IMPACT OF RANDOME WAVE SPECTRA ON STOKES DRIFT IN COASTAL CURRENT MODELING

Yuki Imai[1]

Junichi NINOMIYA[2]

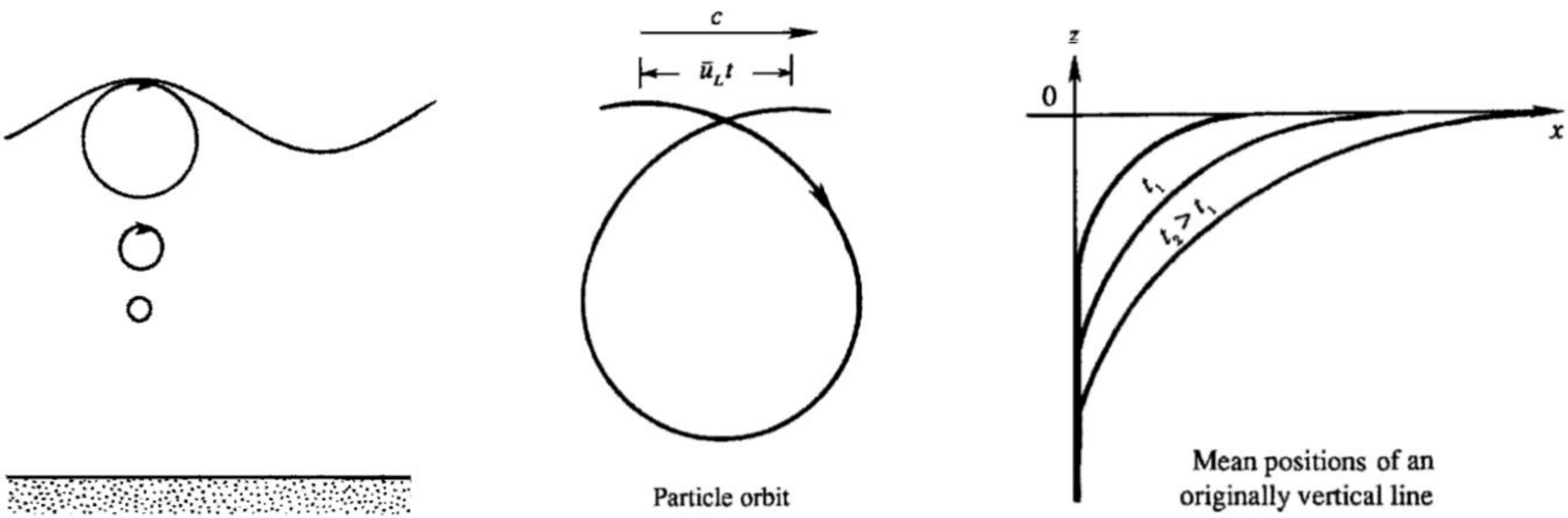
Nobuhito MORI[1]

1. Disaster Prevention Research Institute, Kyoto University
2. Kanazawa University

Introduction

Introduction: Stokes drift velocity

- Stokes drift velocity is the average Lagrangian fluid particle velocity generated from periodic wave motion



(a) first-order (b) second-order (c) mean drift

Figure: The (a) and first and (b) second-order fluid particle trajectories have closed and non-closed orbits respectively. orbits respectively. This difference leads to nonlinear mean drift over time (Kundu and Cohen, 2008)

Motivation

- **Inclusion in Eulerian ocean model**
  - Langmuir circulation
  - Coriolis Stokes forces

  Needs to accurately estimate Stokes drift velocity

- **Stokes drift profile in numerical model**
  - Be replaced by monochromatic profile
  - Uses significant wave height

Try to consider the effect of random waves in calculation of the Stokes drift velocity

Stokes drift on random waves

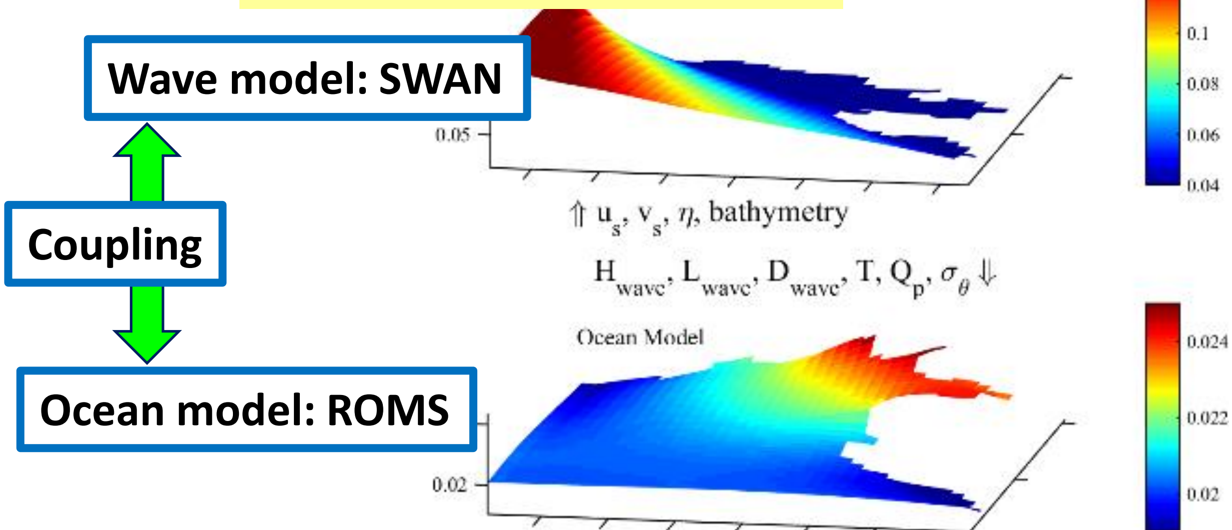
- Discussion: Monochromatic velocity of the Stokes drift in deep water (Tamura et al., 2012, Breivik et al., 2014)
  - Proposes an alternative approximate Stokes drift profile on random waves by assuming the shape of frequency spectrum as Phillips spectrum
  - Reproduces high-frequency contribution to Stokes drift velocity profile in upperhalf meter for deep-water waves
  - Doesn’t mention the impact of Stokes drift on random waves for shallow water

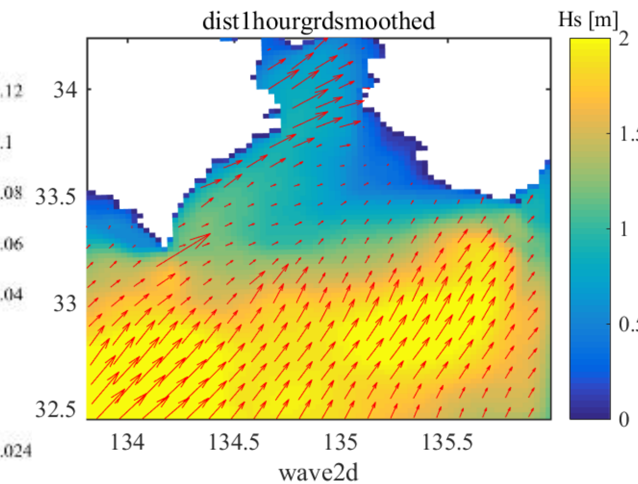
Challenge

☐Outline

- Development of a coupled ocean-wave model considering wave-current interaction on random waves
  - Formulation of Stokes drift on random waves
- Model validation theoretically and empirically
- Coastal current simulation for Tanabe Bay, Japan

Development of a coupled ocean-wave model





Coastal current simulation

Formulation of Stokes drift on random waves

Numerical Model

- **OCEAN: ROMS (Regional Ocean Modeling System)**
  - 3D ocean model
  - Governing equation: Primitive equation using static pressure approximation
  - Coordinate Cartesian coordinate, Vertical: σ coordinate
- **WAVE: SWAN (Simulating WAve Neashore)**
  - Spectral wave model
  - Governing: wave action balance equation
- **Coupling models**
  - Model Coupling Toolkit (Argonne National Laboratory)
  - two-way coupling

ROMS

◻Governing equation

$$\frac{\partial \mathbf{u}}{\boldsymbol{\partial t}} + (\boldsymbol{u} \cdot \boldsymbol{\nabla}_{\perp})\boldsymbol{u} + w\frac{\partial \boldsymbol{u}}{\partial z} + f\hat{\boldsymbol{z}} \times \boldsymbol{u} + \boldsymbol{\nabla}_{\perp}\phi - \boldsymbol{F} = -\boldsymbol{\nabla}_{\perp}\mathrm{H} + \boxed{\mathbf{J}} + \mathbf{F}^{\mathrm{w}}$$

$$\frac{\partial \phi}{z} + \frac{\partial \rho}{\rho_0} = -\frac{\partial H}{\partial z} + \boxed{K}$$

Uchiyama et al., 2010

- $\mathbf{u} = (\mathrm{u}, \mathrm{v})$: the horizontal velocity
- w: vertical velocity
- t: time
- z: vertical coordinate
- $\hat{\boldsymbol{z}}$: vertical unit vector
- ρ: water density
- ρ_0: mean water density
- f: Coriolis parameter
- ϕ: the momentum balance
- $\boldsymbol{F}$: non-wave non-conservation forces
- $\mathbf{F}^{\mathrm{w}}$: wave non-conservation forces
- $(\mathbf{J}, K)$: vortex force and the Stokes-Coriolis
- H: Bernoulli head
- $\boldsymbol{\nabla}_{\perp}$: horizontal differential operator

Formulation of Stokes drift (1/2)

□Stokes drift on random waves（Kenyon et al., 1969）

$$U(z) = \frac{1}{\rho} \int \int_{-\infty}^{\infty} F(\boldsymbol{k}) \frac{\boldsymbol{k}}{\omega(k)} \cdot \left[\frac{2k \cosh 2k(z+h)}{\sinh 2kh} \right] d\boldsymbol{k}$$

- $\boldsymbol{k}$: wave number vector
- k: wave number magnitude
- ω: wave frequency
- θ: wave direction
- ρ ：water density
- F: the two-dimensional energy spectrum
- h: total depth

The directional spectrum

The 2-D Gaussian spectrum

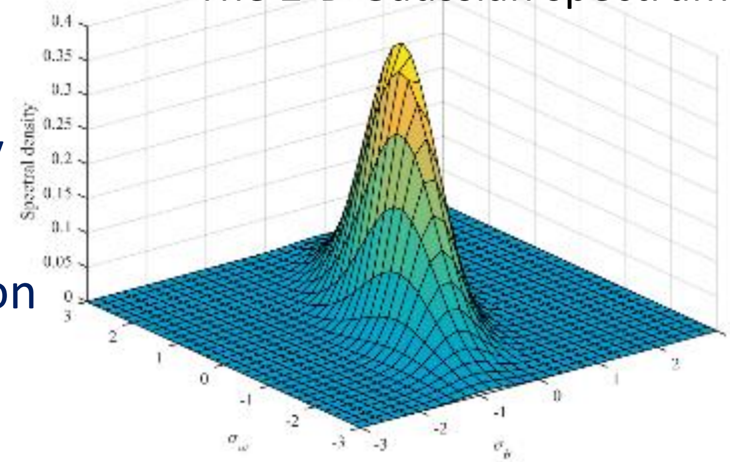
- The spectrum shape is valid near the peak frequency

$$E(\omega, \theta) = \frac{m_0}{2\pi\sigma_\omega\sigma_\theta}\left\{-\frac{1}{2}\left[\left(\frac{\omega-\omega_0}{\sigma_\omega}\right)+\left(\frac{\theta-\theta_0}{\sigma_\theta}\right)\right]\right\}$$

Mori et al., 2011

- ω: wave frequency
- θ: wave direction
- ω_0: the principal frequency
- θ_0: the principal direction
- σ_ω: the standard deviation of frequency
- σ_θ: the standard deviation of direction
- m_0: the variance of the surface elevation

The 2-D Gaussian spectrum



frequency spectrum, directional spectrum

Frequency spectrum

□The Joint North Sea Wave Project (JONSWAP) spectrum

$$S(f) = \beta_J H_{1/3}{}^2 T_P{}^{-4} f^{-5} \exp\left[-1.25\,(T_P f)^{-4}\right] \gamma^{\left[\exp -(T_P f - 1)^2/2\sigma^2\right]}$$

$$\beta_J = \frac{0.0624}{0.230 + 0.0336\gamma - 0.185\,(1.9+\gamma)^{-1}}\left[1.094 - 0.01915 \ln \gamma\right]$$

$$T_P \cong T_{1/3} / \left[1 - 0.132\,(\gamma + 0.2)^{-0.559}\right]$$

- $H_{\frac{1}{3}}$, : $Significant\ wave\ height$, $T_{\frac{1}{3}}$: Significant wave period
- f_p : peak frequency, T_p: peak period

□The relationship between σ_ω and Q_p (Goda's spectral band width)

$$\sigma_\omega = \frac{1}{\sqrt{\pi} Q_p}$$

$$Q_p = \frac{2}{(m_0)^2} \int_0^\infty f S(f)^2 df$$

$$Q_p = -0.015\gamma^2 + 0.60\gamma + 1.37$$

- m_0 : zero moment

Frequency variance σ_ω <-> Frequency variance parameter "Q_p"

Directional spectrum

- Mitsuyasu-type spectrum (Mitsuyasu et al., 1975)

$$D(\theta) = \frac{2^{2S}}{2\pi} \frac{\Gamma(S+1)^2}{\Gamma(2S+1)} \cos^{2S}\left(\frac{\theta - \theta_p}{2}\right)$$

  - S: directional variance parameter

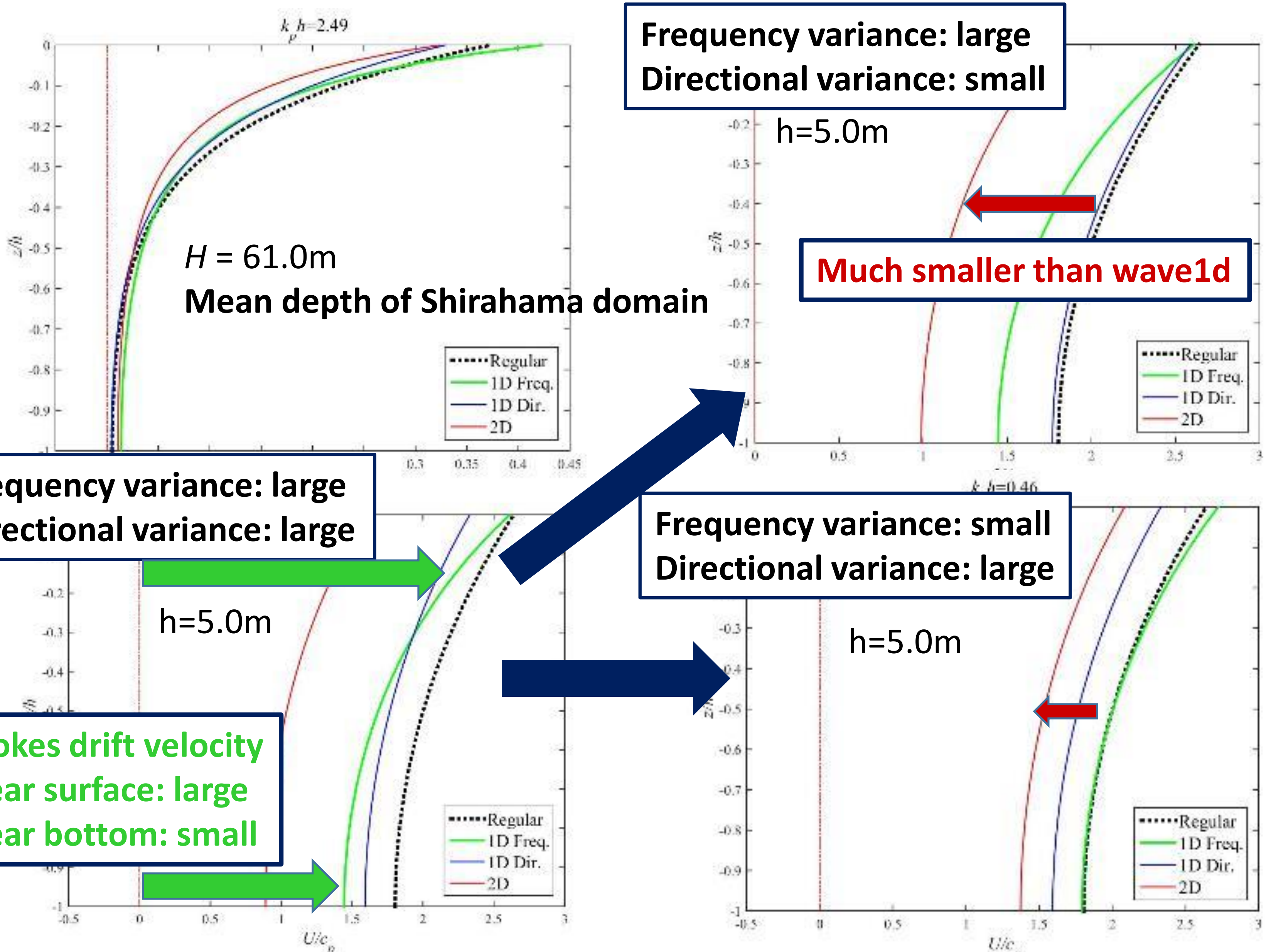
- The relationship between σ_θ and S

$$\sigma_\theta \simeq \sqrt{\frac{2}{1+S}}$$

Directional variance σ_θ <-> directional variance parameter "S"

Model validation

Theoretical validation



Test simulation on ideal simple bathymetry

Models

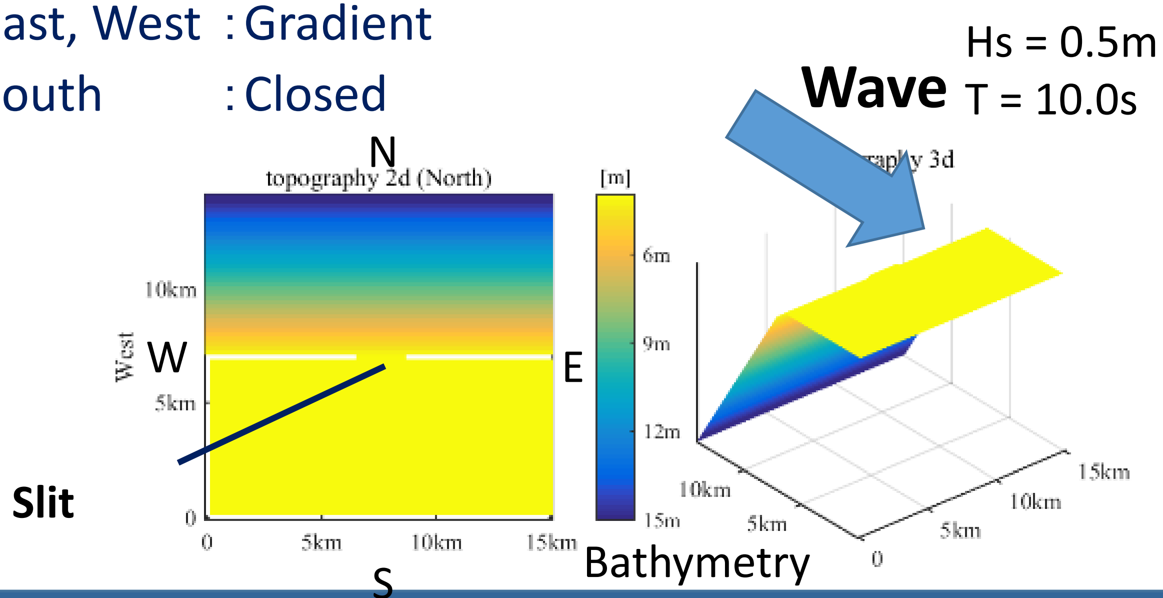
- ROMS ver. 3.5
- SWAN ver. 40.81

Grid

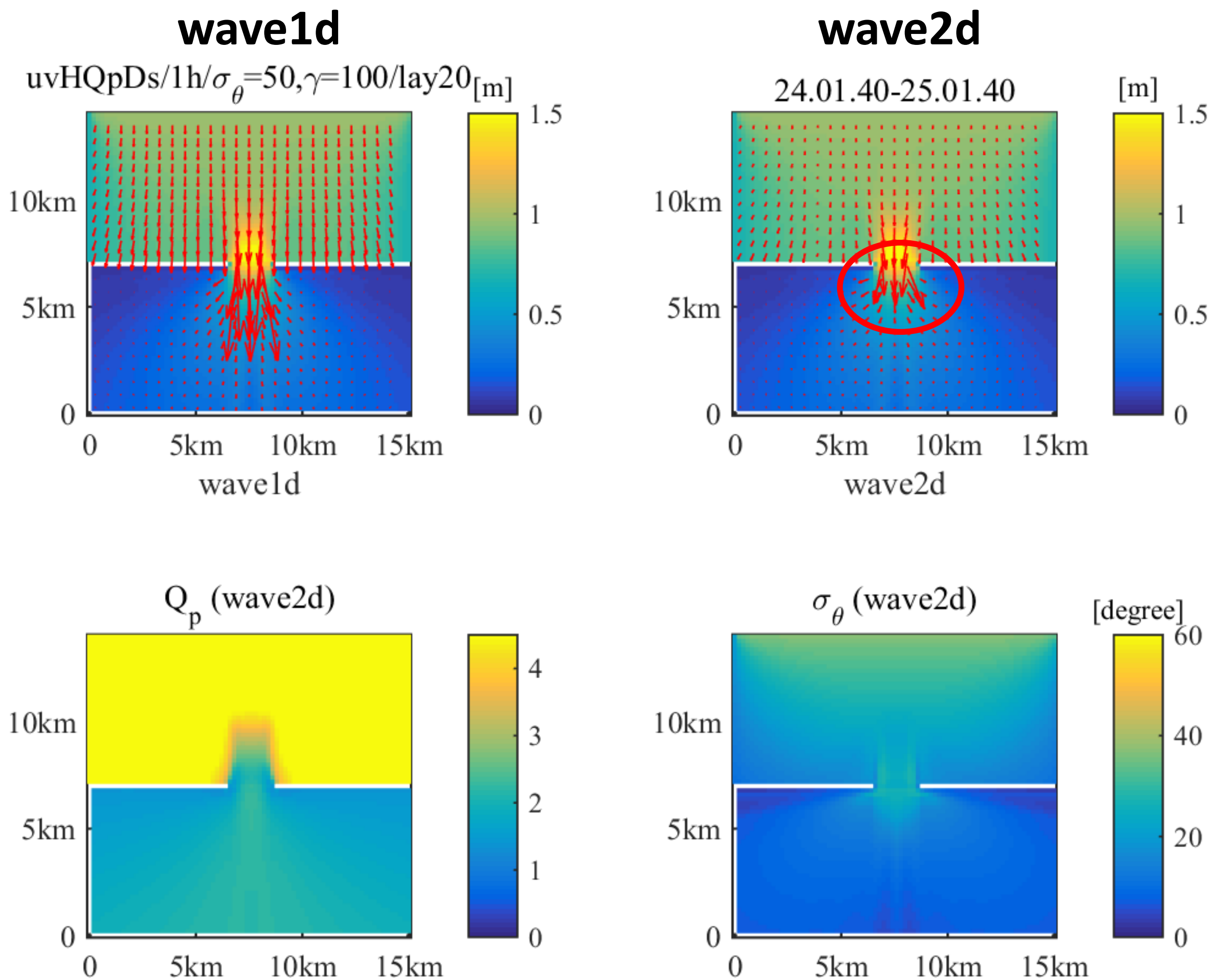
- domain :East-West 15km × North-South 15km
- resolution :Horizontal 200m × Vertical 20 layers

Boundary condition

- North : Fixed (wave coming)
- East, West : Gradient
- South : Closed



Spatial distribution: Stokes drift velocity
Near surface: 1h mean: $\sigma_\theta = 50, \gamma = 10.0$

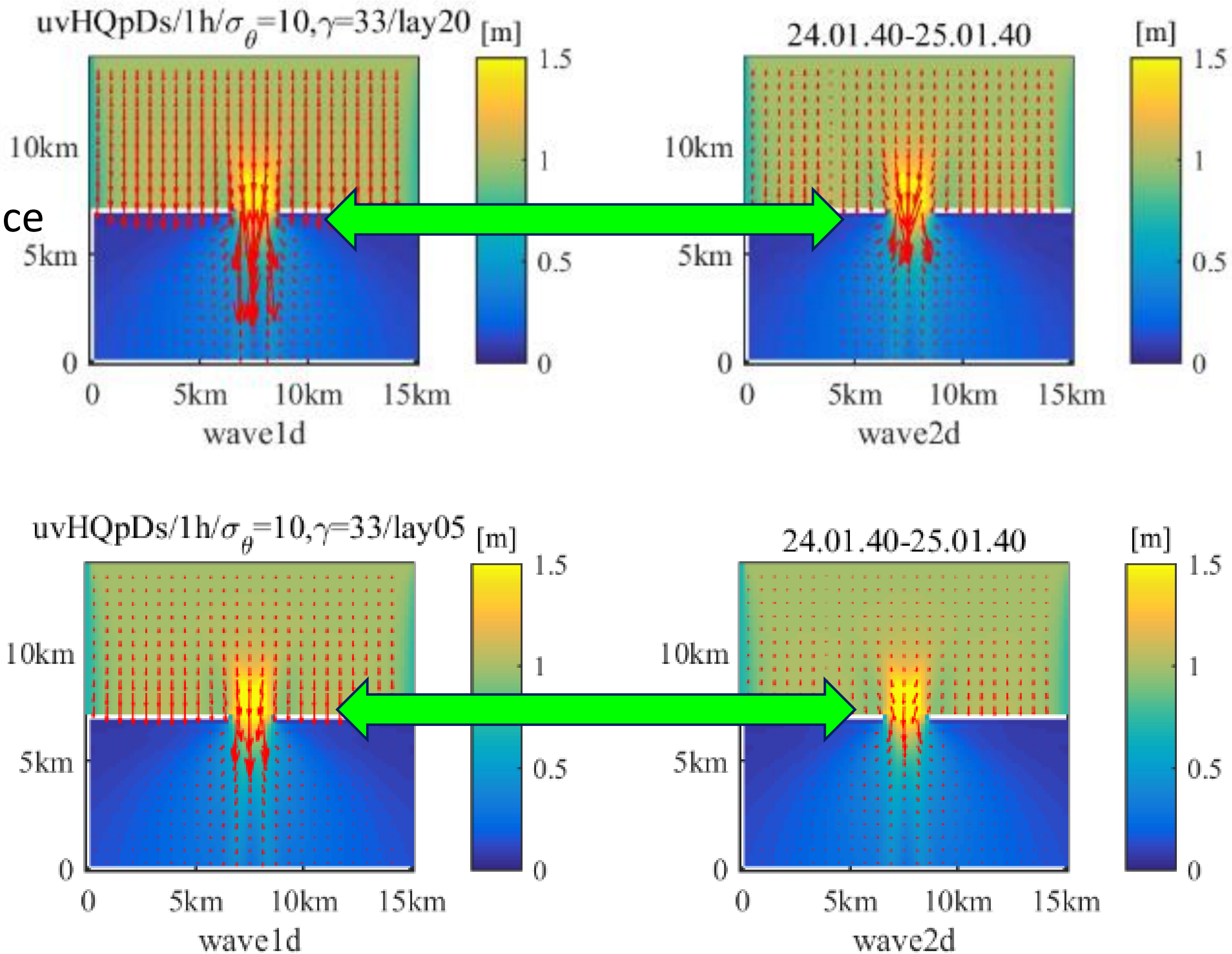


Spatial distribution: Stokes drift velocity 1h mean: $\sigma_\theta = 10, \gamma = 30.0$

wave1d **wave2d**

Near surface

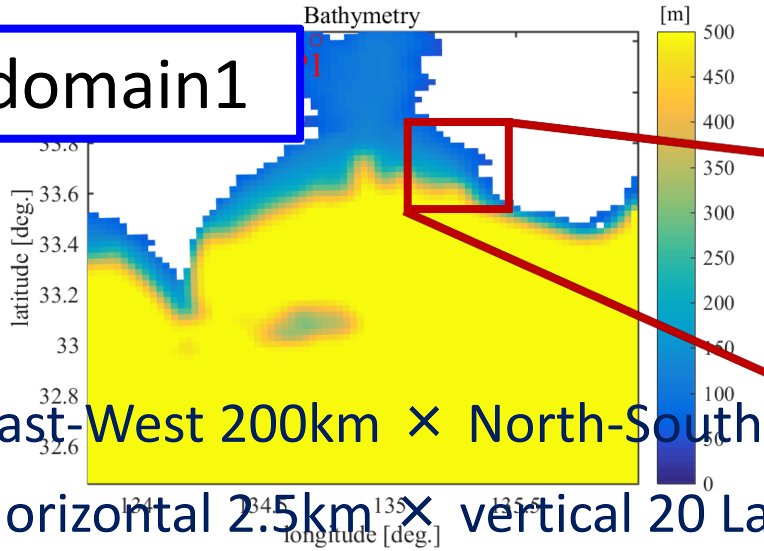
Near bottom (5/20)



Coastal current simulation

Coastal current hindcast: Tanabe Bay, Japan

domain1

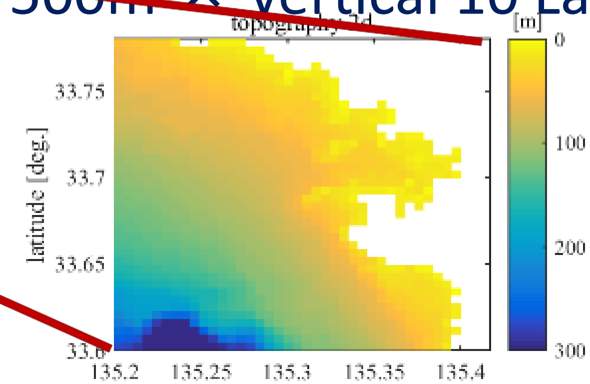


East-West 200km × North-South 200km

Horizontal 2.5km × vertical 20 Layers

East-West 30km × North-South 30km

Horizontal 500m × vertical 10 Layers



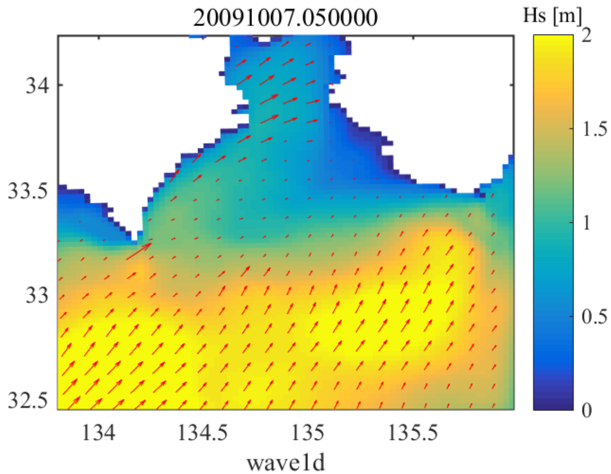
domain2

◻Initial, Boundary Condition

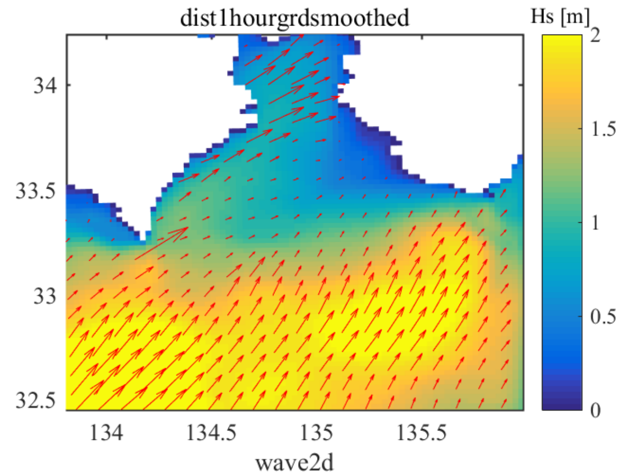
- ➢ Forcing
  - – MSM-GPV data (JMA)
  - – AMeDAS data (JMA) - Long wave radiation
  - – TPXO7.0 (NASA) - Astronomical tide
- ➢ Physics
  - – Surface flux COARE-3.0
  - – Turbulence k-ε model（Generic Length Scheme）
  - – TKE at surface Craig-Banner/Wave dissipation

Mean velocity: Near surface
1 hour mean: domain1

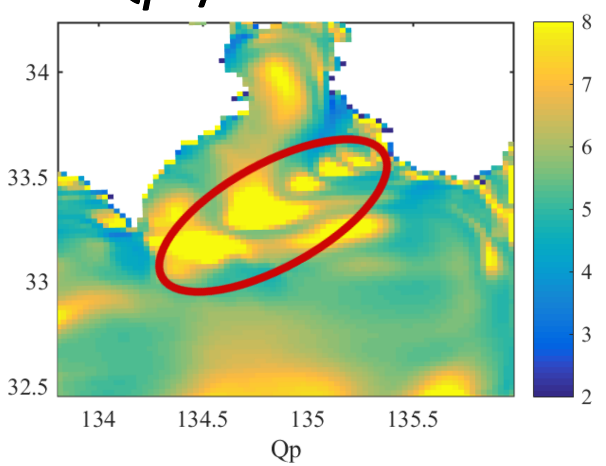
Wave1d



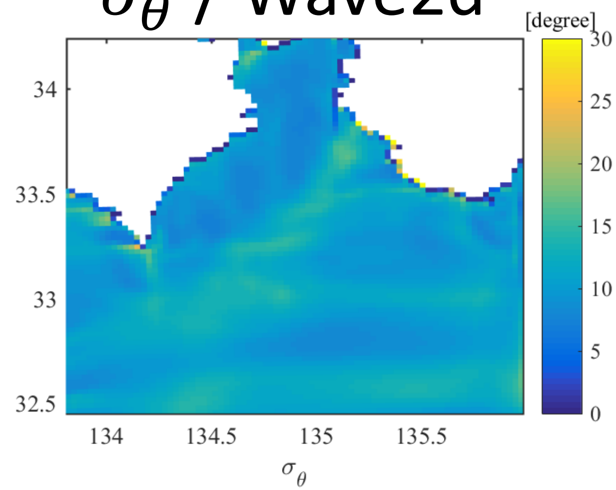
Wave2d



Qp / Wave2d

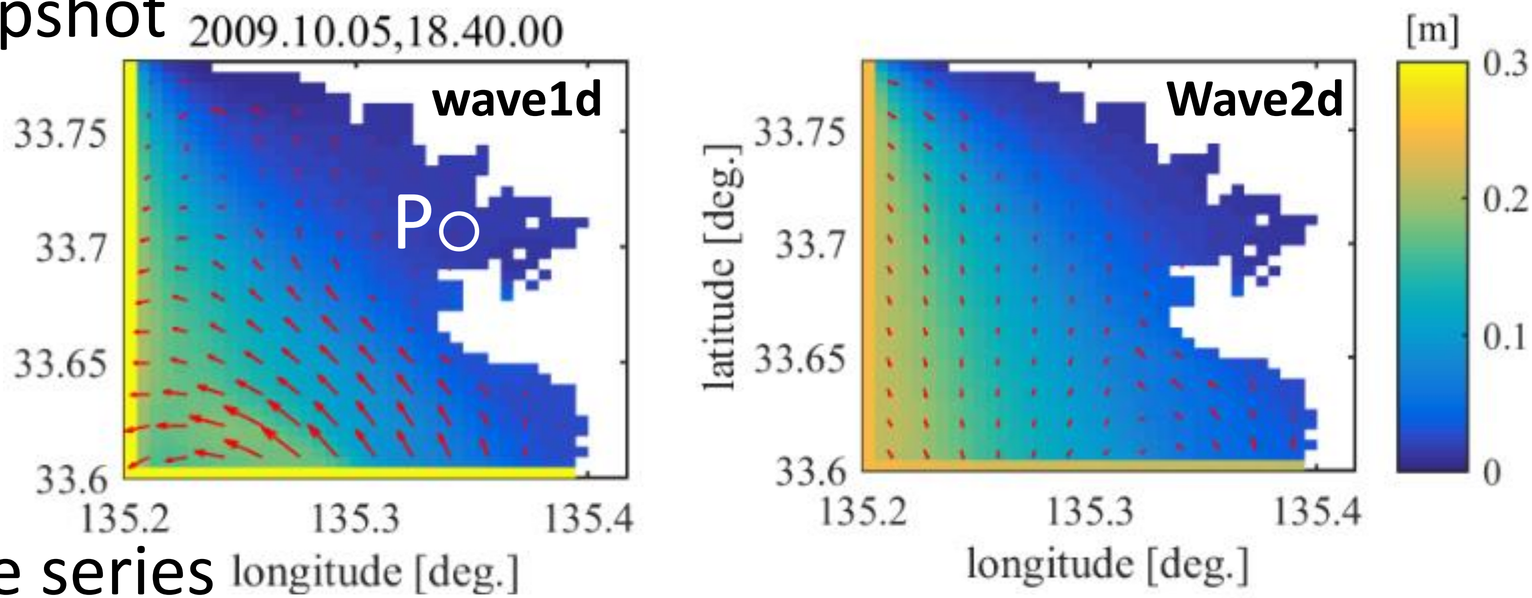


σ_θ / Wave2d

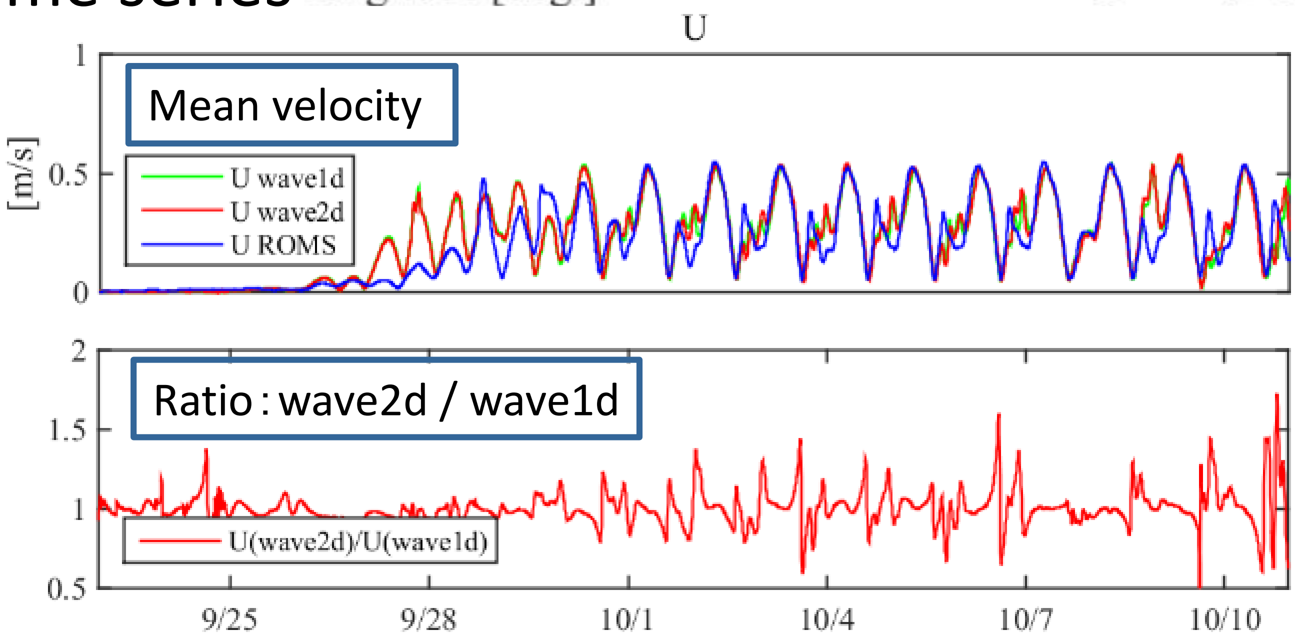


Mean velocity: Near surface: domain2

Snapshot



Time series



- Impact
  - Mean velocity magnitude and direction
  - Significant wave height affected by two-way coupling

Summary

- Development of the coupled ocean-wave model considering wave-current interaction on random waves
- Model Validation
  - Theory: found large frequency variance contribution
  - Test simulation: Stokes drift spreading well behind of the slit
- Costal current simulation
  - Domain1: effect of consideration of Stokes drift on random waves in magnitude of mean velocity
  - Domain2: 20 – 30% difference between wave2d and wave1d in mean velocity

Thank you for your attention!