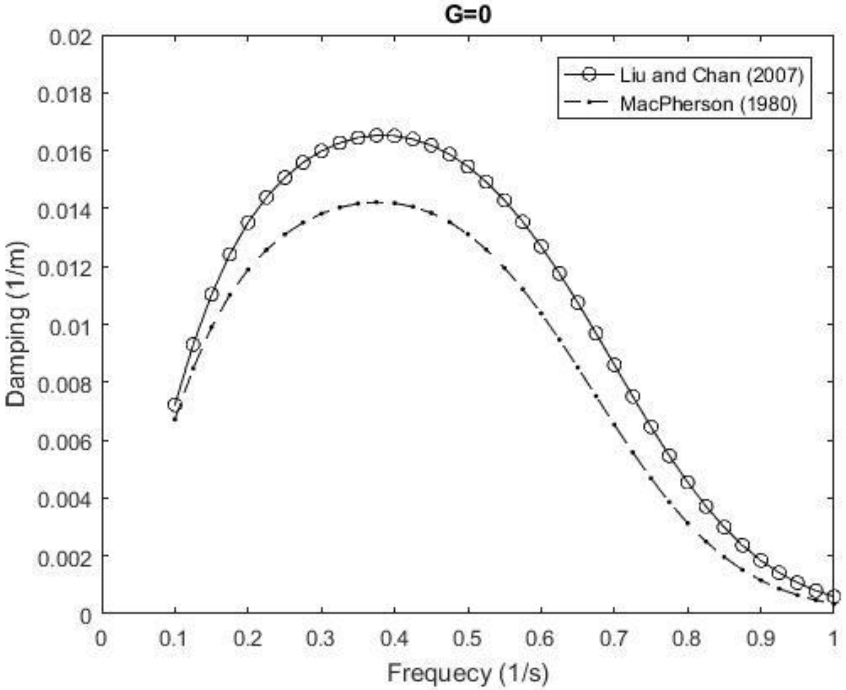
$$k = k_r + iD_{mw}$$

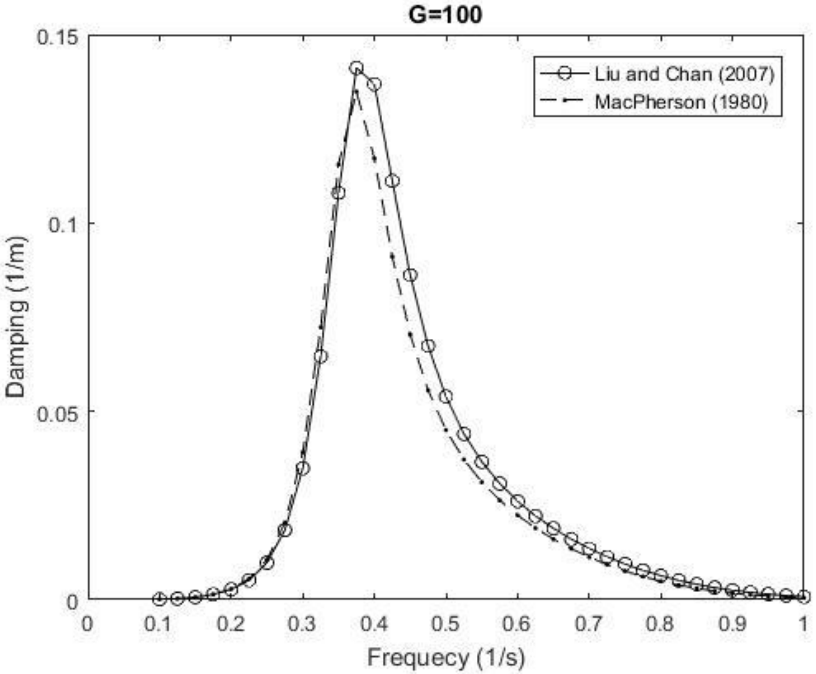
- No assumption is applied on the depth of fluid layer

Results

Variation of surface wave damping rate with frequency for MacPherson (1980) and Liu and Chan (2007) models

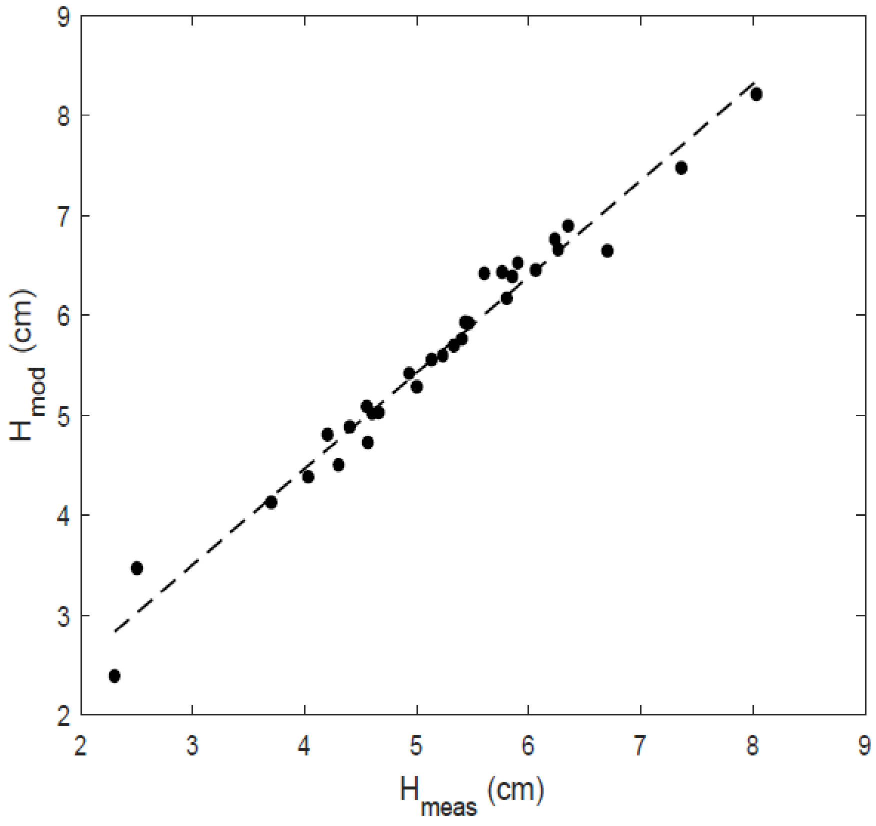
$\zeta = \sqrt{\frac{\vartheta_m}{\vartheta_w}} = 100, h = 1\, m, d_m = 0.2\, m, and\ \rho_m = 1111\, kg/m^3$





Model Validation

➢Comparison of attenuated wave heights from the numerical model and the laboratory experiments of Soltanpour and Samsami (2011)

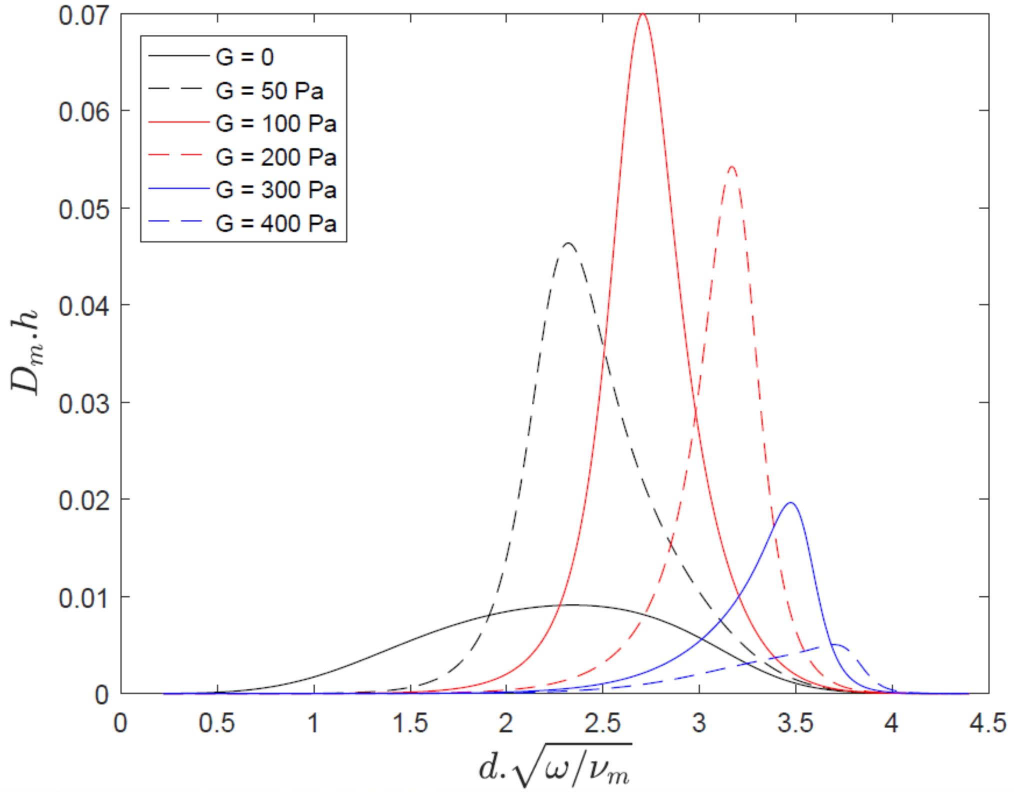


Damping of surface wave over Viscoelastic mud:

- Wave attenuation rate vs. dimensionless frequency for different shear modulus of elasticity (G)

- When G increases ⟹ the peak value of wave damping first increases and then decreases

- It shows that the behavior of wave over mud layer is highly dependent on the initially frequency of wave.



Damping of surface wave over Visco-Elastic mud:

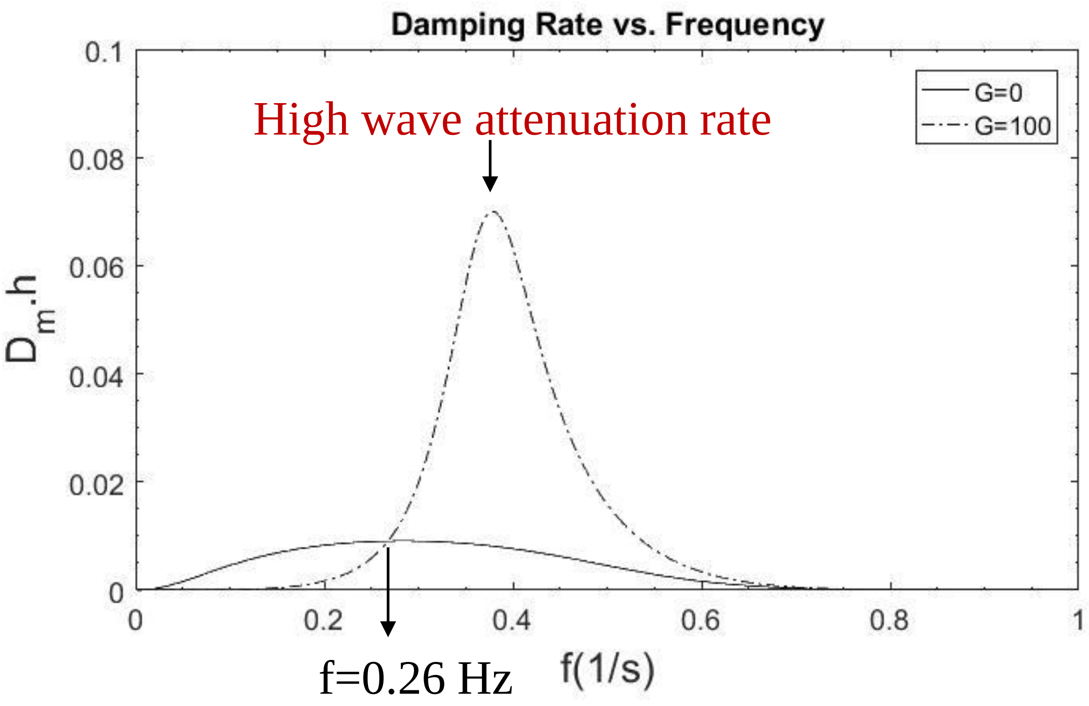
- When f < 0.26 Hz

Damping of Visco-elastic mud < Damping of Viscous mud

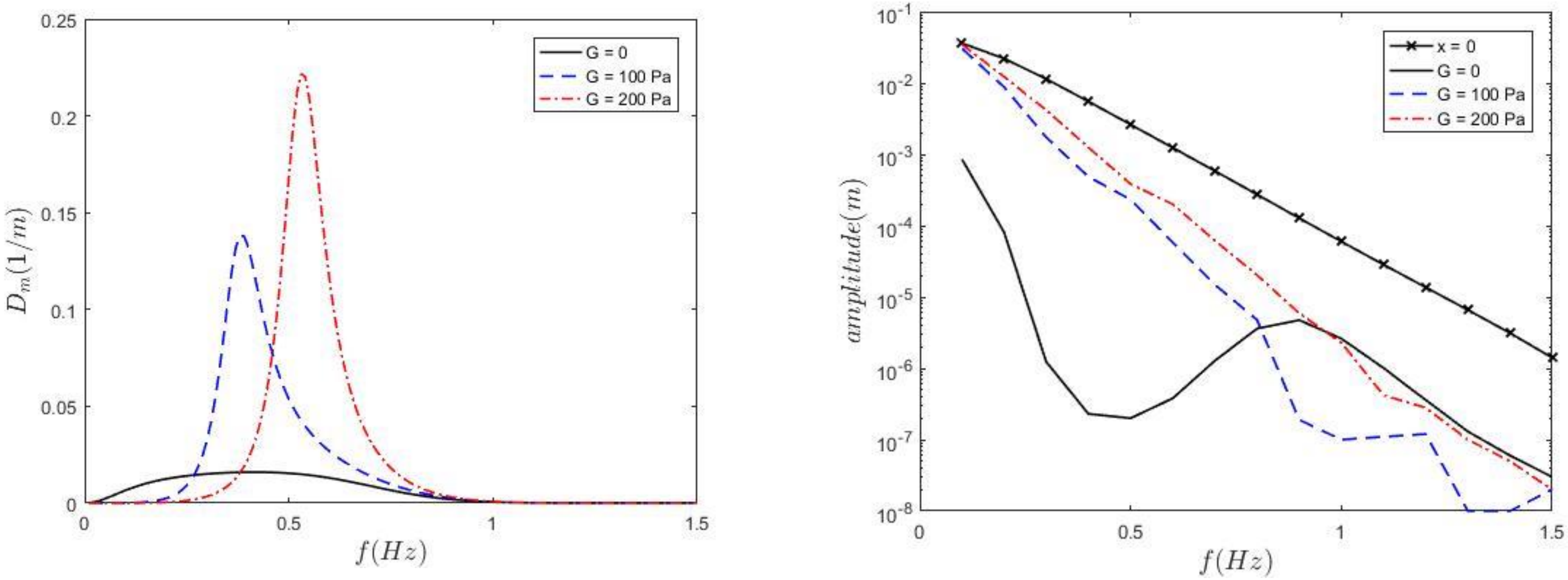
- When f > 0.26 Hz

Damping of Visco-elastic mud > Damping of Viscous mud

- Resonance effect: occurs when the surface wave frequency approaches the natural frequency of oscillation in the mud layer and results in amplification of interfacial waves. As a result, a high shear stress is developed within the mud layer and the surface wave is dissipated at a high rate due to mud viscosity



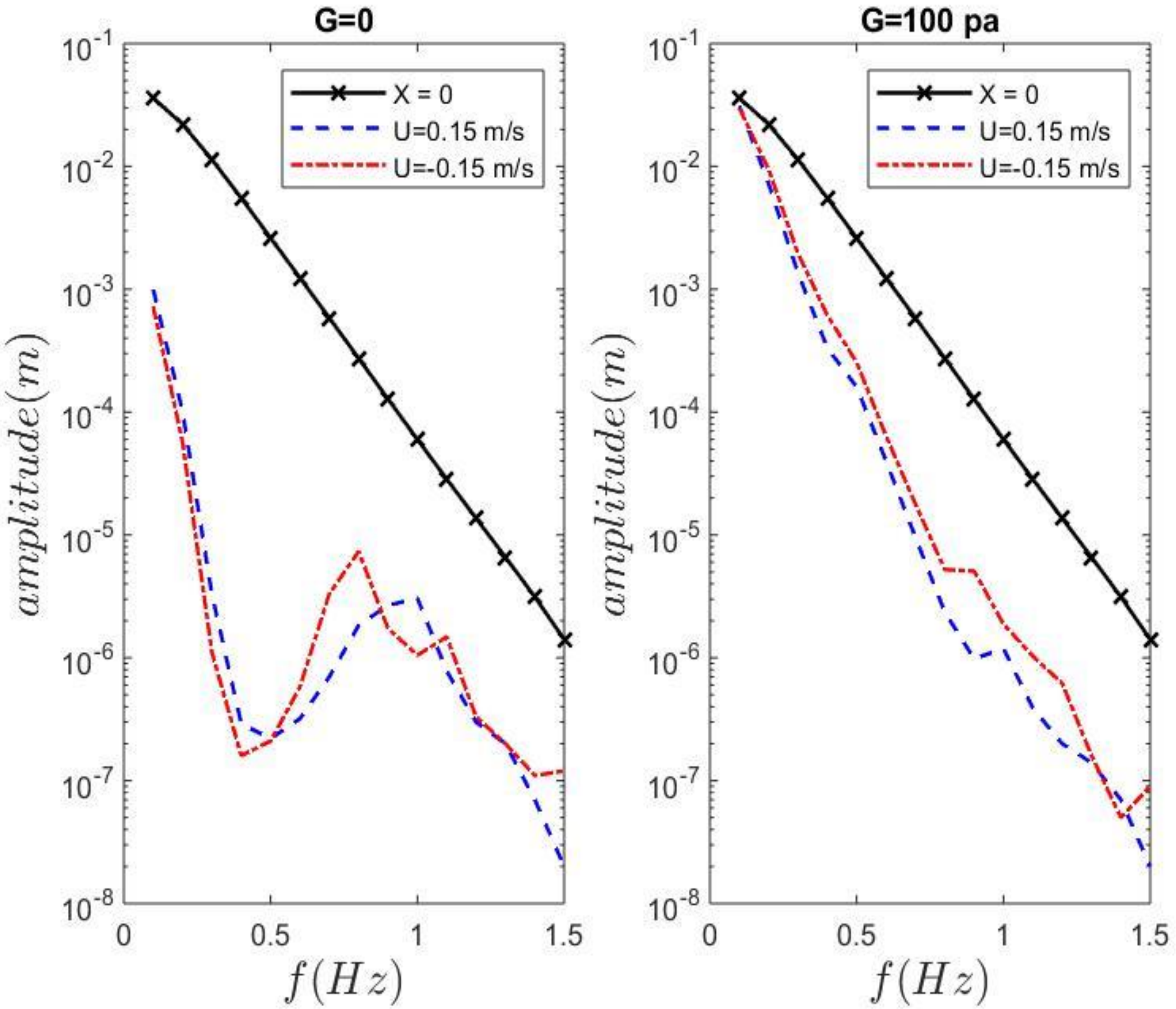
Cnoidal Waves (Without current)



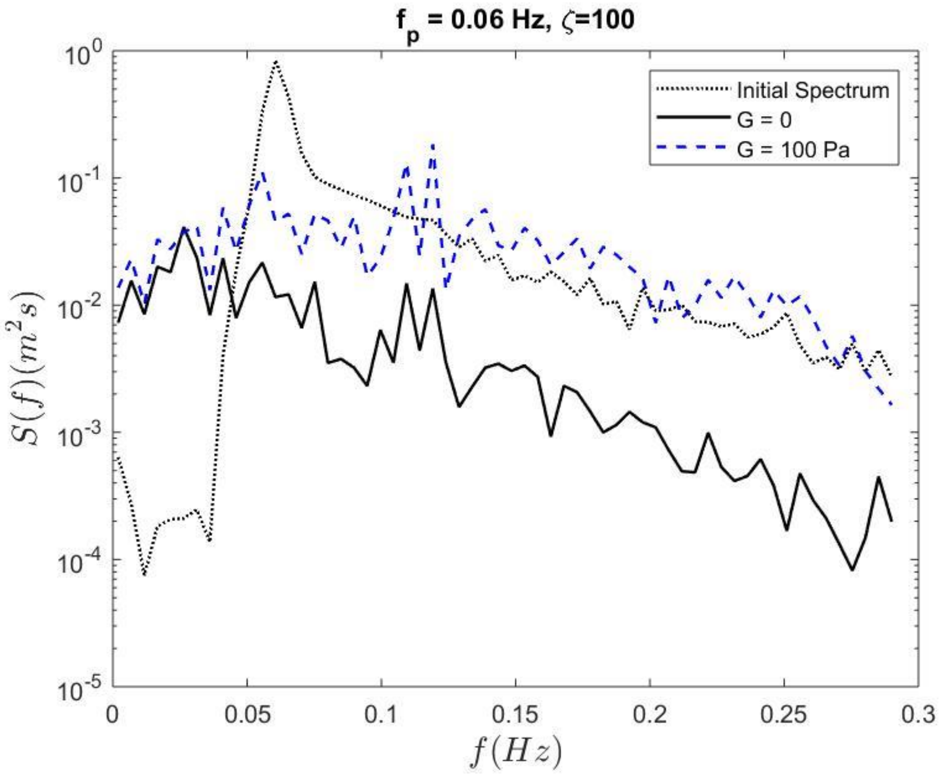
- The figure shows the distinct non-monotonicity with frequency
- There is a significant loss of amplitude at the high frequencies over the mud patch
- subharmonic interactions are responsible for the change in the pattern of damping in low frequencies.

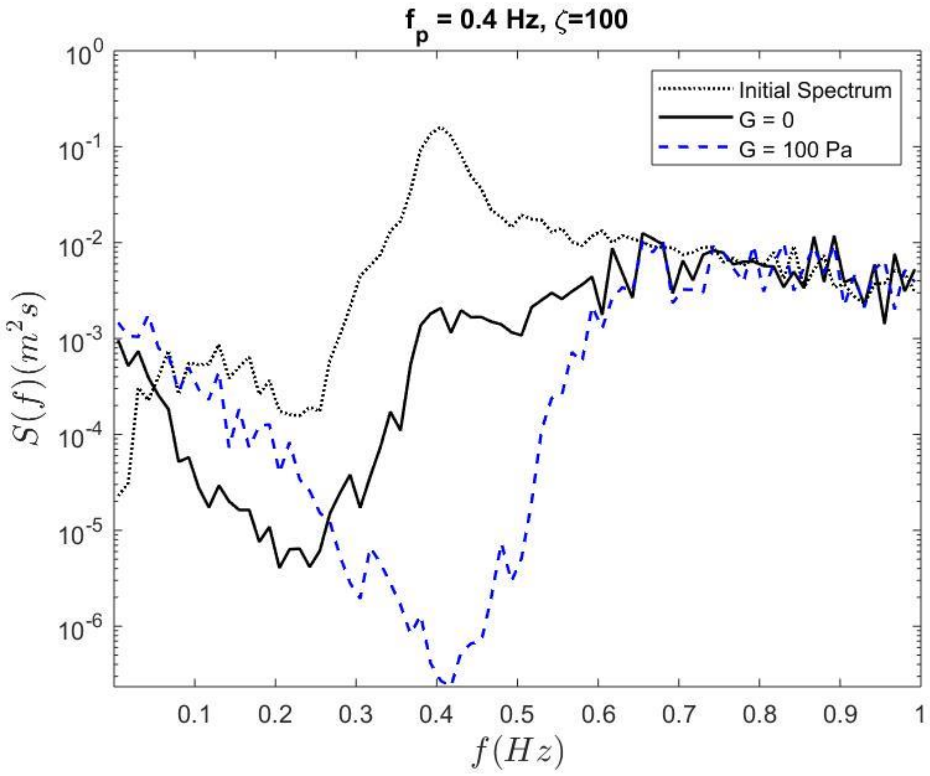
Cnoidal Waves (With current)

- Viscoelastic case:
counter-propagating currents increase wave damping while co-propagating currents reduce it



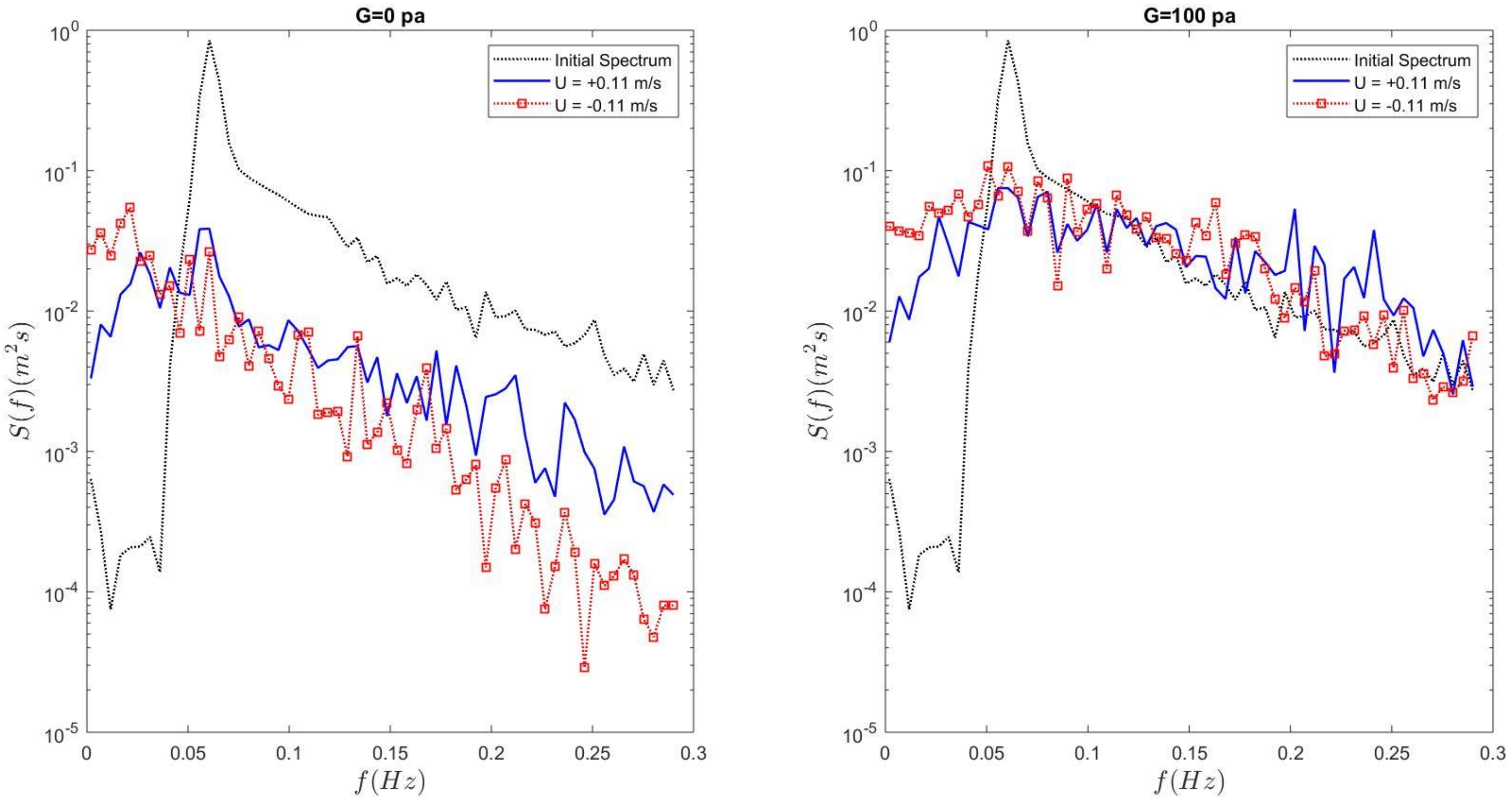
Random Waves (Without current)





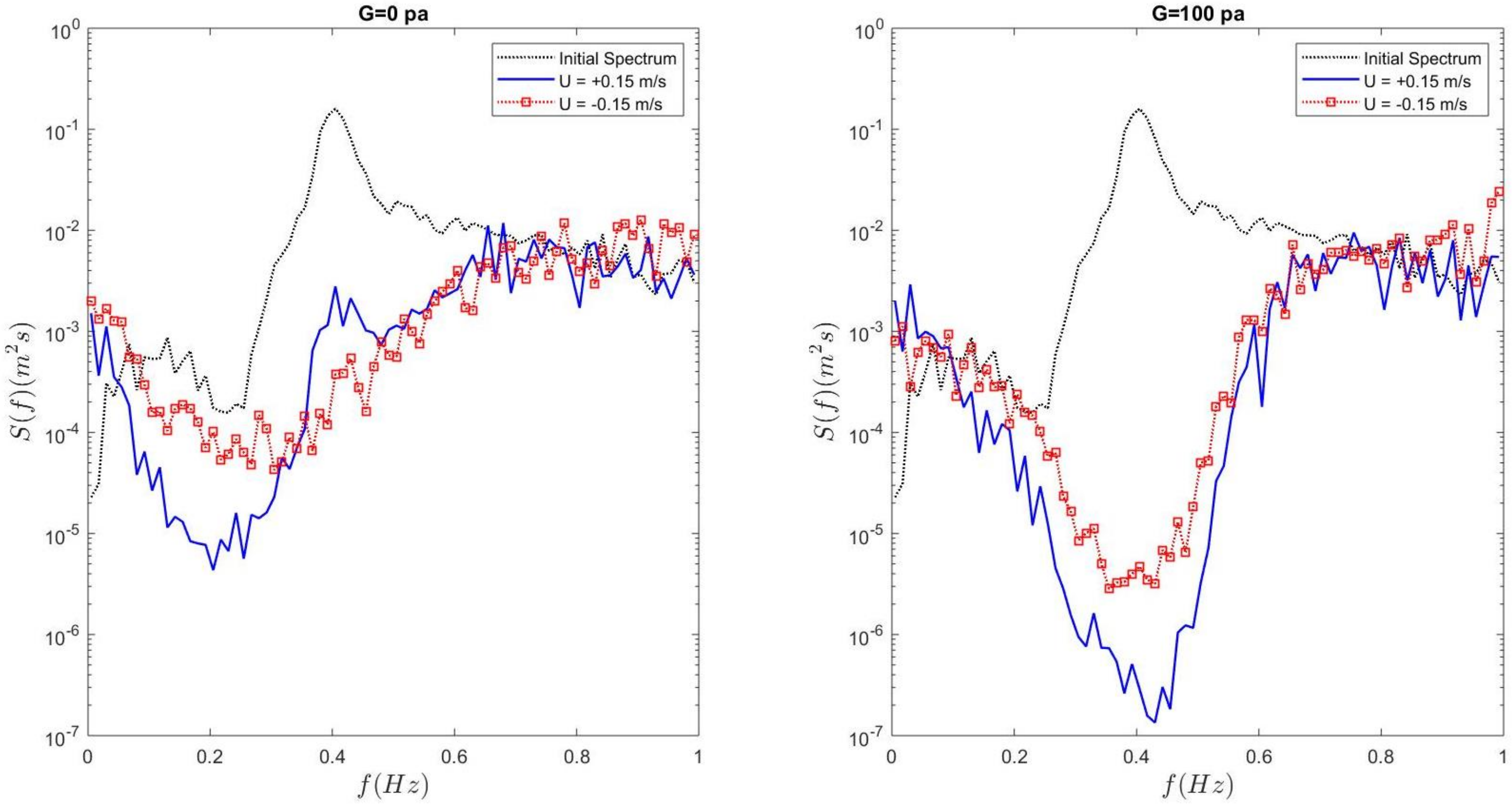
The shape of the spectrum is highly affected by relative magnitude of spectral peak and resonance frequency of mud

Random Waves (With current)



- For the viscous case: counter-propagating currents increase wave damping while co-propagating currents reduce it

Random Waves (With current)



- co-propagating currents increase wave damping while counter-propagating currents reduce it

Summary

- A new phase-resolving model is developed to study wave evolution over viscoelastic mud
- The shape of the spectrum is highly affected by relative magnitude of spectral peak and resonance frequency of mud
- High frequencies are attenuated due to subharmonic interactions regardless of the magnitude of mud shear modulus

Thank you so much for your attention

Any Comments and Questions!