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A two-layer non-hydrostatic landslide model for tsunami generation on irregular bathymetry

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Introduction

❑ Motivation:

1. Landslide (or S(ubmarine) M(ass) F(ailure)s) can have catastrophic impacts on coastal communities and infrastructure;

2. Treatment of SMFs as rigid slides or slumps can lead to overly conservative predictions during hazard analysis;

3. A more representative deformable slide model for SMFs is needed ;

4. Classification is related to the choice of flow rheology in numerical simulation;

5. Type I, **mudflow**;

6. Type II, **granular flow**.

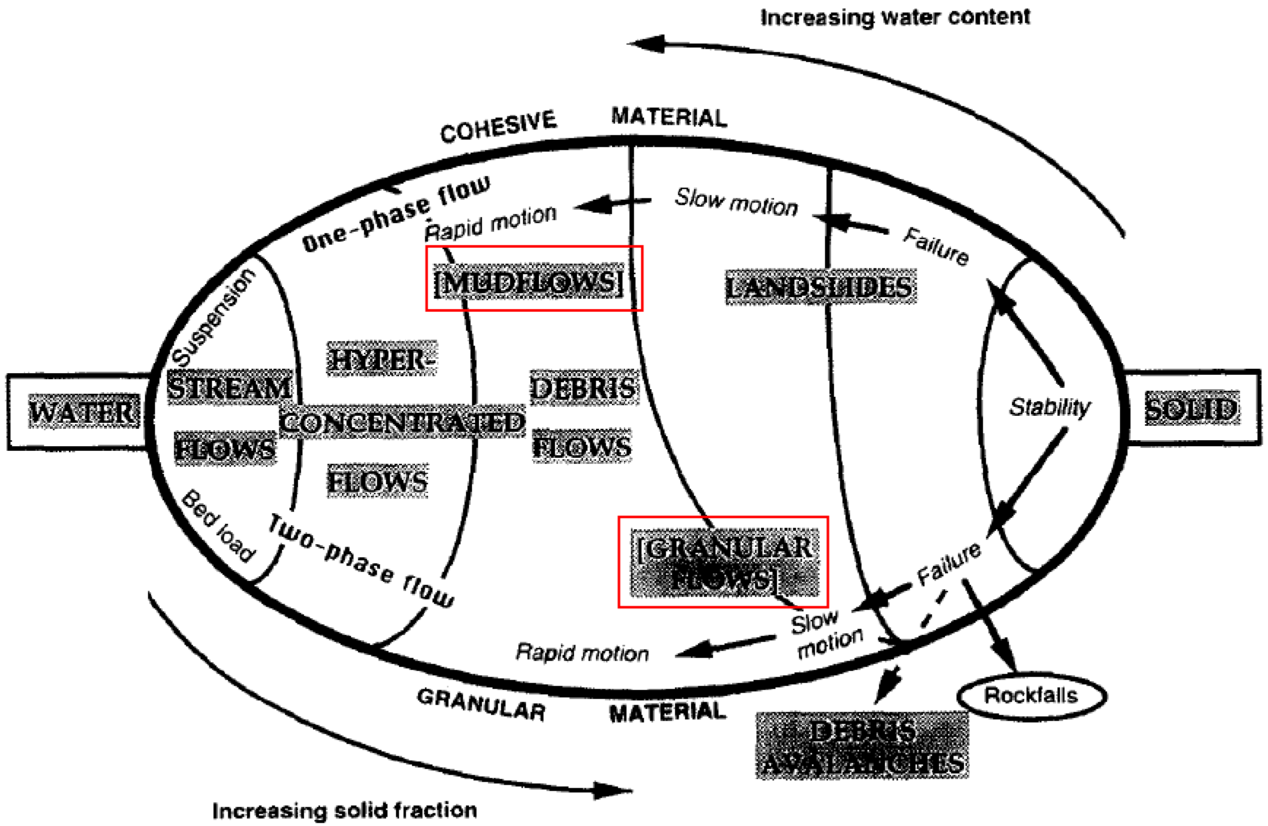


Figure 1: Classification of flowing mass. (From Coussot and Meunier 1996)

Introduction

❑ Two-layer landslide model :

1. The model consists of two models for different layers;

2. Non-hydrostatic wave model NHWAVE is used for upper-layer wave (Ma et al. 2012; Derakhti et al. 2016);

3. Non-hydrostatic shallow-water type model with various rheology closures for lower-layer landslide.

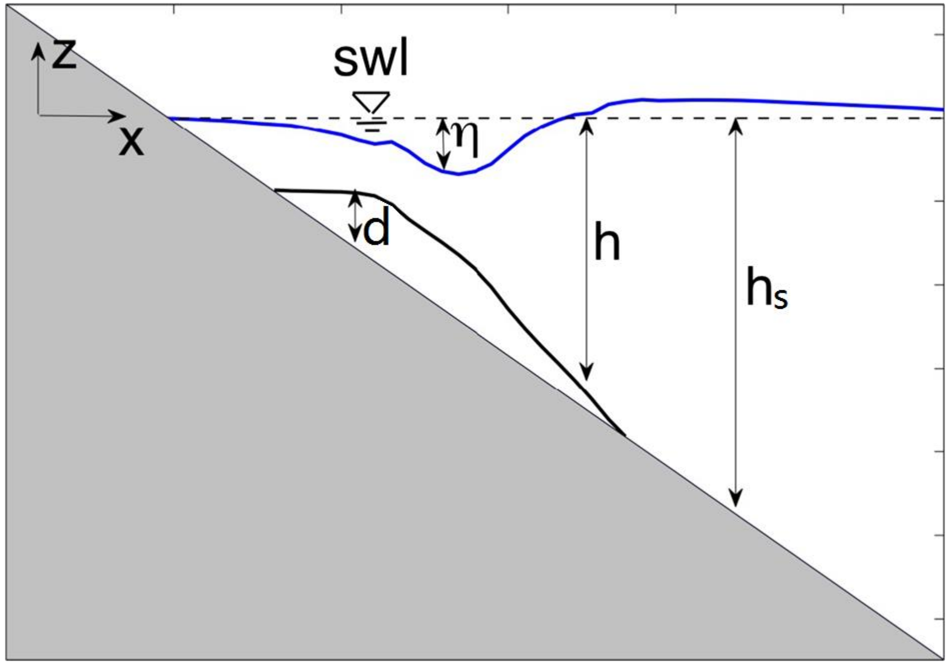


Figure 2: Sketch for two-layer landslide model. (From Grilli et al. 2017)

Introduction

- Shallow water regime:

  1. Flowing mass has fluid-like manner (Savage and Hutter 1989);

  2. Ratio of slide thickness *D* and length *L* is much smaller than1;

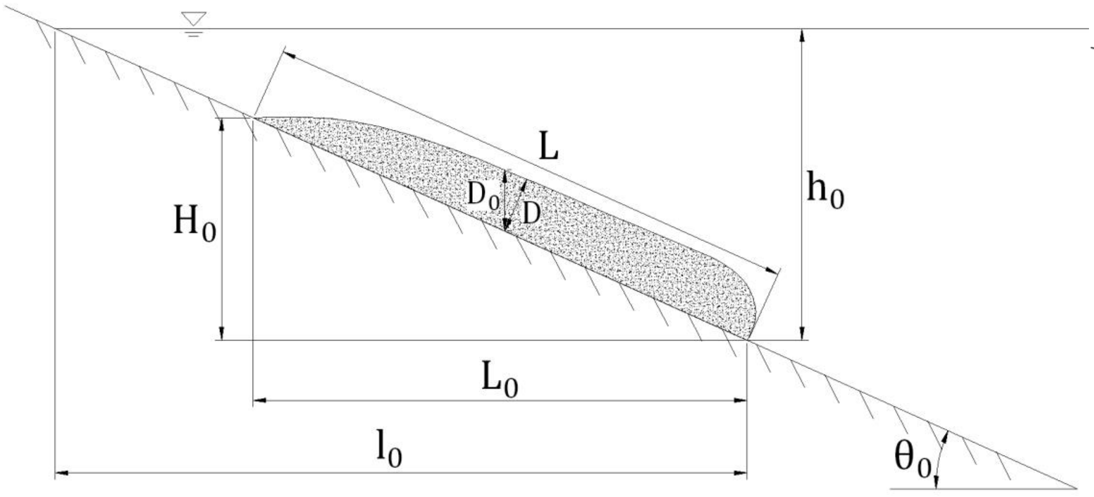


Figure 3: Characteristic lengths of flowing mass.

- Coordinate system:

  1. SMF occurs on irregular bathymetry;

  2. Three coordinate systems;

  3. Regular Cartesian coordinate is adopted;

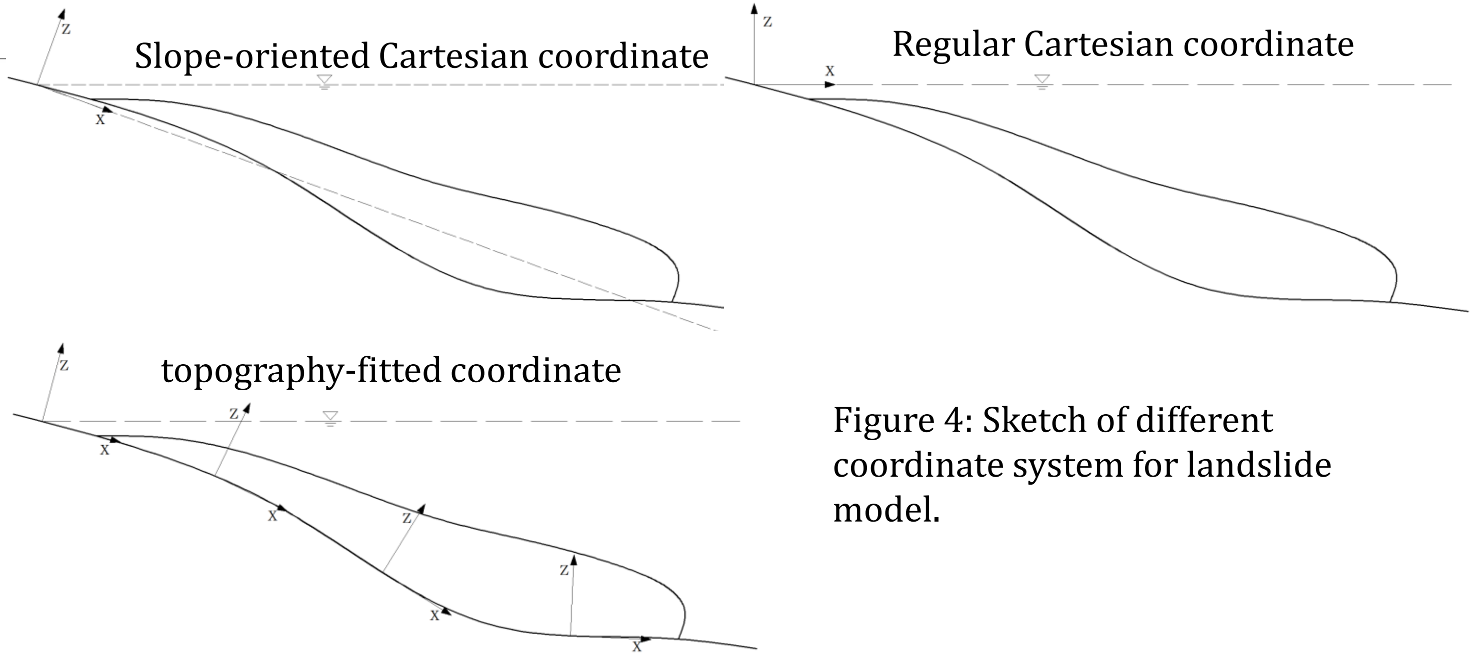


Figure 4: Sketch of different coordinate system for landslide model.

Theoretical Background

- Conservation laws:

1. Governing equations for solid-phase and fluid-phase are:

$$\frac{\partial \rho_s \phi}{\partial t} + \nabla \cdot (\rho_s \phi \mathbf{v}_s) = 0$$

$$\frac{\partial \rho_f (1-\phi)}{\partial t} + \nabla \cdot (\rho_f (1-\phi) \mathbf{v}_f) = 0$$

$$\frac{\partial \rho_s \phi \mathbf{v}_s}{\partial t} + \nabla \cdot (\rho_s \phi \mathbf{v}_s \mathbf{v}_s) = \nabla \cdot \mathbf{T}_s - \nabla \cdot p_s + \rho_s \phi \mathbf{g} + \mathbf{f}$$

$$\frac{\partial \rho_f (1-\phi) \mathbf{v}_f}{\partial t} + \nabla \cdot (\rho_f (1-\phi) \mathbf{v}_f \mathbf{v}_f) = \nabla \cdot \mathbf{T}_f - \nabla \cdot p_f + \rho_f (1-\phi) \mathbf{g} - \mathbf{f}$$

2. Sum them up and obtain equations for mixture mass and momentum conservation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = \nabla \cdot \mathbf{T} - \nabla \cdot p + \rho \mathbf{g}$$

where $\rho = \rho_s \phi + \rho_f (1-\phi)$, $\mathbf{v} = (\rho_s \phi \mathbf{v}_s + \rho_f (1-\phi) \mathbf{v}_f) / \rho$, $\mathbf{T} = \mathbf{T}_s + \mathbf{T}_f$

$\mathbf{v}_f - \mathbf{v}_s$ slip velocity, $p = p_s + p_f$

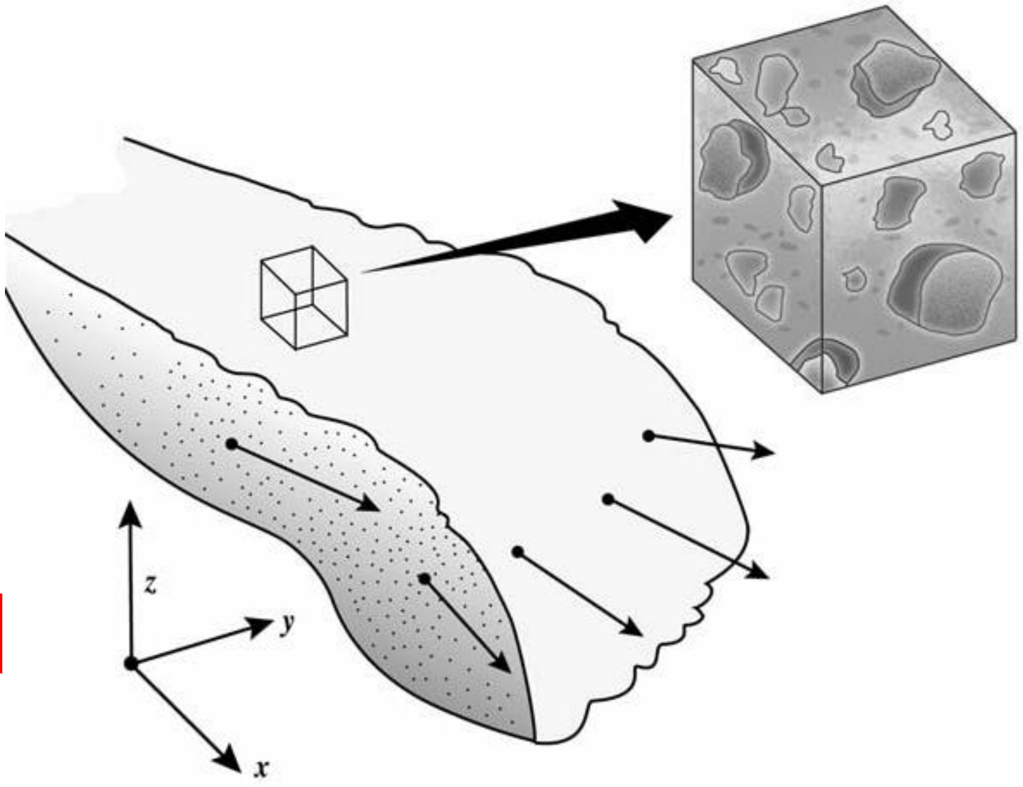


Figure 5: Sketch of landslide flow. (From Iverson 2005).

Theoretical Background

3. In the regular Cartesian coordinates system (*x, y, z*), the conservation equations can be rewritten as

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_\beta}{\partial x_\beta} + \frac{\partial \rho w}{\partial z} = 0$$

$$\frac{\partial \rho u_\alpha}{\partial t} + \frac{\partial \rho u_\alpha u_\beta}{\partial x_\beta} + \frac{\partial \rho u_\alpha w}{\partial z} = \frac{\partial \tau_{\beta\alpha}}{\partial x_\beta} + \frac{\partial \tau_{z\alpha}}{\partial z} - \frac{\partial p}{\partial x_\alpha}$$

$$\frac{\partial \rho w}{\partial t} + \frac{\partial \rho w u_\beta}{\partial x_\beta} + \frac{\partial \rho w^2}{\partial z} = \frac{\partial \tau_{\beta z}}{\partial x_\beta} + \frac{\partial \tau_{zz}}{\partial z} - \frac{\partial p}{\partial z} - \rho g$$

Kinematic boundary conditions (KBCs) for the equations are

$$\frac{\partial h}{\partial t} + u_\beta(-h)\frac{\partial h}{\partial x_\beta} + w(-h) = 0$$

$$\frac{\partial h_s}{\partial t} + u_\beta(-h_s)\frac{\partial h_s}{\partial x_\beta} + w(-h_s) = 0$$

Traction-free boundary condition is used in the current version of model.

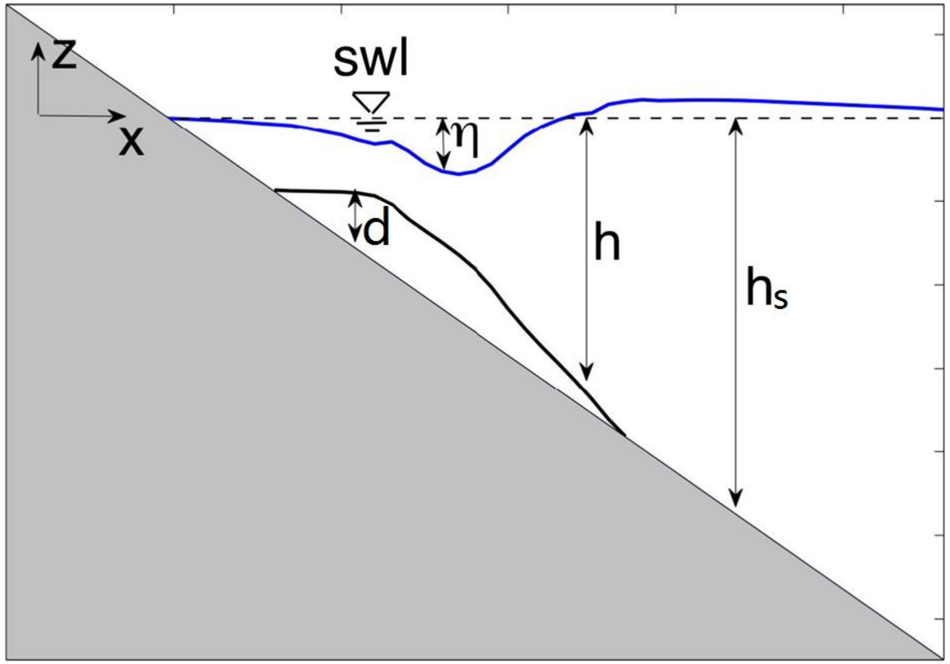


Figure 6: Definition sketch for underwater landslide. (From Grilli et al. 2017)

Theoretical Background

- Assumptions:

1. ρ is vertically uniform;

2. Vertical velocity is linear in vertical;

3. $\bar{q} = 2/3\,(q(-h_s) + q(-h))$ is imposed to ensure a correct linear dispersion relation;

5. In z momentum equation, assume vertical normal stress is dominant and advective effect is small;

6. No water and soil exchange;

7. Constant porosity and mixture density.

Theoretical Background

- Depth-averaged equations:

Mass:

$$\frac{\partial \rho d}{\partial t} + \frac{\partial \rho \bar{u}_\beta d}{\partial x_\beta} = 0$$

***x/y*-momentum:**

$$\frac{\partial \rho \bar{u}_\alpha d}{\partial t} + \frac{\partial \Gamma_{\alpha\beta} \rho \bar{u}_\alpha \bar{u}_\beta d}{\partial x_\beta} = \frac{\partial \bar{\tau}_{\beta\alpha} d}{\partial x_\beta} + \tau_{\beta\alpha}(-h)\frac{\partial h}{\partial x_\beta} + \tau_{z\alpha}(-h) - \tau_{\beta\alpha}(-h_s)\frac{\partial h_s}{\partial x_\beta} - \tau_{z\alpha}(-h_s)$$
$$- \rho_f g d \frac{\partial(\eta + h)}{\partial x_\alpha} - \frac{1}{2} g \frac{\partial \rho d^2}{\partial x_\alpha} + \rho g d \frac{\partial h_s}{\partial x_\alpha}$$
$$- \frac{2}{3}\frac{\partial q(-h) d}{\partial x_\alpha} - q(-h)\frac{\partial h}{\partial x_\alpha} - \frac{2}{3}\frac{\partial q(-h_s) d}{\partial x_\alpha} + q(-h_s)\frac{\partial h_s}{\partial x_\alpha}$$

***z*-momentum:**

$$\frac{\partial \rho \bar{w} d}{\partial t} = \tau_{zz}(-h) - \tau_{zz}(-h_s) - q(-h) + q(-h_s)$$

Rheology Closure

- Laminar flow model (mudflow):

  1. Low solid concentration;

  2. Newtonian fluid at a low Reynolds number;

  3. The influence from solid phase is embodied as an increased viscosity which is significantly higher than water;

  4. Parabolic velocity profile:

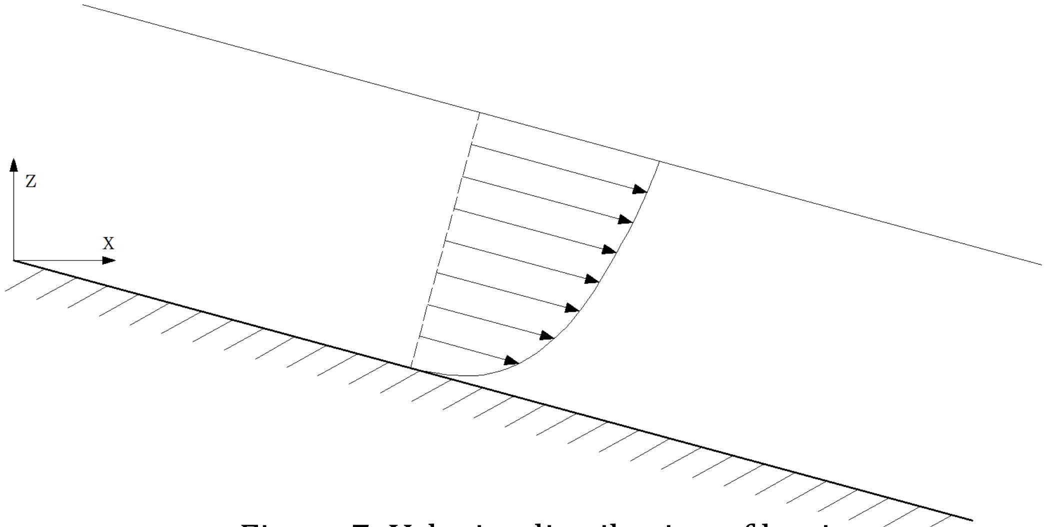


Figure 7: Velocity distribution of laminar open channel flow.

$$\tau_{xx}(-h_s) = 2\mu_e \frac{\partial u}{\partial x}\bigg|_{\sigma=0} = \frac{6\mu_e \bar{u}}{d}\frac{\partial h_s}{\partial x}$$

$$\tau_{yy}(-h_s) = 2\mu_e \frac{\partial v}{\partial y}\bigg|_{\sigma=0} = \frac{6\mu_e \bar{v}}{d}\frac{\partial h_s}{\partial y}$$

$$\tau_{zz}(-h_s) = 2\mu_e \frac{\partial w}{\partial z}\bigg|_{\sigma=0} = -\frac{2\mu_e}{\rho}\frac{\partial \rho}{\partial t} - \frac{6\mu_e}{d}\left(\bar{u}\frac{\partial h_s}{\partial x} + \bar{v}\frac{\partial h_s}{\partial y}\right)$$

$$\tau_{xy}(-h_s) = \tau_{yx}(-h_s) = \mu_e\left(\frac{\partial u}{\partial y}\bigg|_{\sigma=0} + \frac{\partial v}{\partial x}\bigg|_{\sigma=0}\right) = \frac{3\mu_e}{d}\left(\bar{u}\frac{\partial h_s}{\partial y} + \bar{v}\frac{\partial h_s}{\partial x}\right)$$

$$\tau_{xz}(-h_s) = \tau_{zx}(-h_s) = \mu_e\left(\frac{\partial u}{\partial z}\bigg|_{\sigma=0} + \frac{\partial w}{\partial x}\bigg|_{\sigma=0}\right)$$
$$= \frac{3\mu_e \bar{u}}{d} - \mu_e \frac{\partial^2 h_s}{\partial t \partial x} + \mu_e \frac{\partial E_b \xi_b}{\partial x} - \frac{\mu_e}{\rho}\frac{\partial \rho}{\partial t}\frac{\partial h_s}{\partial x} - \frac{3\mu_e}{d}\left(\bar{u}\frac{\partial h_s}{\partial x} + \bar{v}\frac{\partial h_s}{\partial y}\right)\frac{\partial h_s}{\partial x}$$

$$\tau_{yz}(-h_s) = \tau_{zy}(-h_s) = \mu_e\left(\frac{\partial v}{\partial z}\bigg|_{\sigma=0} + \frac{\partial w}{\partial y}\bigg|_{\sigma=0}\right)$$
$$= \frac{3\mu_e \bar{v}}{d} - \mu_e \frac{\partial^2 h_s}{\partial t \partial y} + \mu_e \frac{\partial E_b \xi_b}{\partial y} - \frac{\mu_e}{\rho}\frac{\partial \rho}{\partial t}\frac{\partial h_s}{\partial y} - \frac{3\mu_e}{d}\left(\bar{u}\frac{\partial h_s}{\partial x} + \bar{v}\frac{\partial h_s}{\partial y}\right)\frac{\partial h_s}{\partial y}$$

Rheology Closure

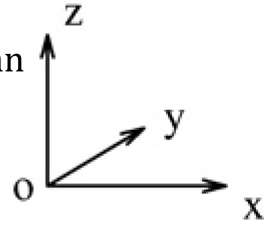
- Coulomb friction model (granular flow):

  1. High solid concentration, low Reynolds number;

  2. Shows no particular stress-strain rate relation, and inter-granular stresses satisfy the Coulomb rule;

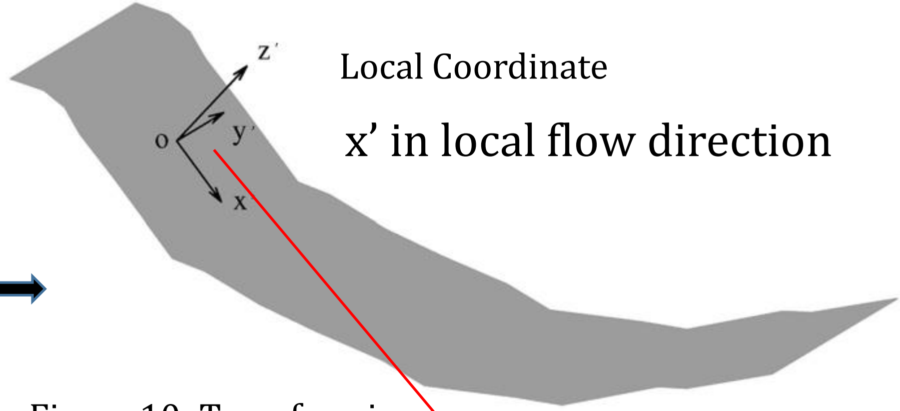
  3. Two coordinate systems:

Regular Cartesian Coordinate



$$\mathbf{C} = \begin{bmatrix} \frac{\bar{u}}{B} - \frac{D}{A^2B}h_{s,x} & \frac{h_{s,y}\bar{w}-\bar{v}}{AB} & \frac{h_{s,x}}{A} \\ \frac{\bar{v}}{B} - \frac{D}{A^2B}h_{s,y} & \frac{-h_{s,x}\bar{w}+\bar{u}}{AB} & \frac{h_{s,y}}{A} \\ \frac{\bar{w}}{B} - \frac{D}{A^2B} & \frac{h_{s,x}\bar{v}-h_{s,y}\bar{u}}{AB} & \frac{1}{A} \end{bmatrix}$$

$$\mathbf{T}' = \mathbf{C}^T \cdot \mathbf{T} \cdot \mathbf{C}$$



4. In Local coordinates, only consider normal stress $\tau_{ex'x'}$, $\tau_{ey'y'}$ and $\tau_{ez'z'}$ and dominant shear stresses $\tau_{ez'x'}$ and $\tau_{ex'z'}$, and neglect other components.

$$\tau_{ex'x'}(-h_s) = K_{x'_{act/pas}} \cdot \tau_{ez'z'}(-h_s)$$

$$\tau_{ey'y'}(-h_s) = K^{x'_{act/pas}}_{y'_{act/pas}} \cdot \tau_{ez'z'}(-h_s)$$

$$\tau_{ez'x'}(-h_s) = \tau_{ex'z'}(-h_s) = -\tau_{ez'z'}(-h_s) \cdot \tan\varphi_{bed}$$

$$\tau_{ez'y'}(-h_s) = \tau_{ey'z'}(-h_s) = \tau_{ex'y'}(-h_s) = \tau_{ey'x'}(-h_s) = 0$$

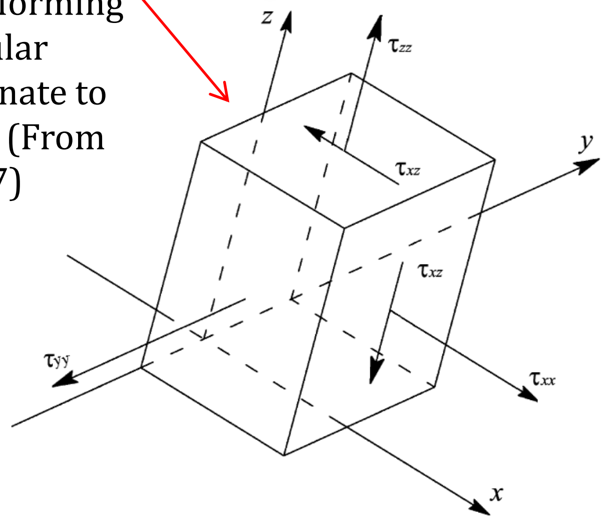


Figure 10: Transforming tensor from regular Cartesian coordinate to local coordinate. (From Pirulli et al. 2007)

Numerical Approach

1. Governing equations in written in conservative form;

2. Two-step projection method used to update horizontal velocities;

3. TVD Finite volume scheme used to evaluate spatial derivatives;

4. Strong Stability Preserving (SSP) Runge-Kutta scheme used for time-stepping.;

5. Vertical velocity is determined using KBBC at bottom, and determined using z momentum equation at surface;

6. Non-hydrostatic pressure correction determined using a Poisson equation, and velocities updated in second step of projection scheme. (This step neglected in hydrostatic model)

$$\left(\frac{L_{4,i,j}}{\Delta y^2} + \frac{L_{5,i,j}}{2\Delta y}\right) q_{b,i,j+1}^{n+1} + \left(\frac{L_{1,i,j}}{\Delta x^2} + \frac{L_{2,i,j}}{2\Delta x}\right) q_{b,i+1,j}^{n+1} + \left[(L_3 + L_6 + L_7)_{i,j} - \frac{2L_{1,i,j}}{\Delta x^2} - \frac{2L_{4,i,j}}{\Delta y^2}\right] q_{b,i,j}^{n+1}$$
$$+ \left(\frac{L_{1,i,j}}{\Delta x^2} - \frac{L_{2,i,j}}{2\Delta x}\right) q_{b,i-1,j}^{n+1} + \left(\frac{L_{4,i,j}}{\Delta y^2} - \frac{L_{5,i,j}}{2\Delta y}\right) q_{b,i,j-1}^{n+1} = \mathrm{RHS}_{i,j}$$

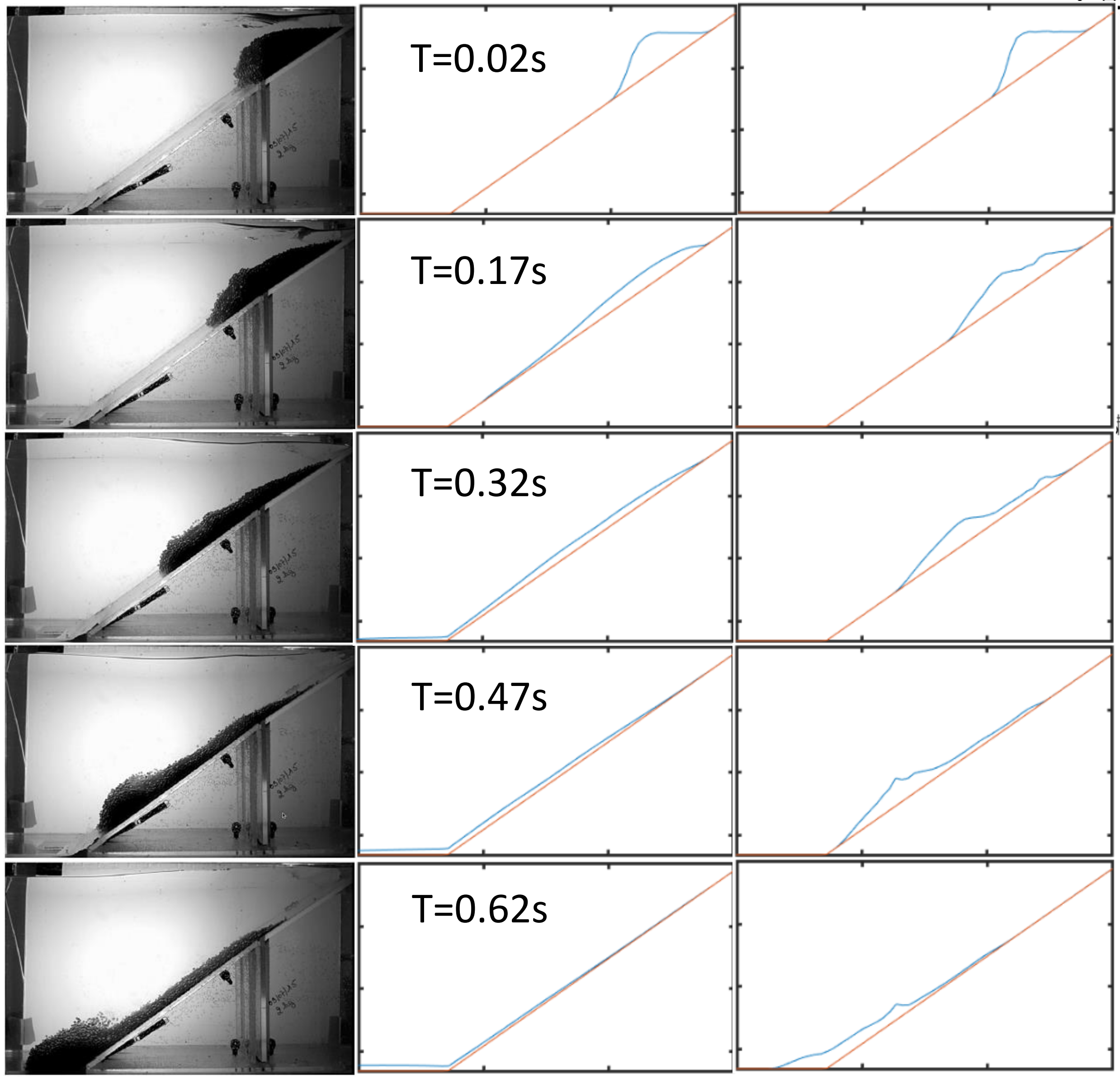
Verification and Validation

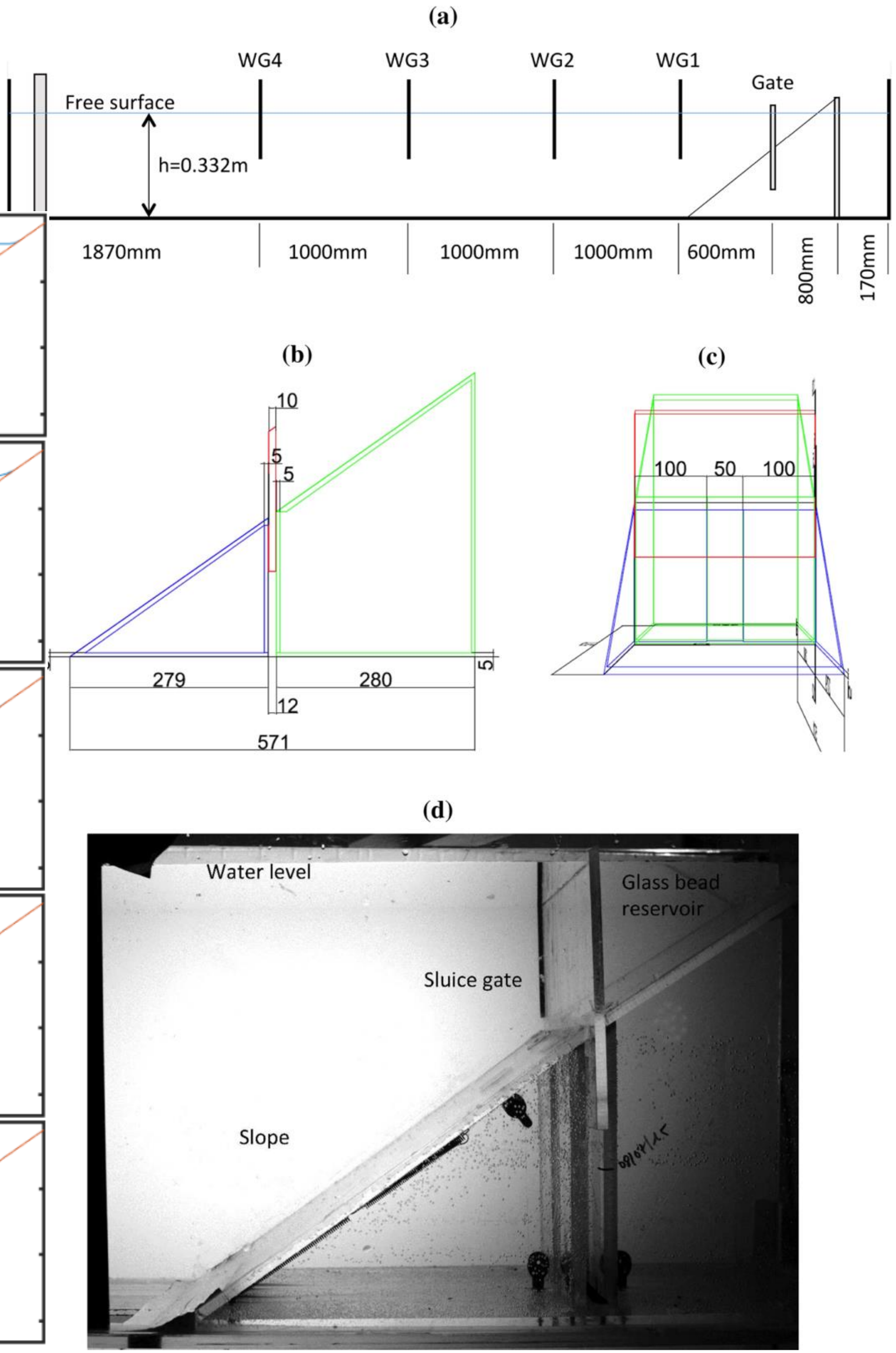
1. Tsunami generation by 2D submarine landslide (Grilli et al., 2017) :

Parameter:
Δx=0.01m
9 sigma layers
CFL=0.5

Viscous model:
ρ=1,951kg/m^3
μ_e=0.01 kg/(m.s)

Granular model:
ρ_s=2,500kg/m^3
ρ_f=1,000kg/m^3
φ = 63.4%
Φ_{int}=34^o
Φ_{bed}=12^o

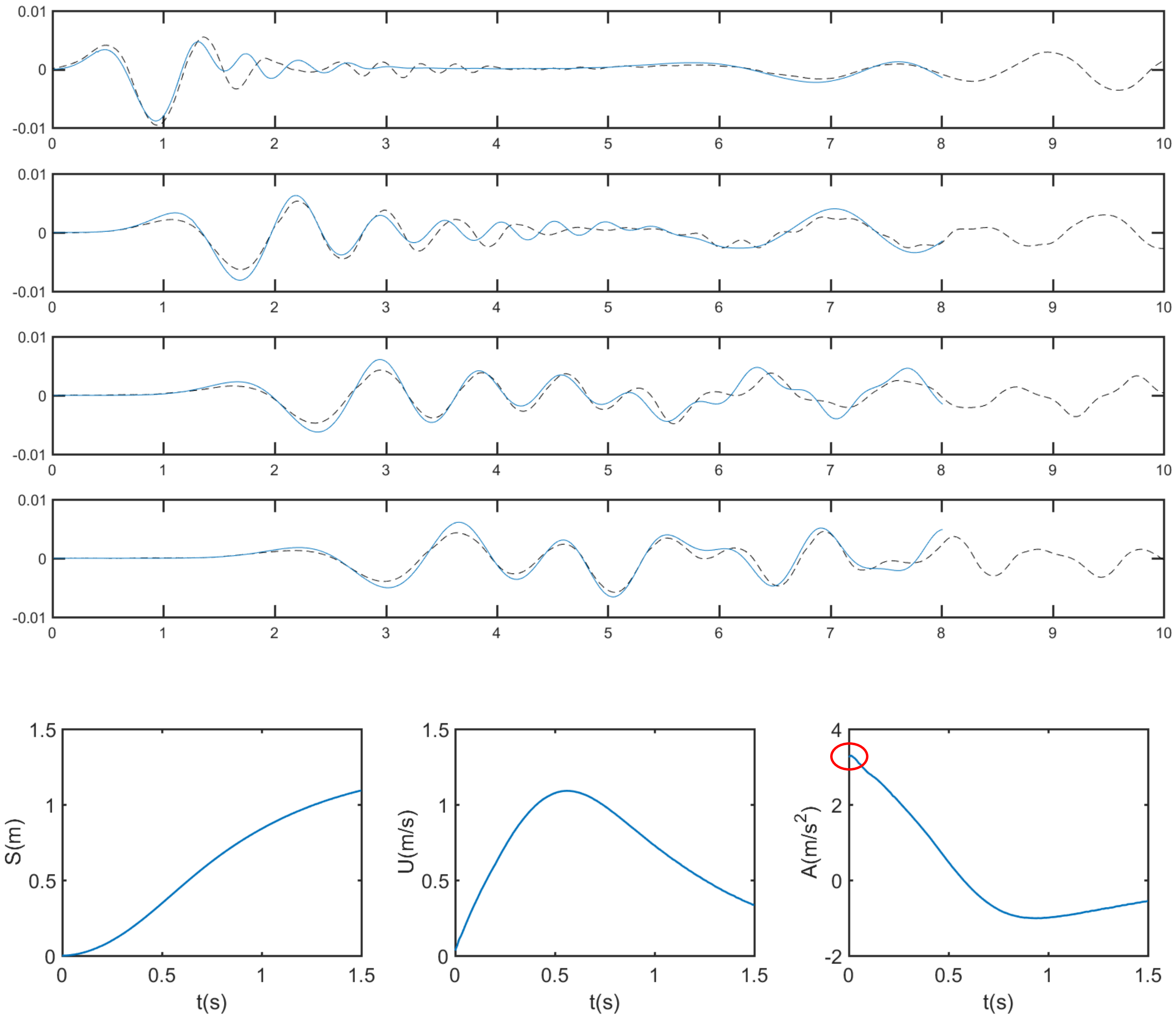




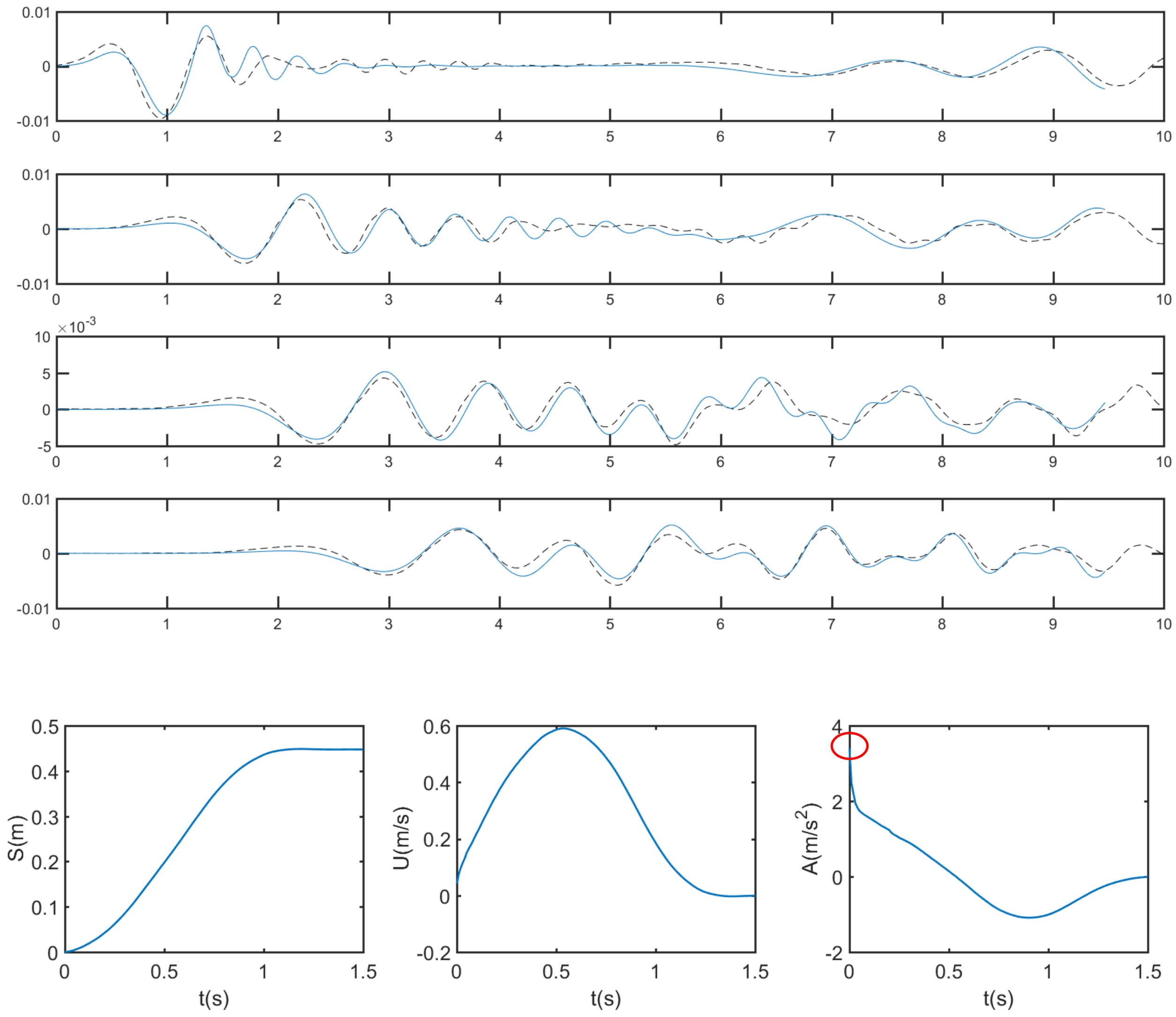
Verification and Validation

2. Tsunami generation by 2D submarine landslide (Grilli et al., 2017) :

Viscous model



Granular model

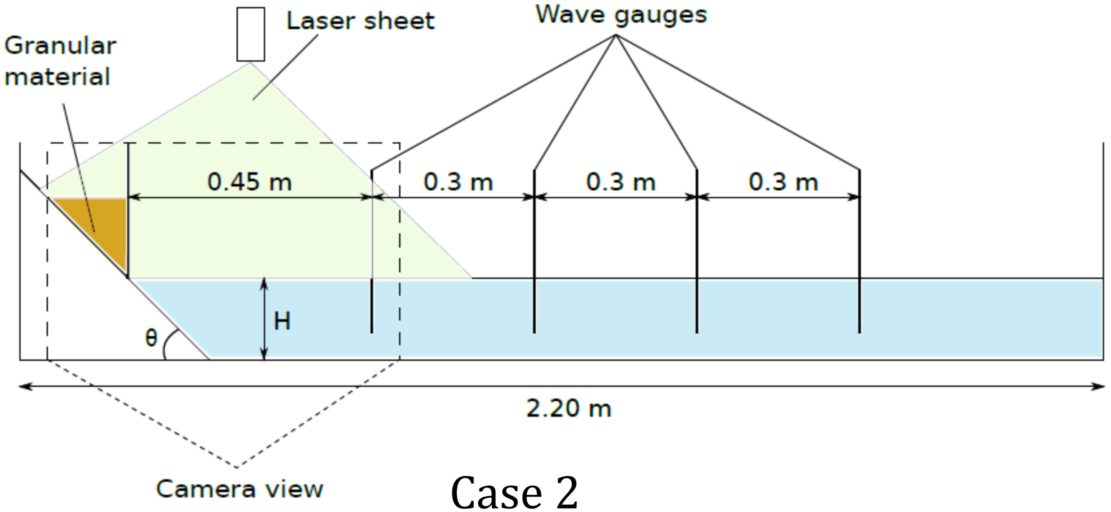


Verification and Validation

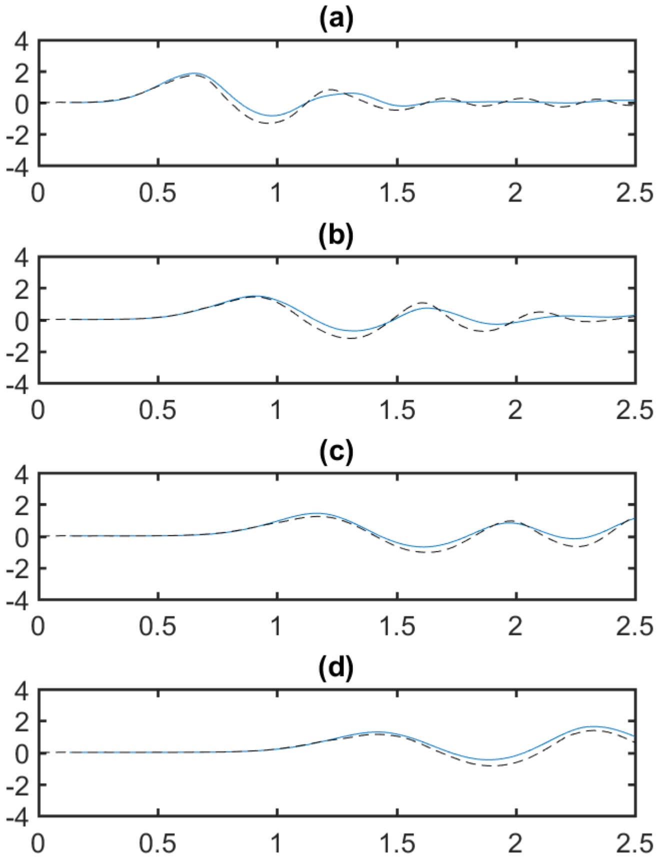
2. Tsunami generation by 2D subaerial landslide (Viroulet et al., 2014) :

Parameter:

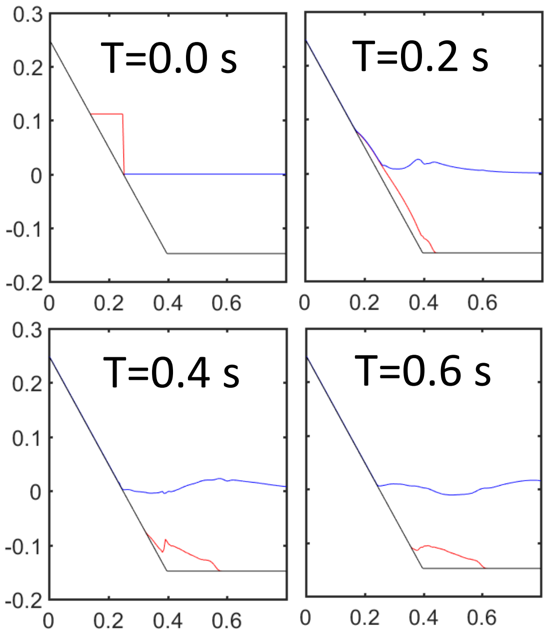
Δx=0.005m	CFL=0.5	ρ_s=2,500kg/m^3
3 sigma layers	φ = 60%	ρ_f=1,000kg/m^3

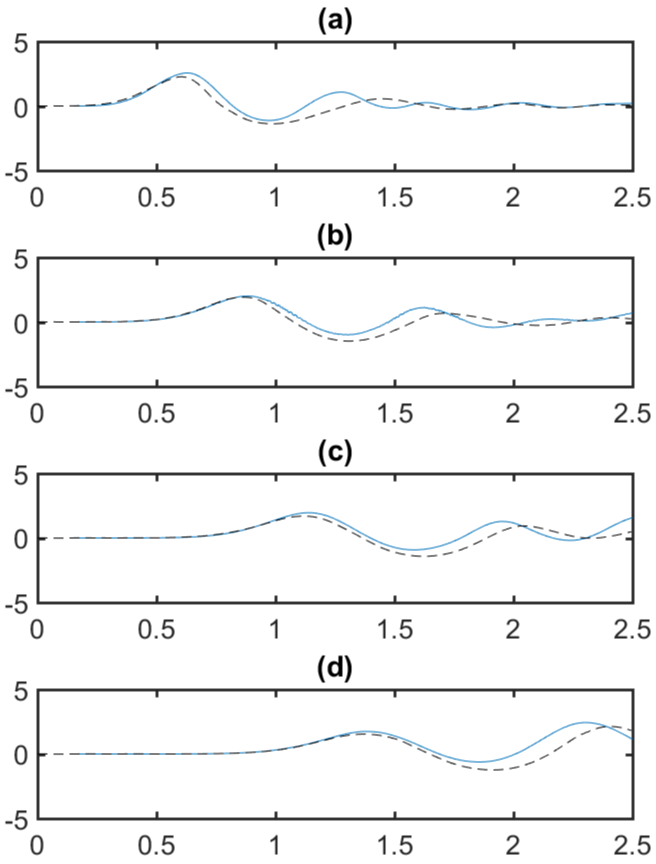


Case 2

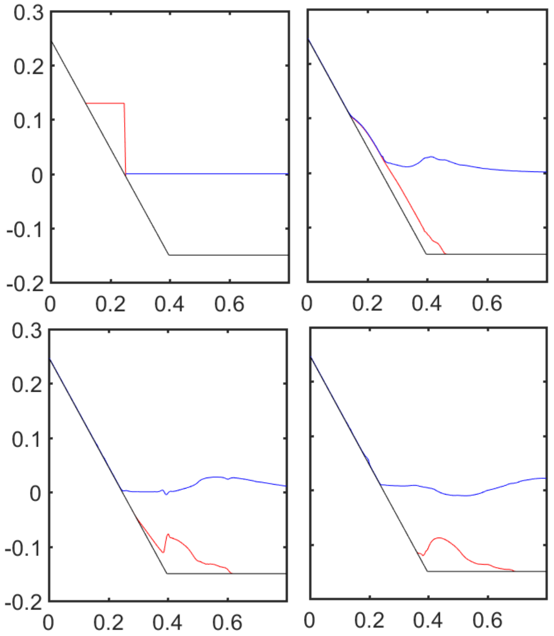


Case 1:
H=14.8cm Φ_{int}=41°
L=11cm Φ_{bed}=25.7°



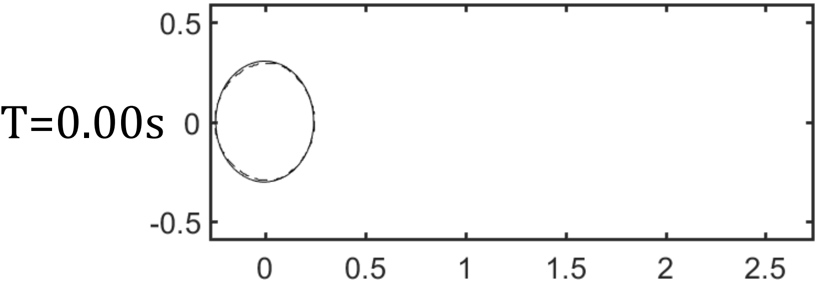


Case 2:
H=15cm Φ_{int}=41°
L=13.5cm Φ_{bed}=21.3°



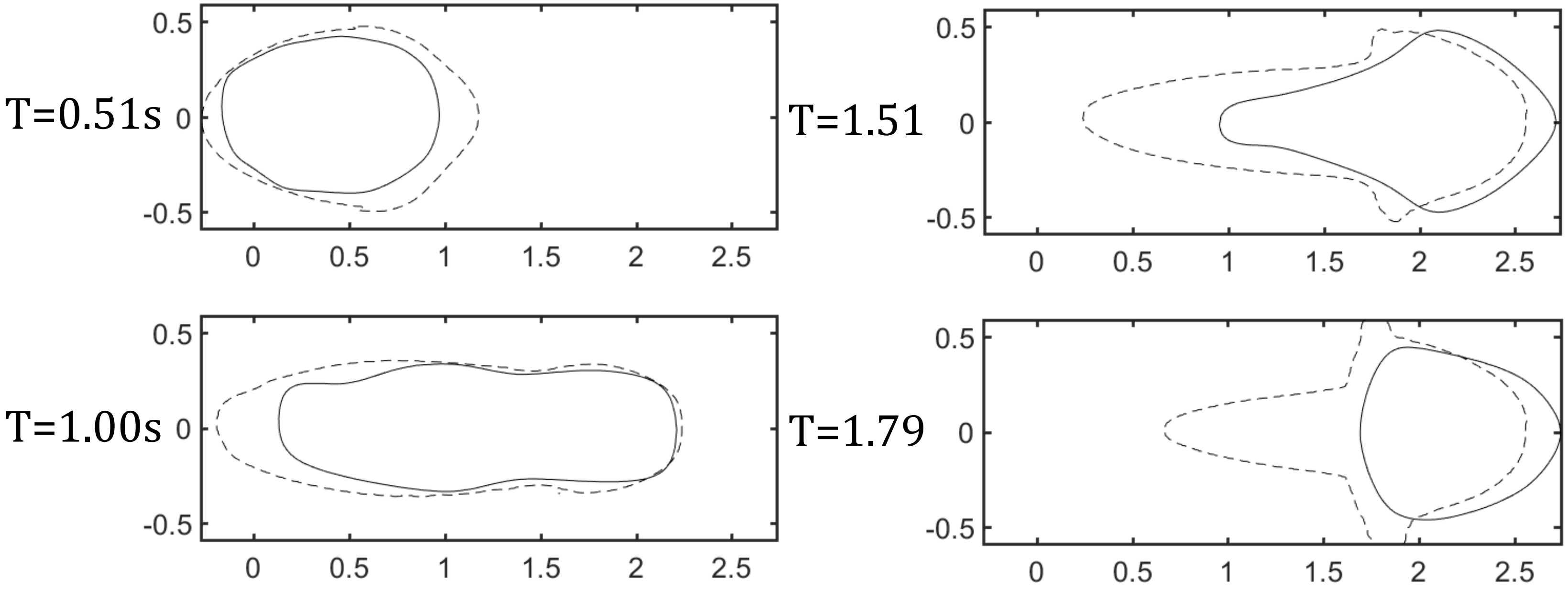
Verification and Validation

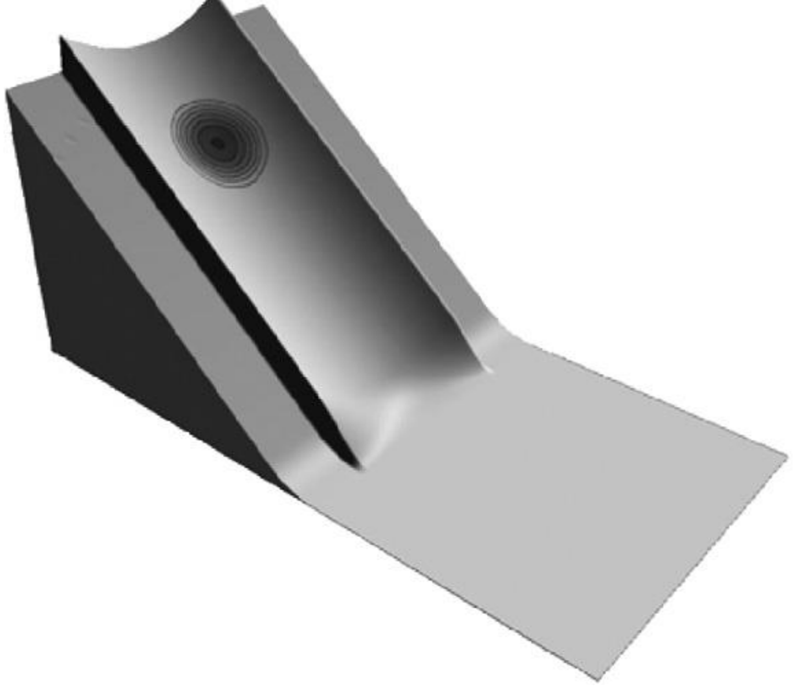
3. 3D granular flow move over irregular bathymetry (Gray et al., 1999) :

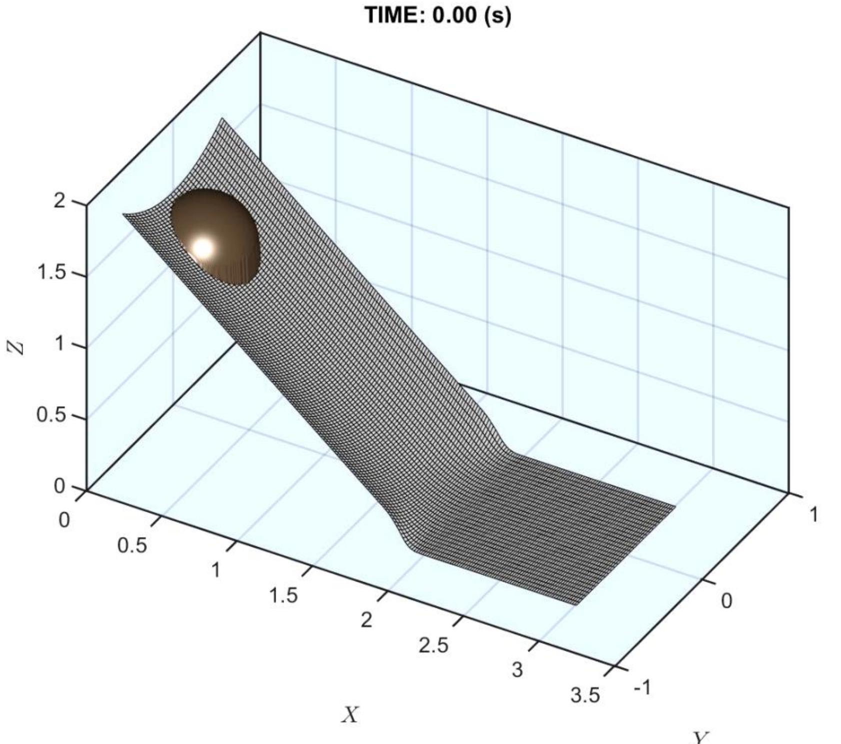


Parameter:
Δx=0.0073658m
Δy=0.0073658m
CFL=0.5

ρ_s=2,500kg/m^3
ρ_f=0kg/m^3
φ = 60.0%
Φ_{int}=40^o
Φ_{bed}=30^o







Thank you!

Any questions?

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