HARMONICS GENERATION BY COASTAL FORESTS

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INTRODUCTION

The linear model on shallow-water wave propagations through a coastal forest has been developed by Mei et al. (2011). In this study, we extend the linear model and consider weakly nonlinear effects. Because of the weakly nonlinear effects, higher harmonics will be generated inside the forest. Wave attenuation is investigated and the new results are compared with those of the linear model. The effects of different physical parameters are also discussed.

THEORETICAL MODEL

Following Mei et al. (2011), the homogenization theory (multi-scale perturbation) is applied to separate the micro-scale flow around cylinders from the macro-scale wave dynamics. To estimate wave dissipation by forests, a bulk value of eddy viscosity within a homogeneous area is assumed and can be determined by an empirical formula. To decompose each harmonic (m) component, we express the velocity and free surface elevation as

$$u_i = \frac{1}{2}\sum_{m=-\infty}^{\infty}\tilde{u}_{i,m}e^{-imt}\,,\ \eta = \frac{1}{2}\sum_{m=-\infty}^{\infty}\tilde{\eta}_m e^{-imt} \qquad (1)$$

in which $\tilde{u}_{i,-m}$ and $\tilde{\eta}_{-m}$ are the complex conjugates of $\tilde{u}_{i,m}$ and $\tilde{\eta}_m$, respectively. The governing equations for both micro- and macro-scale problems and the numerical algorithms used to solve them are introduced as follows.

MICRO-SCALE (CELL) PROBLEM

Focusing on the flow motion in a micro-scale cell, the governing equations for each harmonic yield

$$\partial\tilde{u}^{(0)}_{i,m}/\partial x_i = 0 \qquad (2)$$

and

$$-im\tilde{u}^{(0)}_{i,m} + \frac{\alpha_n}{2}\sum_{m_1=-\infty}^{\infty}\tilde{u}^{(0)}_{j,m_1}\frac{\partial\tilde{u}^{(0)}_{i,m-m_1}}{\partial x_j} = -\frac{\partial\tilde{\eta}^{(0)}_m}{\partial X_i} - \frac{\partial\tilde{\eta}^{(1)}_m}{\partial x_i} + \sigma\frac{\partial^2\tilde{u}^{(0)}_{i,m}}{\partial x_j^2} \qquad (3)$$

in which the forcing term $\partial\tilde{\eta}^{(0)}_m/\partial X_i$ is given by the macro-scale solutions. The parameter $\alpha_n = (A/h)/(k\ell)$ is assumed to be $\mathcal{O}(1)$, indicating weak nonlinearity. Here we employ a modified pressure correction method (Anderson 1995) to solve the nonlinear cell problem, requiring a proper initial guess of both pressure and velocity fields for iterative solutions. Within each iteration, the corrections on pressure and velocity are constructed until converged results are achieved.

MACRO-SCALE PROBLEM

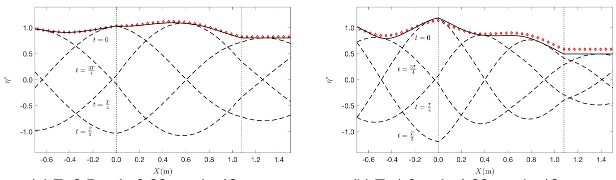
The cell-averaged equations governing macro-scale wave dynamics are derived:

$$\frac{\partial^2\tilde{\eta}^{(0)}_m}{\partial X_i^2} + m^2\tilde{\eta}^{(0)}_m = -\frac{\alpha_n}{n}\frac{\partial\tilde{M}_m}{\partial X_i} - \frac{1}{n}\frac{\partial\tilde{N}_m}{\partial X_i} + \frac{\sigma}{n}\frac{\partial\tilde{Q}_m}{\partial X_i} \qquad (4)$$

where $\tilde{M}_m$, $\tilde{N}_m$ and $\tilde{Q}_m$ are all obtained from the solutions for cell problem: $\tilde{M}_m = \frac{1}{2\Omega}\iint_{\Omega_f}\left[\sum_{m_1=-\infty}^{\infty}\left(\tilde{u}^{(0)}_{j,m_1}\,\partial\tilde{u}^{(0)}_{i,m-m_1}/\partial x_j\right)\right]d\Omega$, $\tilde{N}_m = \frac{1}{\Omega}\iint_{\Omega_f}\left(\partial\tilde{\eta}^{(1)}_m/\partial x_i\right)d\Omega$ and $\tilde{Q}_m = \frac{1}{\Omega}\iint_{\Omega_f}\sigma\left(\partial^2\tilde{u}^{(0)}_{i,m}/\partial x_j^2\right)d\Omega$, in which Ω represents a unit cell and Ω_f denotes the fluid part within Ω. As indicated, the macro-scale pressure gradient serves as the driven force for the cell problem. Therefore, another iterative scheme is introduced to solve this coupled system.

NUMERICAL RESULTS

Here we consider shallow-water waves through a homogeneous forest belt with an infinite length. Assuming normal incident waves of simple harmonic, higher harmonic waves are generated inside the forest region and radiated into open water as reflected and transmitted waves. In Fig.1, two cases with different values of nonlinearity are presented. The linear model results are also plotted for comparison. It can be observed that higher nonlinearity brings about more dissipation of wave energy. Each harmonic component can also be found in Fig.2, which shows that the first mode is dominant while higher modes present relatively smaller amplitude. Different nonlinearity and the effects on reflection and transmission coefficients are also investigated as shown in Fig.3. More analyses will be presented in the conference.



(a) T=2.5 s, A=0.33 cm, h=12 cm (b) T=1.9 s, A=1.22 cm, h=12 cm

Figure 1 - Comparison of wave amplitude variation. Dashed lines show snapshots of dimensionless free surface while solid lines denote the upper wave envelopes. Diamonds present the linear model solutions.

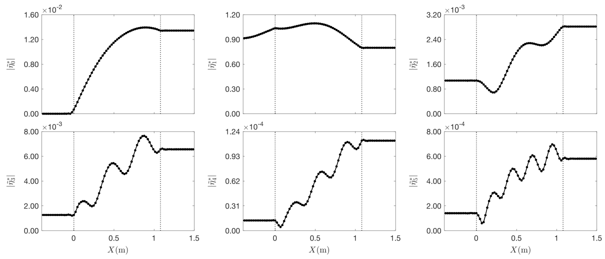


Figure 2 - Wave amplitude variation for each mode (T=2.5 s, A=0.33 cm, h=12 cm)

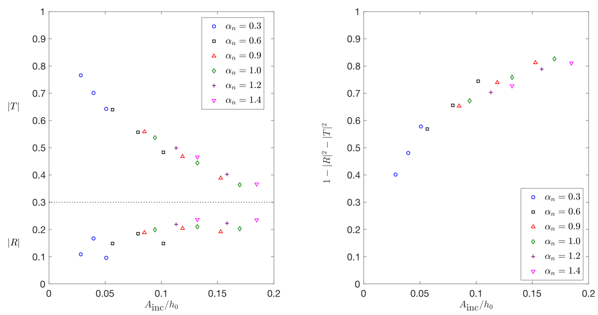


Figure 3 - Comparisons of reflection $|R|$ and transmission coefficients $|T|$. $1-|R|^2-|T|^2$ shows the percentage of energy dissipation by the forest.

REFERENCES

Mei, Chan, Liu, Huang and Zhang (2011): Long waves through emergent coastal vegetation, Journal of Fluid Mechanics, CAMBRIDGE, vol. 687, pp. 461-491.

Anderson (1995): Computational Fluid Dynamics - The Basics with Applications, McGraw-Hill Inc.