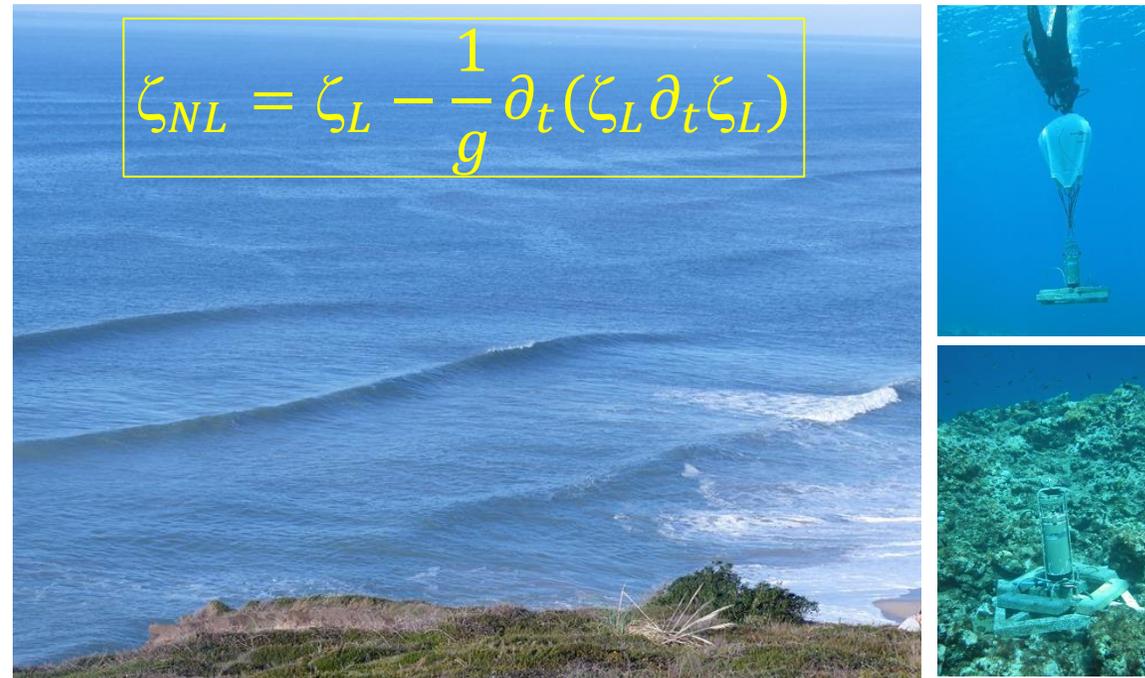


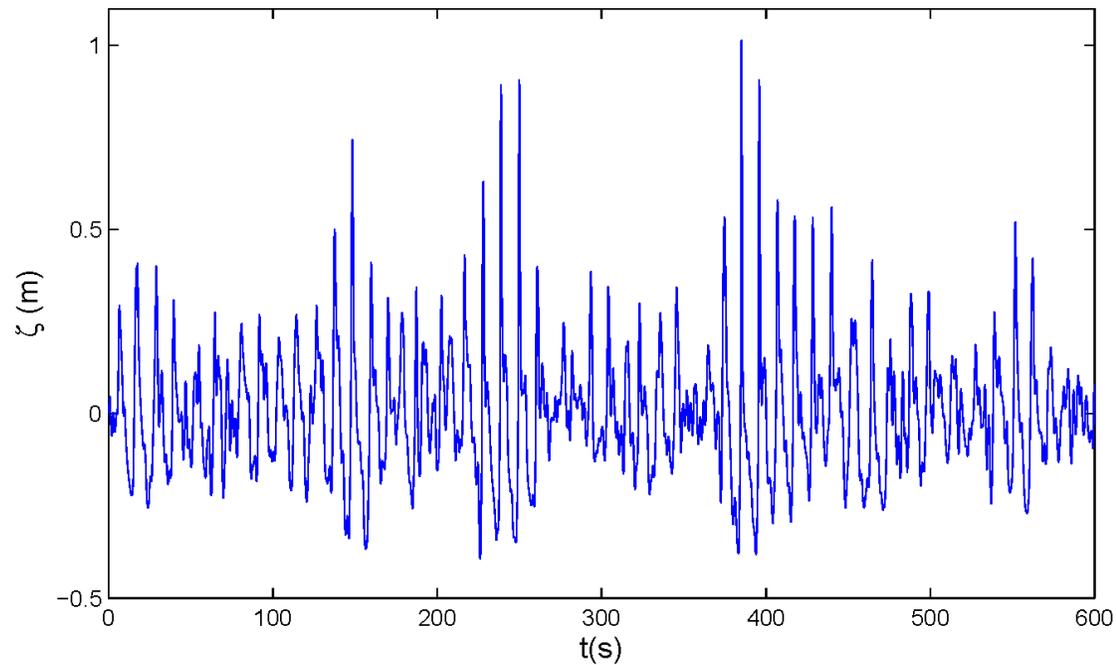
A simple and accurate nonlinear method for recovering the surface wave elevation from pressure measurements

Bonneton P.¹, Mouragues A.¹, Lannes D.¹, Martins K.², Michallet H.³

¹Bordeaux Univ., ²Bath Univ., ³Grenoble Univ.



Accurate measurements of surface wave elevation ζ are crucial for many coastal applications:

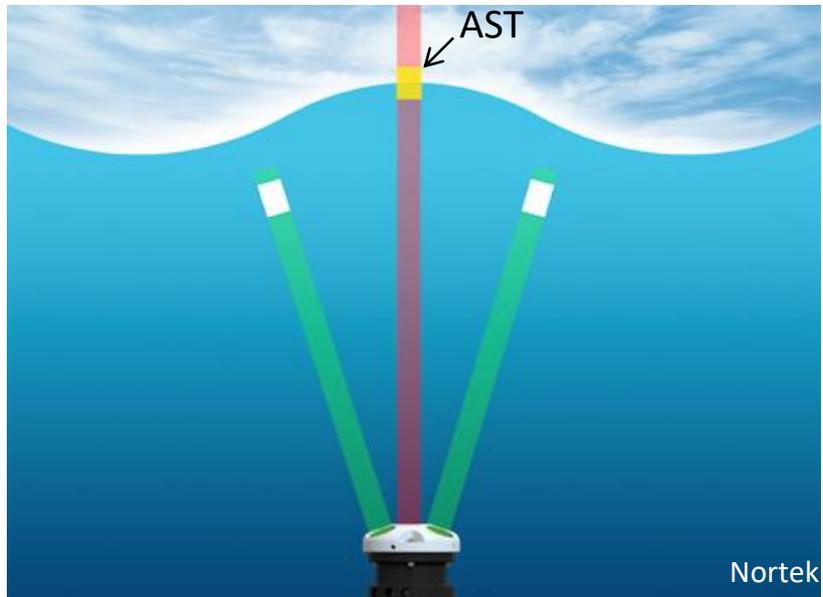


- wave overtopping and submersion
- navigation and platform safety
- wave-induced sediment transport

Direct measurement of the surface elevation

→ highly accurate measurement

Acoustic surface tracking



Lidar scanning

*Martins et al 2018*

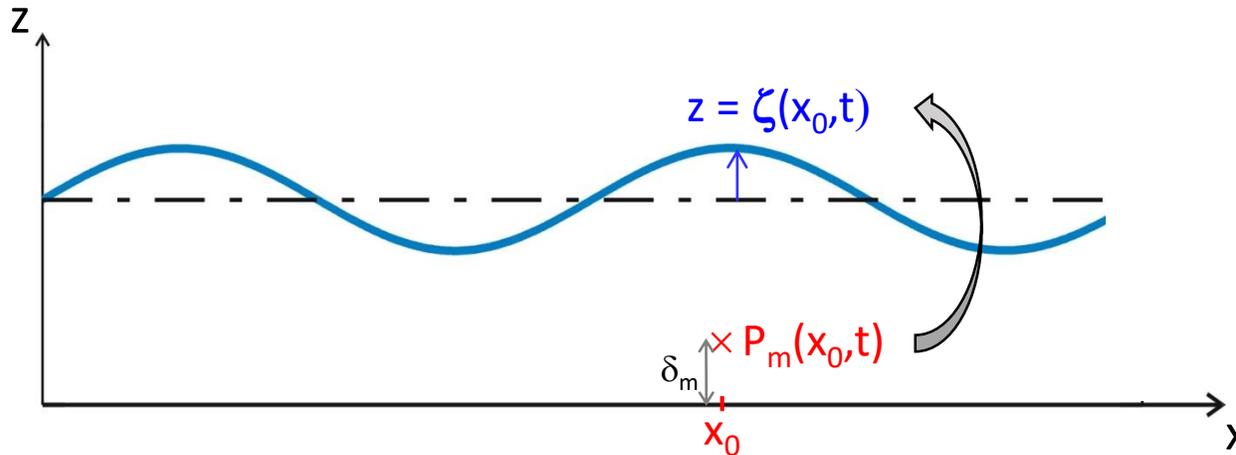
- ✓ expensive
- ✓ difficult to deploy and fragile
- ✓ AST sensitive to air bubbles and turbidity
- ✓ lidar requires the presence of nearshore structures

Pressure sensors are still a very useful tool for measuring waves

Pressure sensor



Ocean Sensor Systems



- ✓ cheap
- ✓ easy to deploy
- ✓ robust (storms, bottom trawling, ...)
- ✓ not sensitive to air bubbles and turbidity

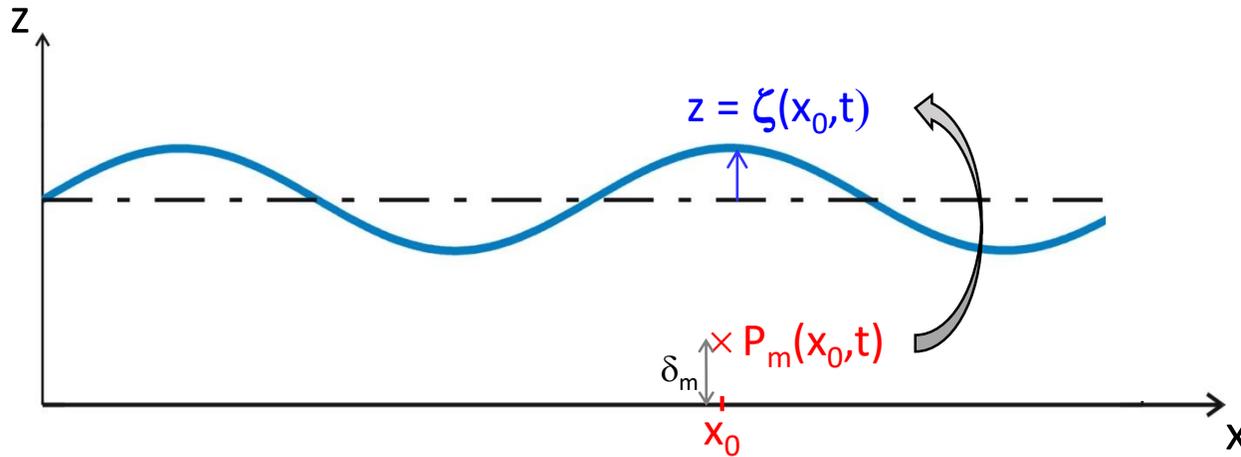
not a direct measurement of ζ → methods for recovering ζ

Commonly used reconstruction methods and their limitations

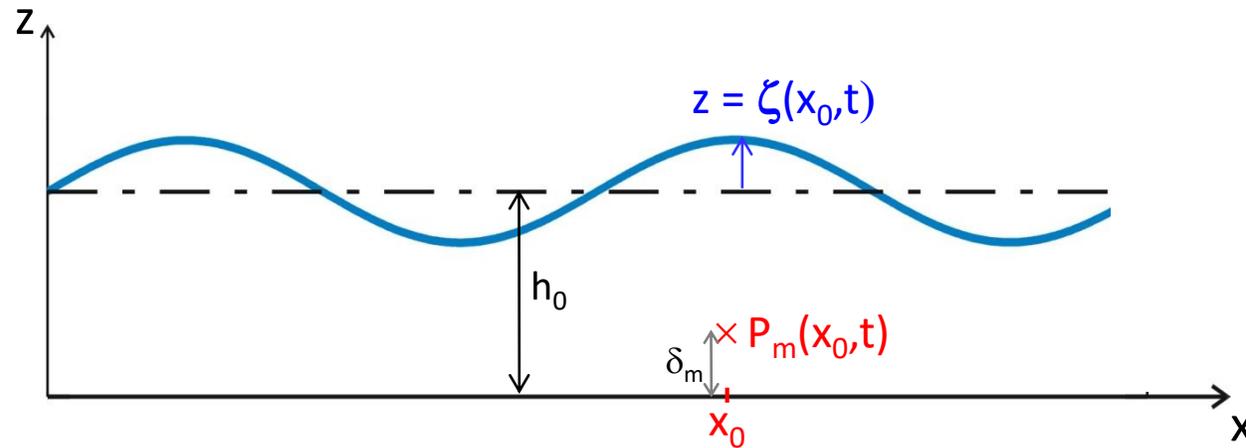
Pressure sensor



Ocean Sensor Systems



long waves: tsunamis, tides, ... → hydrostatic reconstruction



$$\frac{\partial P}{\partial z} = -\rho_0 g \Rightarrow h_H(x_0, t) = \frac{P_m - P_a}{\rho_0 g} + \delta_m$$

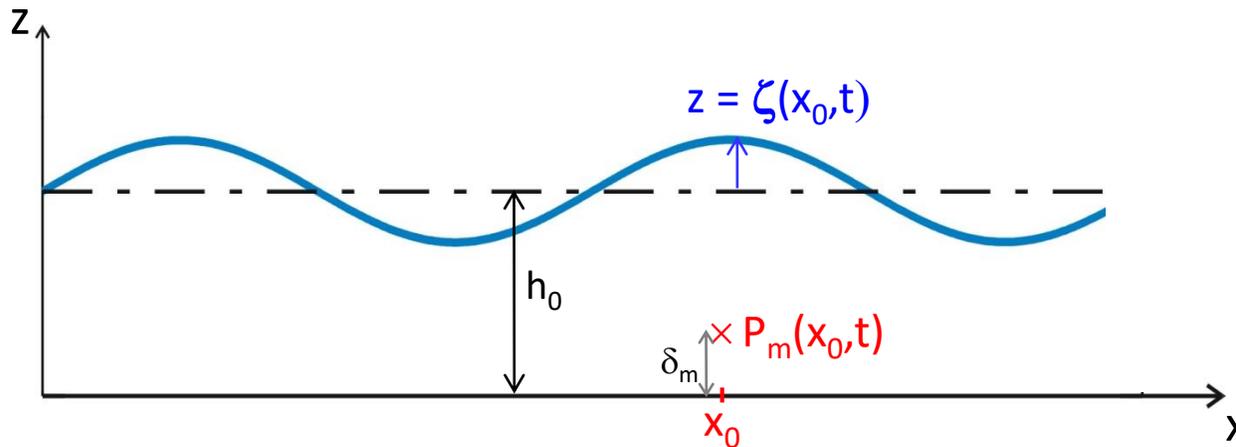
$$\zeta_H = \frac{P_m - P_a}{\rho_0 g} + \delta_m - h_0$$

swell, wind-generated waves → non-hydrostatic reconstruction



swell, wind-generated waves → non-hydrostatic reconstruction

recover the wave field by means of a transfer function based on linear theory “*transfer function method*”



pressure response factor

$$\tilde{\zeta}_L(\omega) = \frac{\cosh(h_0|k|)}{\cosh(\delta_m|k|)} \tilde{\zeta}_H(\omega)$$

$$\zeta_H = \frac{P_m - P_a}{\rho_0 g} + \delta_m - h_0$$

$$\omega^2 = g|k| \tanh(h_0|k(\omega)|)$$

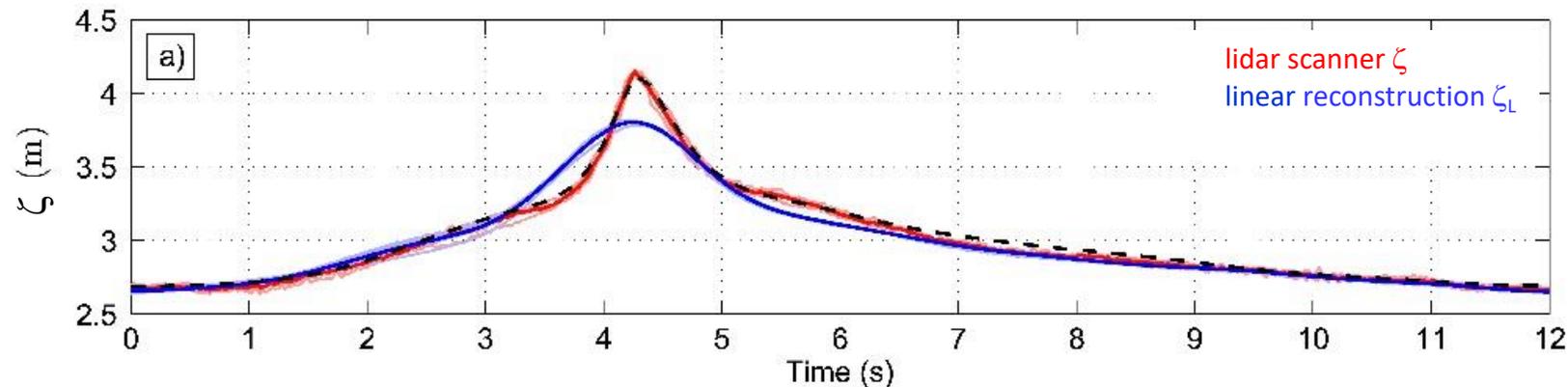
swell, wind-generated waves → non-hydrostatic reconstruction

“transfer function method”

- gives reasonable estimates for bulk wave parameters such as H_s

Guza and Thornton 1980, Bishop and Donelan 1987, Tsai et al. 2005, ...

- fails to describe the shape of nonlinear nearshore waves



Martins et al. 2017, BARDEX II, $h_0=1.17$ m, $T_p=12.1$ s

→ provides a poor description of the peaky and skewed shape of nonlinear waves

→ underestimates the individual wave height by up to 30 %

There is a critical need for nonlinear reconstruction methods

❑ *Constantin 2012, Deconinck et al. 2012, Olivears et al. 2012, Clamond, 2013*

→ periodic waves of permanent form

❑ *Oliveras et al. 2012*

→ heuristic approximation for irregular waves

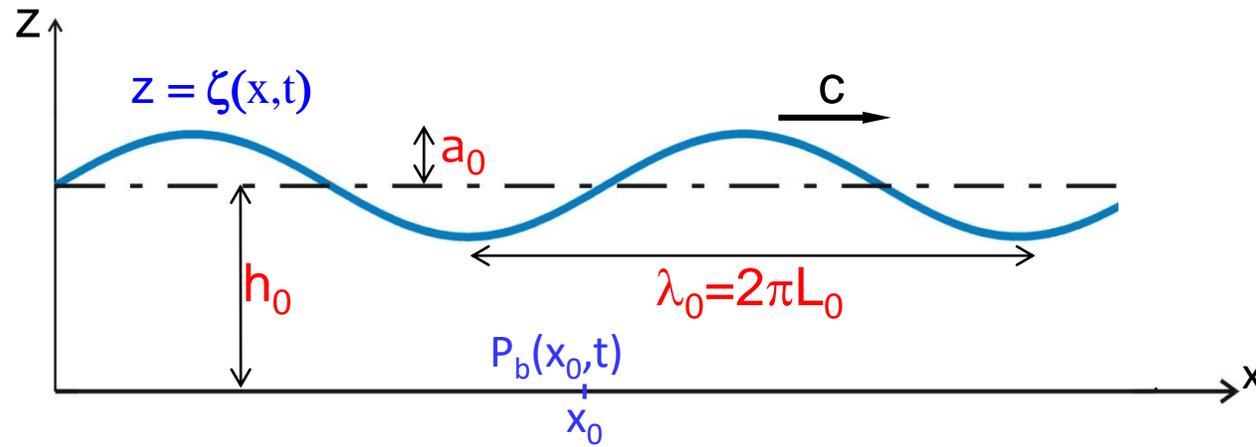
Two nonlinear reconstruction methods for irregular waves in the field

□ *Bonneton, Lannes, Martins, Michallet, Coastal Eng. 2018*

→ weakly dispersive

□ *Bonneton and Lannes, JFM 2017*

→ fully dispersive



$$\varepsilon = \frac{a_0}{h_0} = 0(1) \quad \mu = \left(\frac{h_0}{L_0}\right)^2 \lesssim 1$$

$$\sigma = \frac{a_0}{L_0} = \varepsilon\sqrt{\mu} \ll 1$$

Asymptotic expansion of the Euler equations in terms of $\sigma \rightarrow \phi = \phi_0 + \sigma\phi_1 + O(\sigma^2)$

$$\zeta_{NL} = \zeta_L - \sqrt{\mu}\sigma \partial_t(\zeta_L \partial_t \zeta_L)$$

Bonneton and Lannes (JFM 2017)

$$\hat{\zeta}_L(k) = \cosh(\sqrt{\mu}|k|) \hat{\zeta}_H(k)$$

In variables with dimension

$$\zeta_{NL} = \zeta_L - \frac{1}{g} \partial_t(\zeta_L \partial_t \zeta_L)$$

$$\hat{\zeta}_L(k) = \cosh(h_0|k|) \hat{\zeta}_H(k)$$

Waves in shallow water ($\mu \ll 1$)

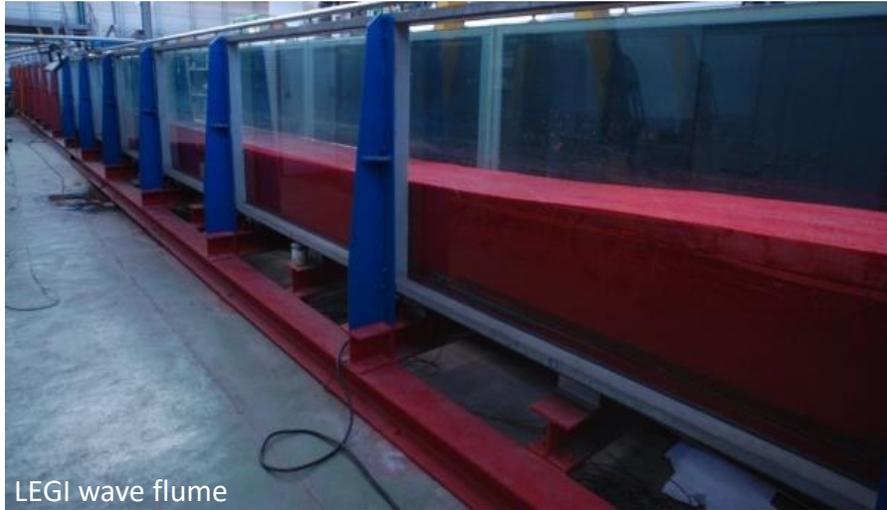
Asymptotic expansion of the Euler equations in terms of $\mu \rightarrow \phi = \phi_0 + \mu\phi_1 + O(\mu^2)$

$$\zeta_{SNL} = \zeta_{SL} - \frac{1}{g} \partial_t (\zeta_{SL} \partial_t \zeta_{SL})$$

Bonneton et al., Coastal Eng. 2018

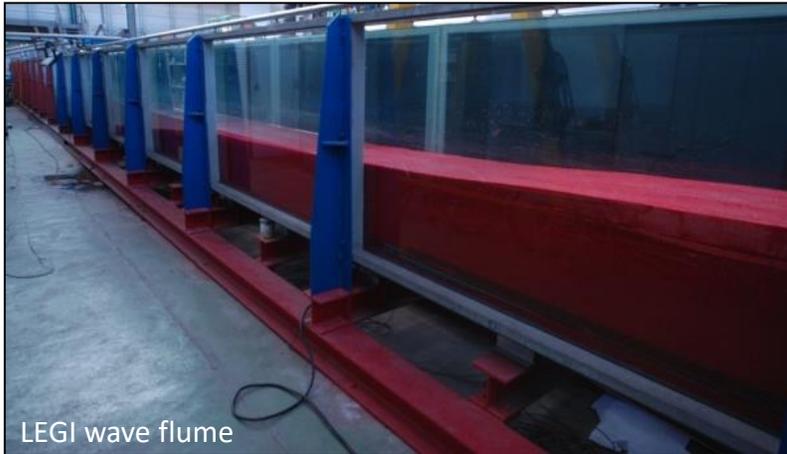
$$\zeta_{SL} = \zeta_H - \frac{h_0}{2g} \partial_t^2 \zeta_H$$

Laboratory experiments



Field measurements

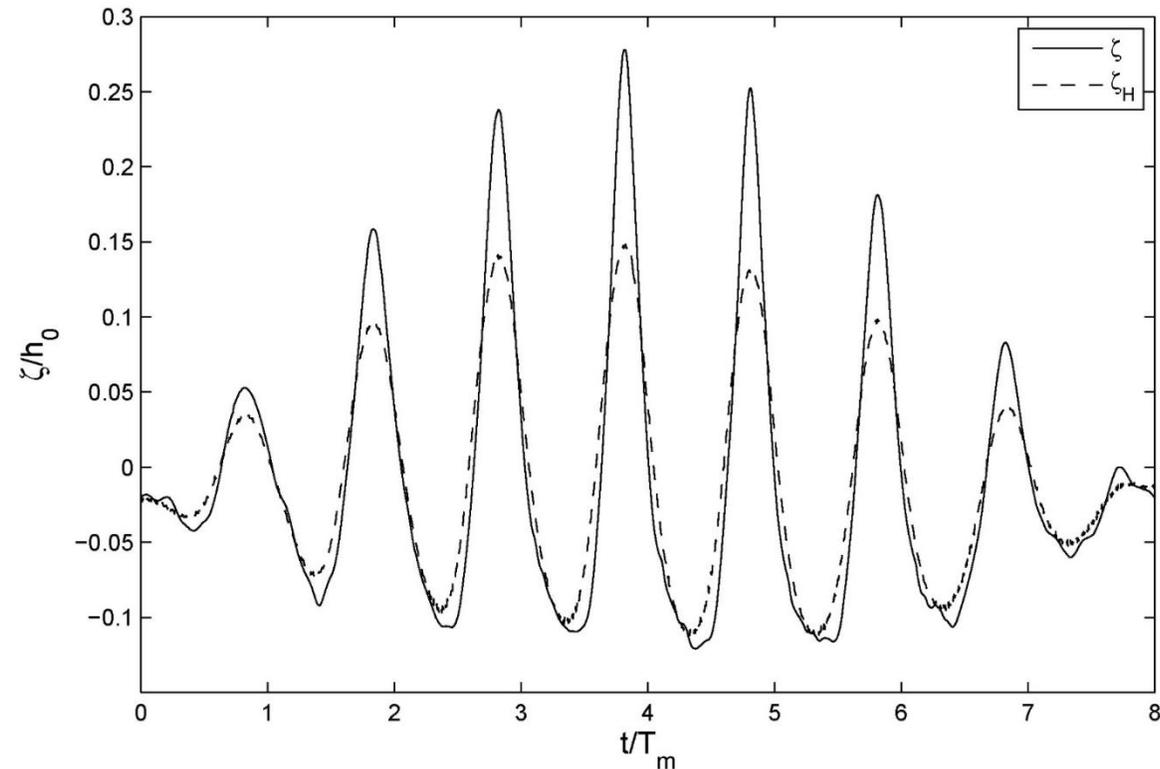


LEGI wave flume experiments, *Michallet et al. 2017*

LEGI wave flume

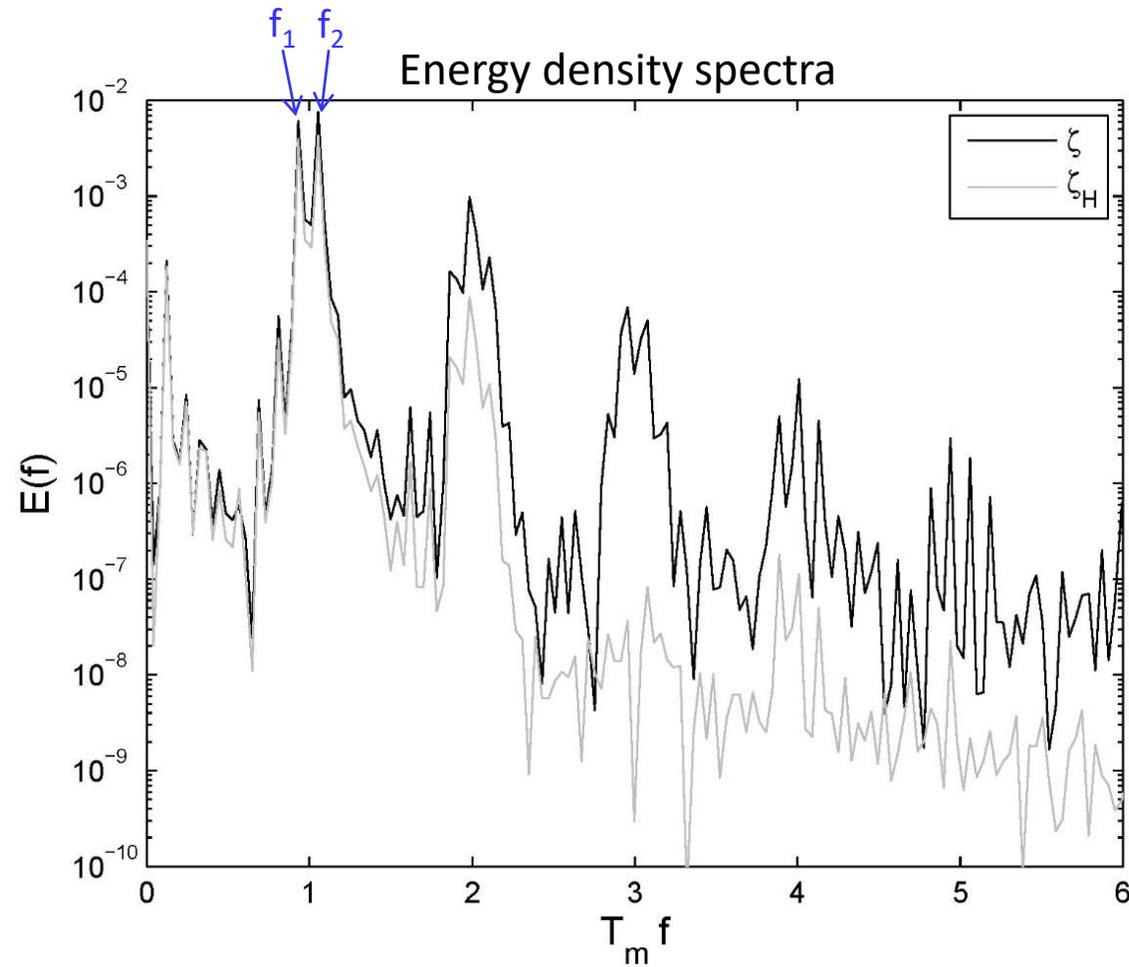
36 m long, 0.55 m wide

bichromatic waves propagating over a gently sloping (1/20) movable bed



ζ and P_m were synchronously measured in the shoaling zone, at 18.5m from the wave maker; $h_0=0.326$ m, $T_m=1.7$ s, $\mu=0.53$

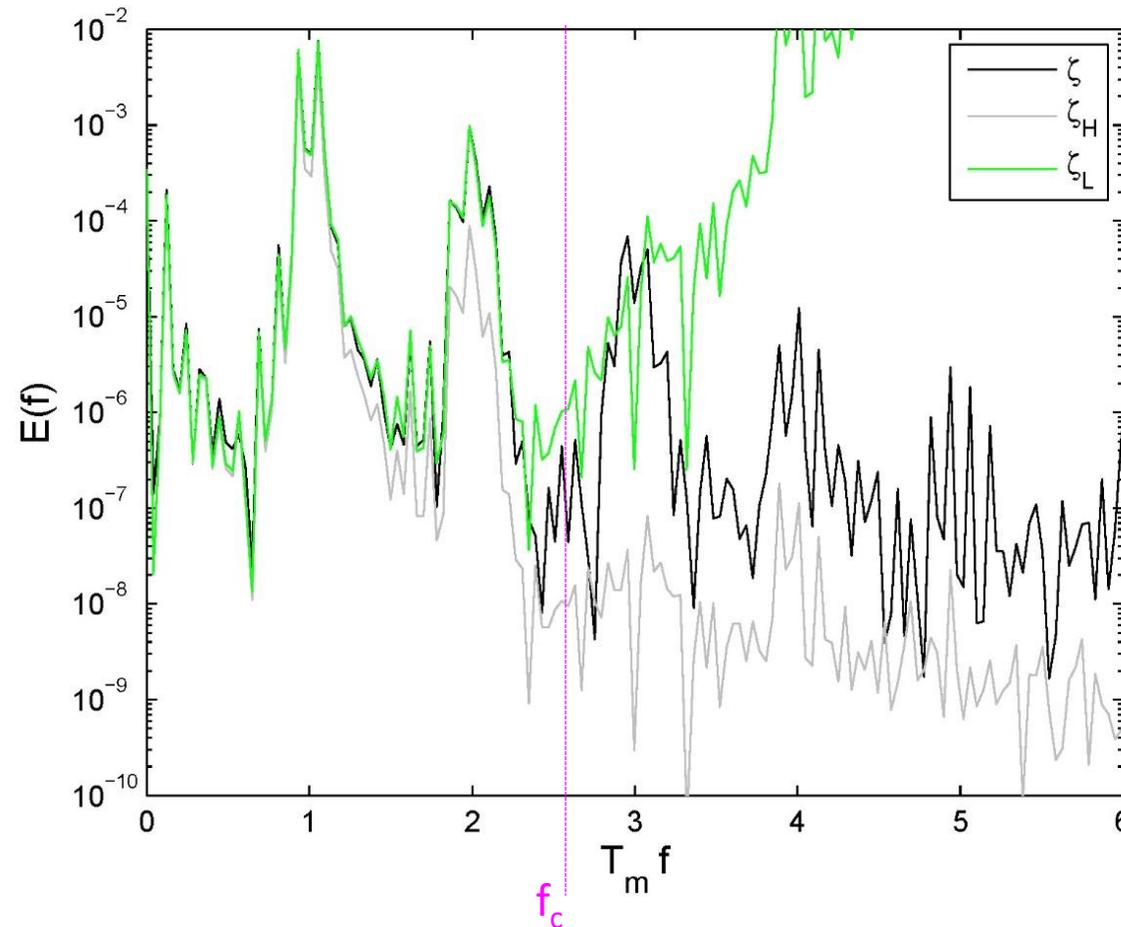
bichromatic waves propagating over a gently sloping movable bed



$$\tilde{\zeta}_L(\omega) = \cosh(h_0 |k(\omega)|) \tilde{\zeta}_H(\omega)$$

linear reconstruction

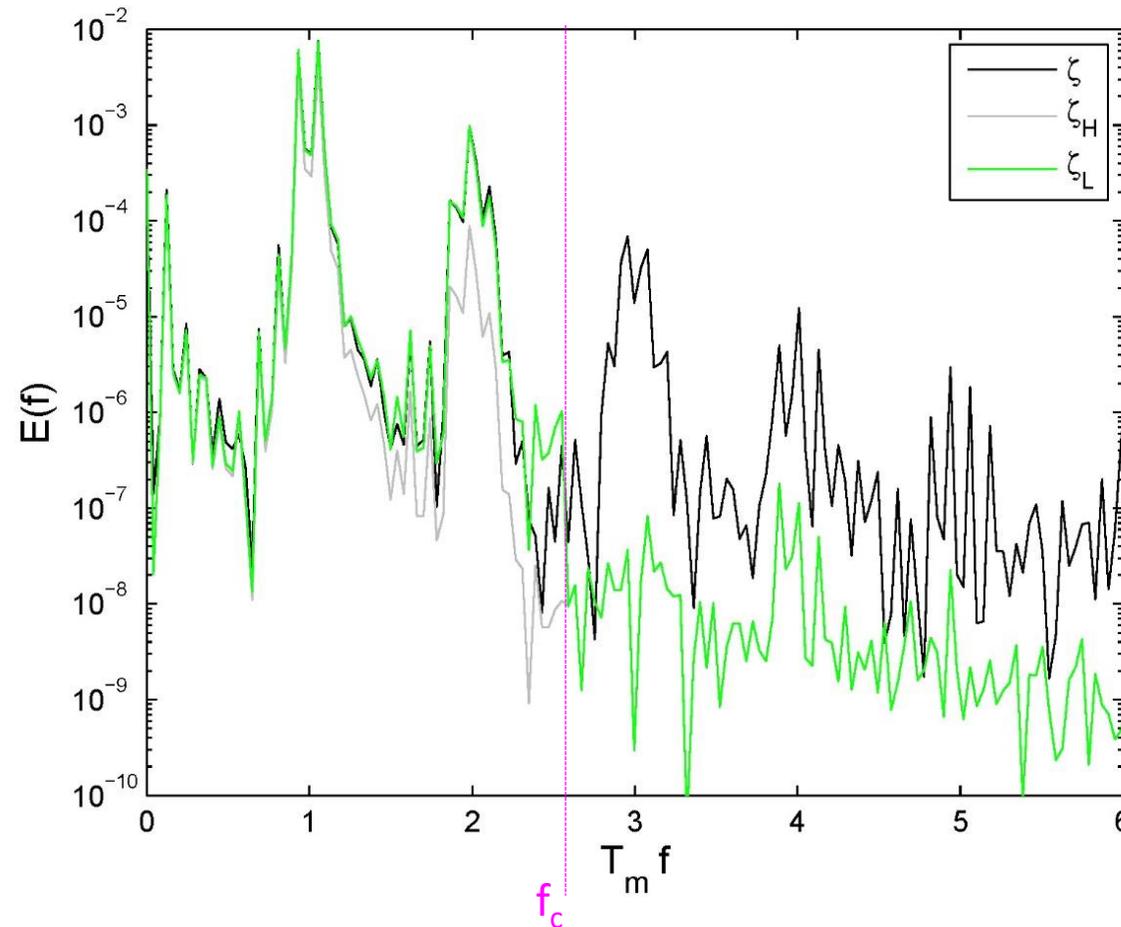
$$\omega^2 = g|k| \tanh(h_0 |k(\omega)|)$$



$$\tilde{\zeta}_L(\omega) = \cosh(h_0 |k(\omega)|) \tilde{\zeta}_H(\omega)$$

$$\omega^2 = g|k| \tanh(h_0 |k(\omega)|)$$

linear reconstruction



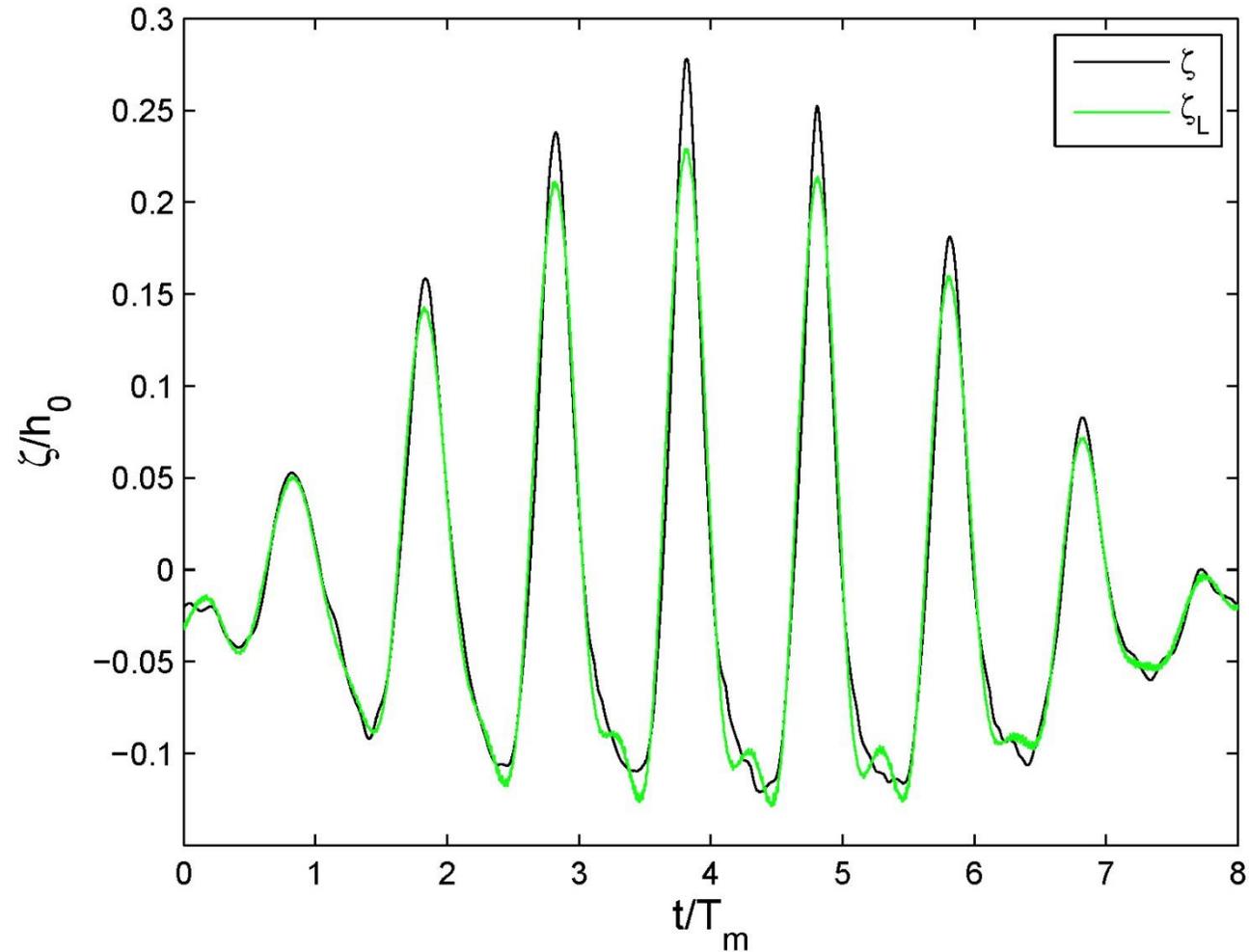
$$f > f_c$$

$$\tilde{\zeta}_L(\omega) = \tilde{\zeta}_H(\omega)$$

$$\tilde{\zeta}_L(\omega) = \cosh(h_0 |k(\omega)|) \tilde{\zeta}_H(\omega)$$

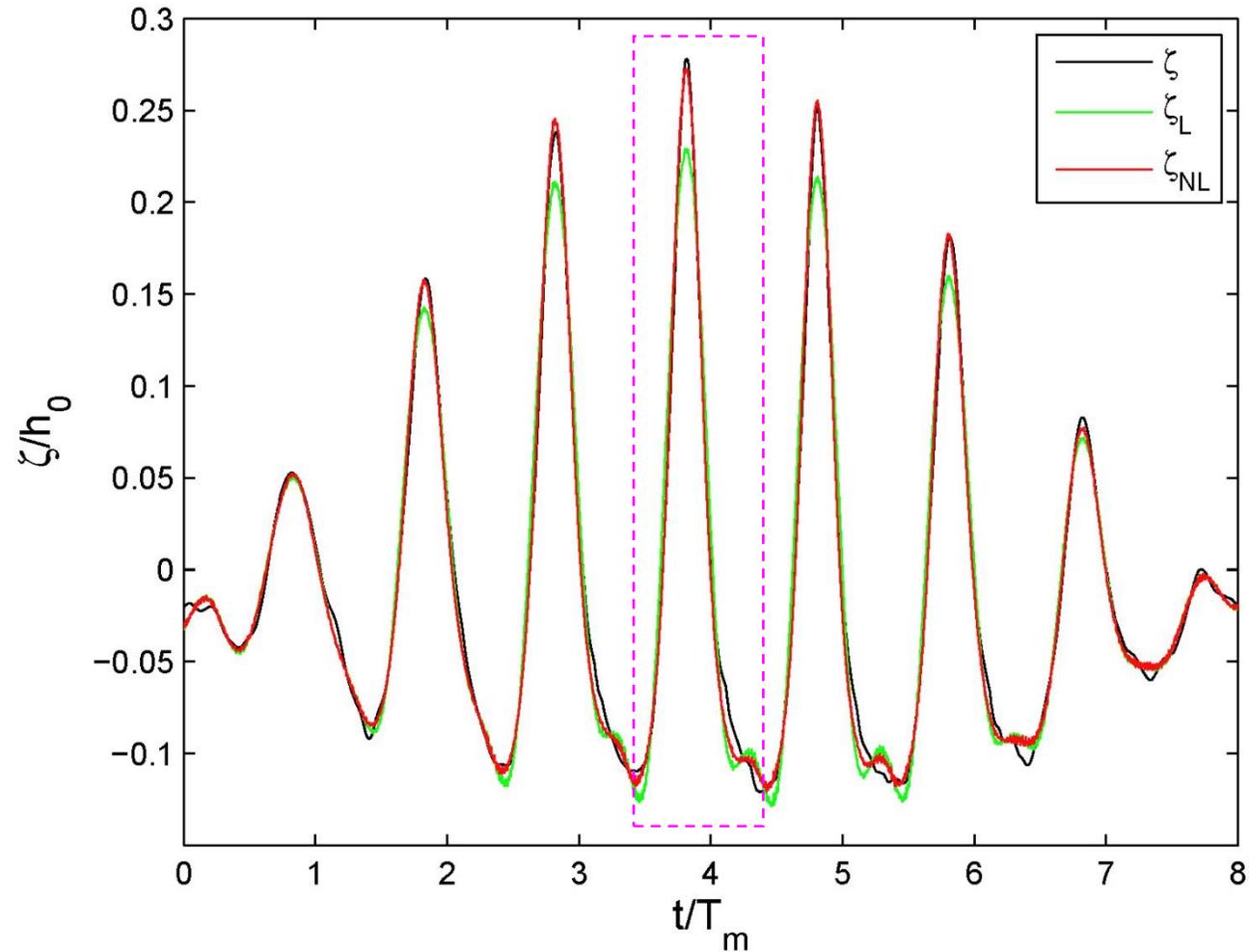
linear reconstruction

$$\omega^2 = g|k| \tanh(h_0 |k(\omega)|)$$



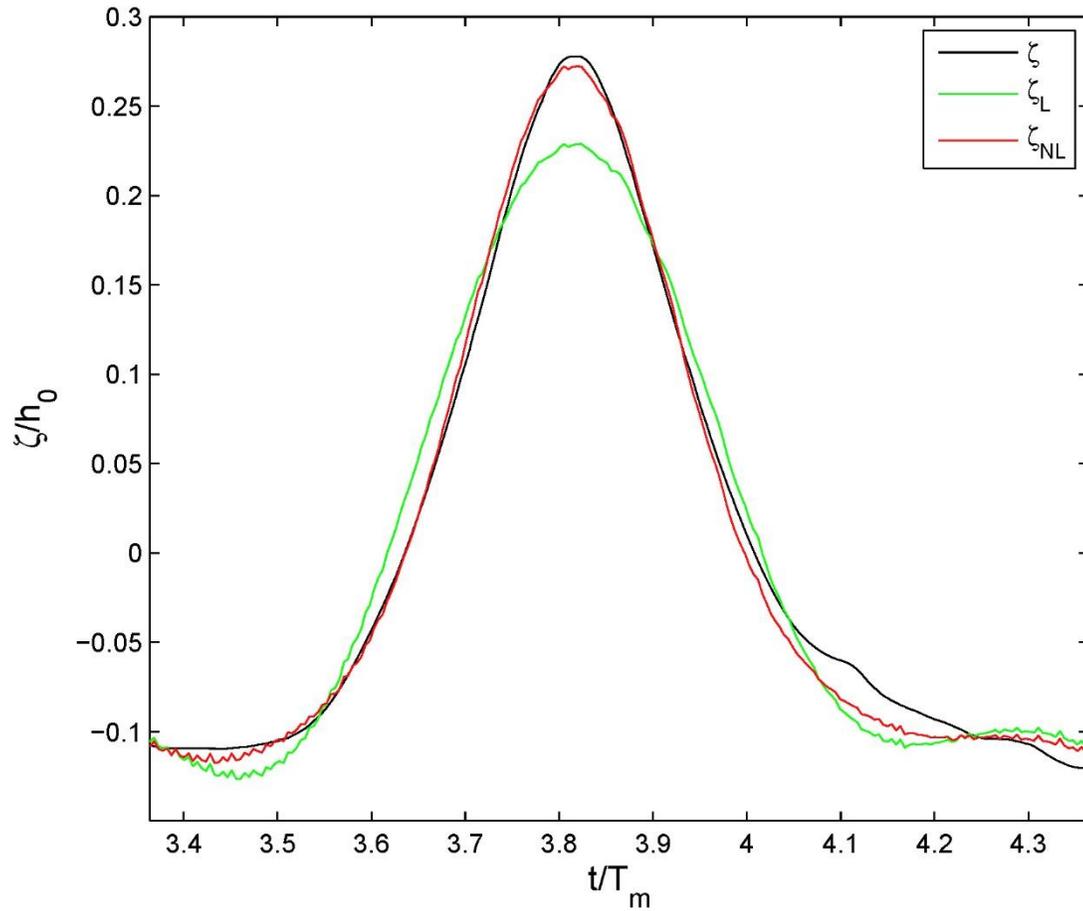
$$\zeta_{NL} = \zeta_L - \frac{1}{g} \partial_t (\zeta_L \partial_t \zeta_L)$$

non-Linear reconstruction



$$\zeta_{NL} = \zeta_L - \frac{1}{g} \partial_t (\zeta_L \partial_t \zeta_L)$$

non-Linear reconstruction

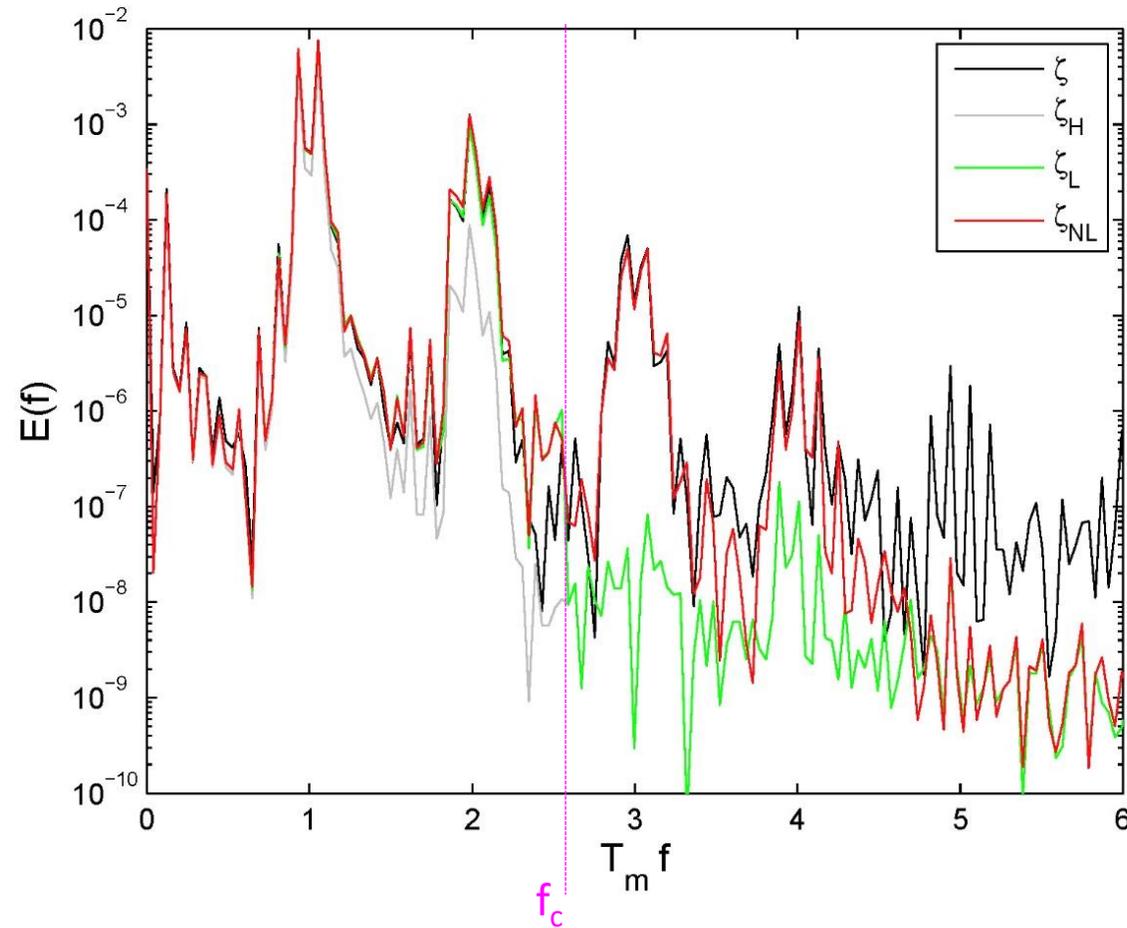


$$S_k = \frac{\langle (\zeta - \langle \zeta \rangle)^3 \rangle}{\langle (\zeta - \langle \zeta \rangle)^2 \rangle^{3/2}}$$

	ζ direct measurement	ζ_L	ζ_{NL}
S_k	0.93	0.70	0.96
S_k error	—	25%	3%

$$\zeta_{NL} = \zeta_L - \frac{1}{g} \partial_t (\zeta_L \partial_t \zeta_L)$$

non-Linear reconstruction



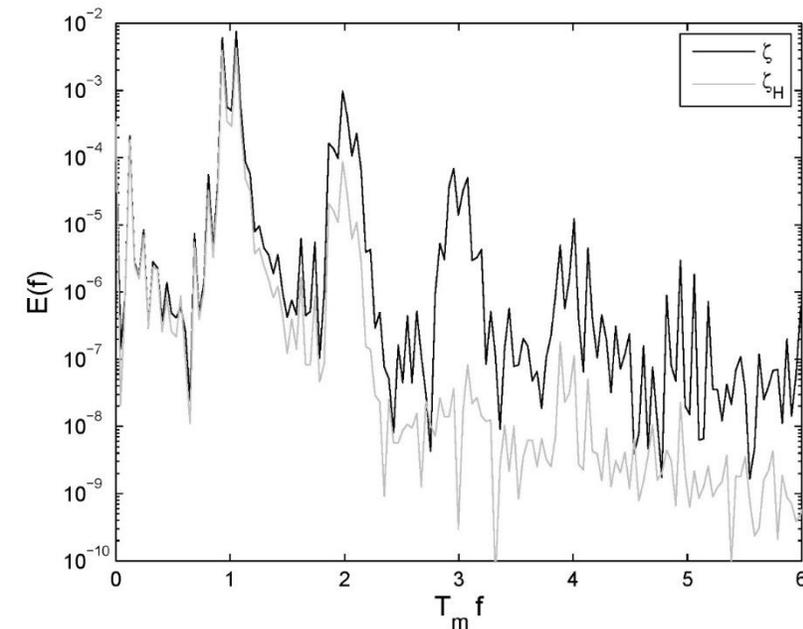
Highest wave nonlinearities generally occur in shallow water close to the onset of breaking

nonlinear weakly dispersive reconstruction

$$\zeta_{SNL} = \zeta_{SL} - \frac{1}{g} \partial_t (\zeta_{SL} \partial_t \zeta_{SL})$$

$$\zeta_{SL} = \zeta_H - \frac{h_0}{2g} \partial_t^2 \zeta_H$$

- local in time
- no frequency cut-off f_c
- reconstruction of the whole elevation density spectrum
- $\mu < 0.3$



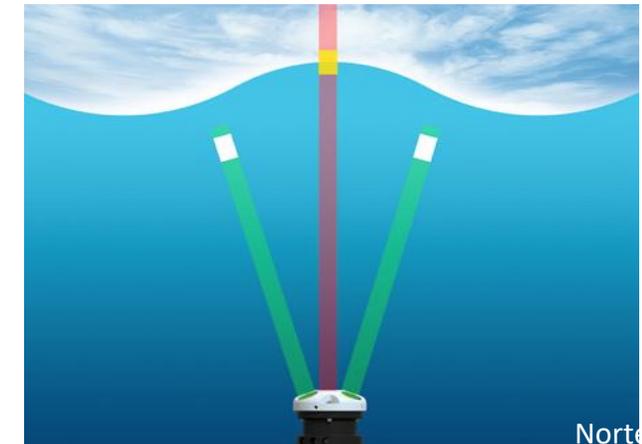
Field campaign (April 13-14, 2017)

La Salie beach, southern part of the French Atlantic coast

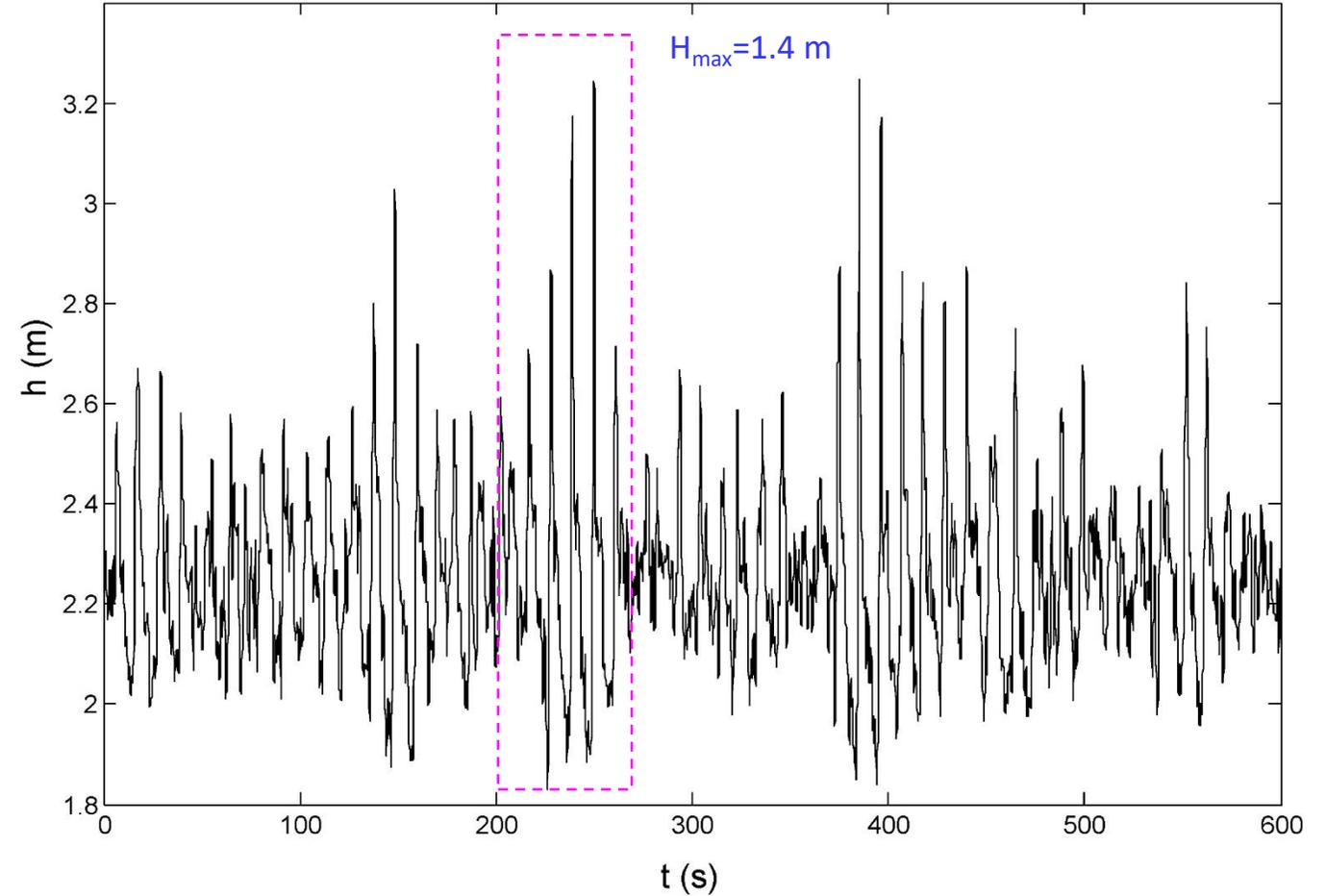


Instruments were deployed at low tide:

- pressure transducers (Ocean Sensor Systems)
- Nortek ADCP, Signature 1000 → direct measurement of ζ from the vertical beam of the ADCP



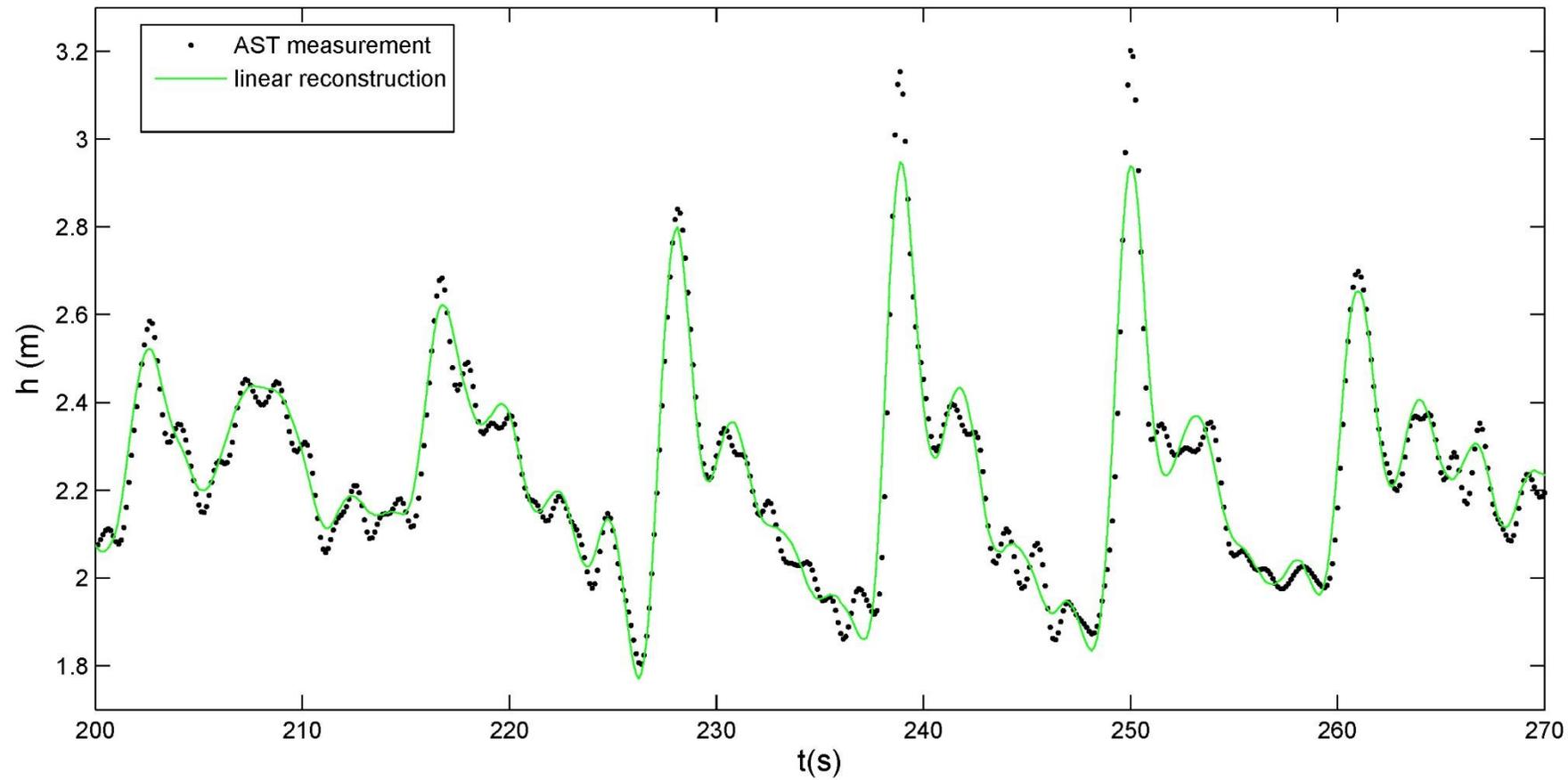
Field campaign → nonlinear waves in the shoaling zone just prior to breaking



$h_0 = 2.25$ m, $H_s = 0.70$ m, $T_p = 11.1$ s

$\mu = 0.075 \rightarrow$ weakly dispersive waves

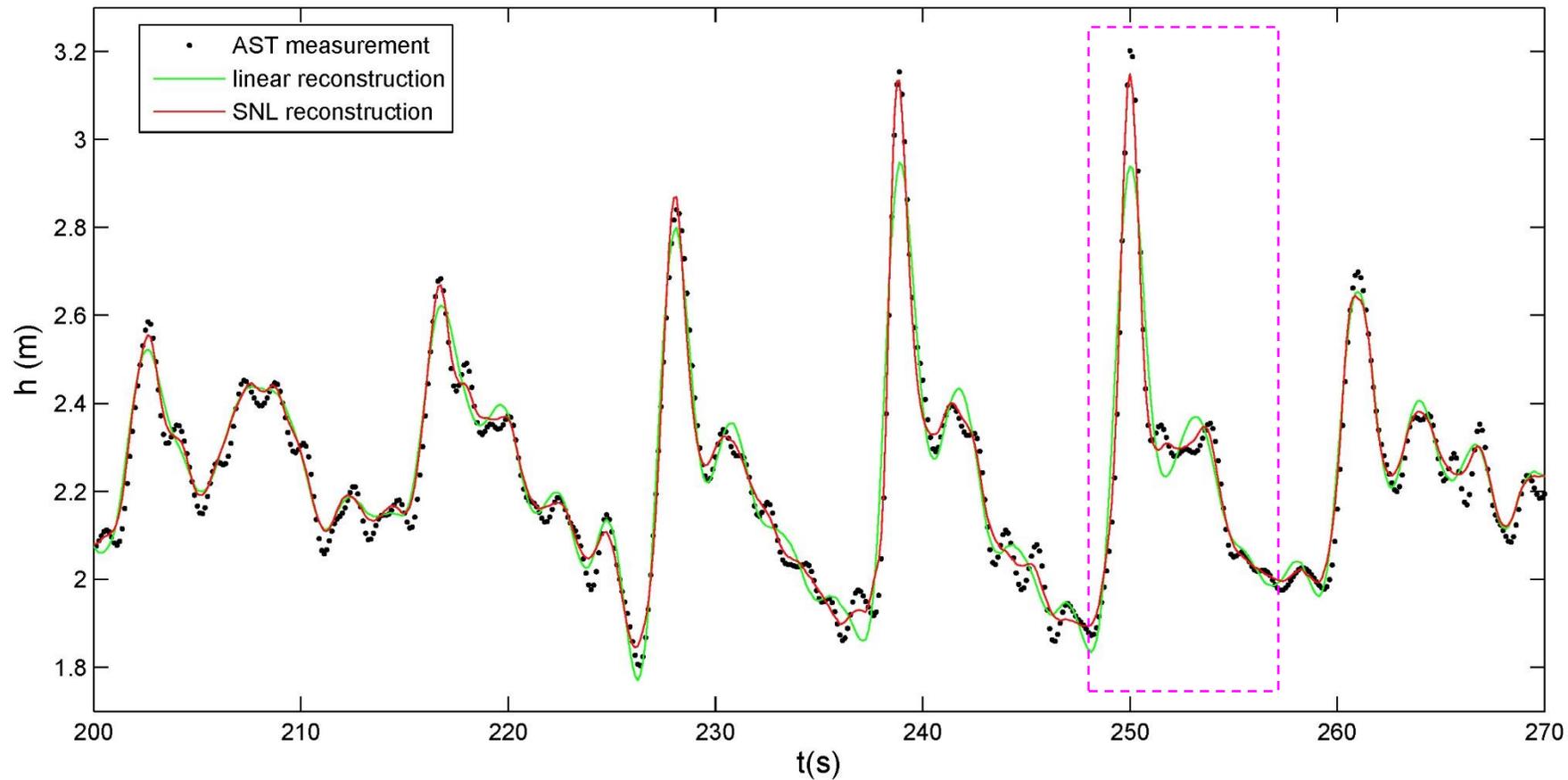
Field campaign



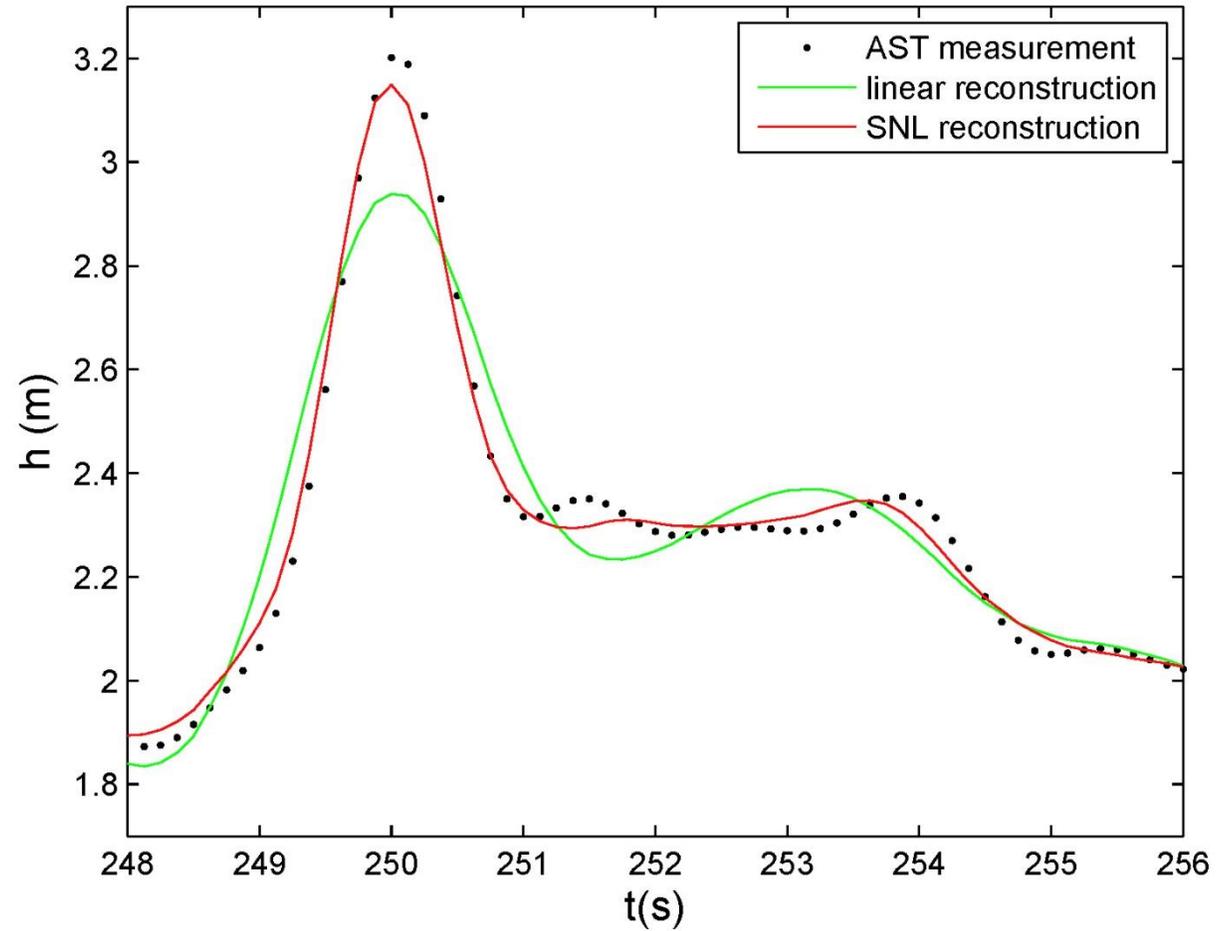
Field campaign

$$\zeta_{SNL} = \zeta_{SL} - \frac{1}{g} \partial_t (\zeta_{SL} \partial_t \zeta_{SL})$$

$$\zeta_{SL} = \zeta_H - \frac{h_0}{2g} \partial_t^2 \zeta_H$$

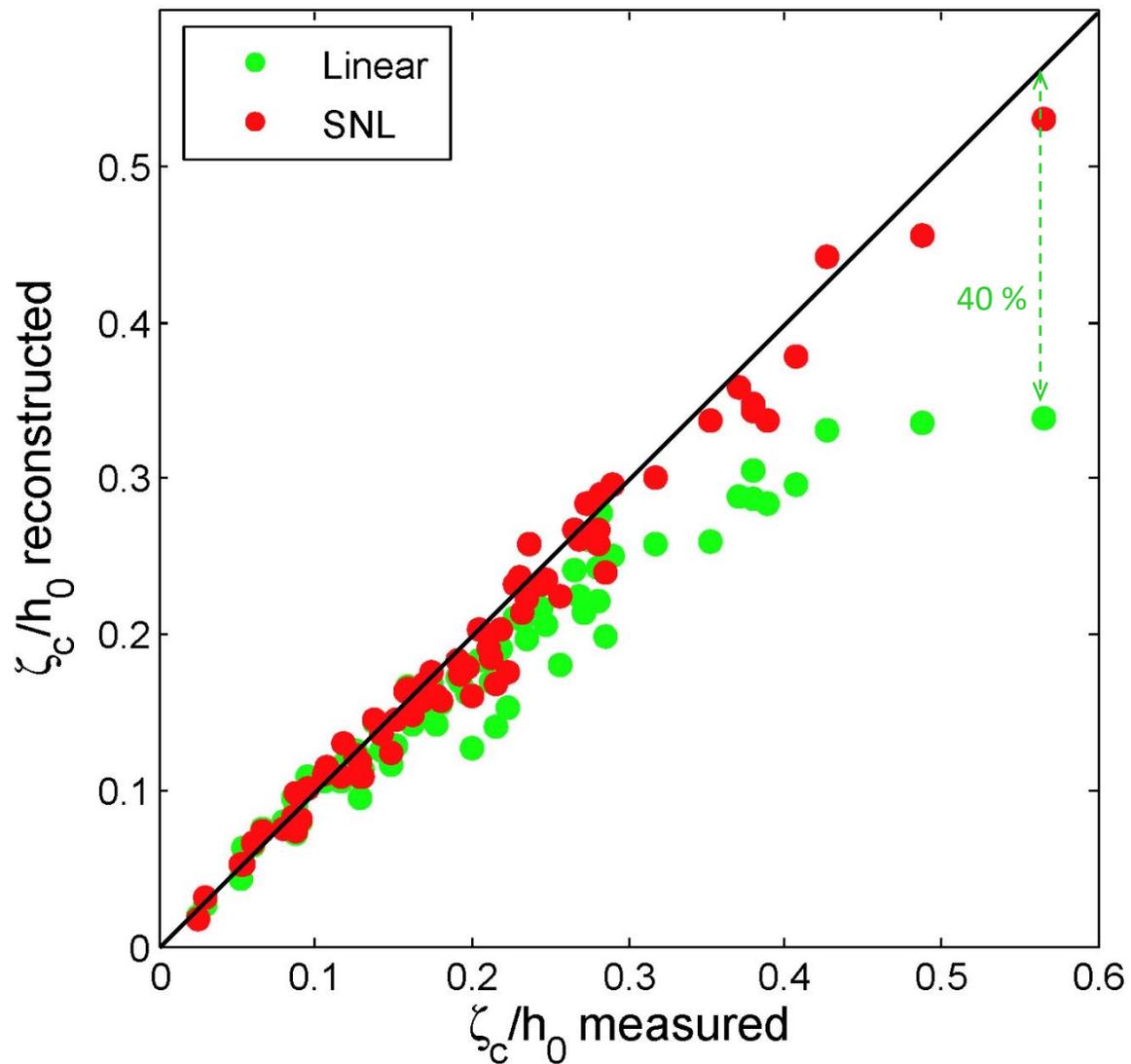
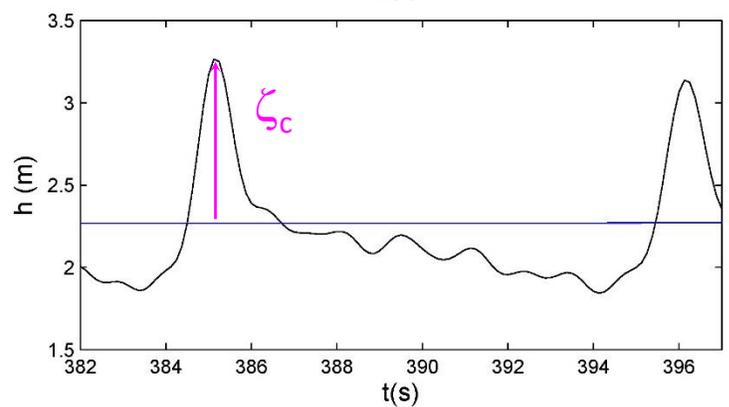
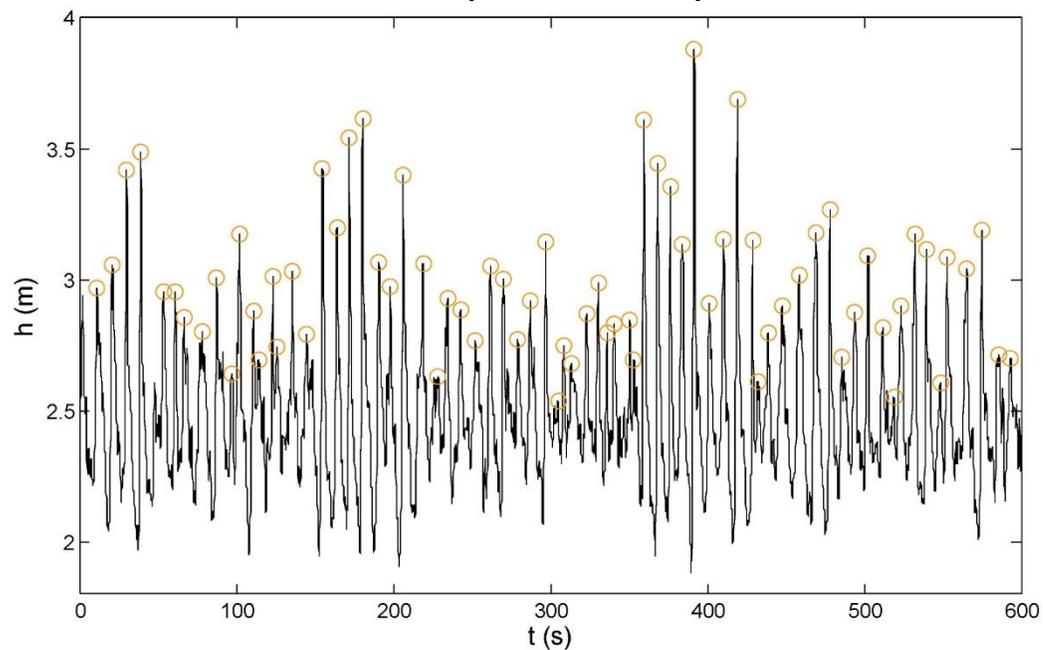


Field campaign



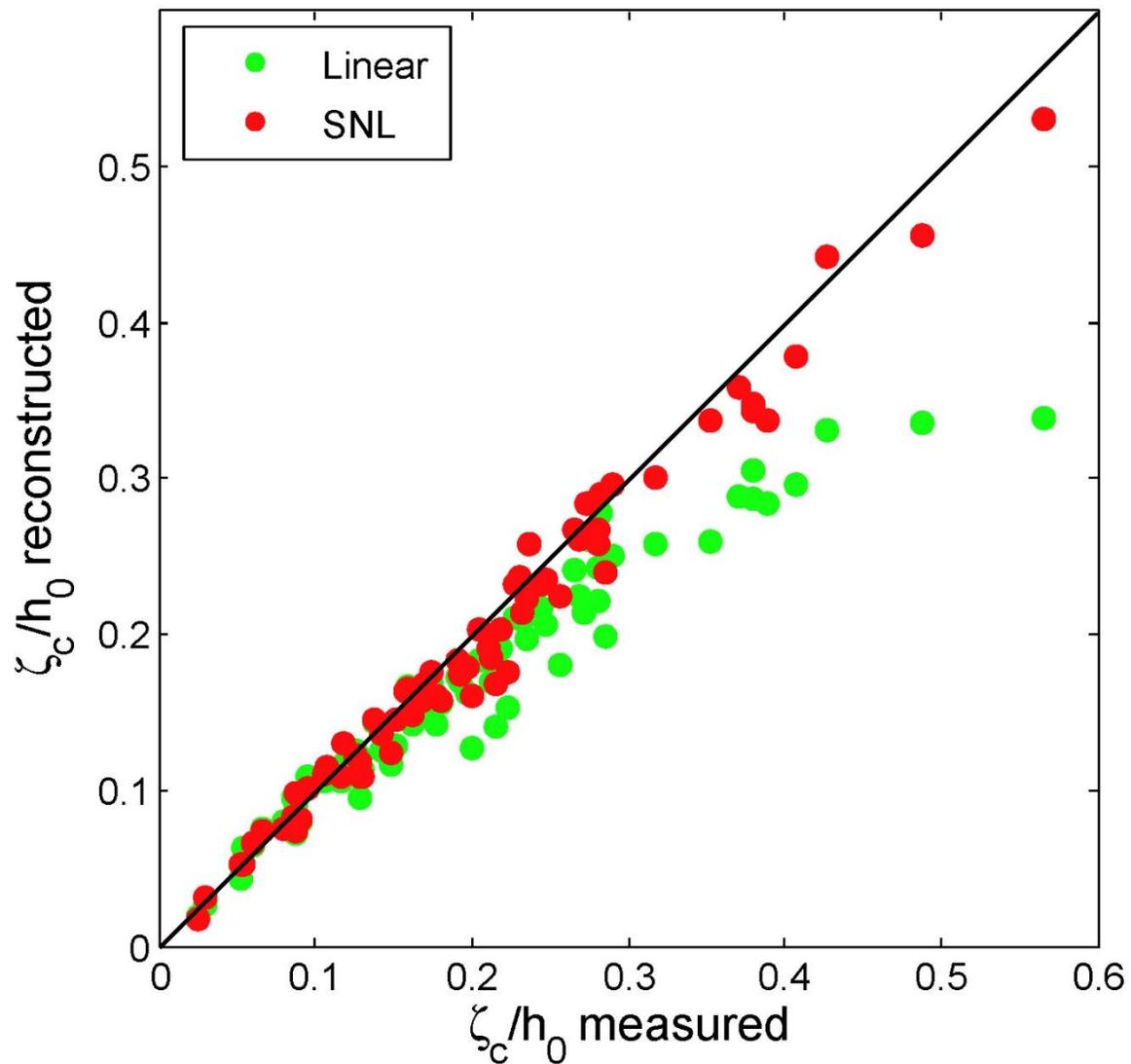
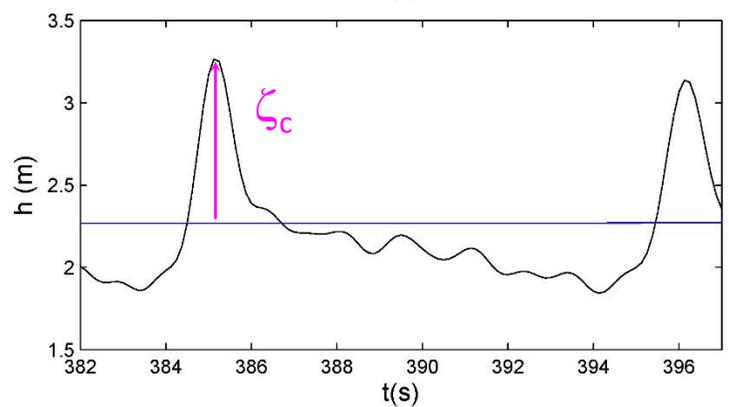
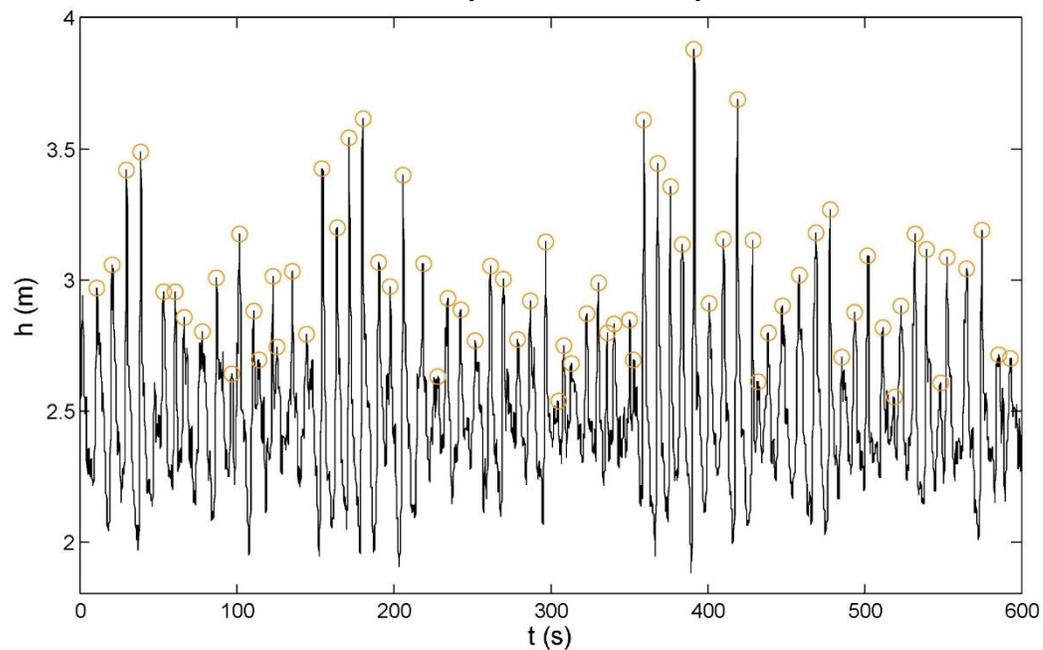
Field campaign

wave by wave analysis



Field campaign

wave by wave analysis

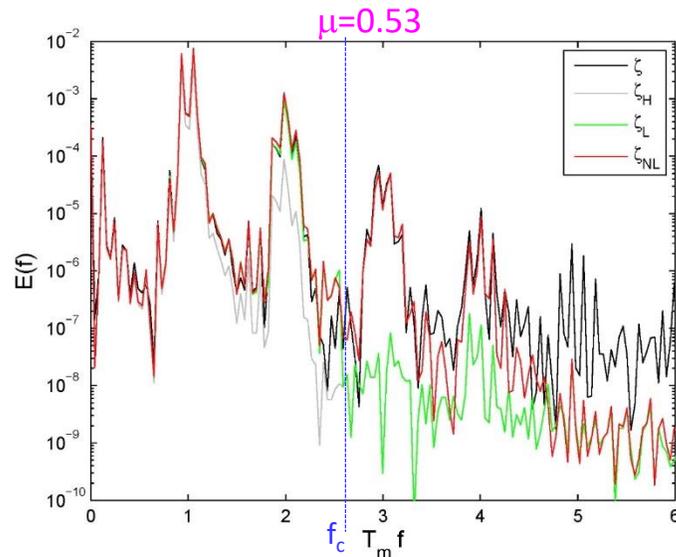


Conclusion

Two novel nonlinear methods for recovering the surface wave elevation from pressure measurements

□ general fully dispersive method

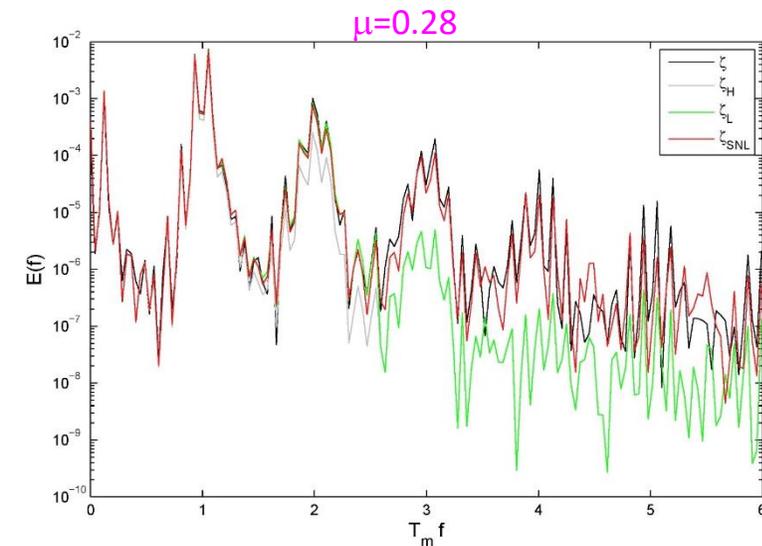
$$\zeta_{NL} = \zeta_L - \frac{1}{g} \partial_t (\zeta_L \partial_t \zeta_L)$$



□ weakly dispersive method, $\mu \leq 0.3$

$$\zeta_{SNL} = \zeta_{SL} - \frac{1}{g} \partial_t (\zeta_{SL} \partial_t \zeta_{SL})$$

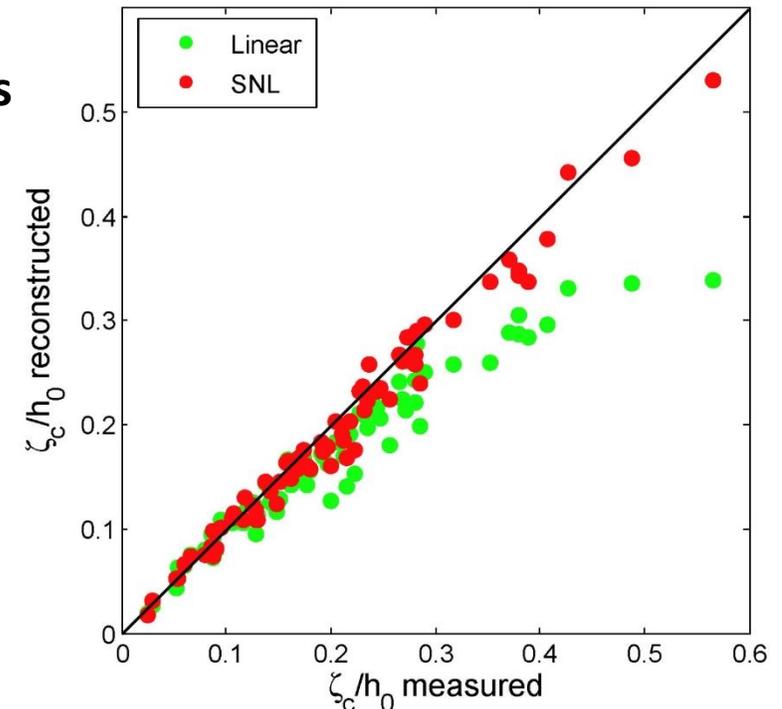
$$\zeta_{SL} = \zeta_H - \frac{h_0}{2g} \partial_t^2 \zeta_H$$



Conclusion

Two novel nonlinear methods for recovering the surface wave elevation from pressure measurements

- ❑ provide much better results compared to the transfer function method commonly used in coastal applications
- ❑ are very simple and easy to use
- ❑ represent an economic alternative to direct wave elevation measurement methods (AST and Lidar)
- ❑ are a valuable tool for accurately characterizing extreme waves in shallow and intermediate water depths



Thank you for your attention



Field campaign

