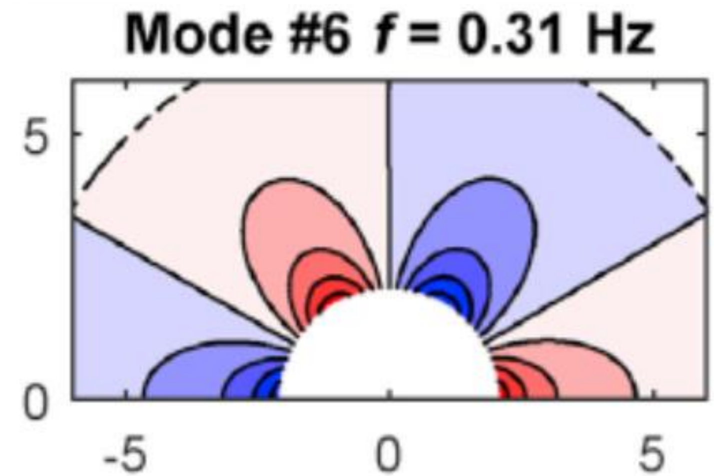
Empirical and numerical modal analysis of coastal areas prone to tsunami risk



G. Bellotti[1], A. Romano[2]

[1] Roma Tre University, [2] Sapienza Università di Roma

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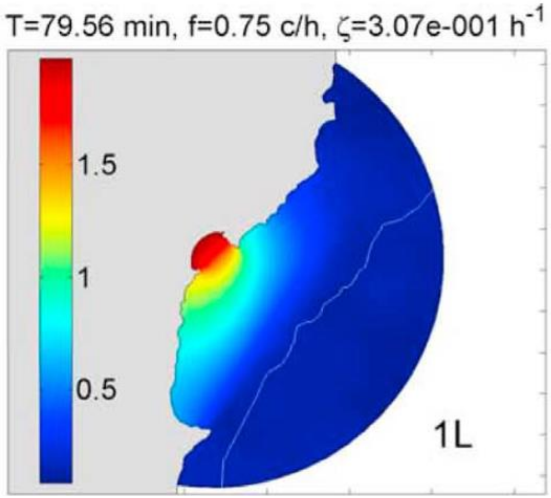
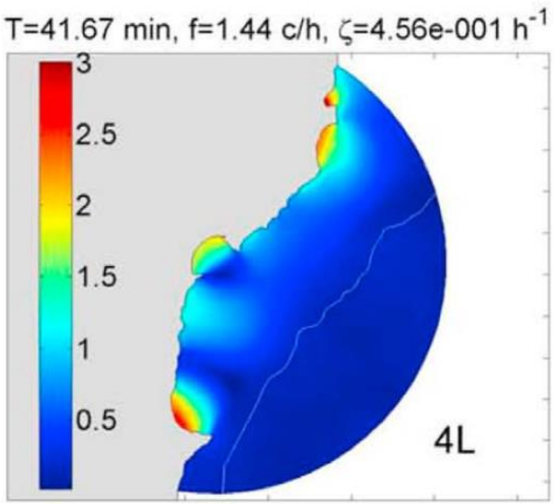
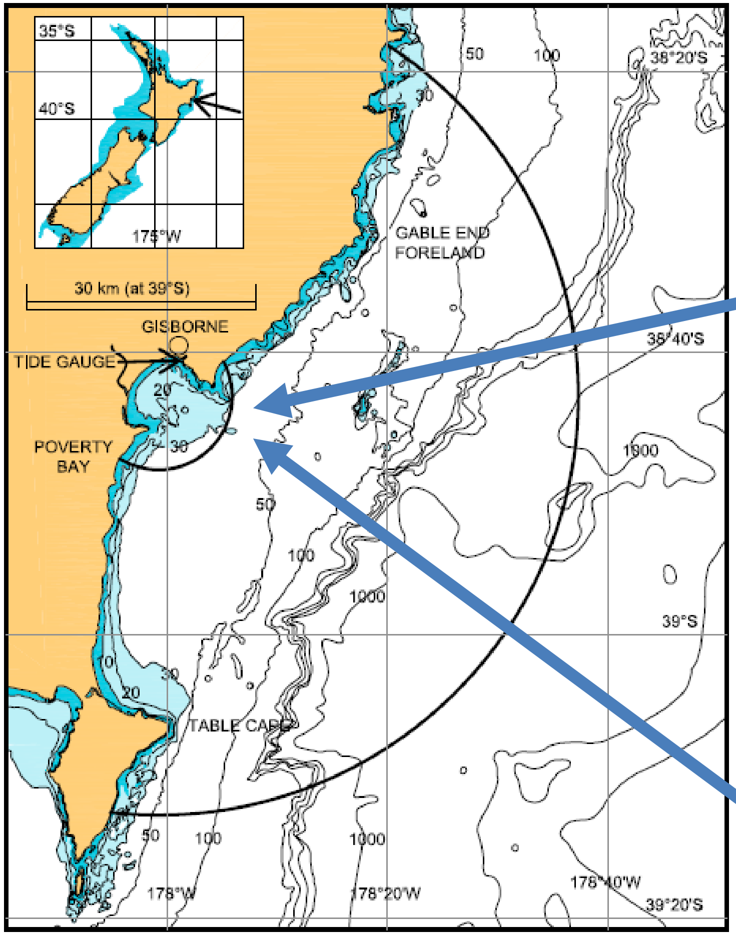
The importance of the modal response (natural modes) on tsunami amplification

- Shelf effects

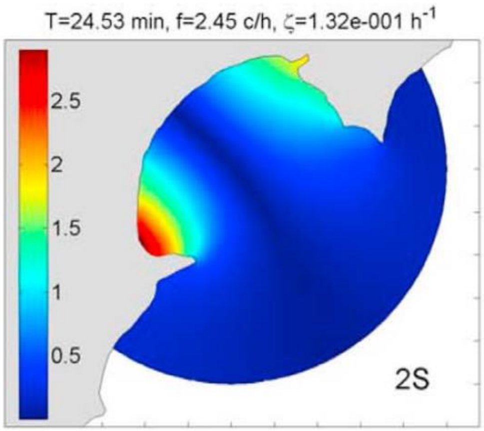


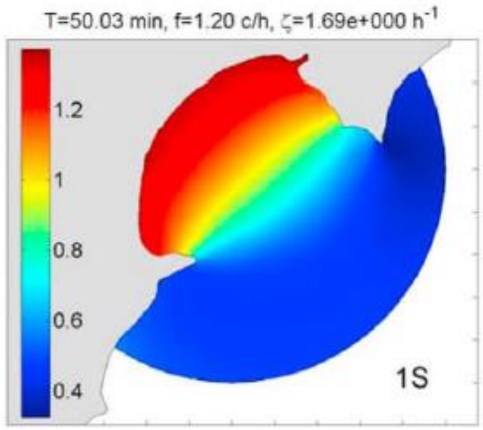




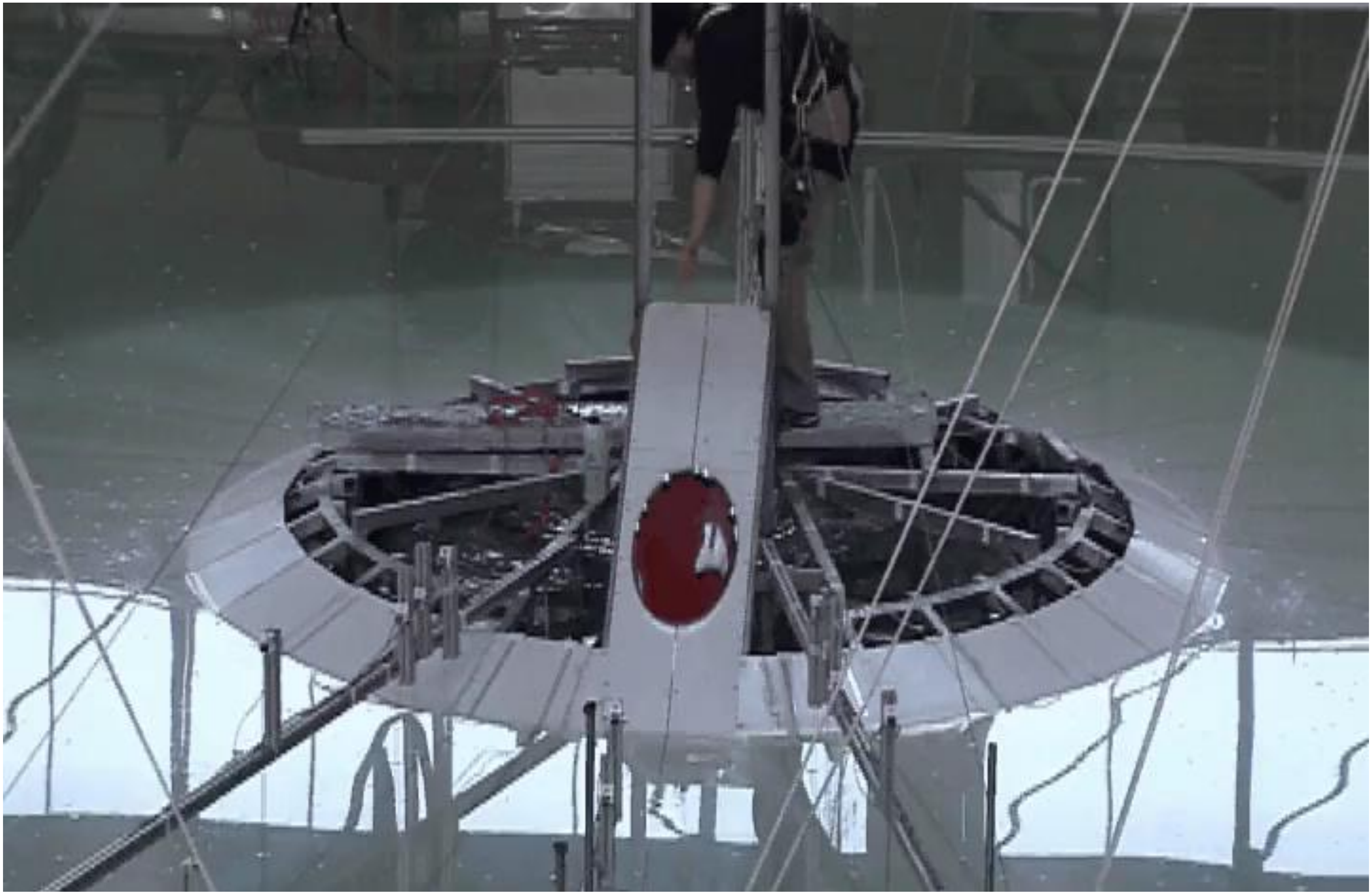
Poverty Bay (NZ)

- Bay effects



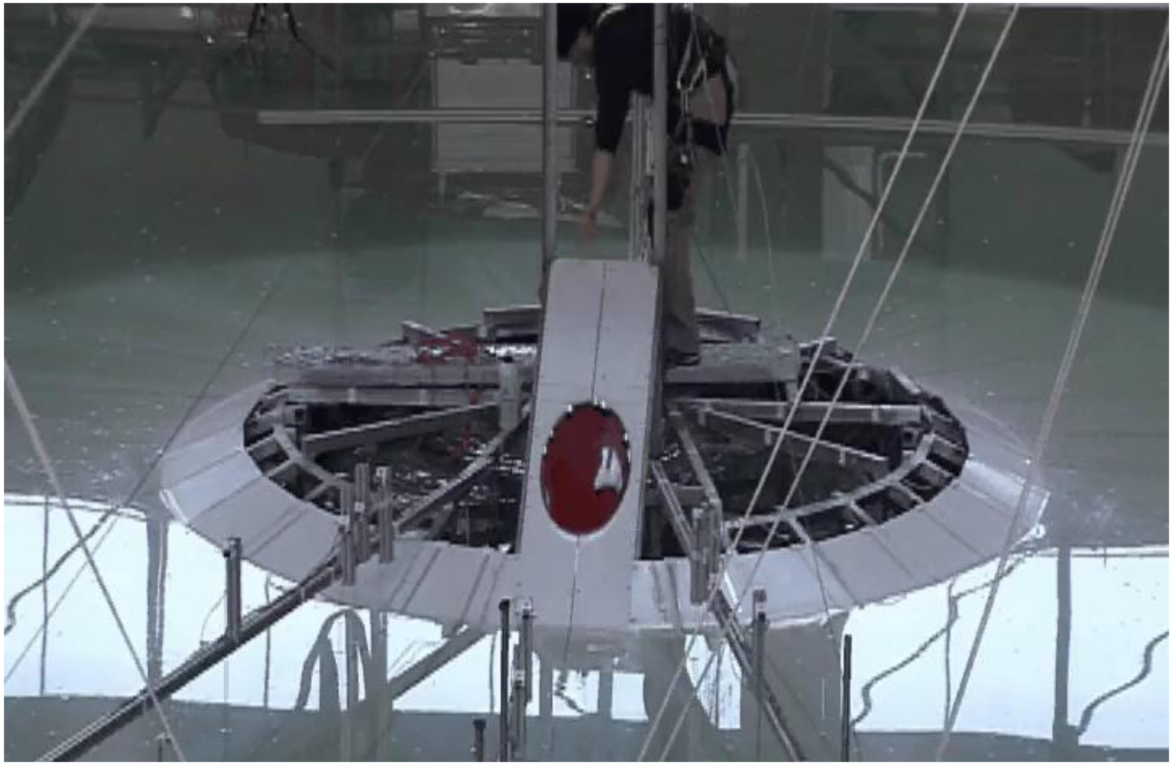


Edge waves generated by landslides?



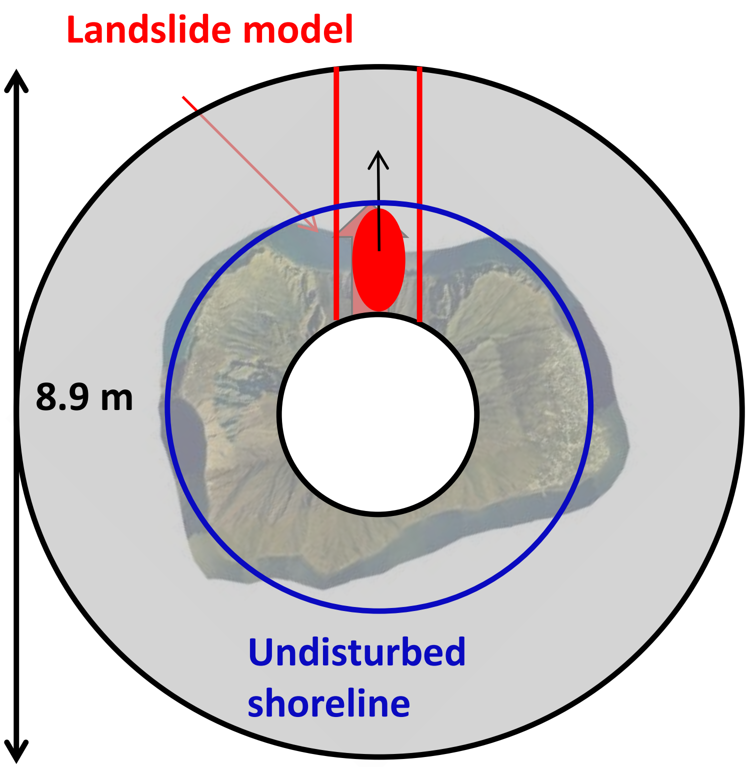
Index of the presentation

- Brief description of the experimental layout
    - <u>Laboratory experiments with moveable wave gauges</u>
- k-f analysis for the trapped waves
- EOF analysis for the trapped waves
- Numerical modal analysis



The conical island: a small simplified Stromboli

The physical model represents a truncated conical island (base radius 4.45 m). The slope of the island's flanks is 1:3 (1 vertical, 3 horizontal). A flat slope (0.5 m wide) allows the model to slide along the flank and to enter the water. The physical model roughly reproduces the small volcanic island Stromboli (South Tyrrenian Sea, Italy) in a Froude law scale (1:1000).



Stromboli island

Technical University of Bari (Italy)

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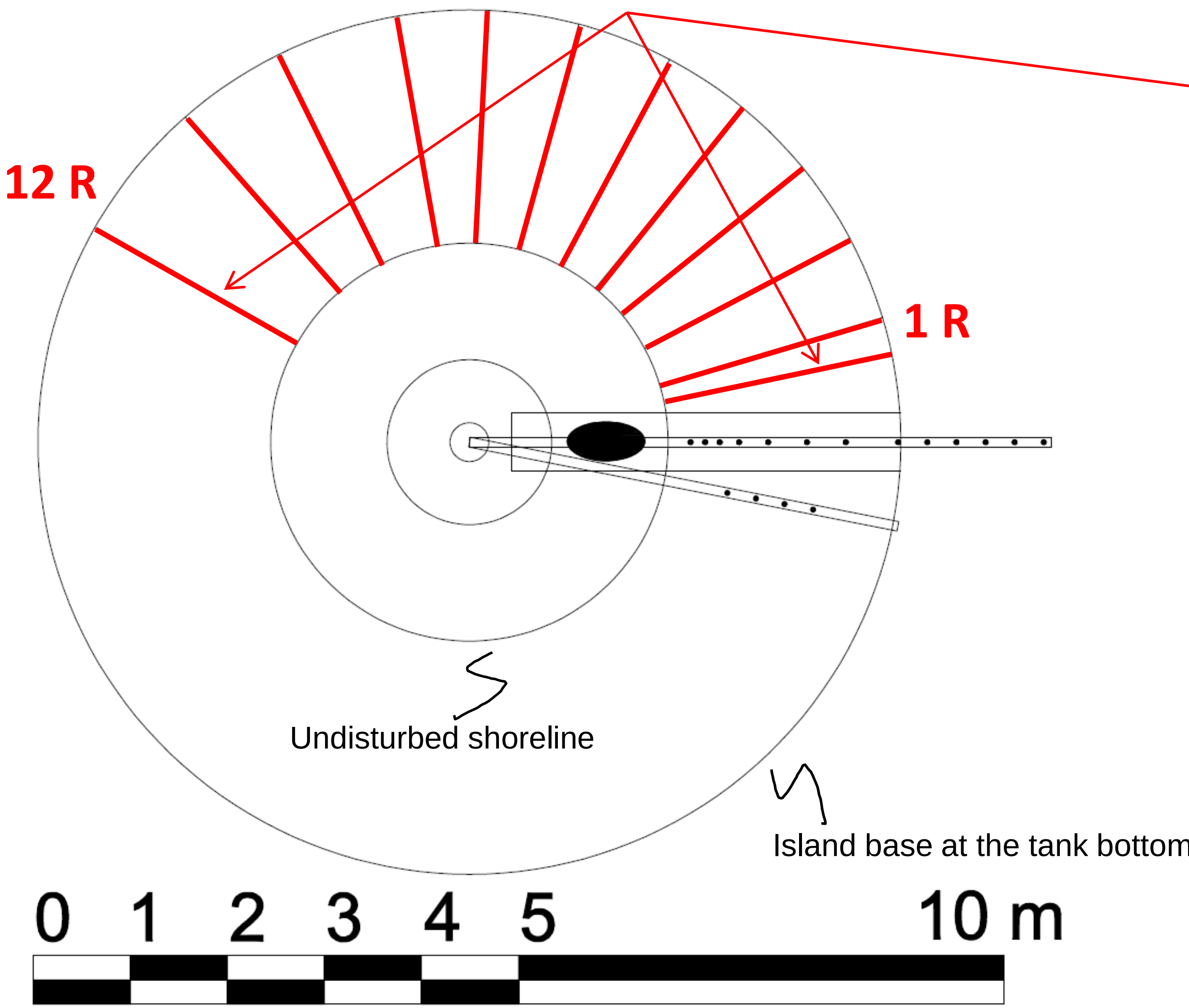
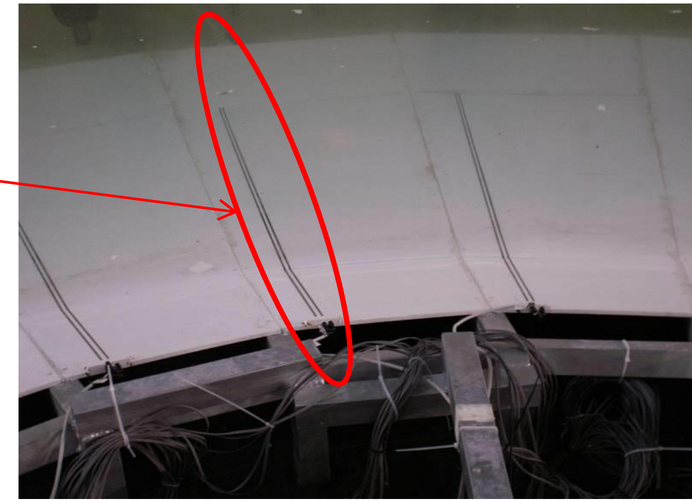
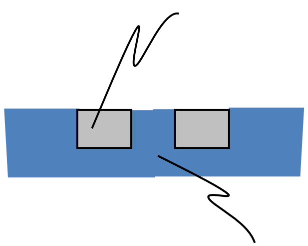
The run-up gauges

12 run-up gauges embebbed in the PVC sheets

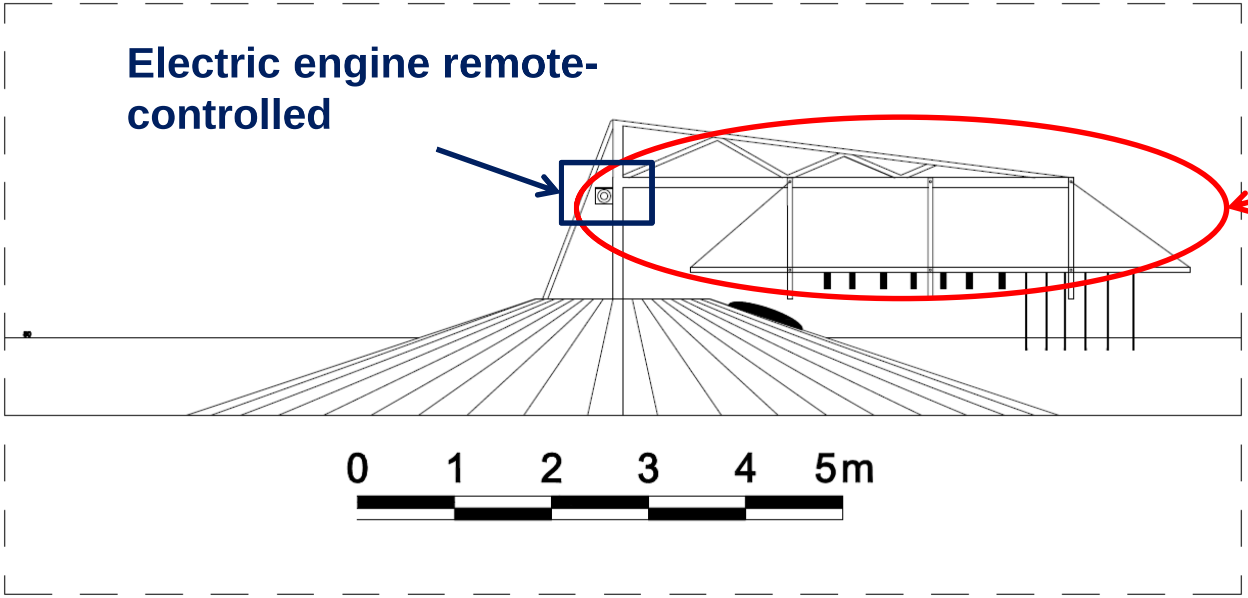




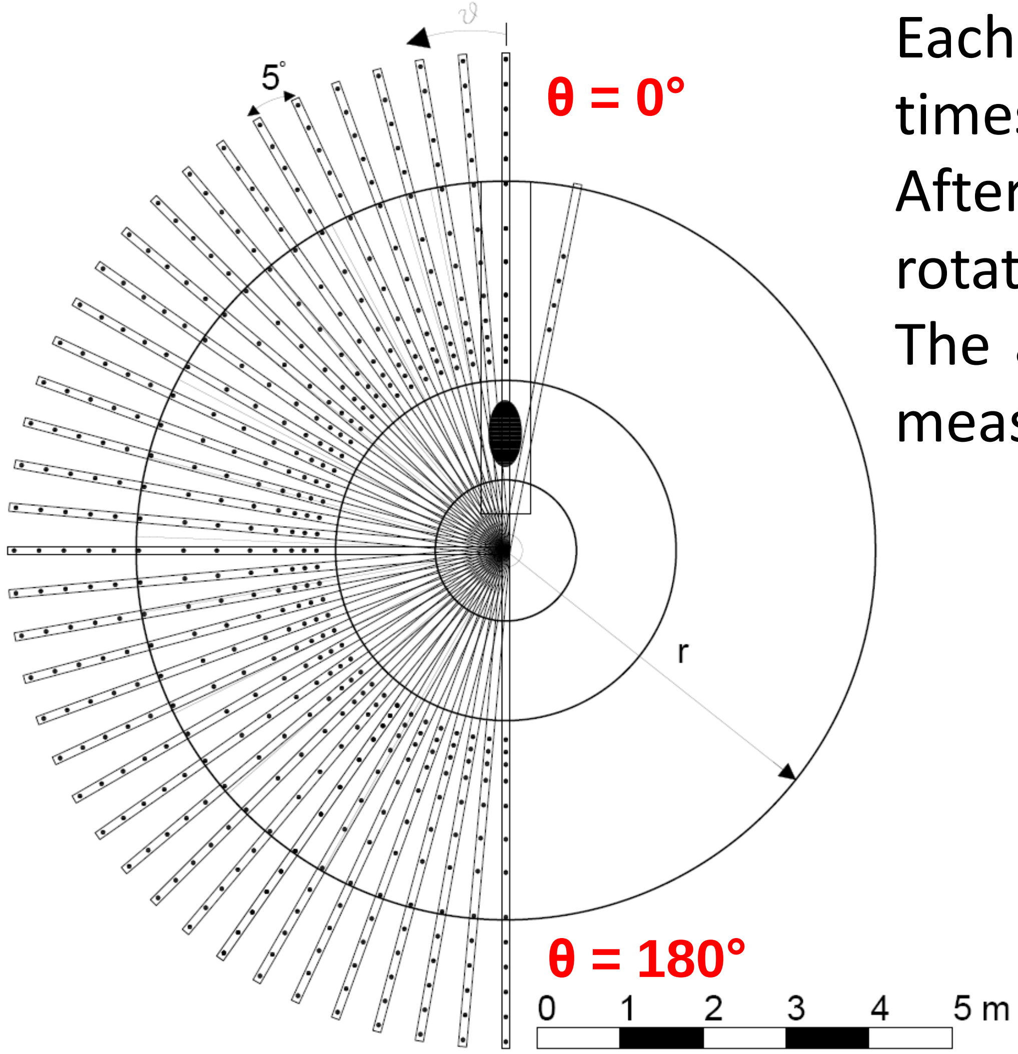


A movable arm to change the position of wave gauges

13 wave gauges are installed on a rotating arm, which is placed at the island center. An electric engine, remotely-controlled rotates it along an angular sector of **180°**, with steps of **5°**.



A movable arm to change the position of wave gauges



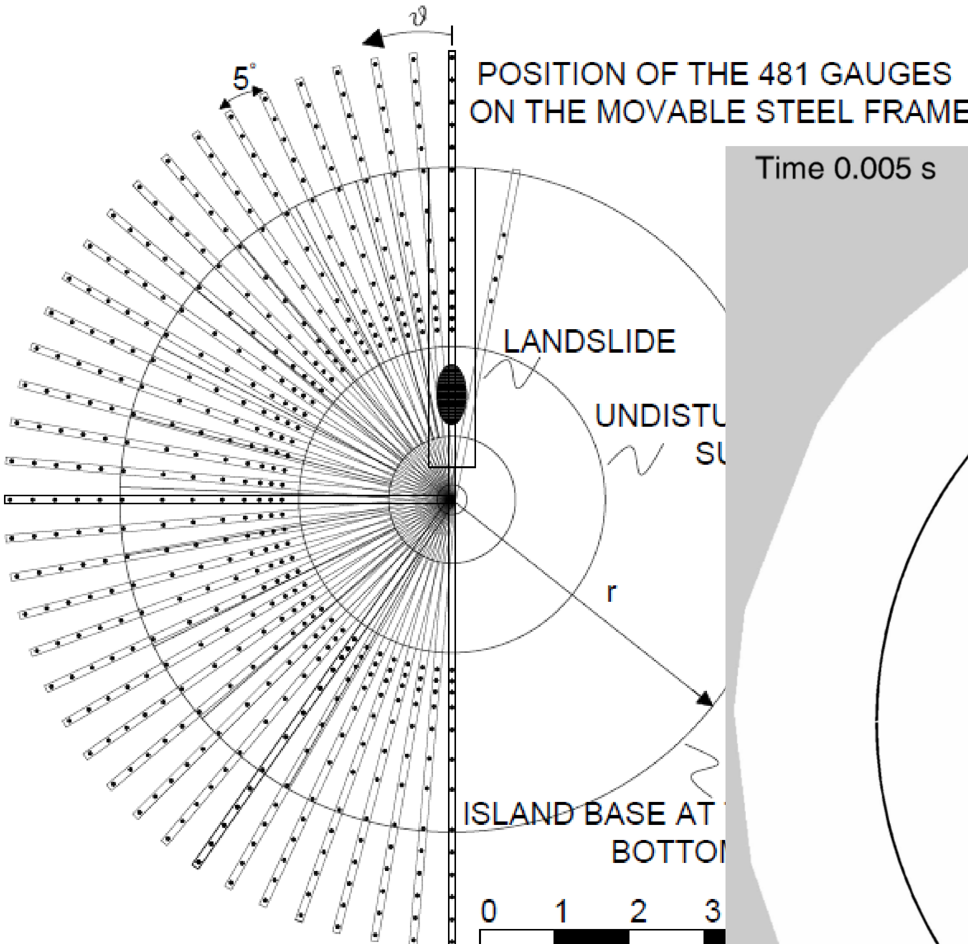
θ = Angle between the path of the landslide and the rotating arm

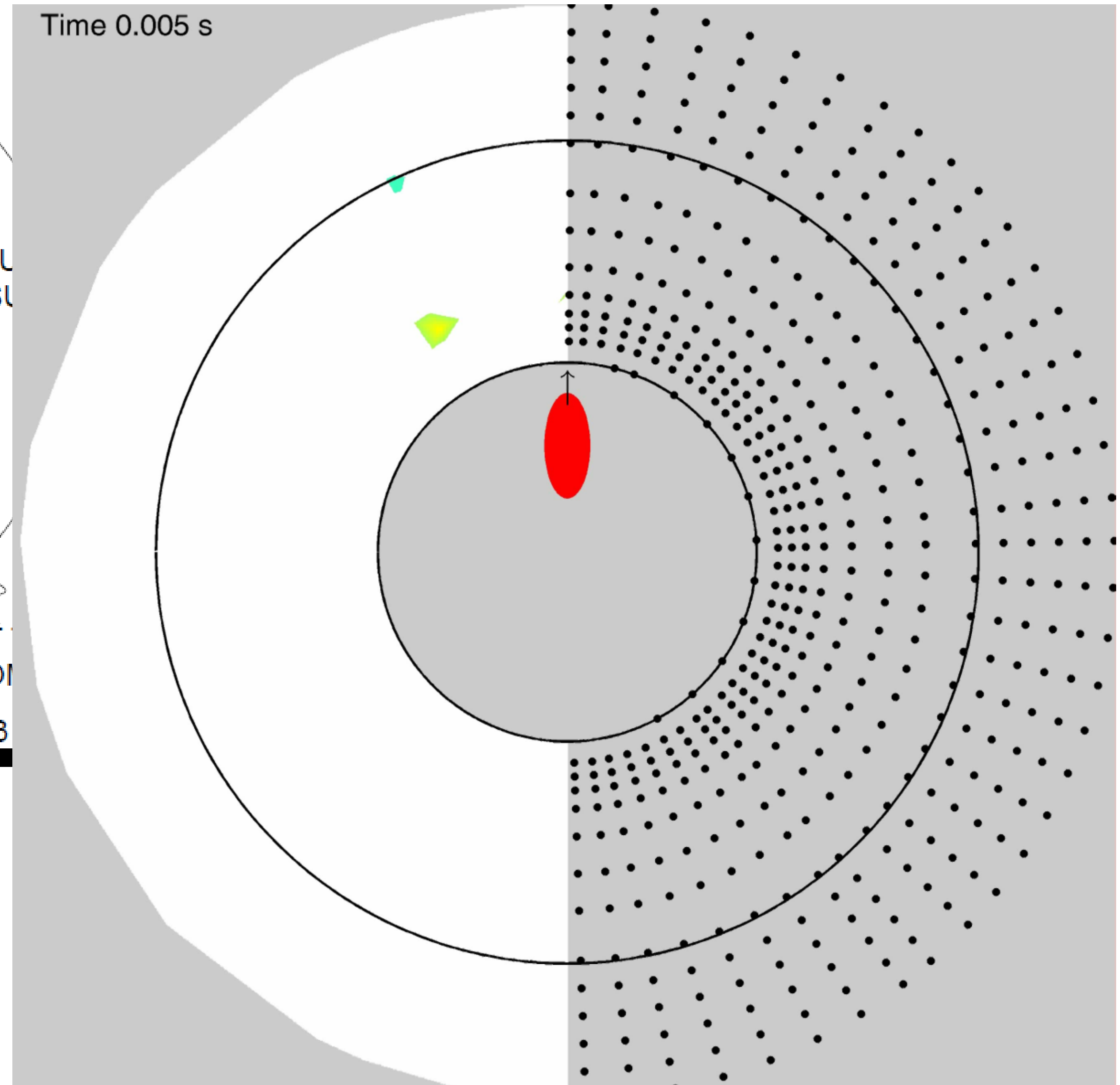
Each experiment is repeated 37 times.
After each test the arm has been rotated by 5°.
The arm position has been carefully measured by a theodolite.

⇩

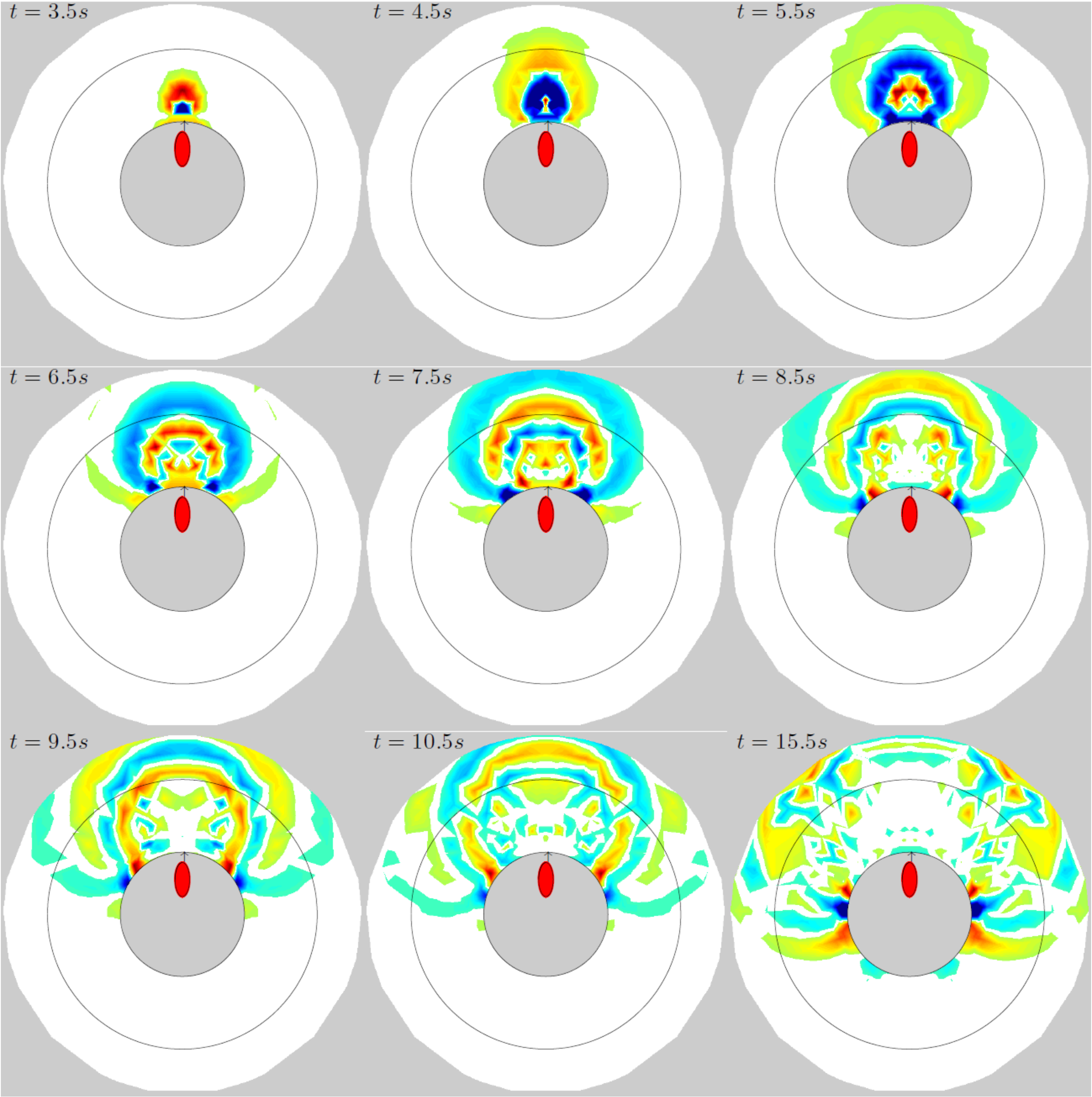
Once the repeatibility of the experiments is ensured, more than **500** punctual free surface elevation time series are available

Animation of the results

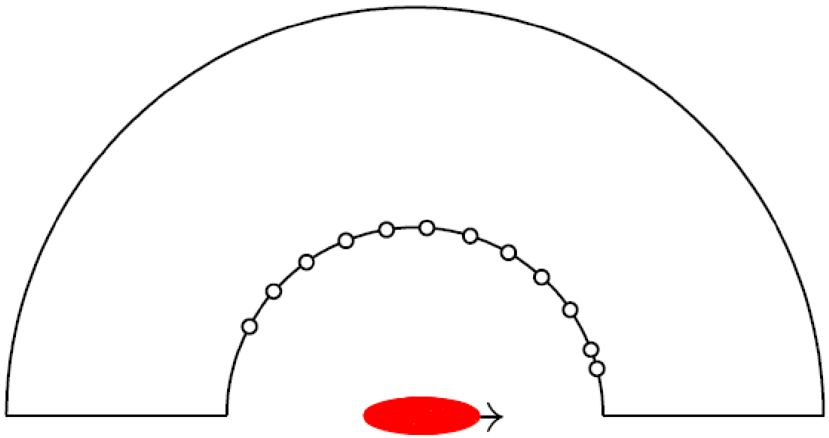




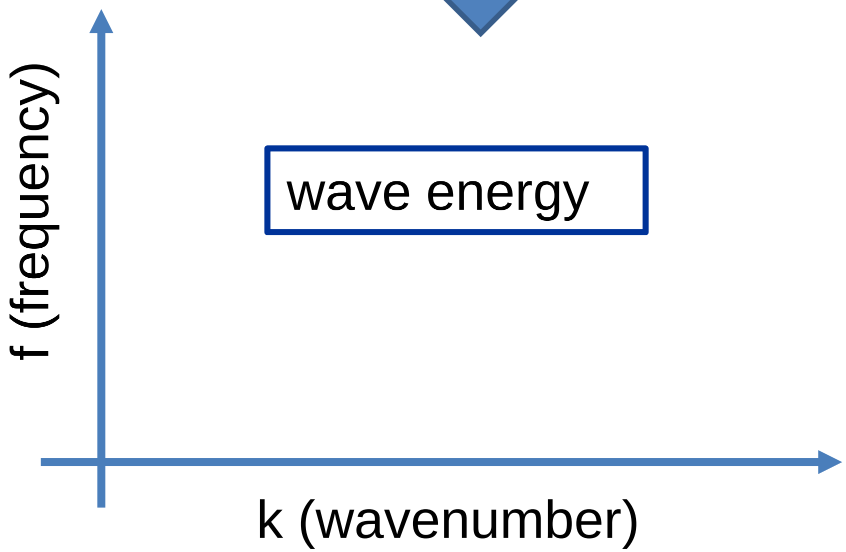
Edge waves travelling around the island?



Wavenumber-frequency (k-f) analysis of the results



The run-up time series are processed to obtain

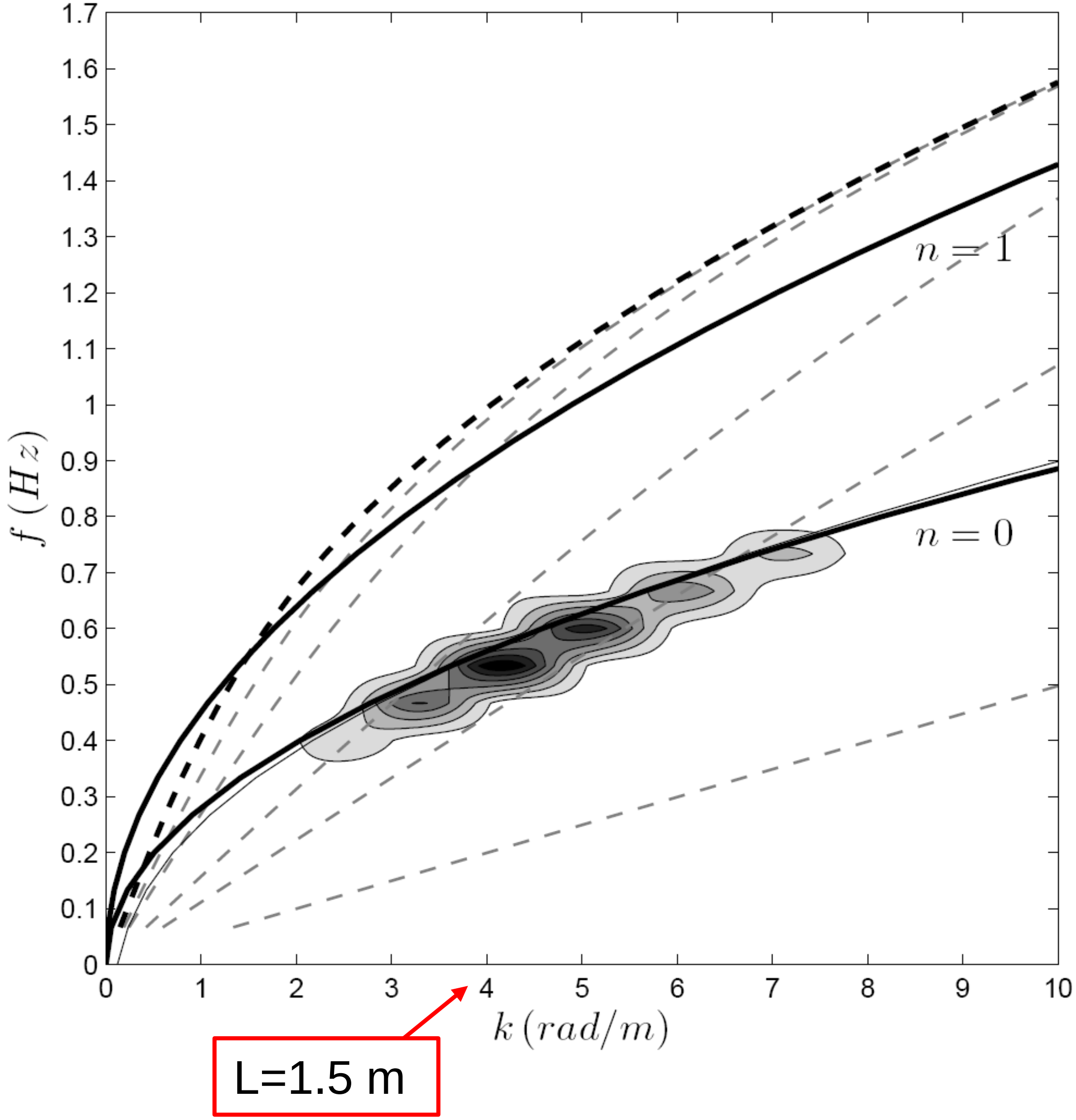


The wave energy distribution in the k-f domain

Romano A., Bellotti G., Di Risio M. (2013). Wavenumber–frequency analysis of the landslide-generated tsunamis at a conical island. Coastal Engineering, vol. 81, pp. 32-43.

Wavenumber-frequency (k-f) analysis of the results

The 1D k-f, applied to the run-up time series, shows that the waves propagate along the shore as a **0th-order edge waves packet** or Stokes edge waves (*Ursell, 1952*).



Dispersion relationships:

- Deep water limit
- Edge waves (n = 0, 1)
- Edge waves (circular shoreline)
- Small amplitude waves

Edge waves (*Ursell, 1952*)

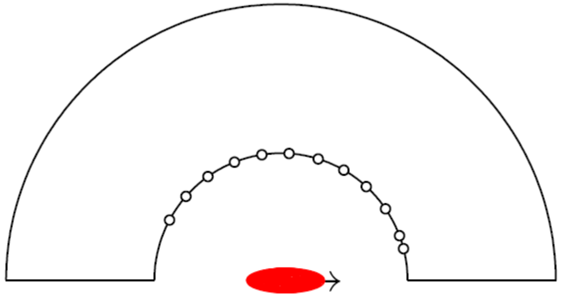
$$\omega^2 = g\,k\,\sin\left[(2n+1)\,\beta\right]$$

Edge waves (circular shoreline, *Smith & Sprinks, 1975*)

$$\omega^2 = g\,k\,\tan\beta\left[1-\frac{1}{4}(k\,r_s)^{-1}\right]^2$$

Small amplitude waves

$$\omega^2 = g\,k\,\tanh\,(kh)$$



The empirical orthogonal function method

$$\eta(x, y, t) = \sum_{n=1}^{N} \alpha_n(t)\, \Gamma_n(x, y)$$

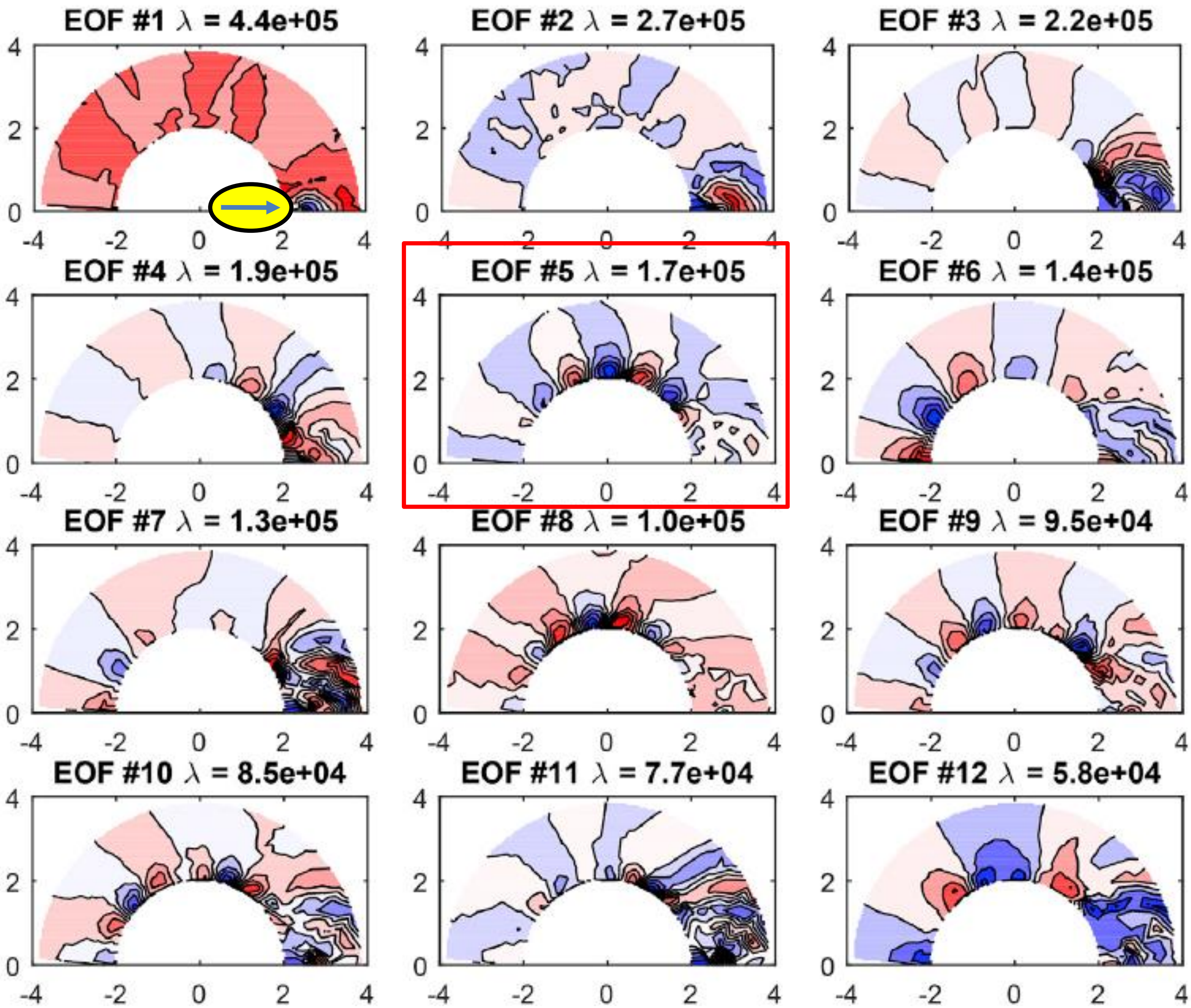
Surface elevation

Time function of the n^{th} mode

n^{th} spatial mode

It is possible to calculate the amount of variance λ_n explained by each mode

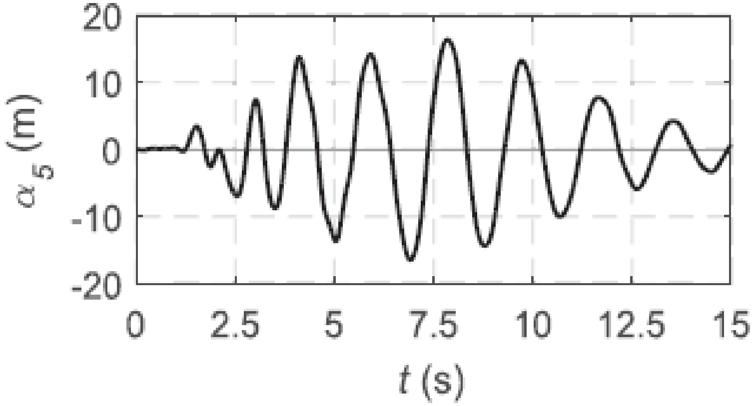
The spatial modes calculated by the EOF method

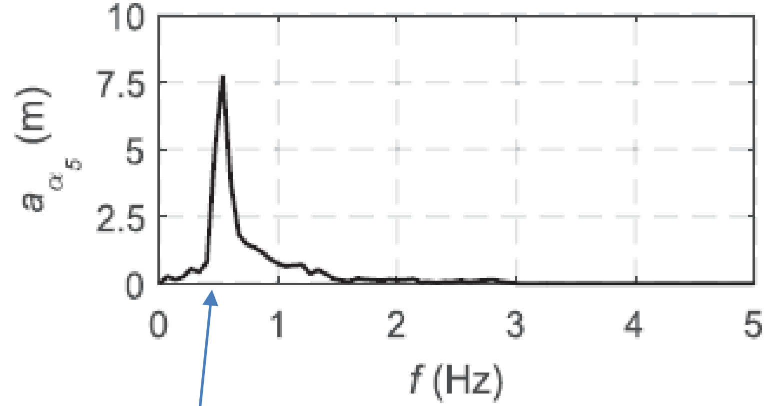


Further processing the EOF modes

Time function for the mode n=5

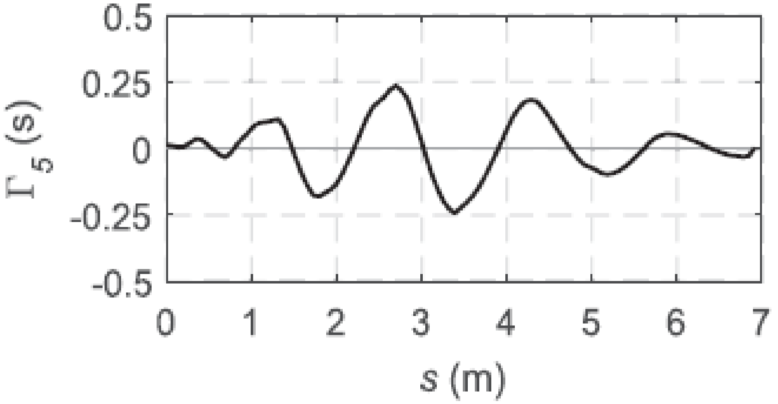
Frequency spectrum

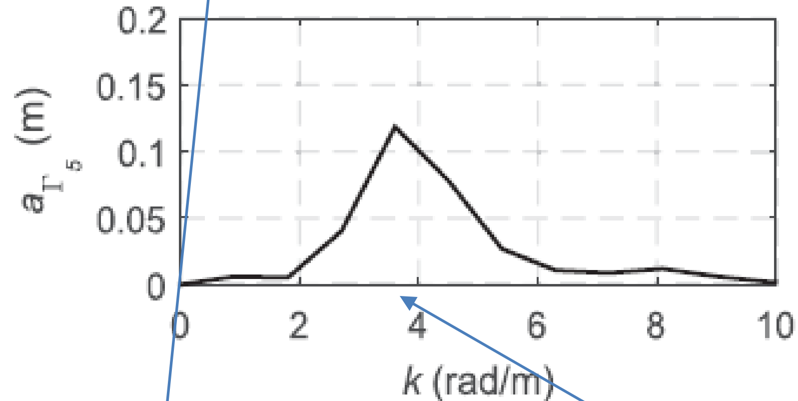




Spatial mode elevation along the shoreline

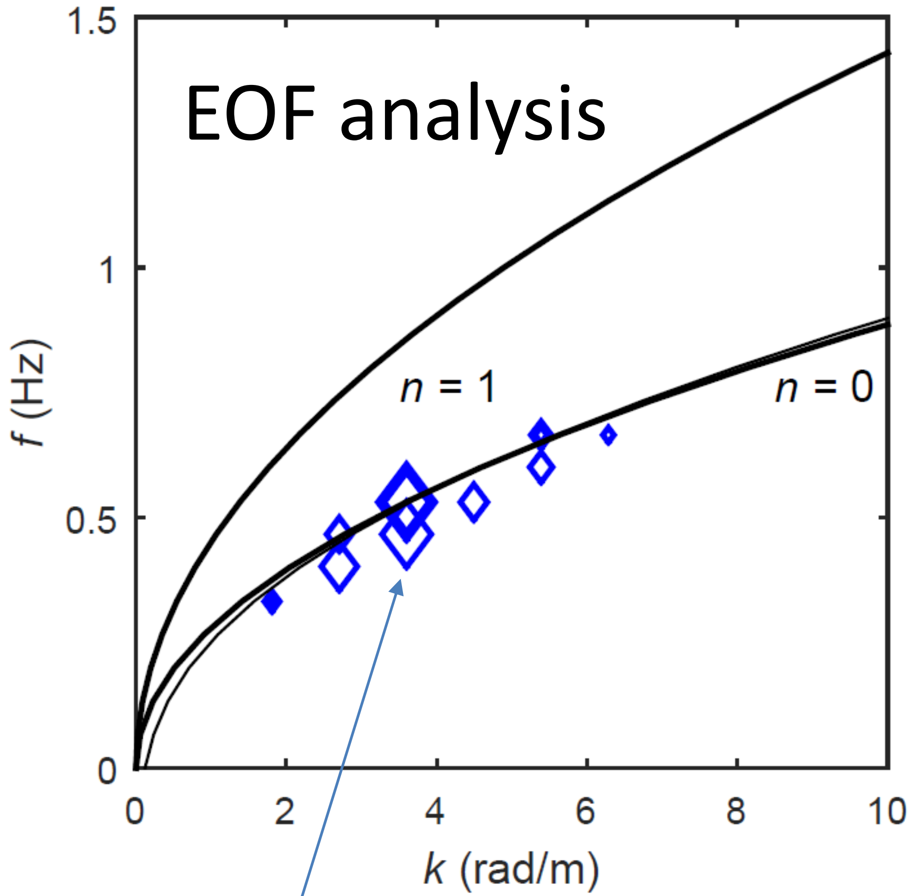
Wavenumber spectrum

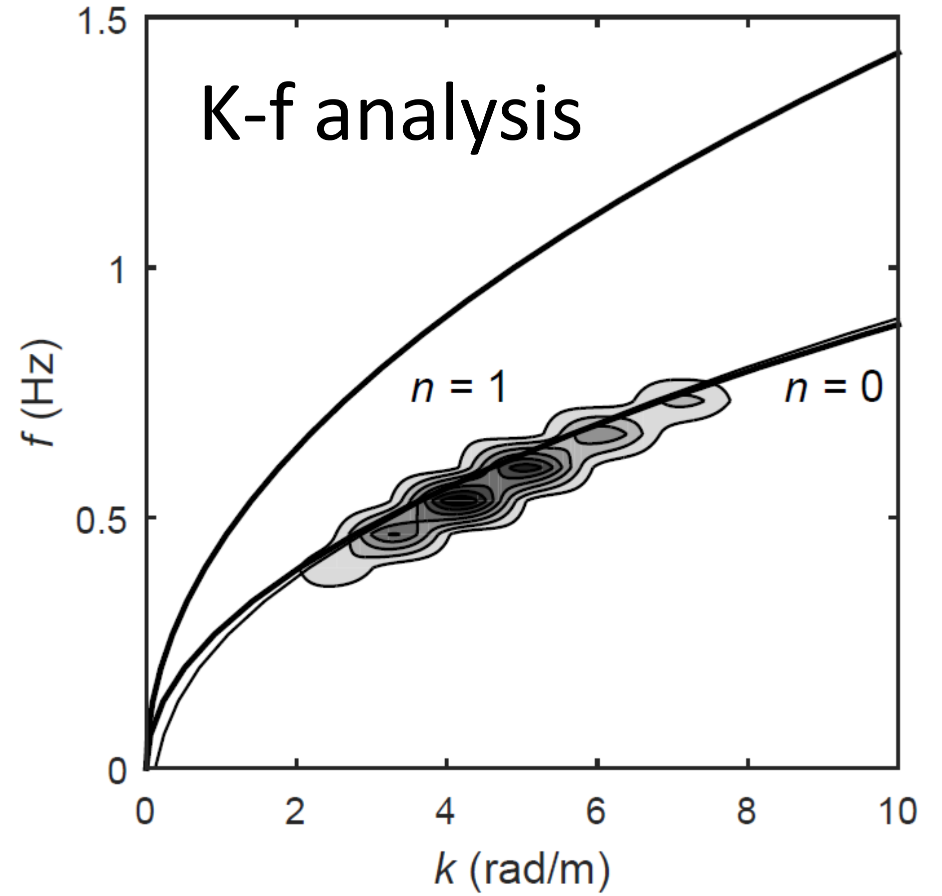




Each EOF mode can be associated to one peak frequency and one peak wavenumber

EOF and k-f results



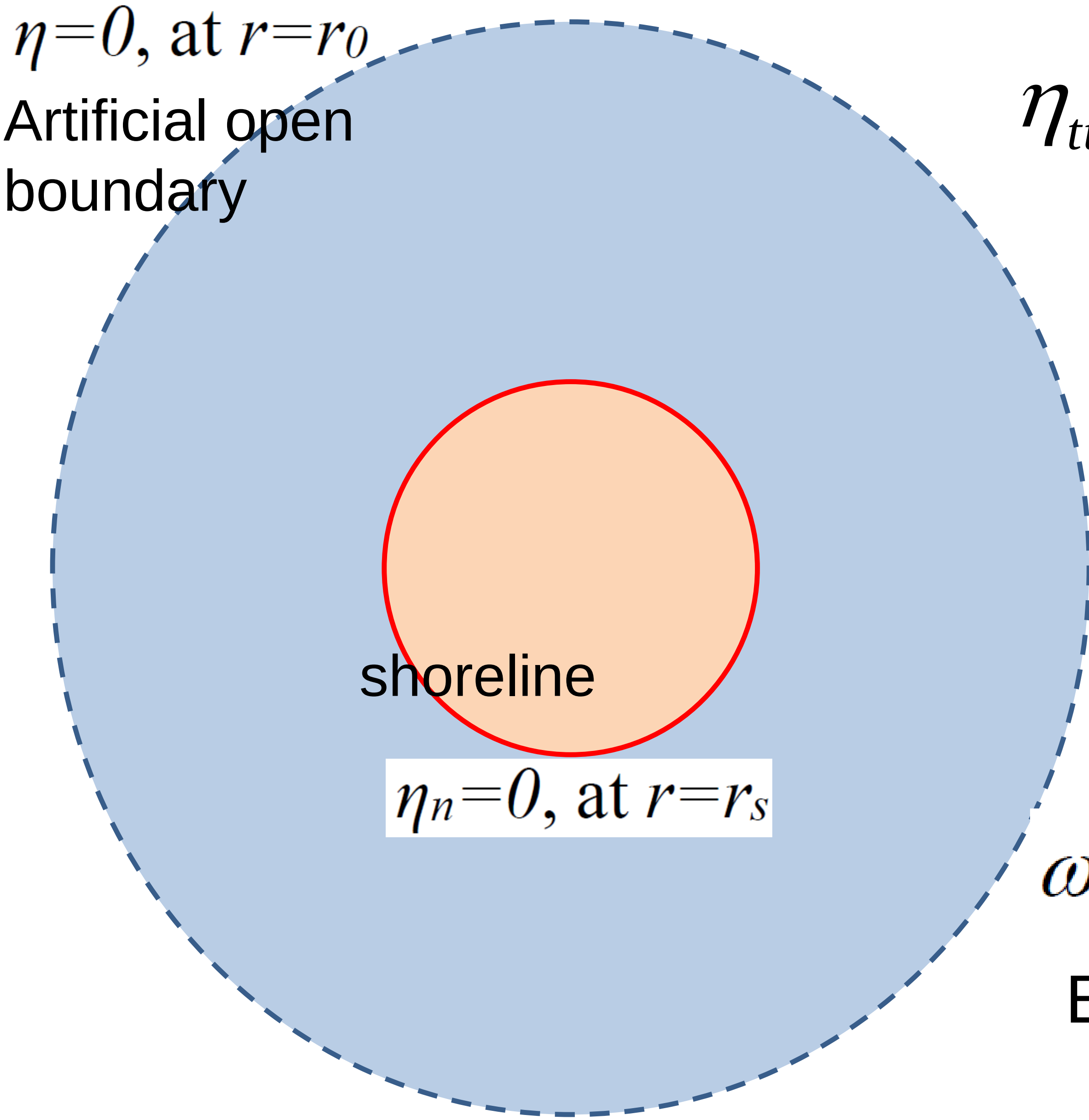


Each EOF mode is represented by a marker
The size of the marker scales with λ_n

Dispersion relationships:

- Edge waves (n = 0, 1)
- Edge waves (circular shoreline)

Numerical modal analysis



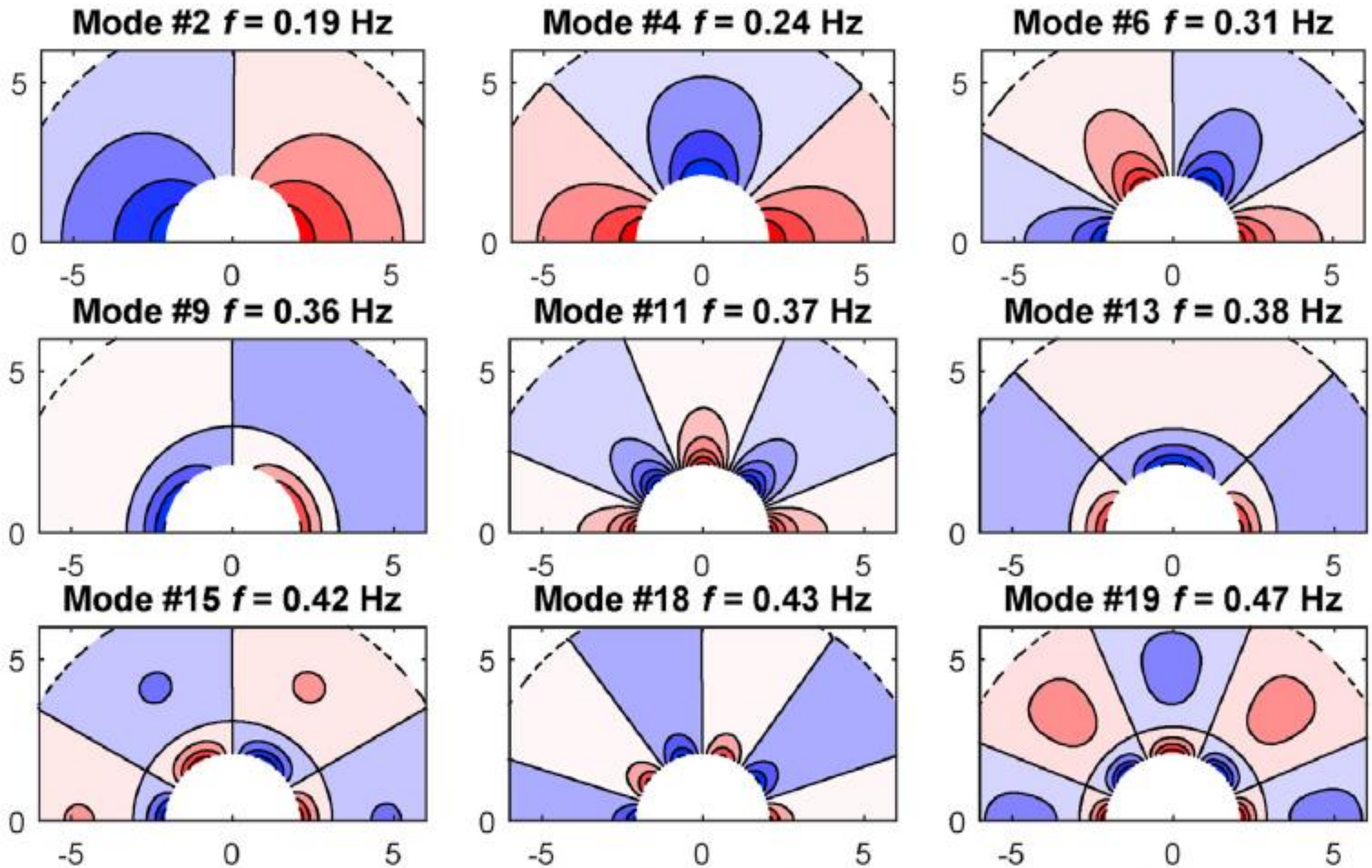
$$\eta_{tt} - \nabla\left(gh\nabla\eta\right) = 0$$

$$\eta = \mathrm{Re}\left[X(x, y)e^{-i\omega t}\right]$$

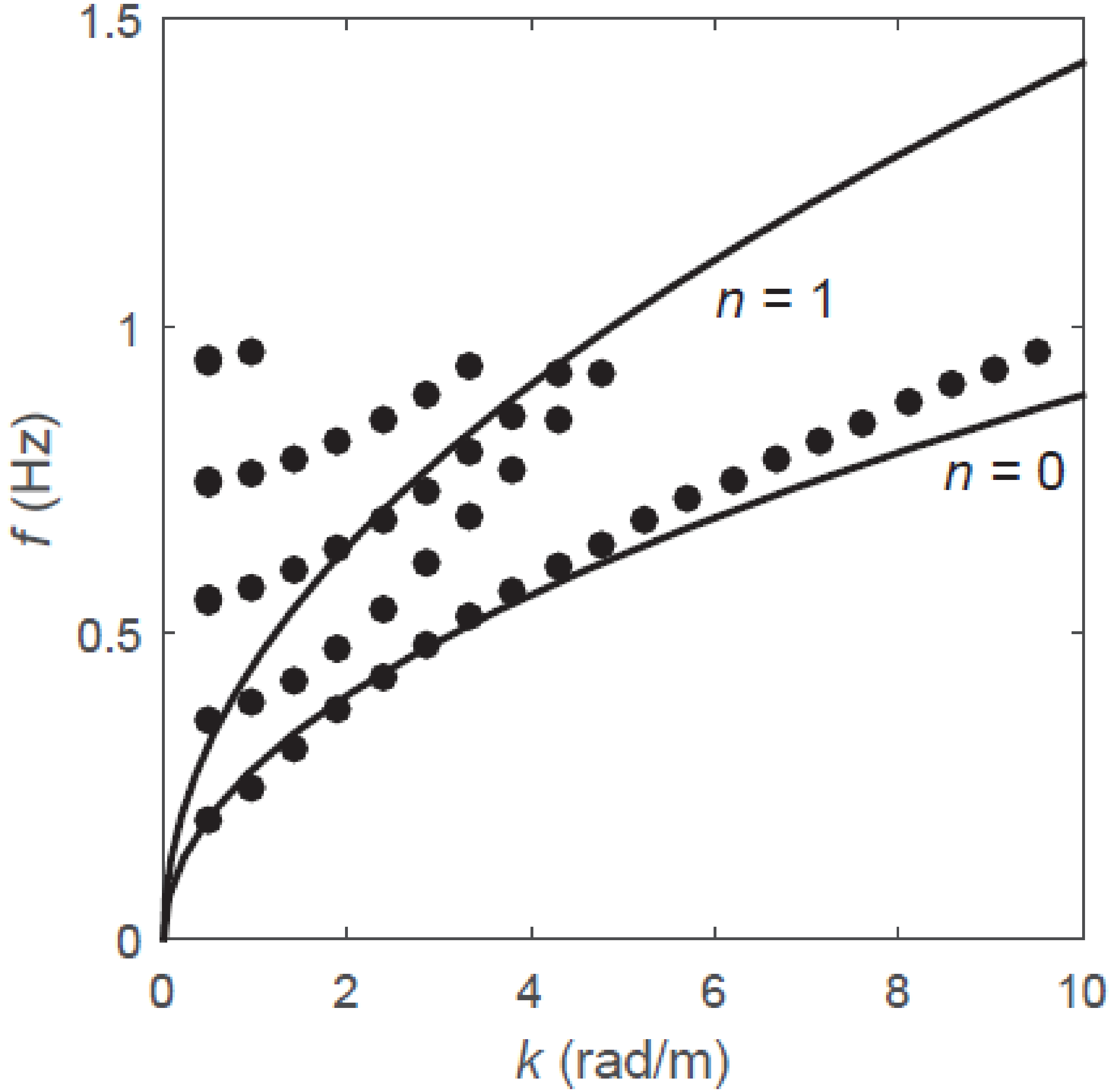
$$\omega^2 X - \nabla\left(gh\nabla X\right) = 0$$

Eigenvalue problem

Numerical modal analysis results

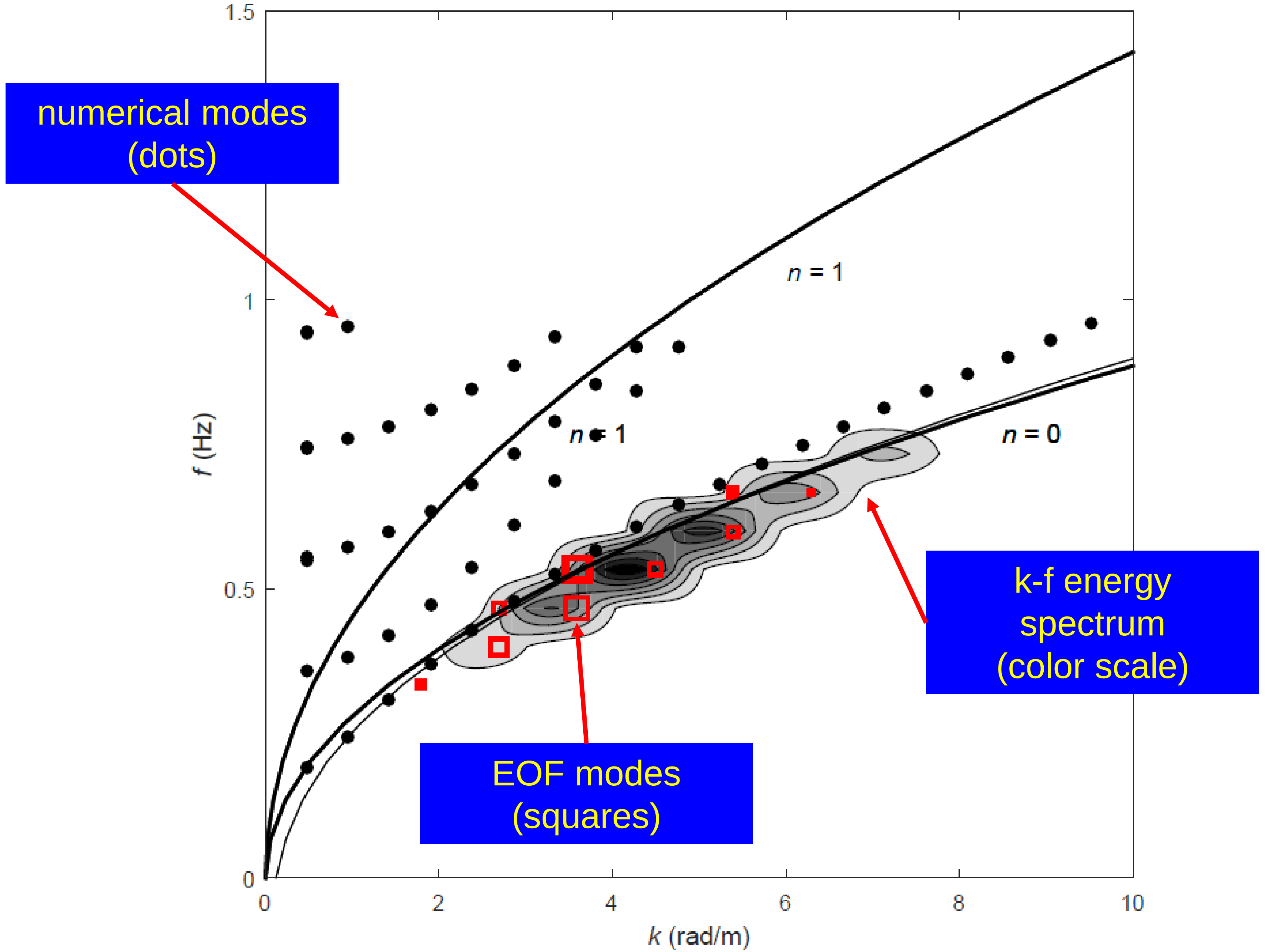


Numerical modal analysis results processed and plotted in the k-f plane



Each numerical mode is represented by a point. Here no information on the size/importance of the modes as the model simulates free (unforced) oscillations

Comparison of the methods



Conclusions

- Three methods for modal identification applied to experimental landslide tsunamis:
  - K-f spectral analysis, non information on the spatial shape of the modes
  - EOF spatial shape, k, f and relative importance (i.e. variance explained)
  - Numerical modal analysis: free response, no information on the importance of the modes
- Results in good agreement
- Edge waves (zeroth mode) dominate the wave propagation around the island

References

Movable arm experiments:

Romano A., M. Di Risio, G. Bellotti, M. G. Molfetta, L. Damiani, P. De Girolamo (2016). Tsunamis generated by landslides at the coast of conical islands: experimental benchmark dataset for mathematical model validation. Landslides, pp. 1-15.

Benchmark data available

k-f analysis:

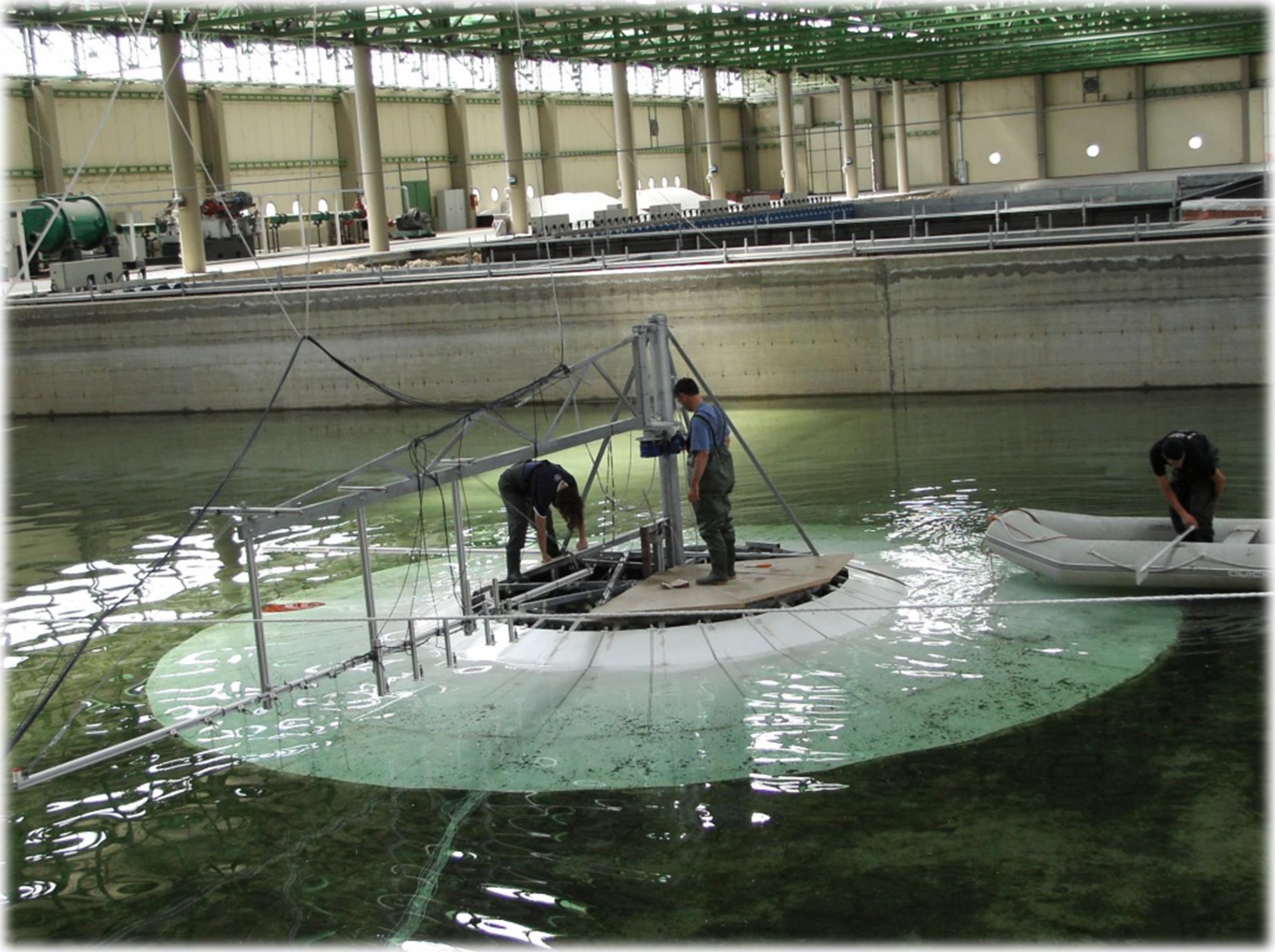
Romano A., Bellotti G., Di Risio M. (2013). Wavenumber–frequency analysis of the landslide-generated tsunamis at a conical island. Coastal Engineering, vol. 81, pp. 32-43.

EOF analysis:

Bellotti G., Romano A. (2017). Wavenumber-frequency analysis of landslide-generated tsunamis at a conical island. Part
II: EOF and modal analysis. Coastal Engineering, vol. 128, pp. 84-91.

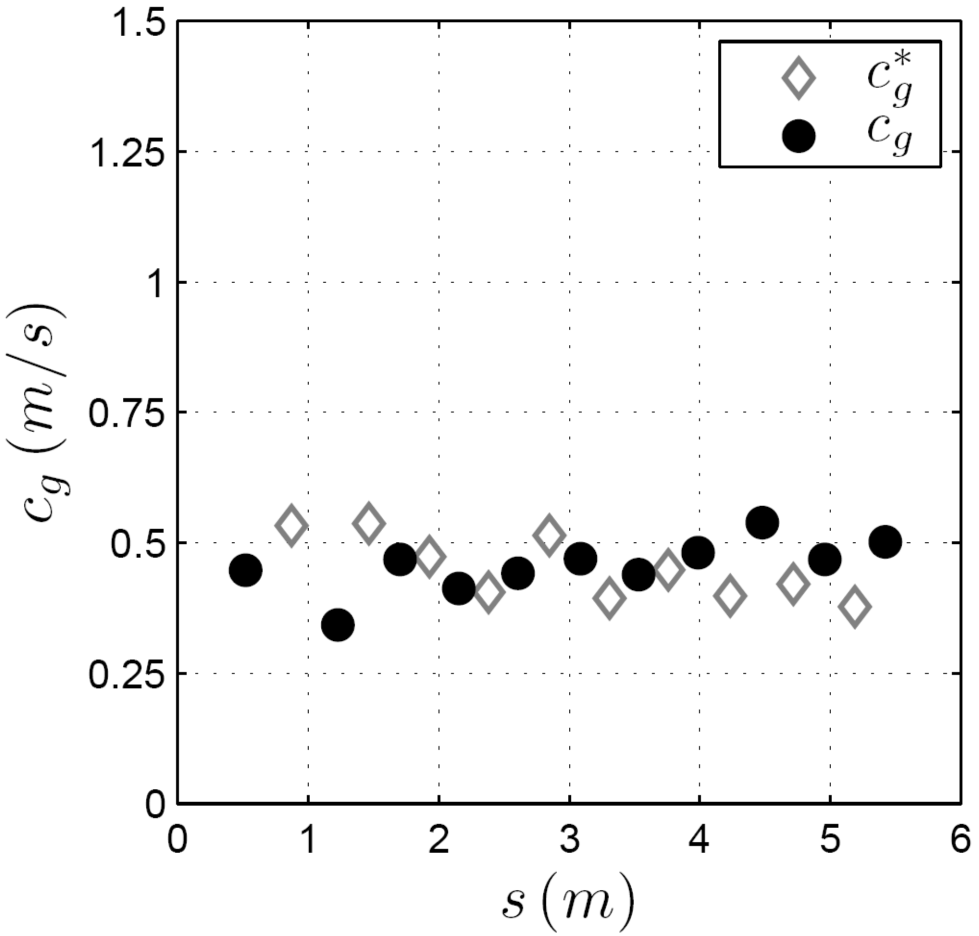
Submerged landslides:

Tsunamis generated by submerged landslides at a conical island: experimental and numerical analysis (in preparation)



Estimate of the wave group celerity

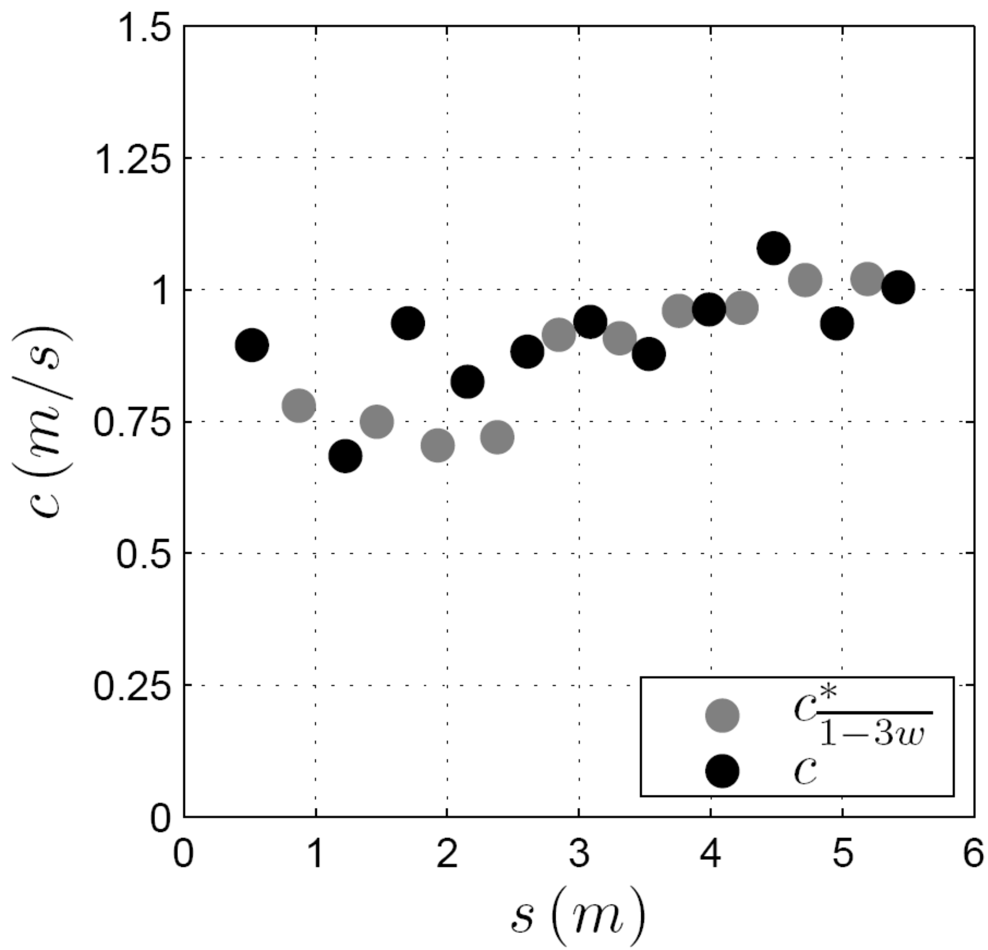
The theoretical group wave celerity, as from the edge waves theory for the 0-th mode, is in very good agreement with the experimental one.



$$s = r\,\theta$$

Estimate of the wave phase celerity

The experimental phase wave celerity of the first three waves that form the packet (c^*_{1w}, c^*_{2w}, c^*_{3w}) has been calculated as from the zero-crossing analysis, while the theoretical one has been obtained (*c*) by the edge waves dispersion relation.



$$s = r\,\theta$$

represents the distance from the impact point, measured along the shoreline