

# PREDICTION OF FORMATION OF RECURVED SAND SPIT USING BG MODEL



**Masumi SERIZAWA** Coastal Engineering Laboratory Co., Ltd.

**Takaaki UDA** Public Works Research Center

**Shiho MIYAHARA** Coastal Engineering Laboratory Co., Ltd.

# INTRODUCTION

- When waves are obliquely incident at a large angle over  $45^\circ$  with respect to the direction normal to the shoreline, a sand spit can extend at a location with a large change in coastline orientation (Ashton et al., 2001).
- Usually, a single spit extends. However, when two sets of waves arrive from different directions, the sand spit is recurved (Bird, 2008).





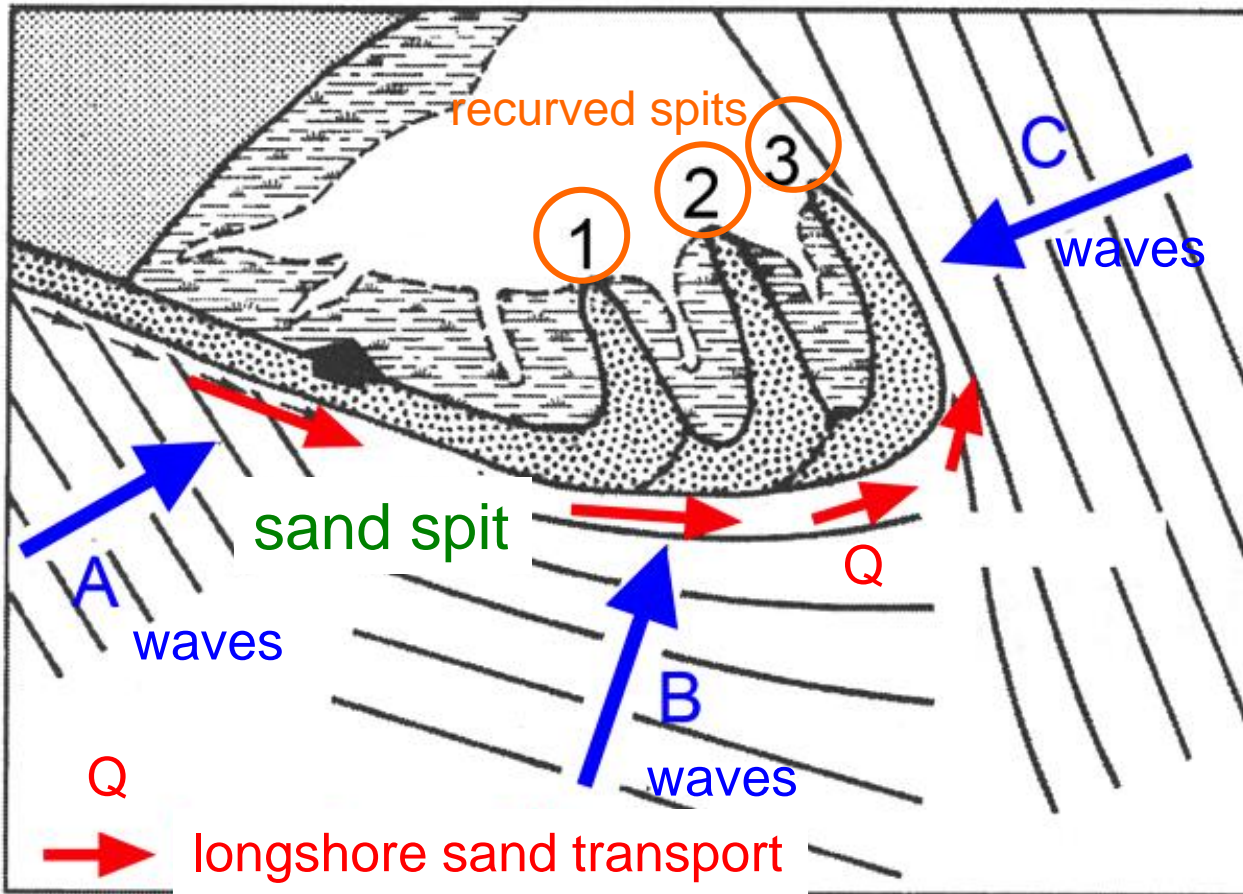
# INTRODUCTION

- In the recurved spit, the previous shoreline often intersects at an acute angle with the overall elongation direction of the sand spit.



# Schematic diagram of development of a recurved sand spit

Zenkovich (1967)



- When waves are obliquely incident to the shoreline from **directions A and B**, **longshore sand transport** toward the tip of the sand spit arises, as a result, **sand spit** extends owing to the deposition of littoral sand.
- However, if additional waves arrive from **direction C** opposing to direction A, a significantly **recurved spit** may be formed.

# INTRODUCTION

- There are few studies on the prediction of the elongation of the recurved sand spit when waves are incident from different directions.
- In this study, the elongation of a recurved spit was predicted using the BG model (a model for predicting three-dimensional beach changes based on Bagnold's concept) (Uda et al. 2016).





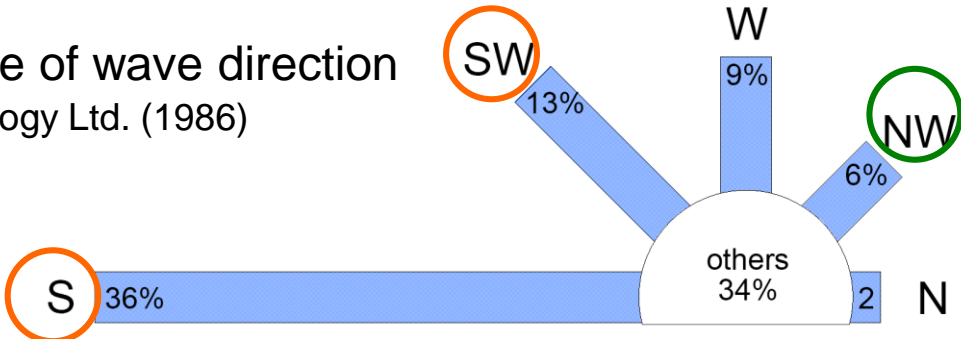
# **Examples of Recurved sand spits**

# Example of recurved sand spit: Namibia

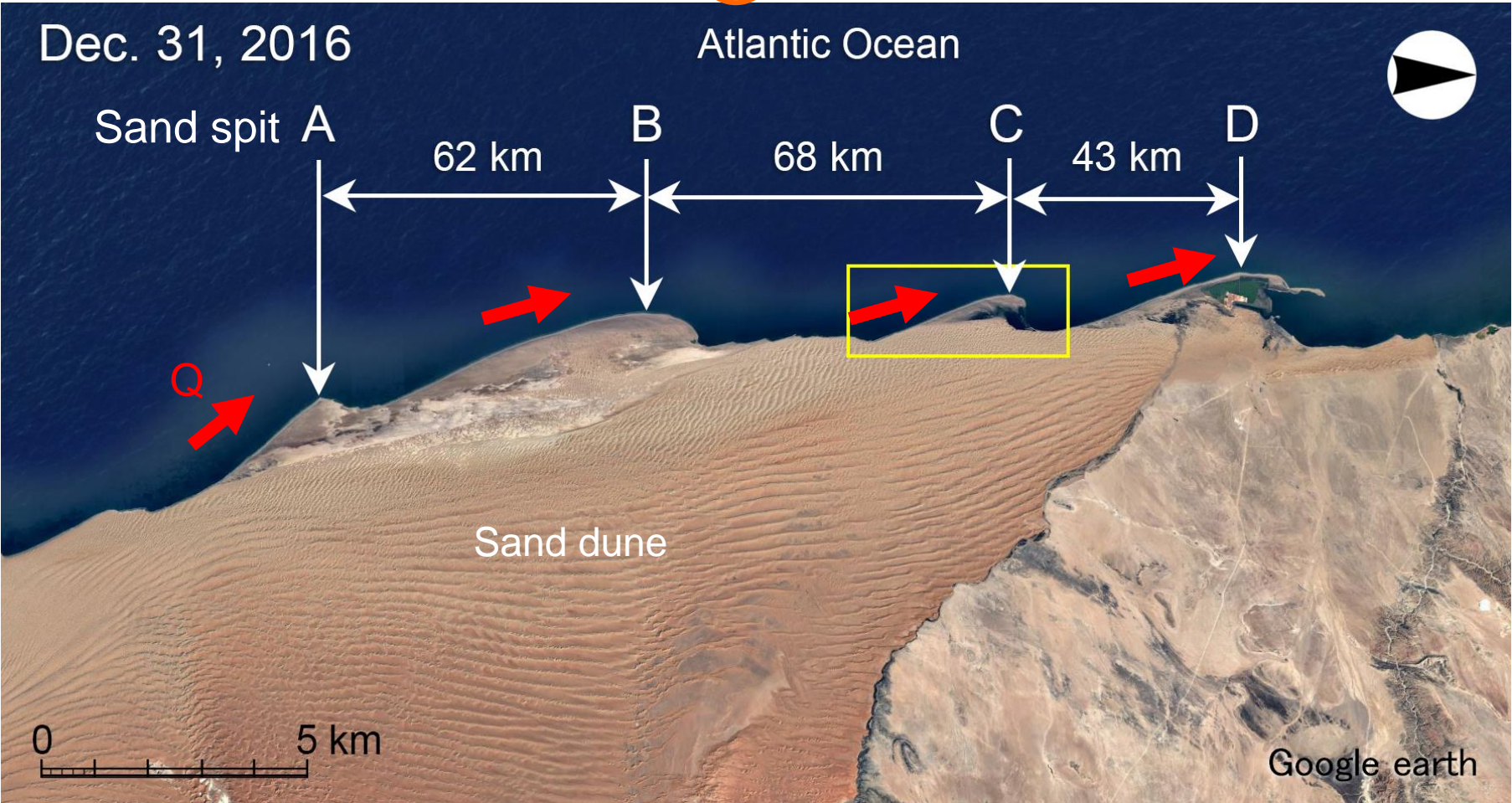


# Example of recurved sand spit: Namibia

Probability of occurrence of wave direction  
British Maritime Technology Ltd. (1986)



**Q**  
→ longshore sand transport





# Example of recurved sand spit: Namibia



Although the sand spit extends northward, many recurved spits are superimposed.

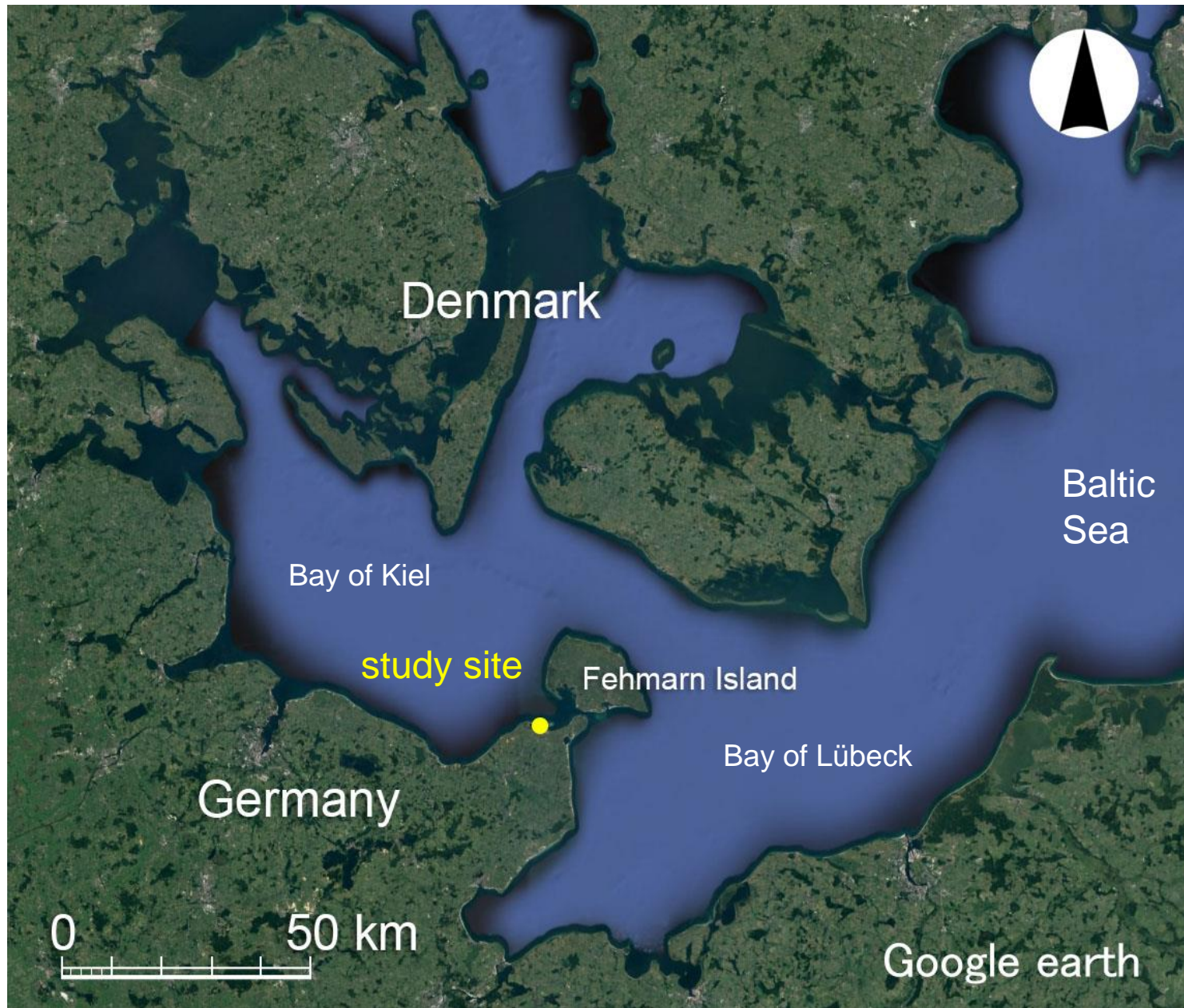
# Example of recurved sand spit: Namibia



Although the sand spit extends northward, many recurved spits are superimposed.

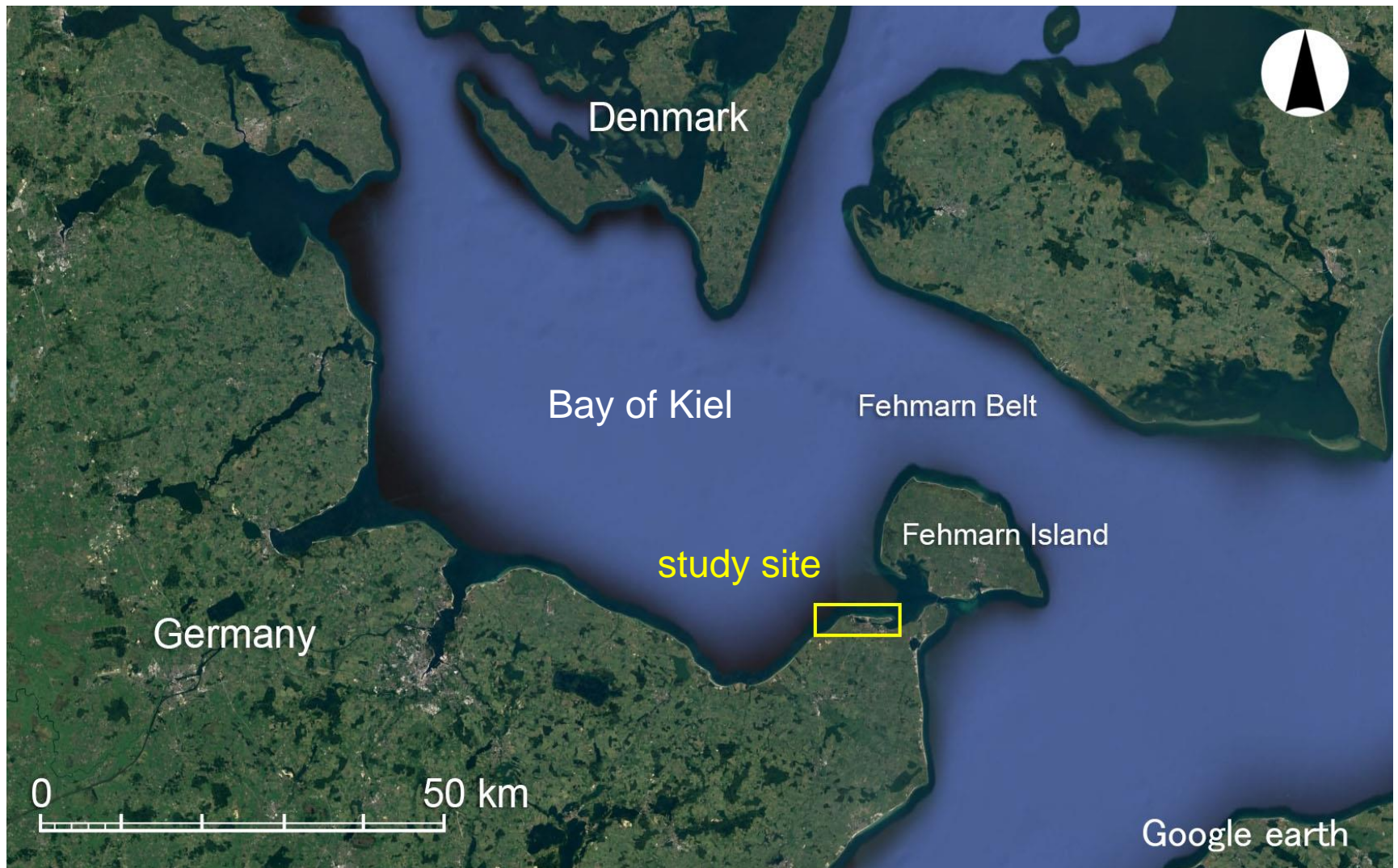


# Graswarder spit (Germany)





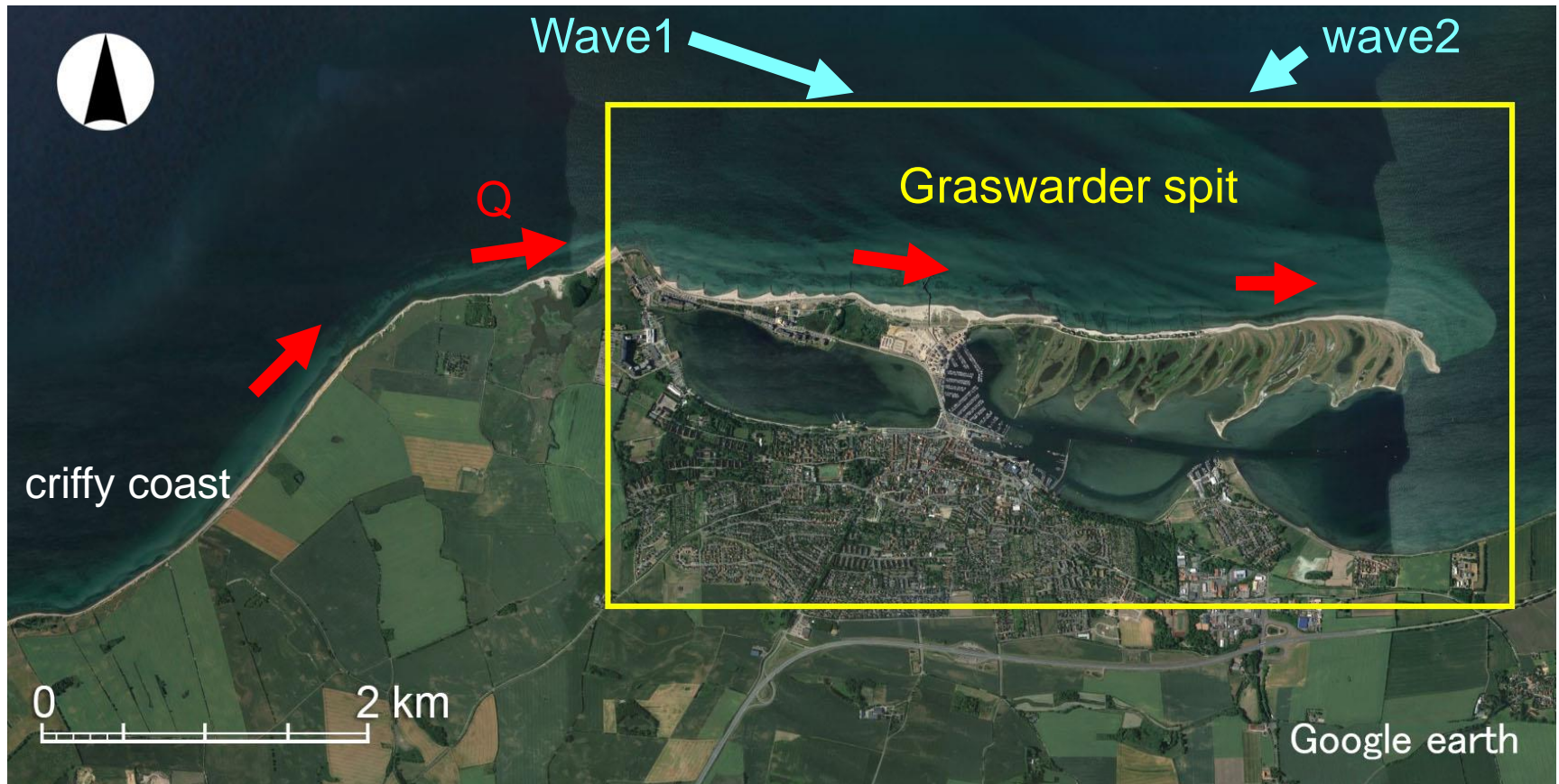
# Graswarder spit (Germany)





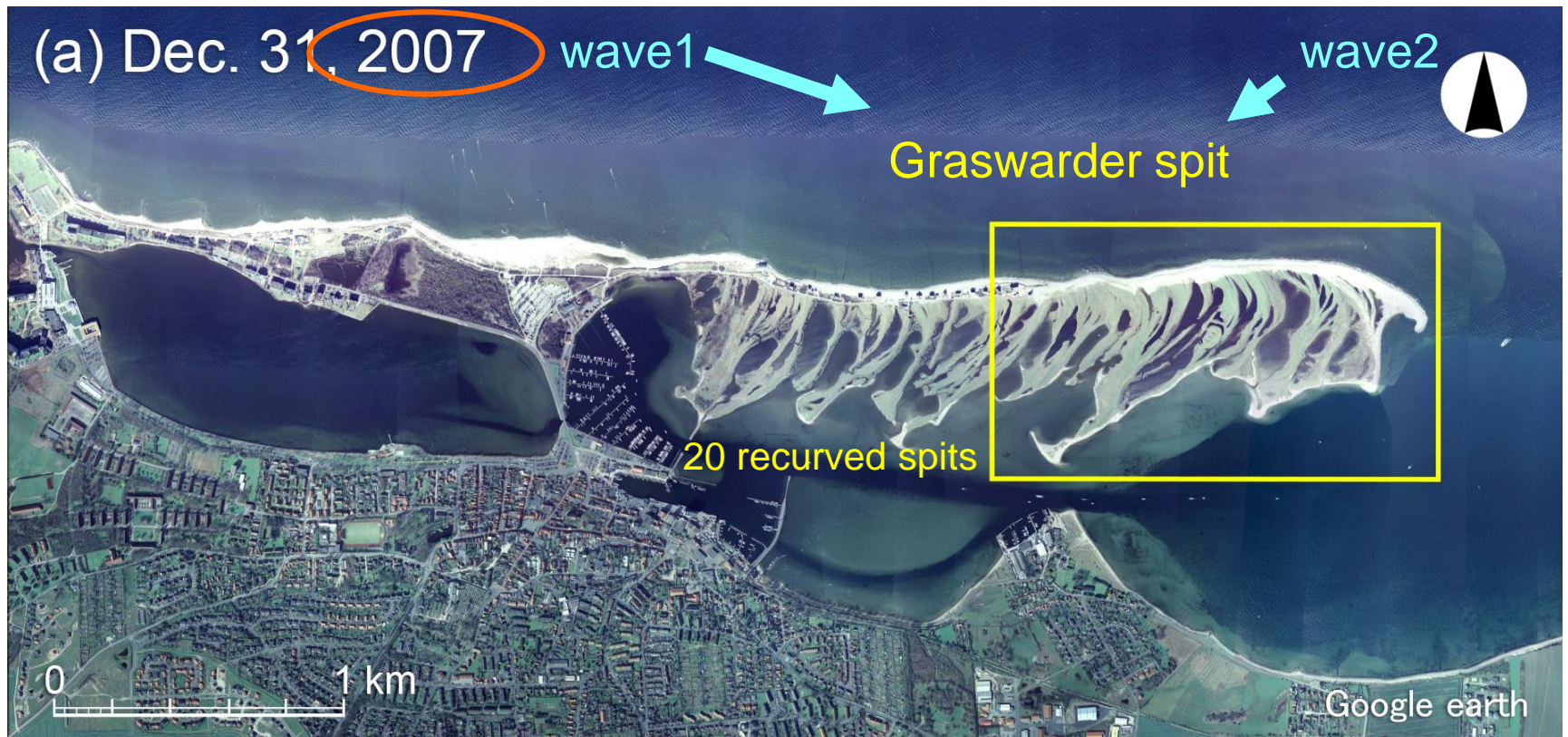
# Graswarder spit (Germany)

$Q$   
→ longshore sand transport



The sand spit extended by eastward longshore sand transport, and a barrier island has grown eastward for about 3,000 years (Bird, 2008).

# Graswarder spit (Germany)



Although the sand spit extends eastward, many recurved spits are superimposed. 20 recurved spits can be seen (Bird, 2008).



# Graswarder spit (Germany)



# Graswarder spit (Germany)





# Graswarder spit (Germany)



# Calculation of elongation of recurved sand spits

**BG Model** (Model for predicting beach changes based on **BaGnold's concept**)

Uda et al. (2016)



# BG Model (Model for predicting beach changes based on Bagnold's concept)

## The sand transport equation

- equilibrium slope : Inman and Bagnold(1963)
- energetic approach : Bagnold(1963)

$$\vec{q} = C_0 \frac{P}{\tan\beta_c} \left\{ \begin{array}{l} K_n (\tan\beta_c \vec{e}_w - |\cos\alpha| \nabla Z) \\ + \left\{ (K_s - K_n) \sin\alpha \frac{K_2}{\tan\beta} \frac{\partial H}{\partial s} \right\} \tan\beta \vec{e}_s \end{array} \right\} \quad \dots(1)$$

$(-h_c \text{ \&#x2D; } Z \text{ \&#x2D; } h_R)$

Ozasa and Brampton (1980)

$P$  = wave energy dissipation by breaking

$$= K \sqrt{g/h} \left[ 1 - (G/g)^2 \right] E \quad \dots(2)$$

$P$  is calculated by using the results of wave field.

## Calculation model of plane wave field:

energy balance equation (Mase, 2001) with dissipation term due to wave breaking (Dally *et al.*, 1984).

$\vec{q} = (q_x, q_y)$  : the net sand transport flux

$Z = (x, y, t)$  : the seabed elevation

$n, s$  : the coordinates of cross-shore and longshore directions,

$\vec{e}_n$  : the unit vector normal to the contour lines (shoreward).

$\vec{e}_s$  : parallel to a contour line

$\nabla Z = \tan\beta \vec{e}_n = (\partial Z / \partial x, \partial Z / \partial y)$  : the gradient vector of  $Z$ .

$\vec{e}_w$  : the unit vector of wave direction

$\theta_w$  : the angle between the  $x$ -axis and the wave direction

$$\tan\beta \vec{e}_s = (-\partial Z / \partial y, \partial Z / \partial x)$$

$\alpha = \theta_w - \theta_n$  : the angle between the wave direction and the normal to the contour line

$\tan\beta = |\nabla Z|$  : the seabed slope

$\tan\beta_c$  : equilibrium slope

$K_s$  : coefficients of longshore sand transport

$K_n$  : cross-shore sand transport

$h_c$  : the depth of closure

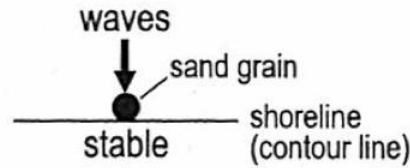
$h_R$  : the berm height

$H$  : the wave height at a local point

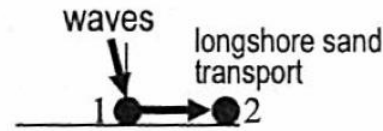
$$C_0 = 1 / \{ (\rho_s - \rho) g (1 - p) \}$$

# Physical meaning of BG model

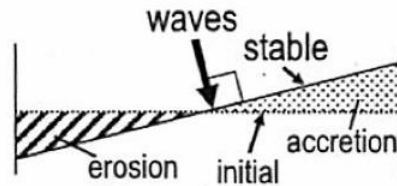
(a) incident waves orthogonal on a contour line



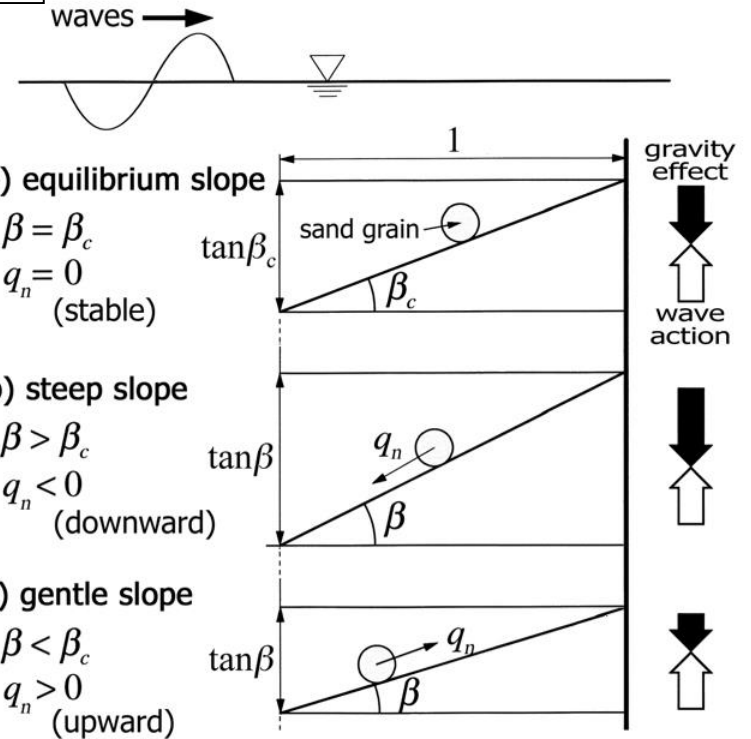
(b) obliquely incident waves



(c) stable contour line



**Fig. Stabilization mechanism of contour lines based on wave directions and longshore sand transport.**



**Fig. Stabilization mechanism of beach profile based on equilibrium between gravity effect and wave action.**



# Calculation results

# Case 1: wave direction $\theta_1 = +45^\circ$ , $\theta_2 = -30^\circ$

## Initial bathymetry

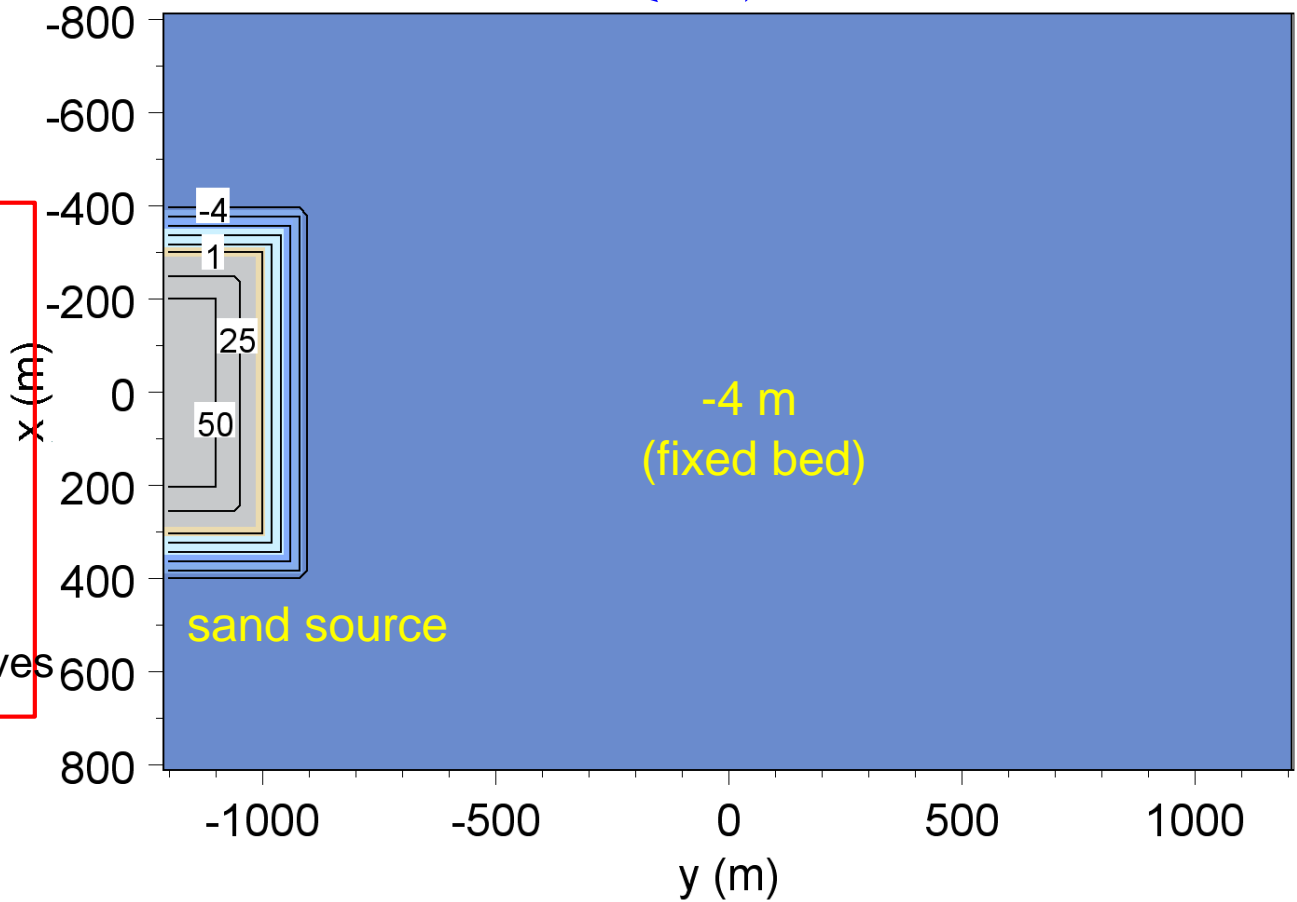
waves:  $H_i = 1$  m,  $T = 4$  s

**2 waves**

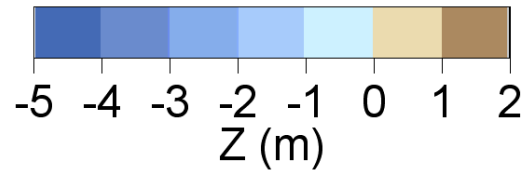
wave 1  $45^\circ$   
wave 2  $-30^\circ$

$P_1 = 0.85$       $P_2 = 0.15$

Ratio of duration of 2 waves



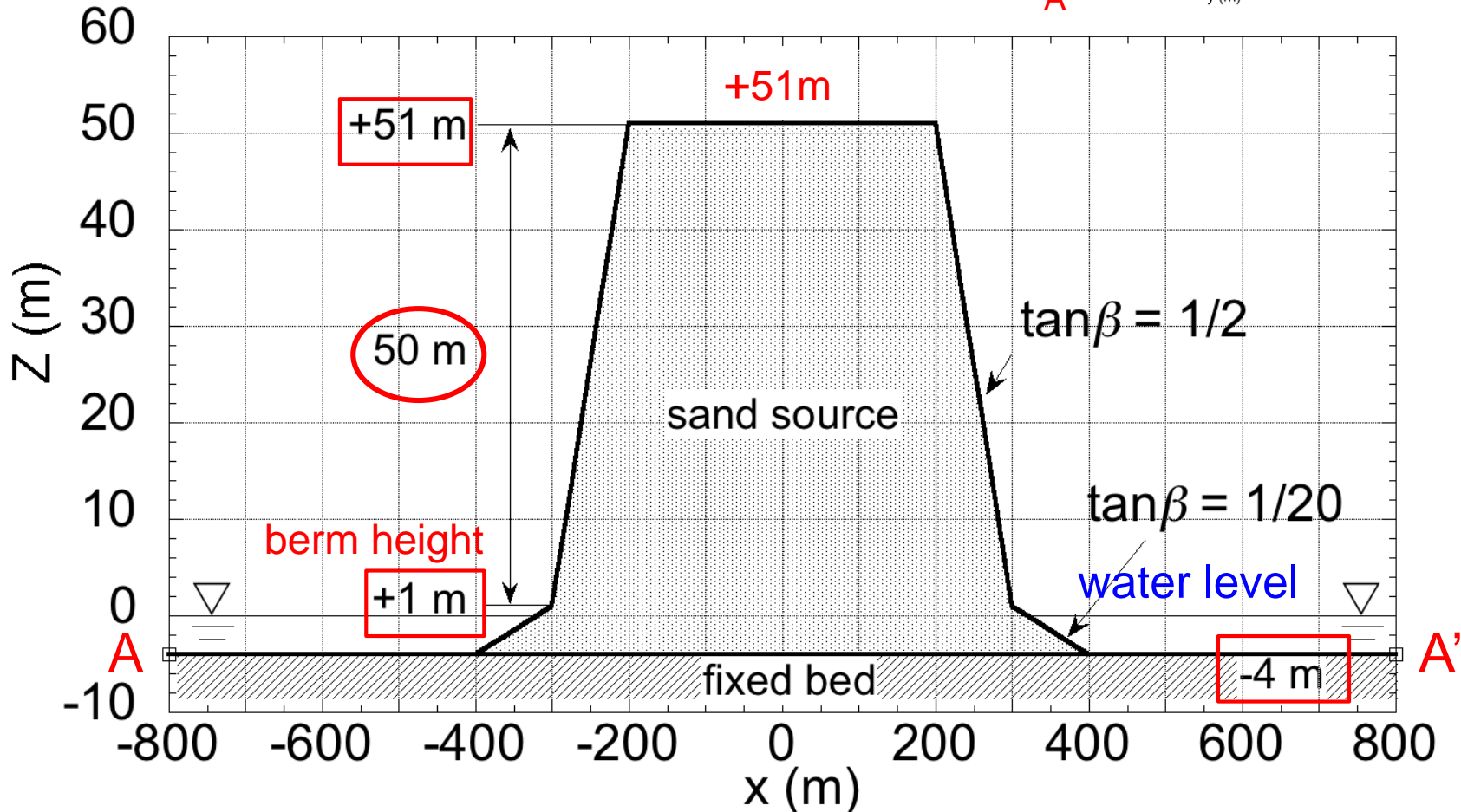
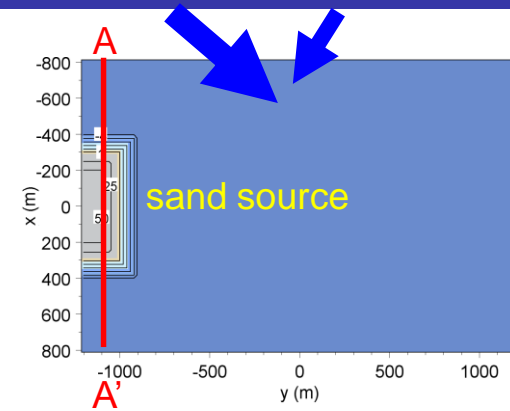
Depth of closure  $h_c = 4$  m  
Berm height  $h_R = 1$  m  
Equilibrium slope:  $\tan\beta_c = 1/20$





# Cross section of sand source.

The reason for setting the high tower as the sand source was that a sufficient amount of sand for the elongation of the sand bar could be supplied.



# Case 1: wave direction $\theta_1 = +45^\circ$ , $\theta_2 = -30^\circ$

Incident wave condition

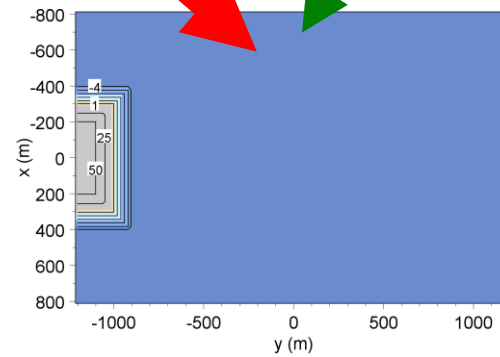
Time series of 2-wave action

wave direction 1 = +45 deg

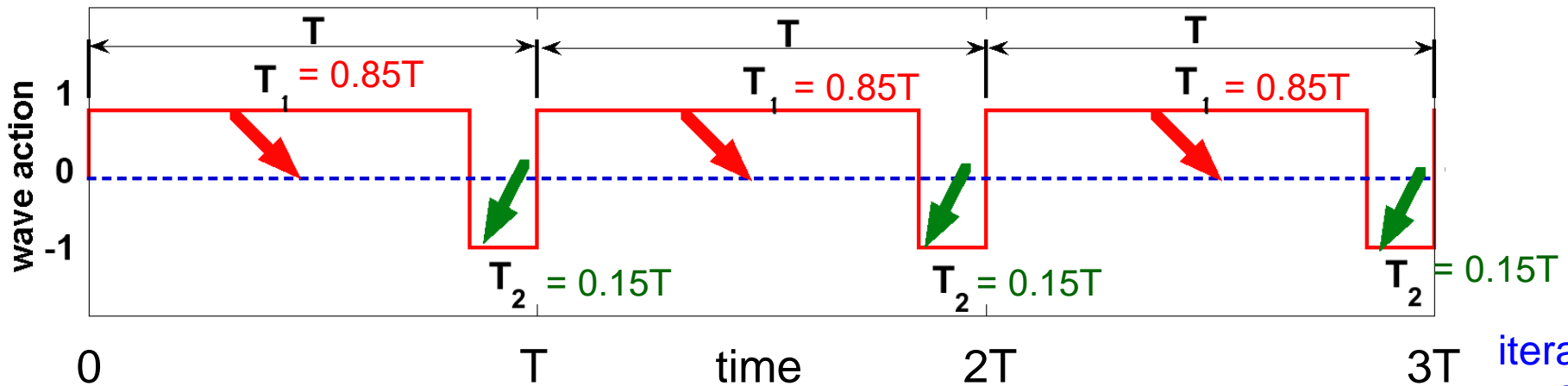
wave direction 2 = -30 deg

wave1

wave2



$H_i = 1$  m,  
 $T = 4$  s



One cycle of the two waves action  $T = 5000$ steps (2500hr)

Waves are incident periodically from two directions with a duration ratio of 0.85: 0.15.

Ratio of duration of 2 waves

$$P_1 = T_1/T = 0.85$$

$$P_2 = T_2/T = 0.15$$



# Case 1: wave direction $\theta_1 = +45^\circ$ , $\theta_2 = -30^\circ$

## Initial bathymetry

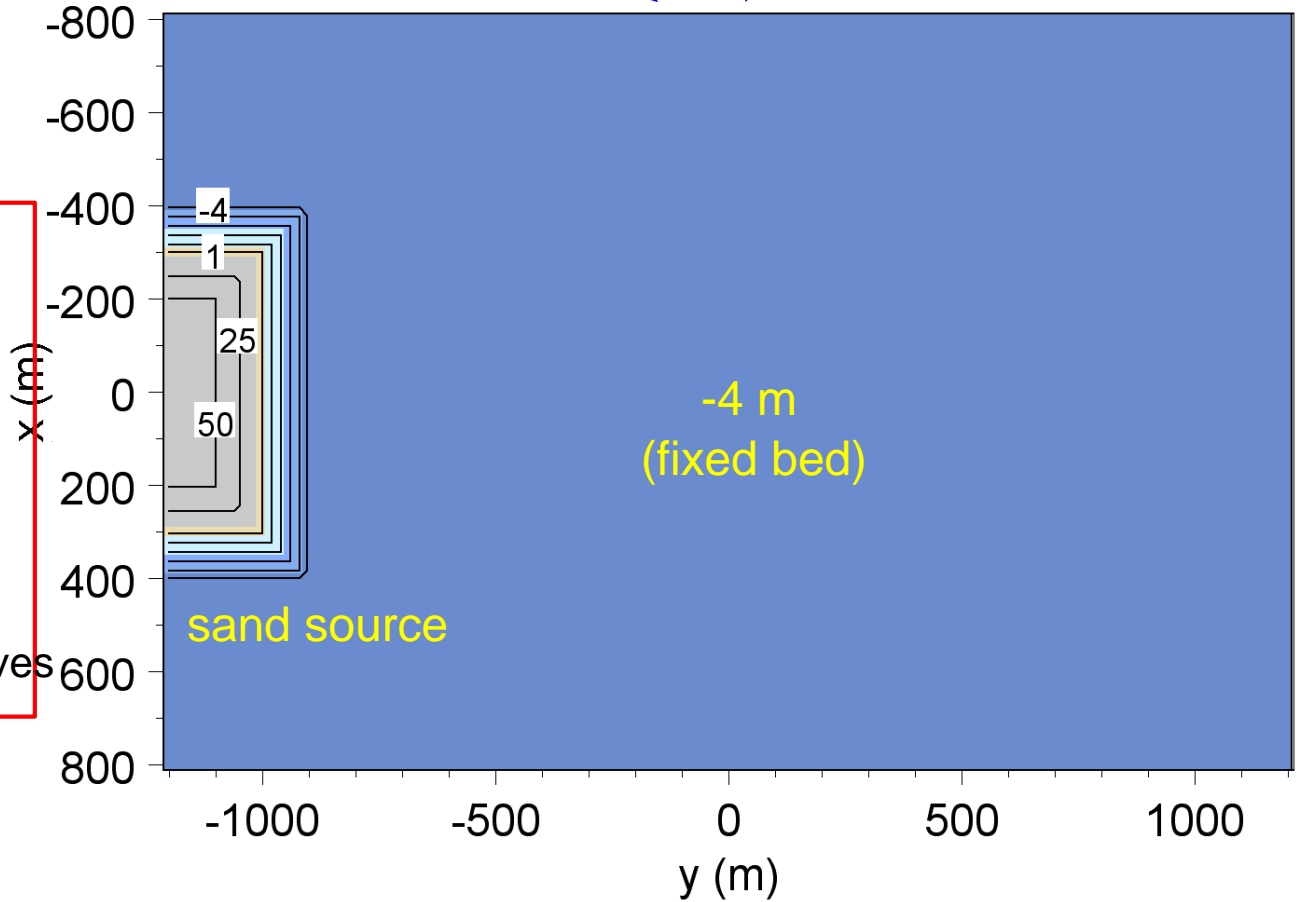
waves:  $H_i = 1$  m,  $T = 4$  s

**2 waves**

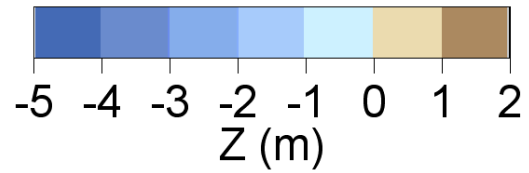
wave 1  $45^\circ$   
wave 2  $-30^\circ$

$P_1 = 0.85$       $P_2 = 0.15$

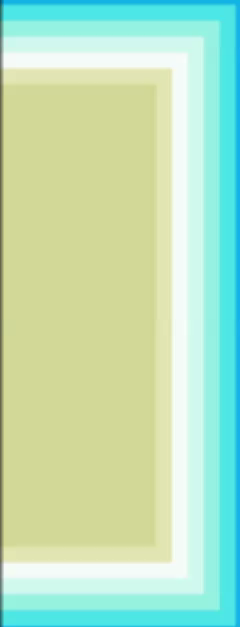
Ratio of duration of 2 waves



Depth of closure  $h_c = 4$  m  
Berm height  $h_R = 1$  m  
Equilibrium slope:  $\tan \beta_c = 1/20$

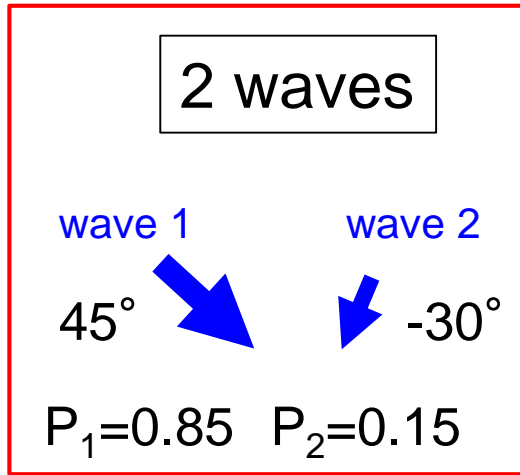


Case 1: wave direction  $\theta_1 = +45^\circ$ ,  $\theta_2 = -30^\circ$

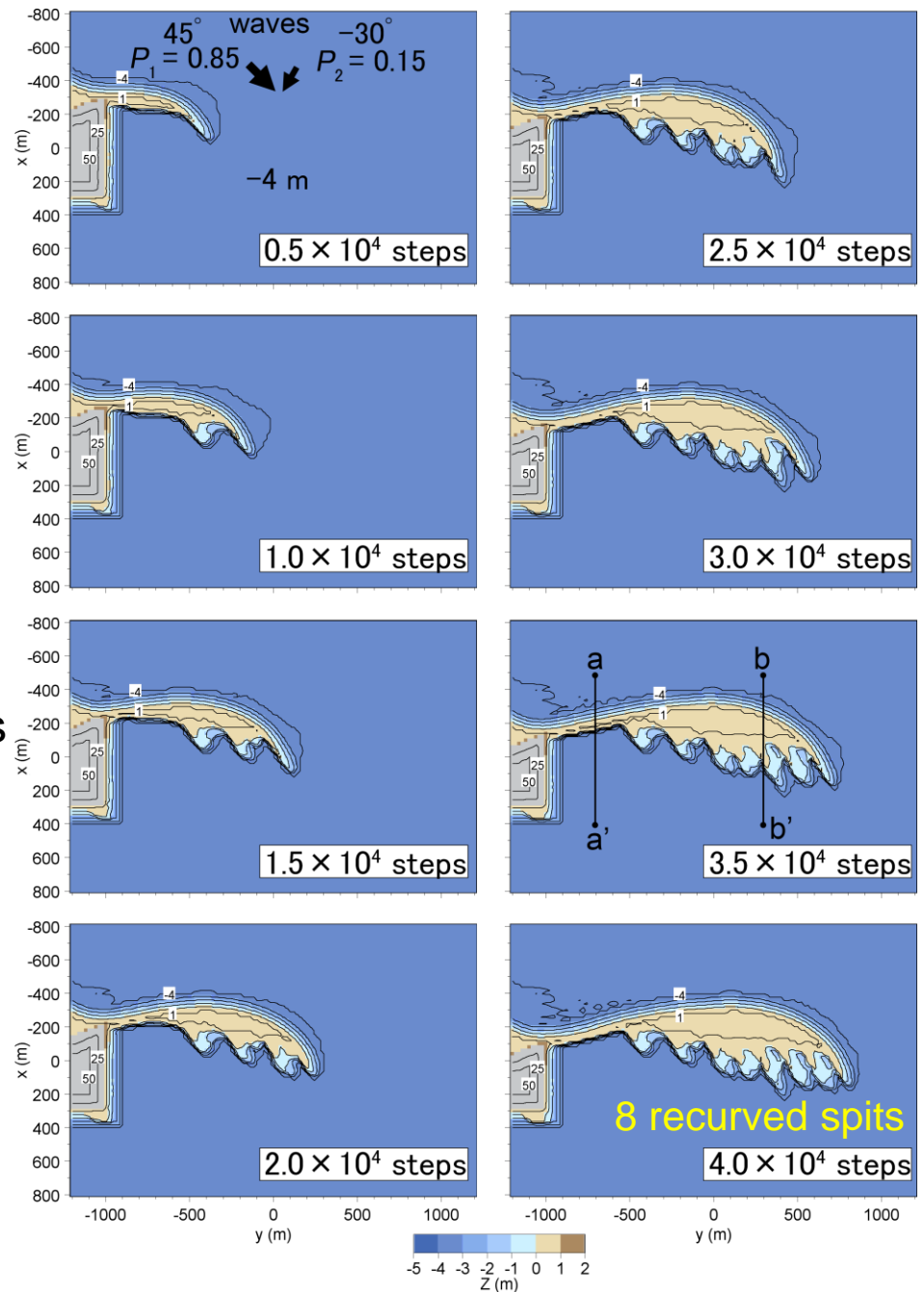




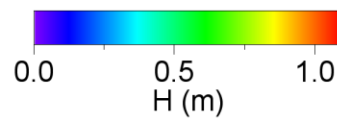
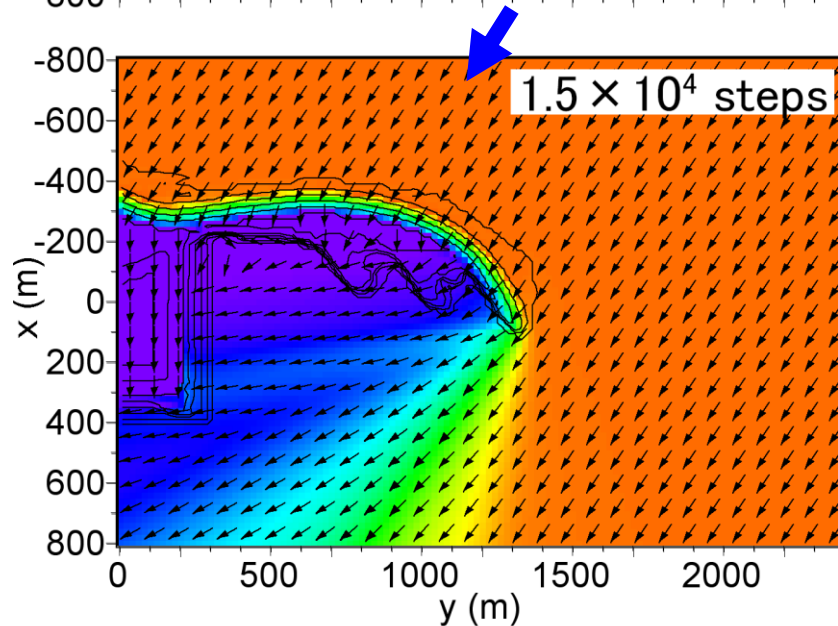
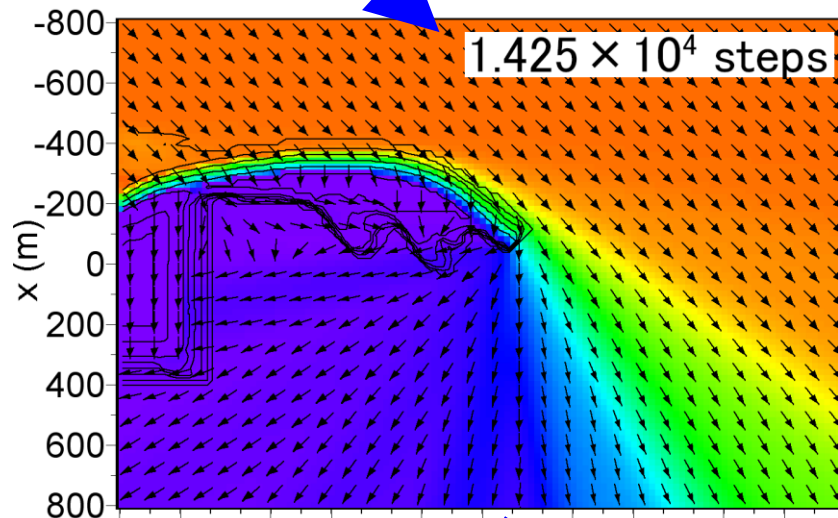
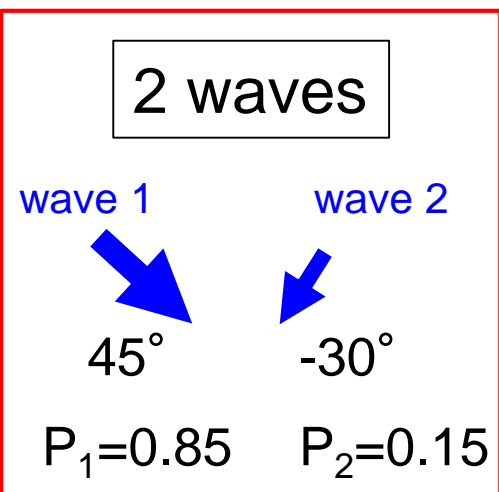
# Case 1: wave direction $\theta_1 = +45^\circ$ , $\theta_2 = -30^\circ$



The number of the recurved sand spits corresponds to the number that the wave direction was changed from  $45^\circ$  to  $-30^\circ$ .



# Case 1: wave direction $\theta_1 = +45^\circ$ , $\theta_2 = -30^\circ$



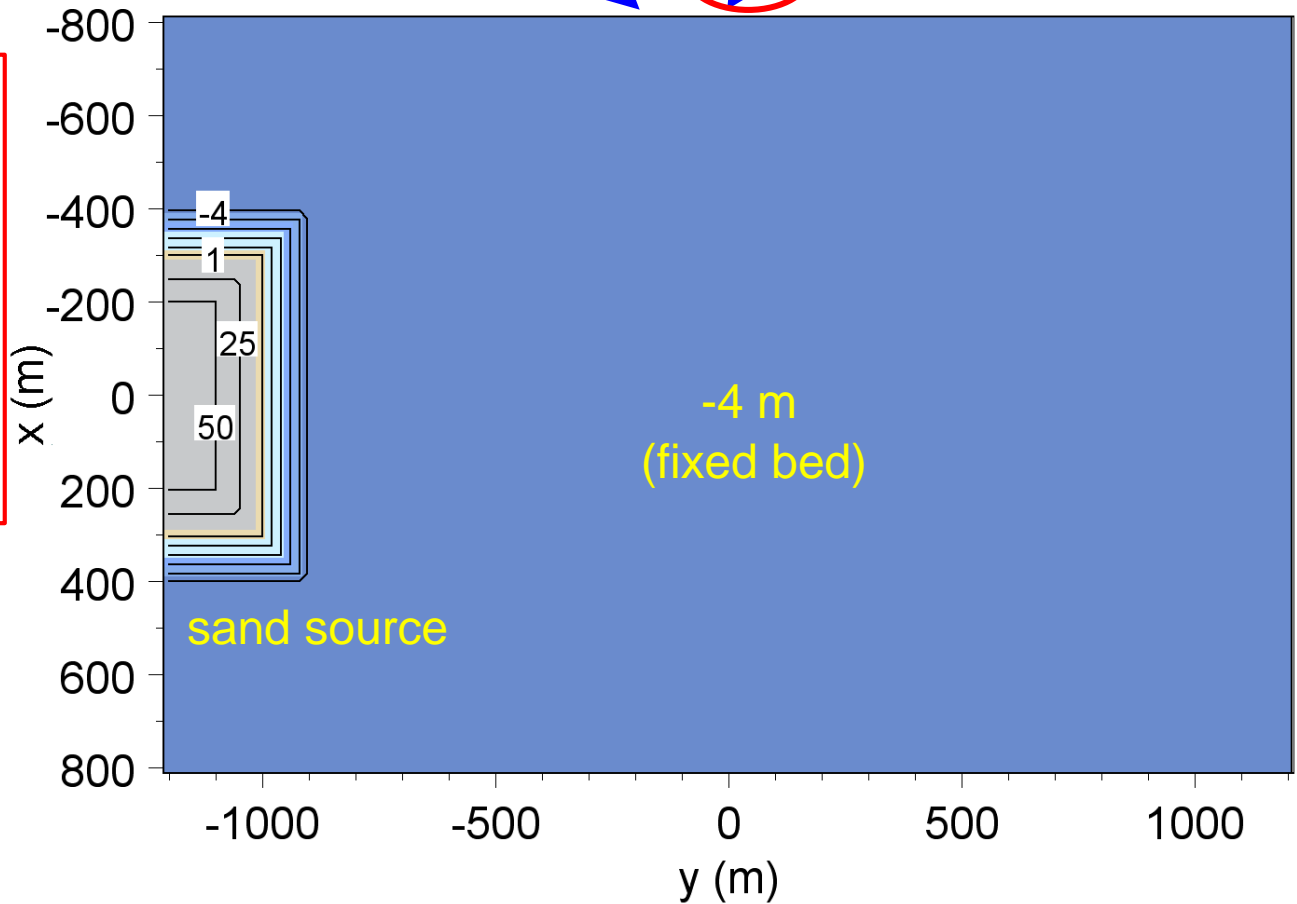
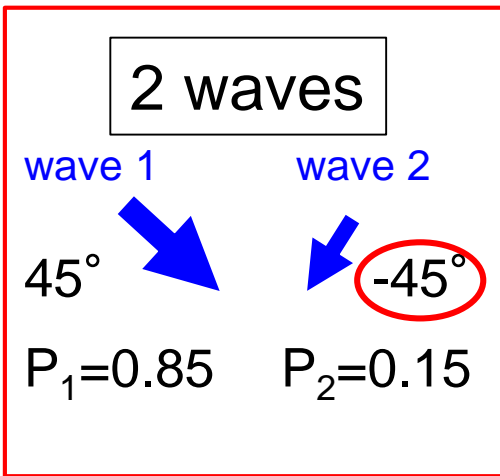
wave height  
and wave  
direction



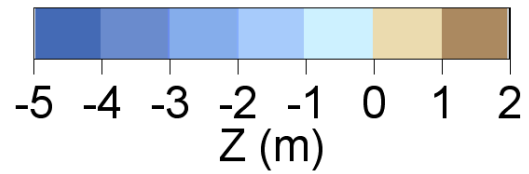
# Case 2: wave direction $\theta_1 = +45^\circ$ , $\theta_2 = -45^\circ$

## Initial bathymetry

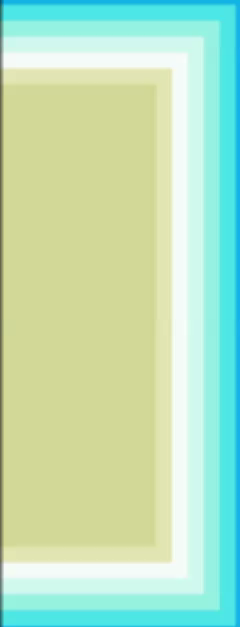
waves:  $H_i = 1$  m,  $T = 4$  s



Depth of closure  $h_c = 4$  m  
Berm height  $h_R = 1$  m  
Equilibrium slope:  $\tan \beta_c = 1/20$

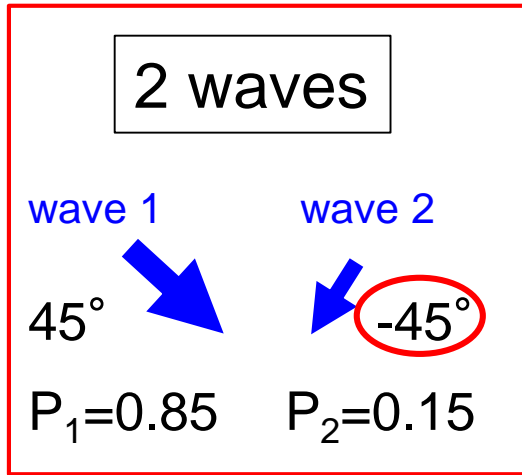


Case 2: wave direction  $\theta_1 = +45^\circ$ ,  $\theta_2 = -45^\circ$

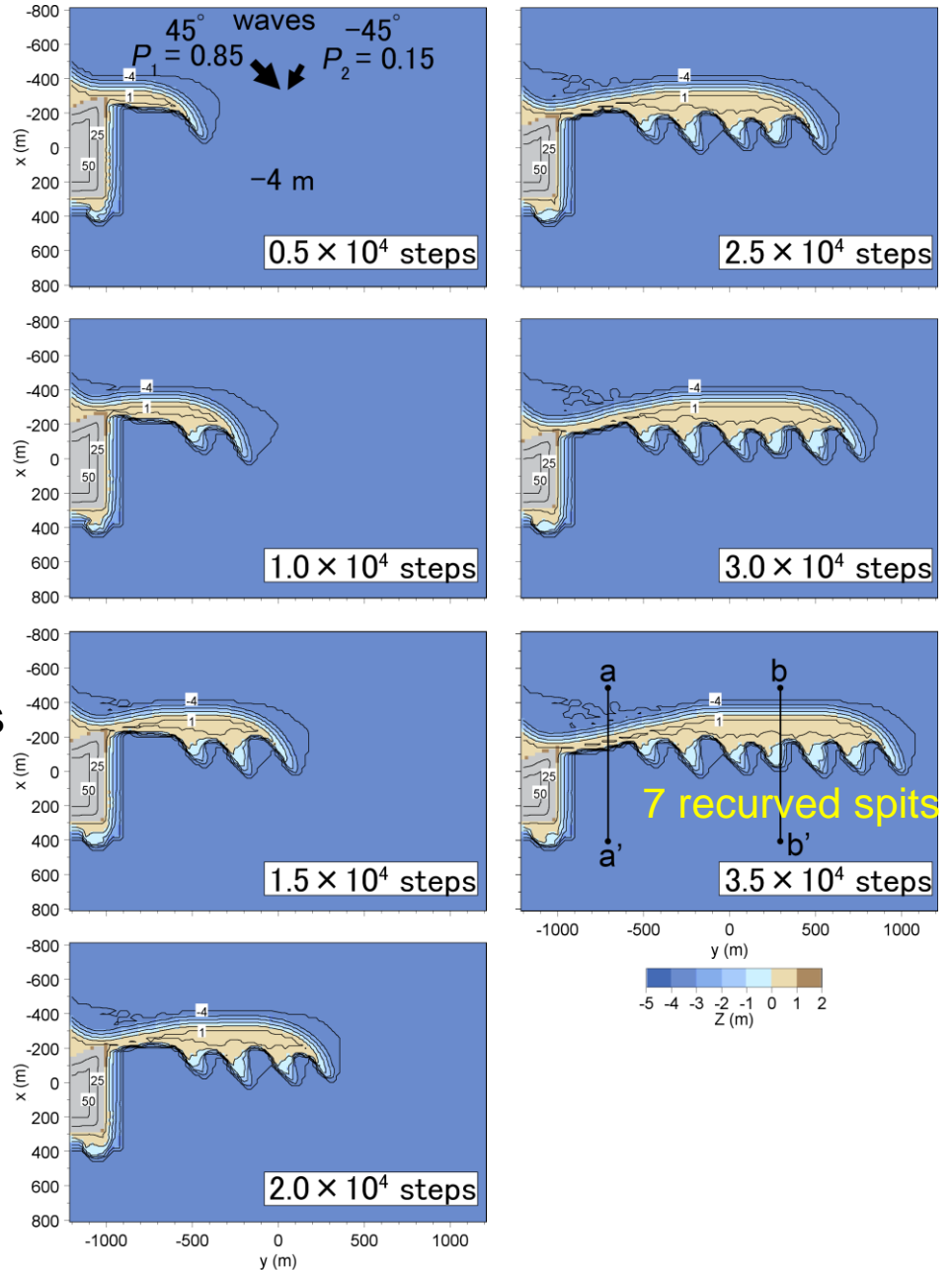




# Case 2: wave direction $\theta_1 = +45^\circ$ , $\theta_2 = -45^\circ$



The number of the recurved sand spits corresponds to the number that the wave direction was changed from  $45^\circ$  to  $-45^\circ$ .



# Case 2: wave direction $\theta_1 = +45^\circ$ , $\theta_2 = -45^\circ$

2 waves

wave 1

wave 2

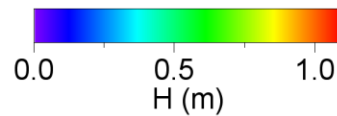
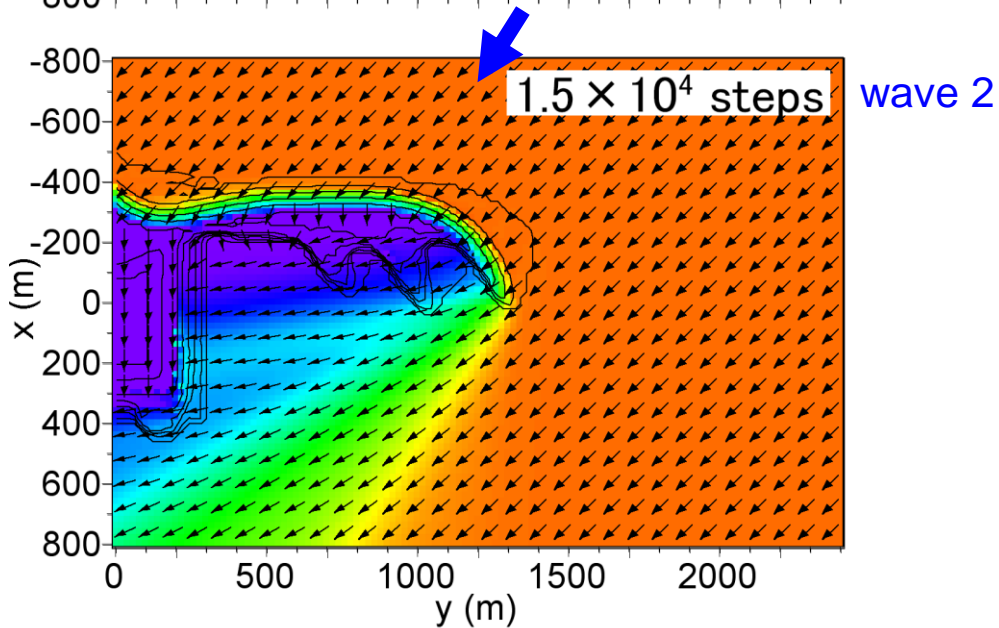
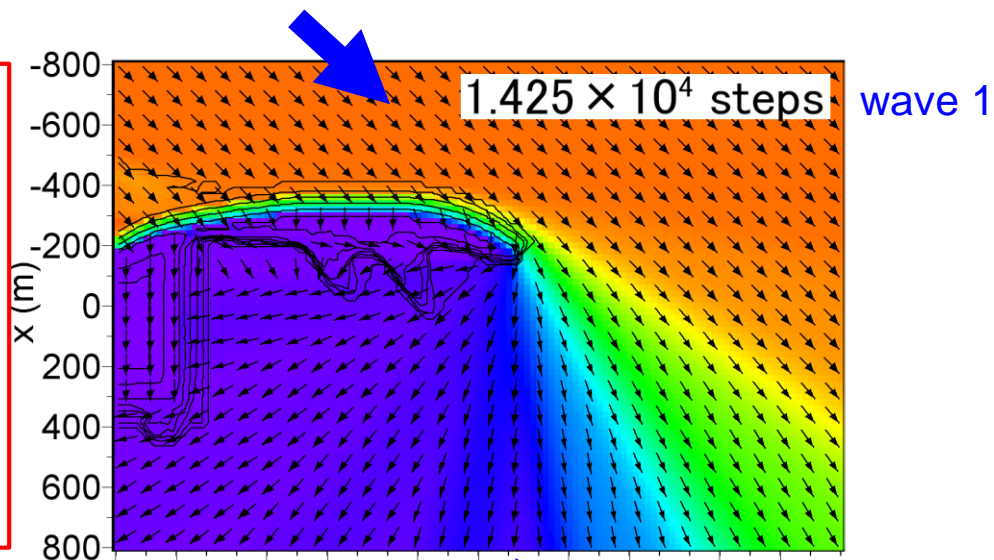
$45^\circ$

$-45^\circ$

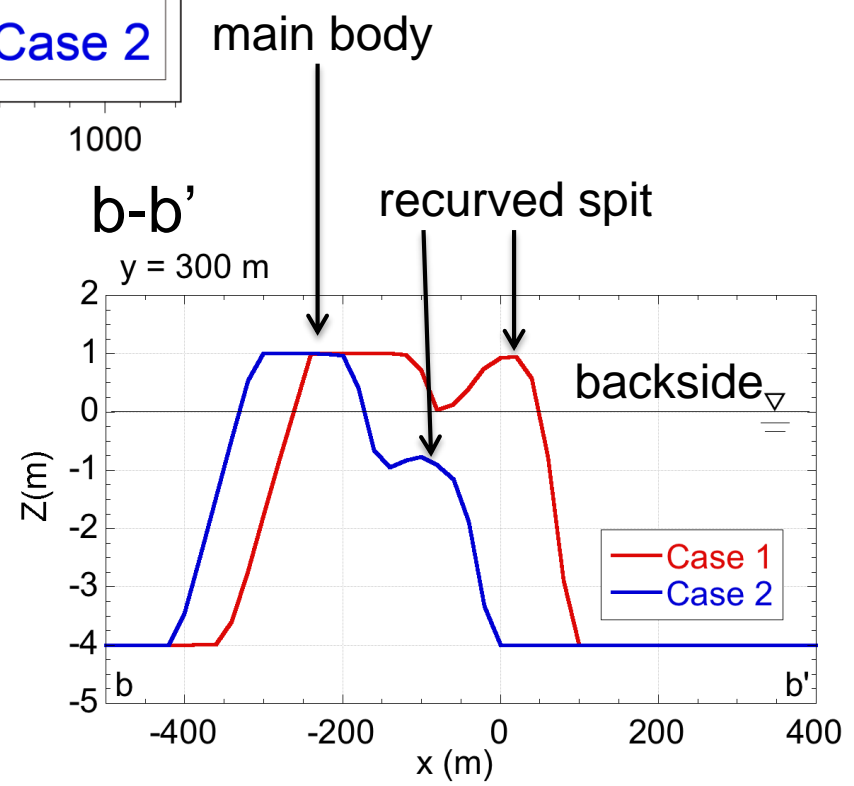
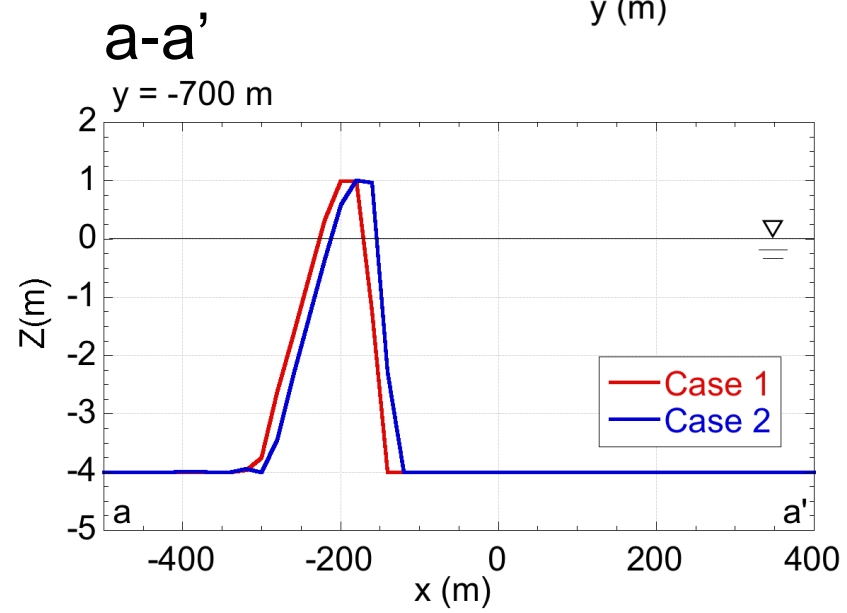
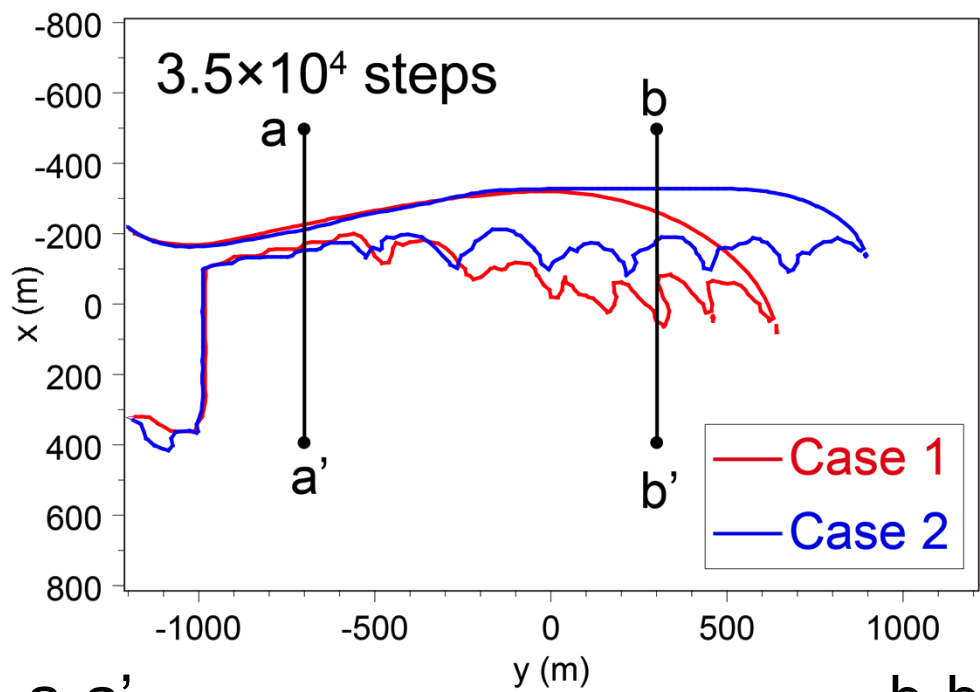
$P_1=0.85$

$P_2=0.15$

wave height  
and wave  
direction



# Shoreline configurations / Longitudinal profiles in Cases 1 and 2





# Comparison with site observation

Site observation



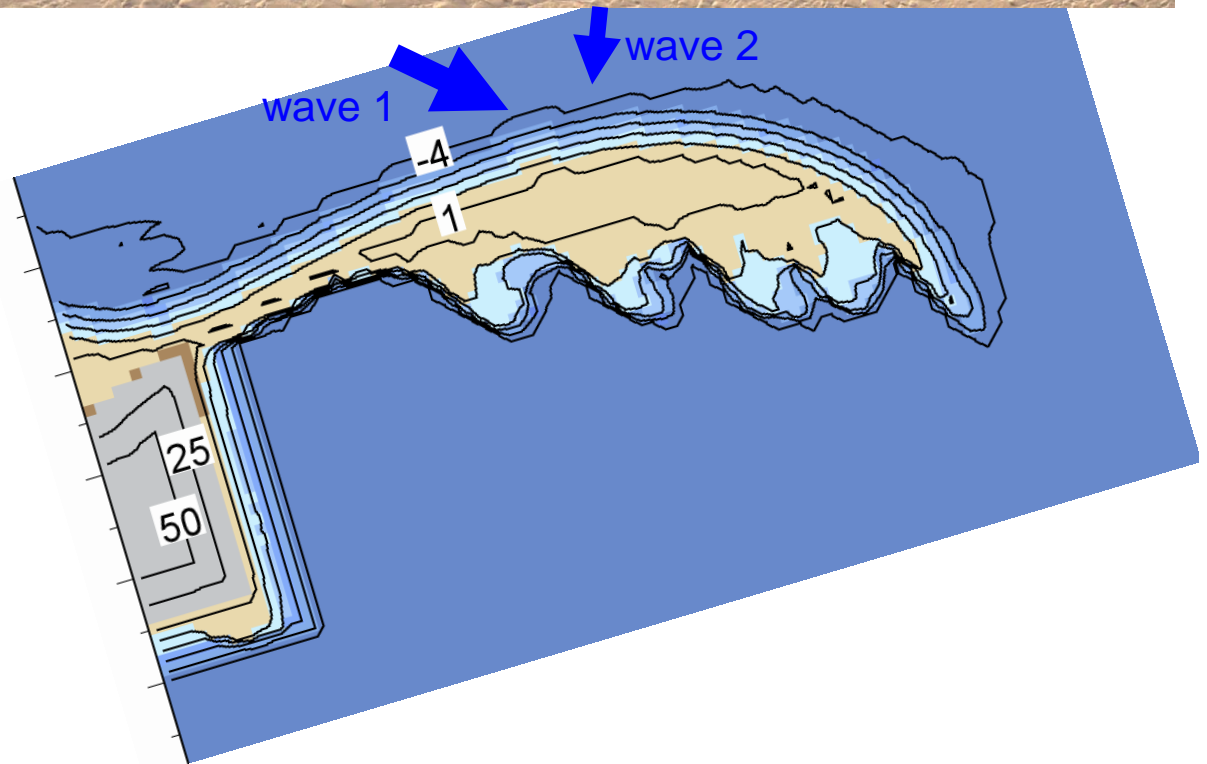
Calculation

Case 1

wave direction

$$\theta_1 = +45^\circ, \theta_2 = -30^\circ$$

$$P_1 = 0.85, P_2 = 0.15$$



# Comparison with site observation

Site observation

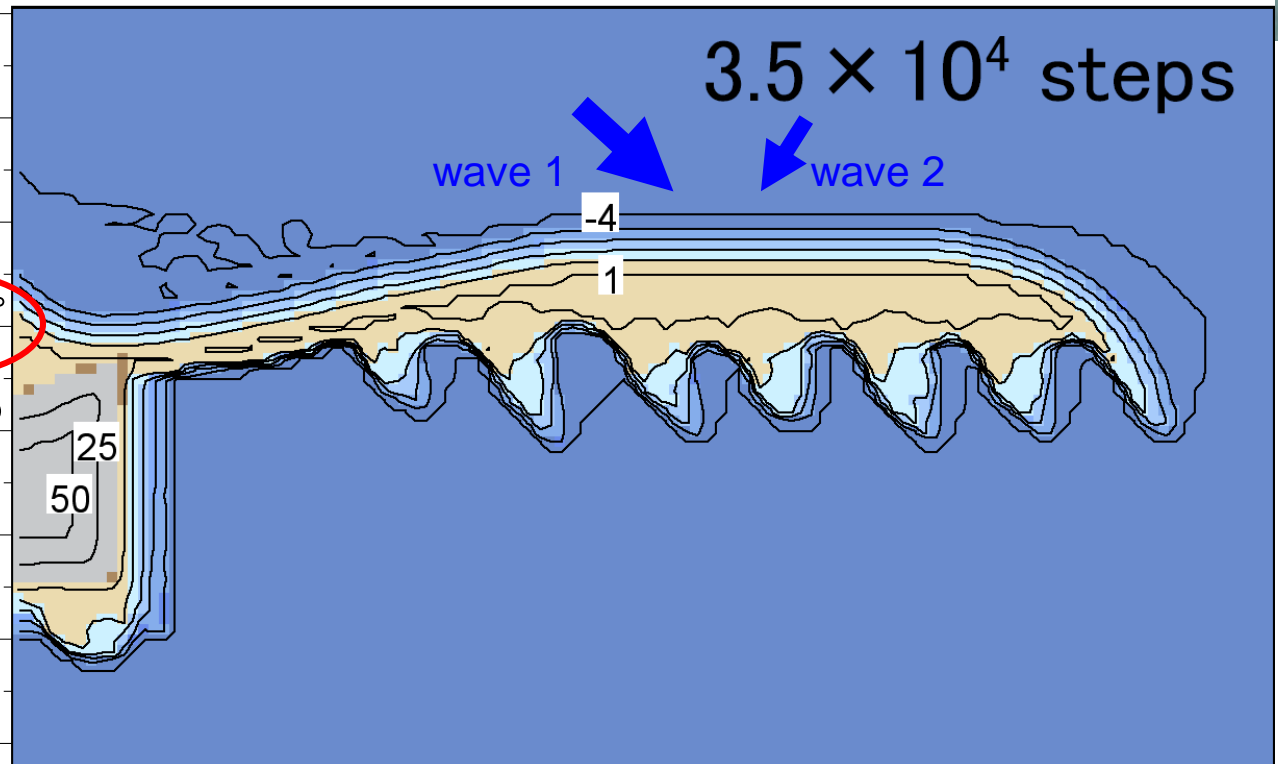


Calculation

Case 2

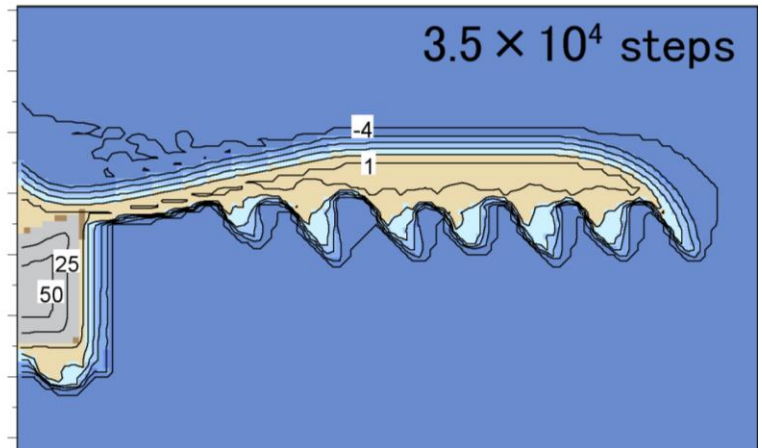
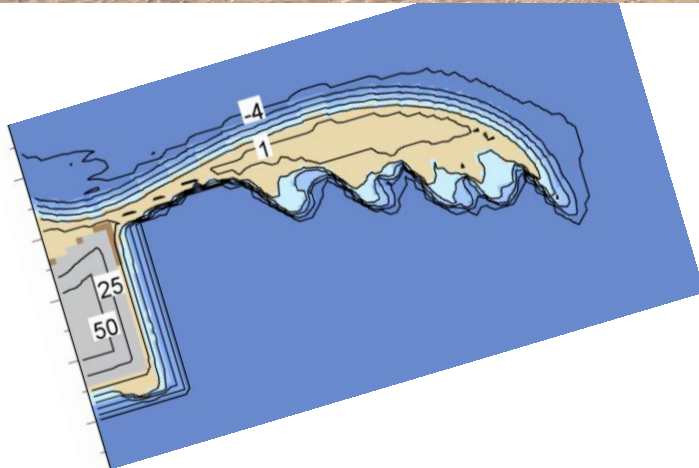
wave direction  
 $\theta_1 = +45^\circ$ ,  $\theta_2 = -45^\circ$

$P_1=0.85$ ,  $P_2=0.15$



# CONCLUSIONS

- In this study, the elongation of the recurved spits was predicted using the BG model.
- It was confirmed that when two sets of waves arrive from different directions, a recurved sand spit can be formed. The measured and calculated characteristics agreed well.

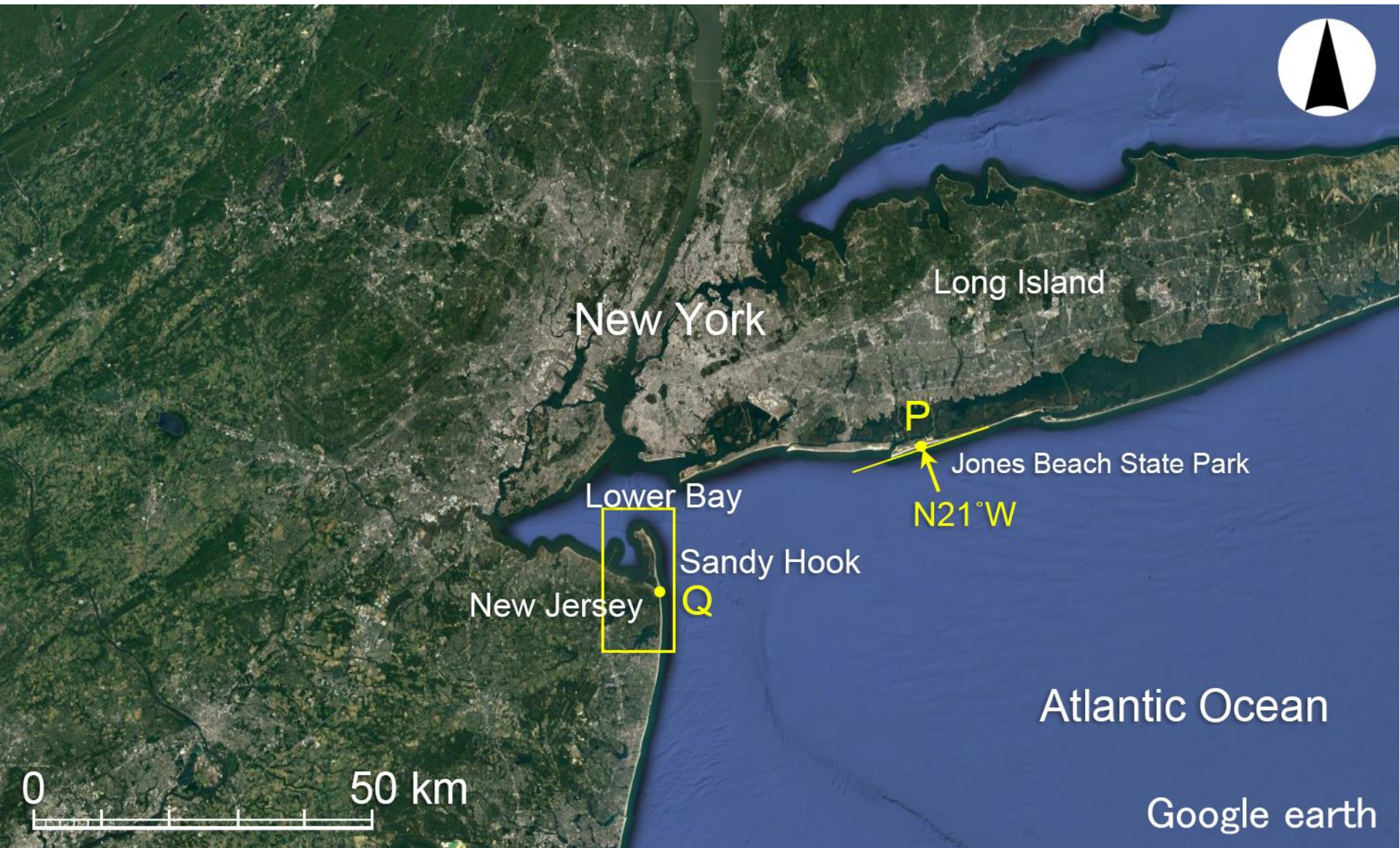




# Calculation conditions

Wave conditions	Incident waves: $H_I = 1$ m, $T = 4$ s, wave direction $\theta_I = 45^\circ$ and $-45^\circ$ (Case 1), $\theta_I = 45^\circ$ and $-30^\circ$ (Case 2) with a duration ratio of 0.85: 0.15 Duration of one cycle: $2.5 \times 10^3$ hr ( $5 \times 10^3$ steps)
Berm height	$h_R = 1$ m
Depth of closure	$h_c = 4$ m
Equilibrium slope	$\tan \beta_c = 1/20$
Coefficients of sand transport	Coefficient of longshore sand transport $K_s = 0.2$ Coefficient of Ozasa and Brampton (1980) term $K_2 = 1.62K_s$ Coefficient of cross-shore sand transport $K_n = K_s$
Mesh size	$\Delta x = \Delta y = 20$ m
Time intervals	$\Delta t = 0.5$ hr
Duration of calculation	$2 \times 10^4$ hr ( $4 \times 10^4$ steps)
Boundary conditions	Shoreward and landward ends: $q_x = 0$ , right and left boundaries: $q_y = 0$
Calculation of wave field	Energy balance equation (Mase, 2001) <ul style="list-style-type: none"> <li>• Term of wave dissipation due to wave breaking: Dally et al. (1984) model</li> <li>• Wave spectrum of incident waves: directional wave spectrum density obtained by Goda (1985)</li> <li>• Total number of frequency components <math>N_F = 1</math> and number of directional subdivisions <math>N_\theta = 8</math></li> <li>• Directional spreading parameter <math>S_{\max} = 25</math></li> <li>• Coefficient of wave breaking <math>K = 0.17</math> and <math>\Gamma = 0.3</math></li> <li>• Imaginary depth between minimum depth <math>h_0</math> (0.5 m) and berm height <math>h_R</math></li> <li>• Wave energy = 0 where <math>Z \geq h_R</math></li> <li>• Lower limit of h in terms of wave decay due to breaking: 0.5 m</li> </ul>

# Location of Sandy Hook on US East Coast

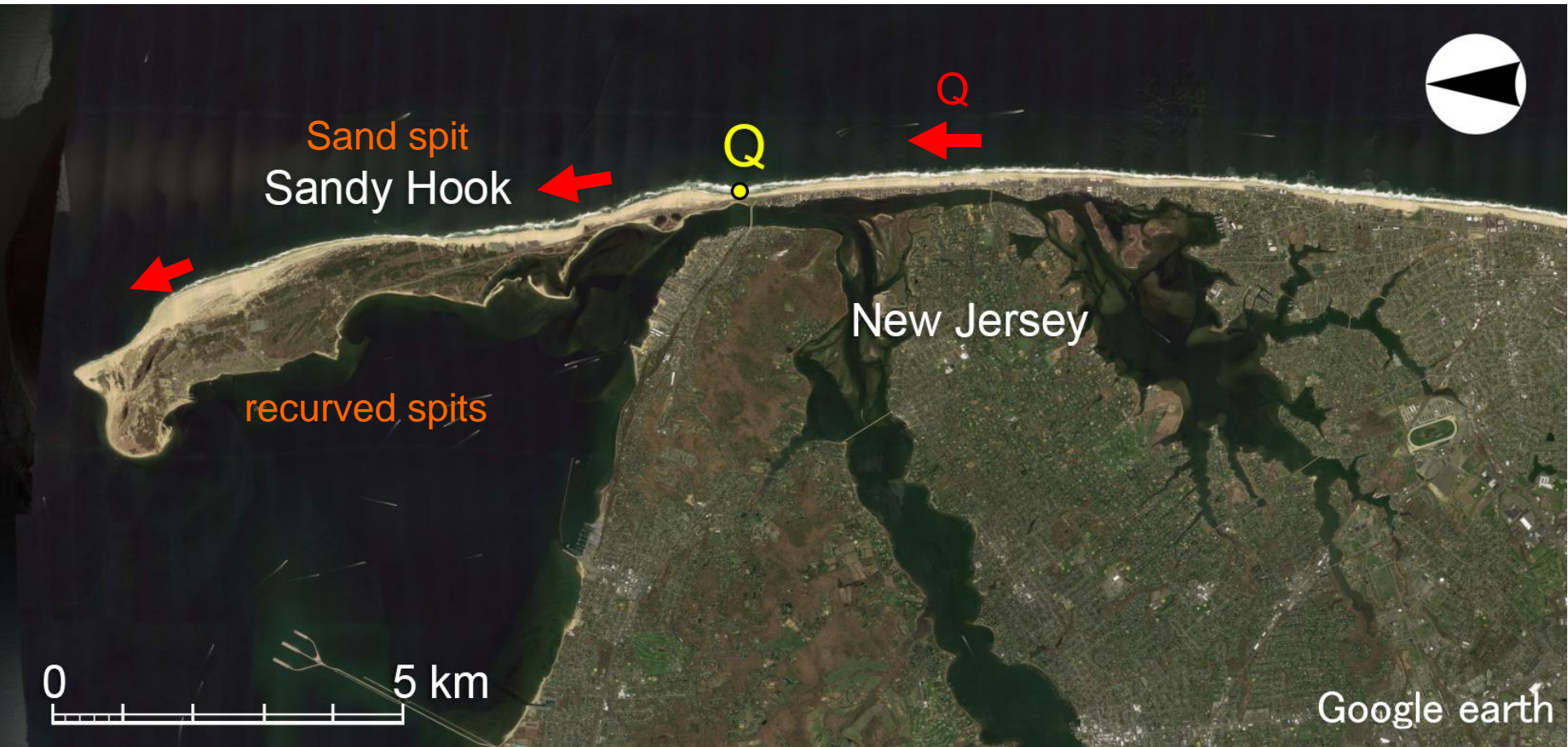
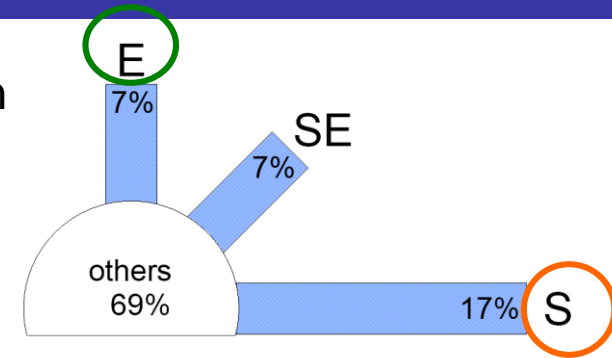




# Sandy Hook

Probability of occurrence of wave direction  
British Maritime Technology Ltd. (1986)

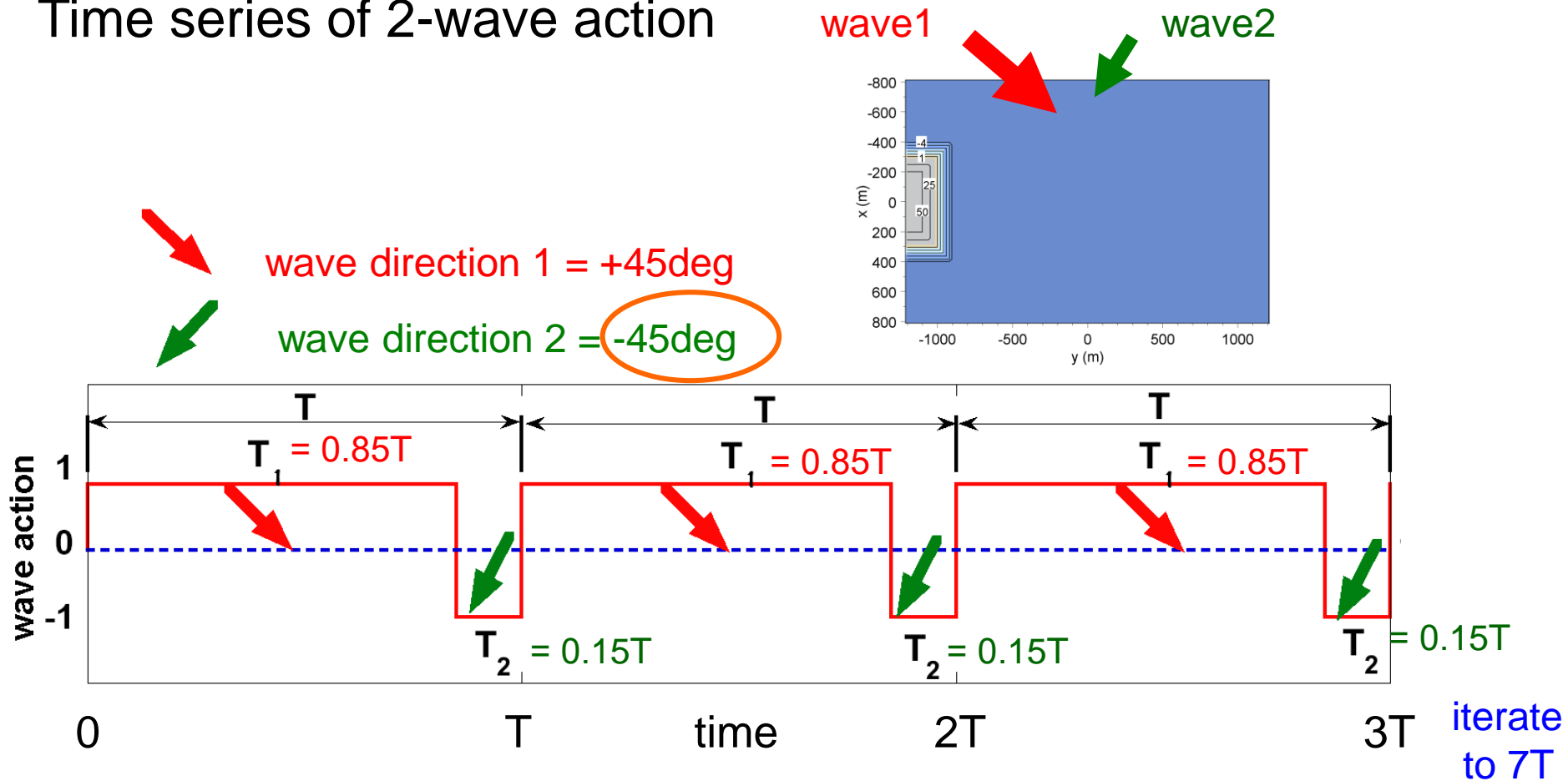
**Q** ← longshore sand transport





# Case 2: wave direction $\theta_1 = +45^\circ$ , $\theta_2 = -45^\circ$

## Time series of 2-wave action



Waves are incident periodically from two directions with a duration ratio of 0.85: 0.15

Ratio of duration of 2 waves

$$P_1 = T_1/T = 0.85$$

$$P_2 = T_2/T = 0.15$$