ICCE 2018
36TH INTERNATIONAL CONFERENCE ON COASTAL ENGINEERING 2018
Baltimore, Maryland | July 30 – August 3, 2018

TIME: 3:50 PM - 4:10 PM
SESSION: Wave Runup
LOCATION: Grand Ballroom I & II

A unified runup formula for breaking solitary and periodic waves on a uniform beach

Yun-Ta Wu, PhD [ceewyt@nus.edu.sg]
Research Fellow, Department of Civil and Environmental Engineering, National University of Singapore

Philip Li-Fan Liu, ScD
Distinguished Professor, Department of Civil and Environmental Engineering, National University of Singapore

Kao-Shu Hwang, PhD
Senior Research Fellow, Tainan Hydraulics Laboratory, National Cheng Kung University, Taiwan

Hwung-Hweng Hwung, PhD
Emeritus Professor, Department of Hydraulic and Ocean Engineering, National Cheng Kung University, Taiwan

NUS National University of Singapore
經濟部水利署 國立成功大學 水工試驗所
Tainan Hydraulics Laboratory
National Cheng Kung University, since 1950
IWDRC,NCKU
國立成功大學國際波動力學研究中心
INTERNATIONAL WAVE DYNAMICS RESEARCH CENTER

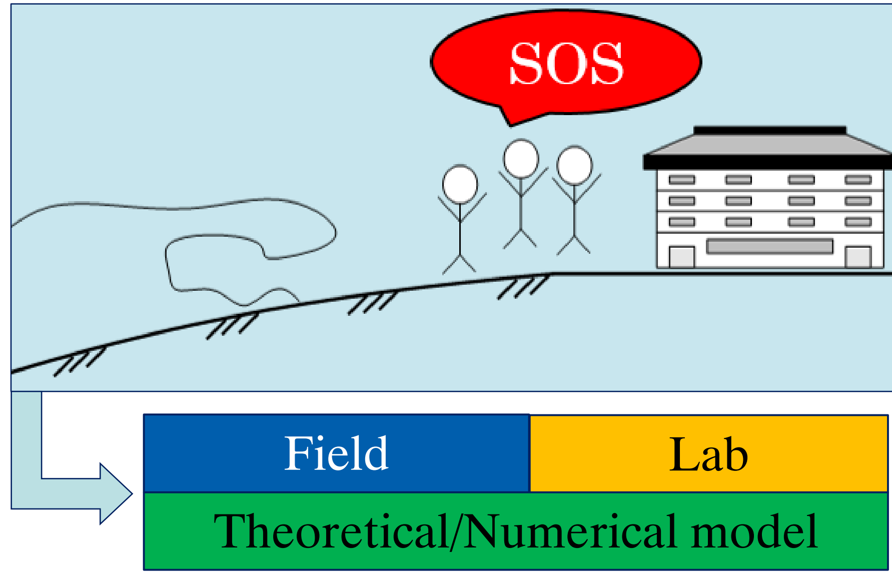
Outline of the presentation

- Motivation of the study
- Surf parameters for solitary and periodic waves
- Solitary wave runup
- Unified runup formula for solitary and periodic waves
- Conclusions

Motivation of the study

- Understanding wave induced runup processes and predicting the maximum runup height are of great importance for coastal management.
- Studying breaking waves on beaches has been an active research area in coastal engineering community.

→Runup of solitary and periodic waves




[http://www.asahi.com/special/saigaishi]


[https://anntw.com/articles/20150807-eJNd]

Surf parameter for solitary waves

- Leading order solution for a solitary wave with wave height, H_0, propagating in a constant depth, h_0, (Boussinesq, 1871):

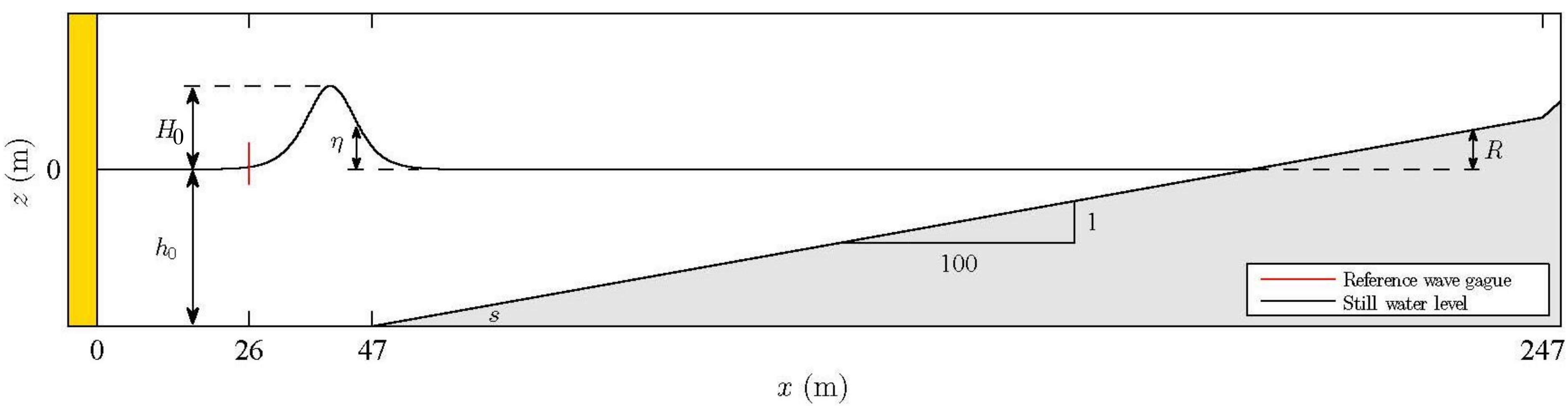
  $\eta(x,t) = H_0 sech^2[K(x-ct)]$, where $K = \sqrt{3H_0/4h_0^3}$ and $C = \sqrt{g(h_0+H_0)}$

- Based on the theory of Carrier and Greenspan (1958), Synolakis (1987) derived an analytical breaking criterion for solitary waves on a uniform slope:

  $\frac{H_0}{h_0} > 0.8183 s^{\frac{10}{9}}$ or $s(\frac{H_0}{h_0})^{-\frac{9}{10}} < (0.8183)^{-\frac{9}{10}} \approx 1.20$

- Lo et al. (2013) defined the surf parameter for solitary waves:

  $\xi_s = s(\frac{H_0}{h_0})^{-\frac{9}{10}}$



Surf parameter for periodic waves

- Following the same approach and the definition of wave breaking for solitary wave proposed by Synolakis (1987), Madsen and Schäffer (2010) derived the wave breaking criterion for sinusoidal waves on a uniform slope as:

✓ $\frac{A_0}{h_0} > \frac{1}{2\sqrt{\pi}}(\frac{\omega^2 h_0}{gs^2})^{-\frac{5}{4}}$ or $s(\frac{H_0}{h_0})^{-\frac{2}{5}}(\frac{\omega^2 h_0}{g})^{-\frac{1}{2}} < (\pi)^{-\frac{1}{5}} \approx 1.26$

- Thus, the surf parameter for periodic waves can be defined as:

✓ $\xi_p = s(\frac{H_0}{h_0})^{-\frac{2}{5}}(\frac{\omega^2 h_0}{g})^{-\frac{1}{2}}$

- The breaking conditions for solitary waves and for periodic waves are consistent, i.e., $\xi_s \approx 1.20$ vs. $\xi_p \approx 1.26$.

- In the shallow water (or long wave) limit $\omega^2 h_0/g \sim (kh_0)^2$ and for solitary waves the Boussinesq approximation requires that $O(h_0/L)^2 \sim O(H_0/h_0)$, where L is the wavelength. Therefore, ξ_p reduces to ξ_s with a coefficient of $1/(2\pi)$ obtained.

Solitary wave runup – available experiments

Reference	H_0/h_0	Slope (s)	Flume material: B & S	Flume material: Slope	Flume dimensions (m) [Length×Width×Height]	Wave generation
Hall and Watts (1953)	0.046 - 0.504	1/1 - 1/11.43	Concrete	Wood	25.91×4.27×1.22	MEC Pusher-type
Saeki et al. (1971)	0.05 - 0.80	1/50 - 1/150	Glass	Plastic	24.0×0.8×NA	MEC Pneumatic
Synolakis (1987) *	0.005 - 0.633	1/19.85	Steel/glass	Aluminum	37.7×0.61×0.93	PGM Piston-type
Briggs et al. (1995) *	0.005 - 0.357	1/30	Glass	NA	42.4×NA×NA	PGM Piston-type
Li and Raichlen (2001) *	0.026 - 0.339	1/2.08	Steel/glass	Aluminum	15.25×0.4×0.61	PGM Piston-type
(2002) *	0.019 - 0.418	1/15			36.6×0.4×0.61	
(2003) *					45.7×0.9×0.9	
Jensen et al. (2003) *	0.12 - 0.665	1/5.37	Glass	Perspex	25.0×0.5×1.0	PGM Piston-type
Hsiao et al. (2008) *	0.011 - 0.338	1/60	Concrete	Concrete	300×5.0×5.2	PGM Piston-type
Chang et al. (2009) *	0.030 - 0.259	1/20	Concrete	Concrete	300×5.0×5.2	PGM Piston-type
Lo et al. (2013) *	0.049 - 0.498	1/2.47	Glass	Glass	34.0×0.6×0.9	PGM Piston-type
	0.005 - 0.139	1/10	Glass	Glass	12.0×0.8×1.0	
	0.105 - 0.198	1/12	Concrete	Concrete	104×3.7×4.6	
	0.005 - 0.417	1/20	Glass	Plastic	34.0×0.6×0.9	
Pedersen et al. (2013) *	0.098 - 0.481	1/5.67	Glass	Plastic	25.0×0.5×1.0	PGM Piston-type
Ting (2013) *	0.70	1/40	Glass	Plexiglas	25.0×0.9×0.75	PGM Piston-type
Pujara et al. (2015) *	0.05 - 0.286	1/12	Concrete	Concrete	104×3.7×4.6	PGM Piston-type
Hafsteinsson et al. (2017) *	0.20 - 0.70	1/9.5 - 1/57.3	Glass	Plastic	14.0×0.50×0.70	PGM Piston-type
Manoj Kumar et al. (2017) *	0.06 - 0.12	1/20	NA	NA	28.0×1.0×NA	PGM Piston-type
Smith et al. (2017) *	0.0986 - 0.4863	1/11.20	Glass	Plastic	25.0×0.5×1.0	PGM Piston-type
Wu et al. (2017) *	0.051 - 0.449	1/20	Steel/glass	Aluminum Plexiglas	22.0×0.50×0.76	PGM Piston-type
Present study *	0.057 - 0.412	1/100	Concrete	Concrete	300×5.0×5.2	PGM Piston-type

Solitary wave runup – new experiments @ THL

Tainan Hydraulics Laboratory (THL)

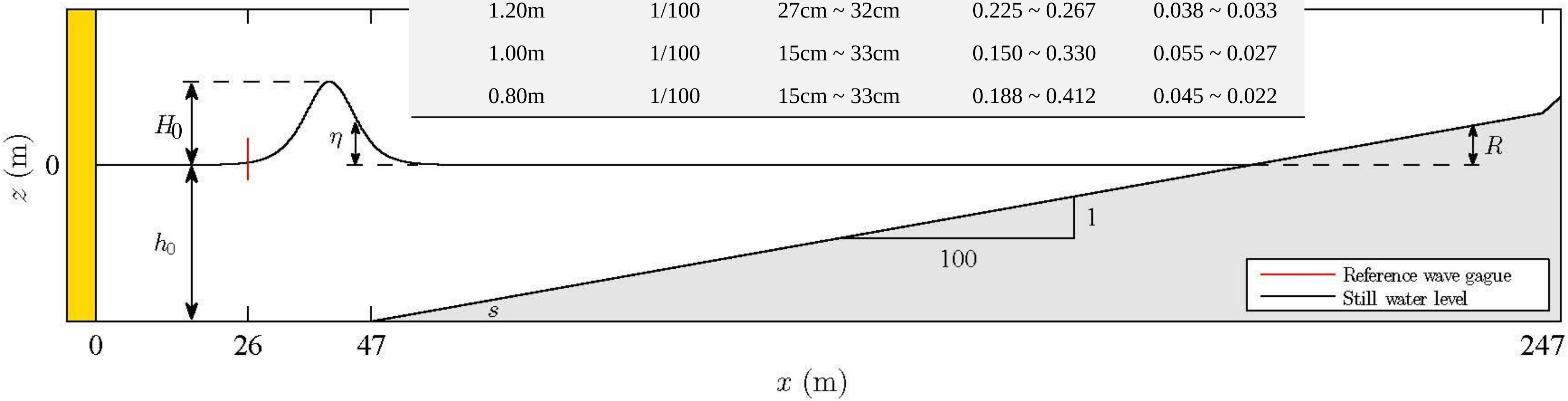
- Dimension = 300m×5.0m×5.0m
- Irregular waves up to 1.0m height
- Re ~ 10^6 (Field ~ 10^7; Lab ~ 10^4)

⇨ Solitary wave with simple geometry

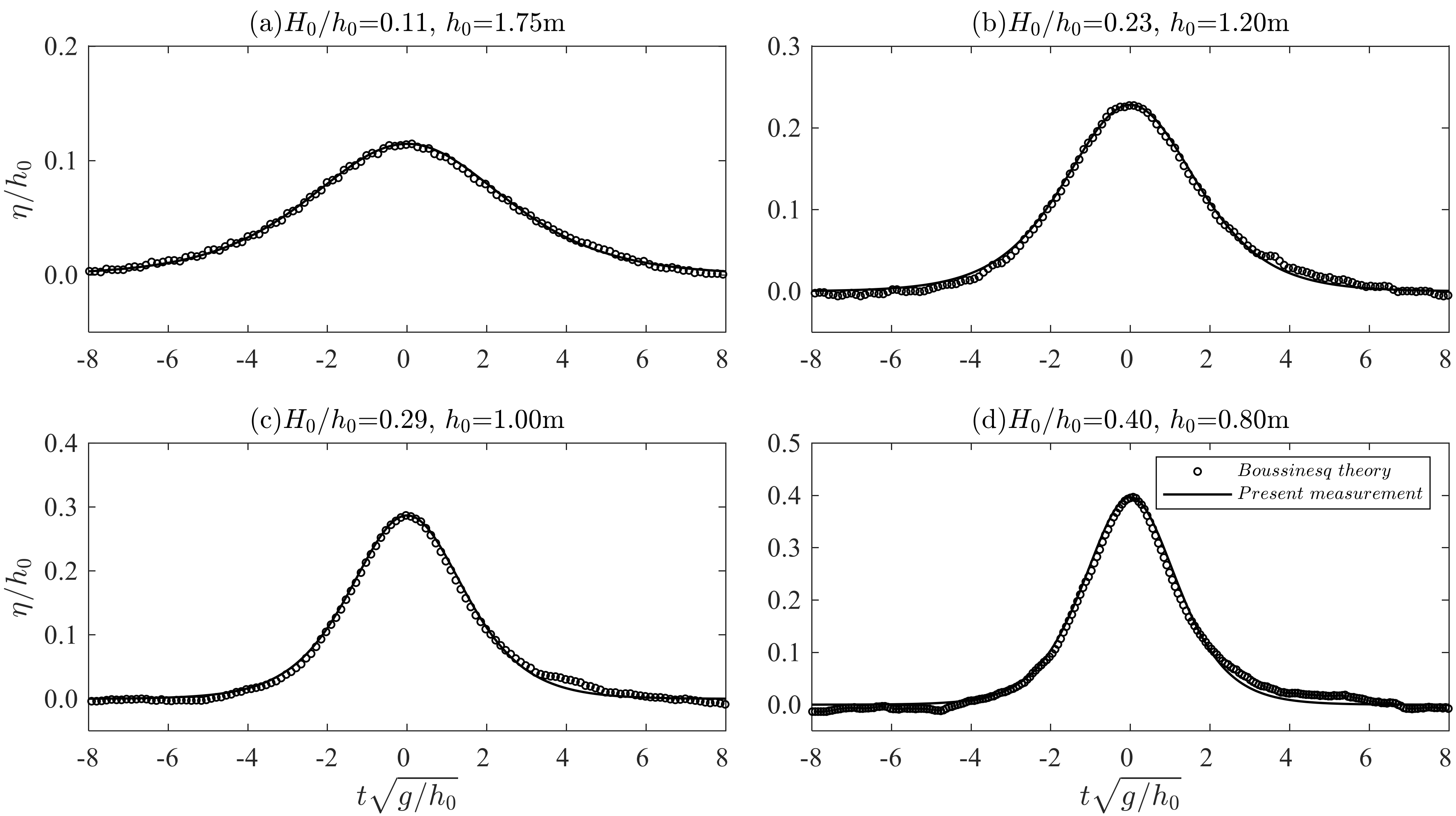
⇨ 1/20, 1/40, 1/60, 1/100 slopes

Solitary wave runup – new experiments @ THL

Water depth (h_0)	Slope (s)	Wave height (H_0)	H_0/h_0	ξ_s
1.75m	1/100	10cm ~ 37cm	0.057 ~ 0.211	0.132 ~ 0.041
1.20m	1/100	27cm ~ 32cm	0.225 ~ 0.267	0.038 ~ 0.033
1.00m	1/100	15cm ~ 33cm	0.150 ~ 0.330	0.055 ~ 0.027
0.80m	1/100	15cm ~ 33cm	0.188 ~ 0.412	0.045 ~ 0.022



Solitary wave runup – new experiments @ THL



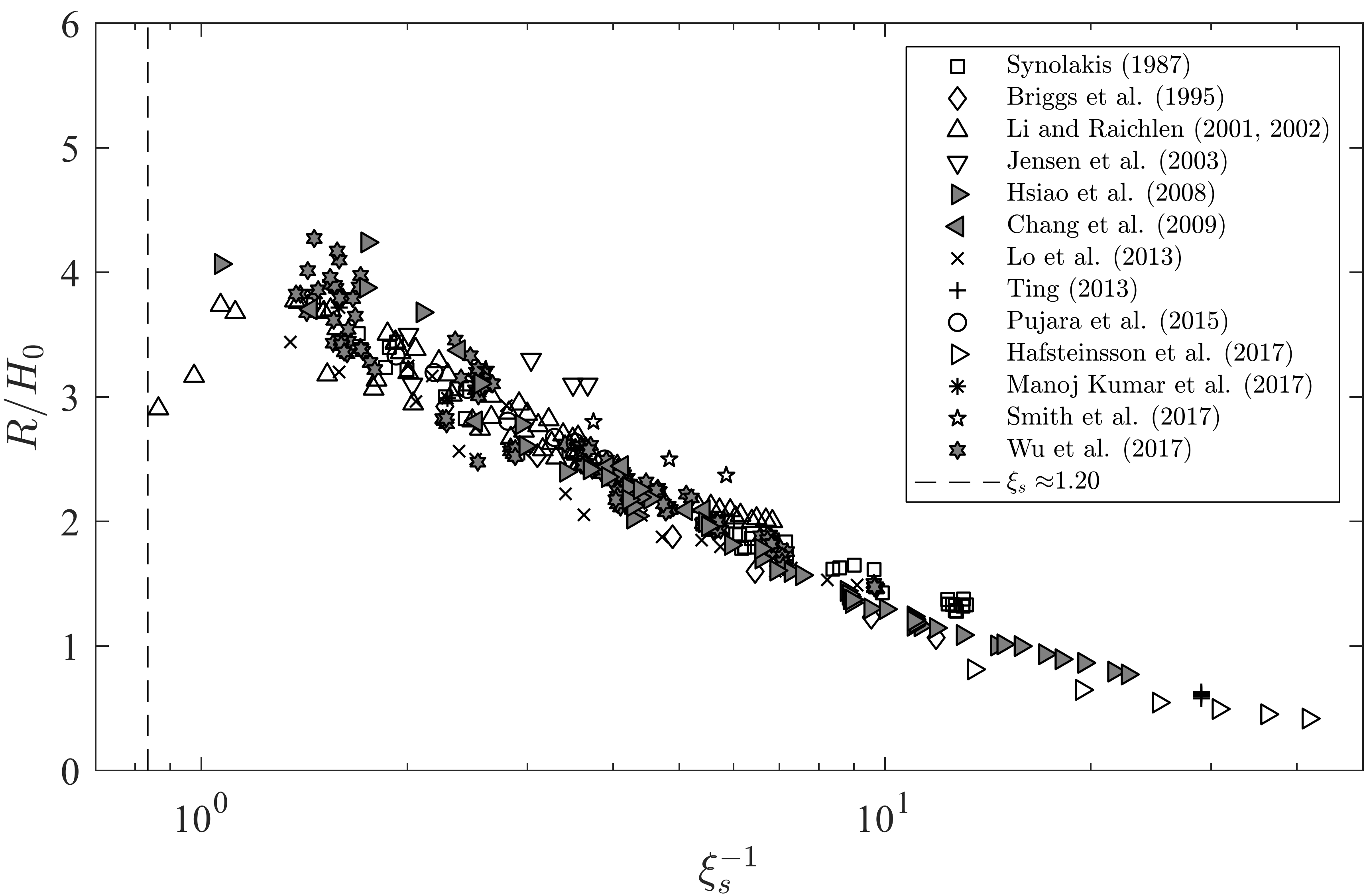
[Comparisons between measurements and theoretical solutions for different H_0/h_0]

Solitary wave runup – new experiments @ THL

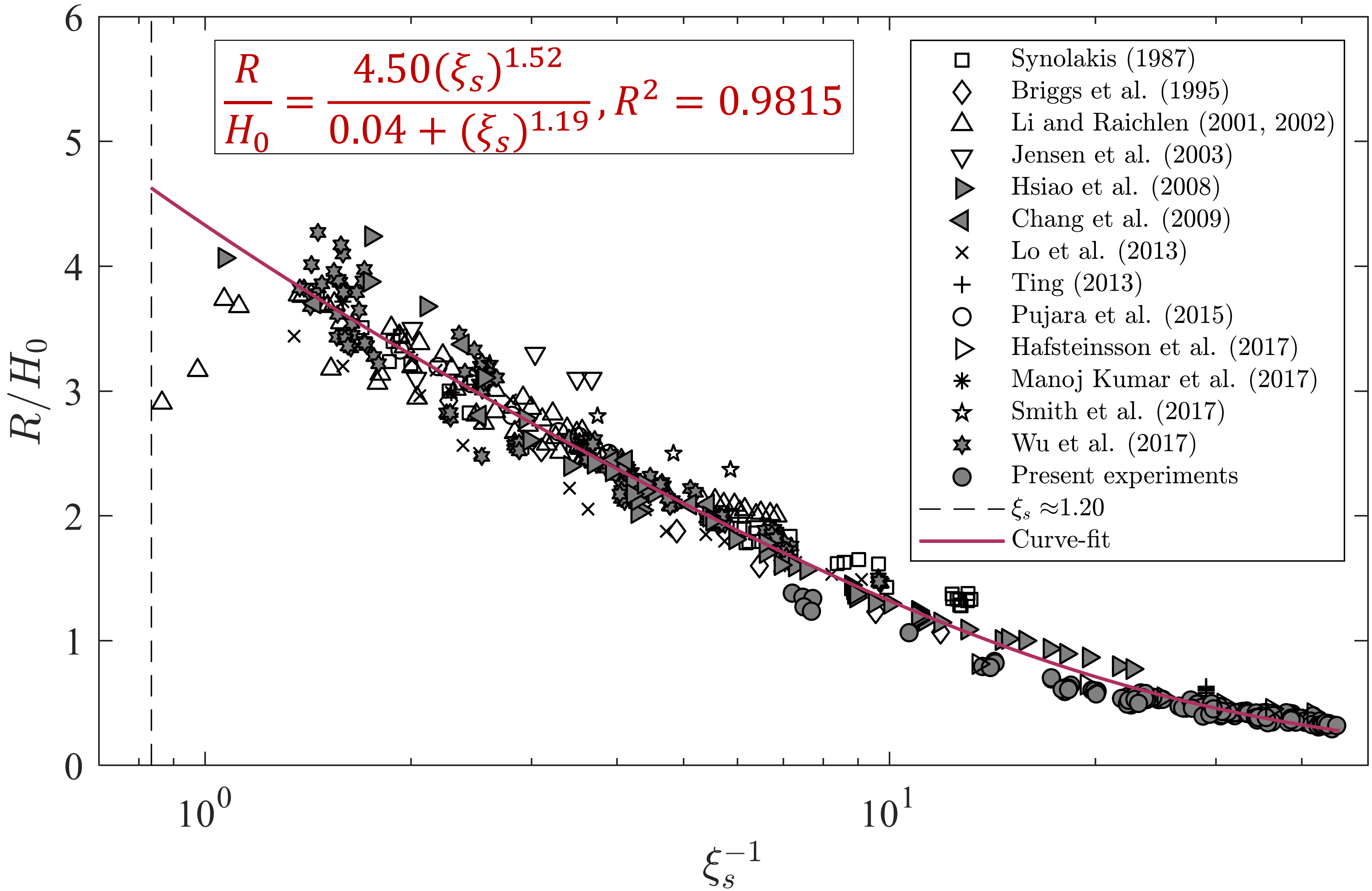
[Snapshot at the moment of maximum runup height]

[Snapshot at the moment of wave overturning of a breaking solitary wave]

Solitary wave runup – available experiments



Solitary wave runup – unified runup formula



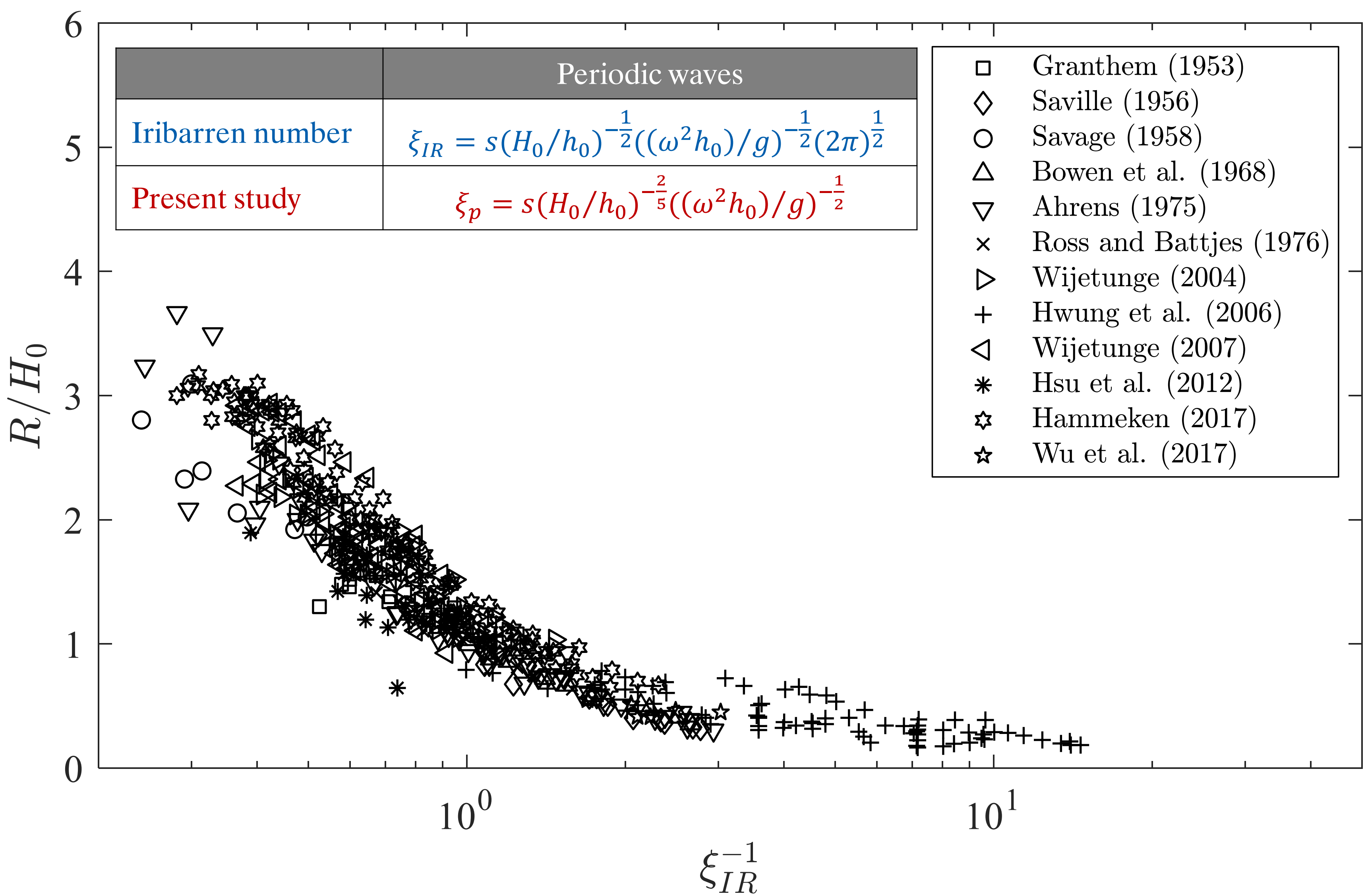
Periodic wave runup – available experiments

Reference	H_0/h_0	T (s)	Slope (s)	Flume material		Flume dimensions (m)	Wave generation
				B & S	Slope	[Length×Width×Height]	
Granthem (1953)	0.077 - 0.349	0.67 - 2.75	1/0.27 - 1/3.73	NA/Glass	NA	18.29×0.30×0.91	NA
Saville (1956) *	0.040 - 0.387	0.61 - 4.70	1/3, 1/10	Steel	NA	29.26×0.46×0.61	Pusher-type
Savage (1958) *	0.013 - 0.168	2.63 - 4.70	1/10	Concrete	Concrete	36.58×1.52×1.52	Plunger-type
Bowen et al. (1968) *	0.051 - 0.313	0.82 - 2.37	1/12	NA/Glass	Wood	40.0×0.50×0.75	NA
Ahrens (1975) *	0.024 - 0.374	0.72 - 4.70	1/10	NA	Fiberglass	22.0×0.46×0.61	Piston-type
Roos and Battjes (1976) *	0.156 - 0.473	1.01 - 1.95	1/3, 1/5, 1/7	NA	Plywood	30.0×0.80×0.60	NA
Neelamani et al. (1999)	0.06 - 0.42	1.5 - 6.0	1/3, 1/4, 1/6	NA	NA	100×2.0×1.2	Flap-type
Wijetunge (2004)	0.090 - 0.383	0.70 - 1.35	1/1.56 - 1/4.13	Perspex	Wood	12.75×0.52×0.70	NA
Hwung et al. (2006) *	0.011 - 0.157	1.6 - 4.4	1/10, 1/20, 1/40	Concrete	Concrete	300.0×5.0×5.2	PGM Piston-type
Wijetunge (2007)	0.066 - 0.255	0.77 - 1.56	1/2	Concrete	Aluminum	40.0×2.00×2.13	Plunger-type
Hsu et al. (2012)	0.043 - 0.182	0.80 - 2.00	1/3, 1/4, 1/5	Steel/glass	Wood	25.0×0.50×0.60	PGM Piston-type
Hammeken (2017) *	0.1 - 0.5	1.00 - 3.33	1/3.73 - 1/8.14	Glass	Metal	20.0×1.20×1.00 13.7×0.45×0.75	PGM Piston-type
Wu et al. (2017) *	0.153 - 0.364	0.8 - 1.5	1/20	Steel/glass	Plexiglas	22.0×0.50×0.76	PGM Piston-type

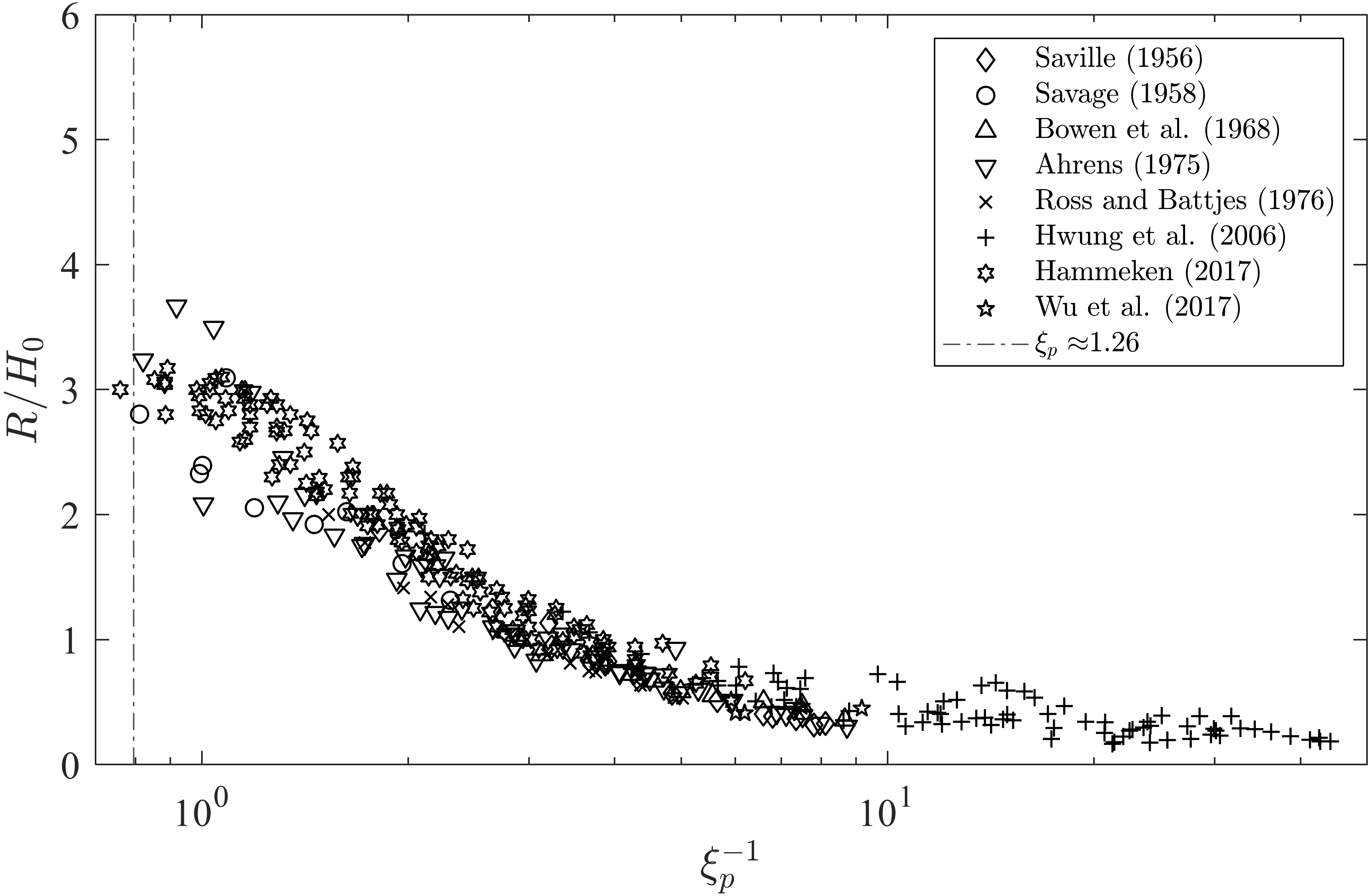
	Periodic waves	Solitary waves
Iribarren number	$\xi_{IR} = s/\sqrt{H_0/L_0} = s(H_0/h_0)^{-\frac{1}{2}}((\omega^2 h_0)/g)^{-\frac{1}{2}}(2\pi)^{\frac{1}{2}}$	$\xi_{IR-s} = s(H_0/h_0)^{-1}$
Present study	$\xi_p = s(H_0/h_0)^{-\frac{2}{5}}((\omega^2 h_0)/g)^{-\frac{1}{2}}$	$\xi_s = s(H_0/h_0)^{-\frac{9}{10}}$

Periodic wave runup – available experiments

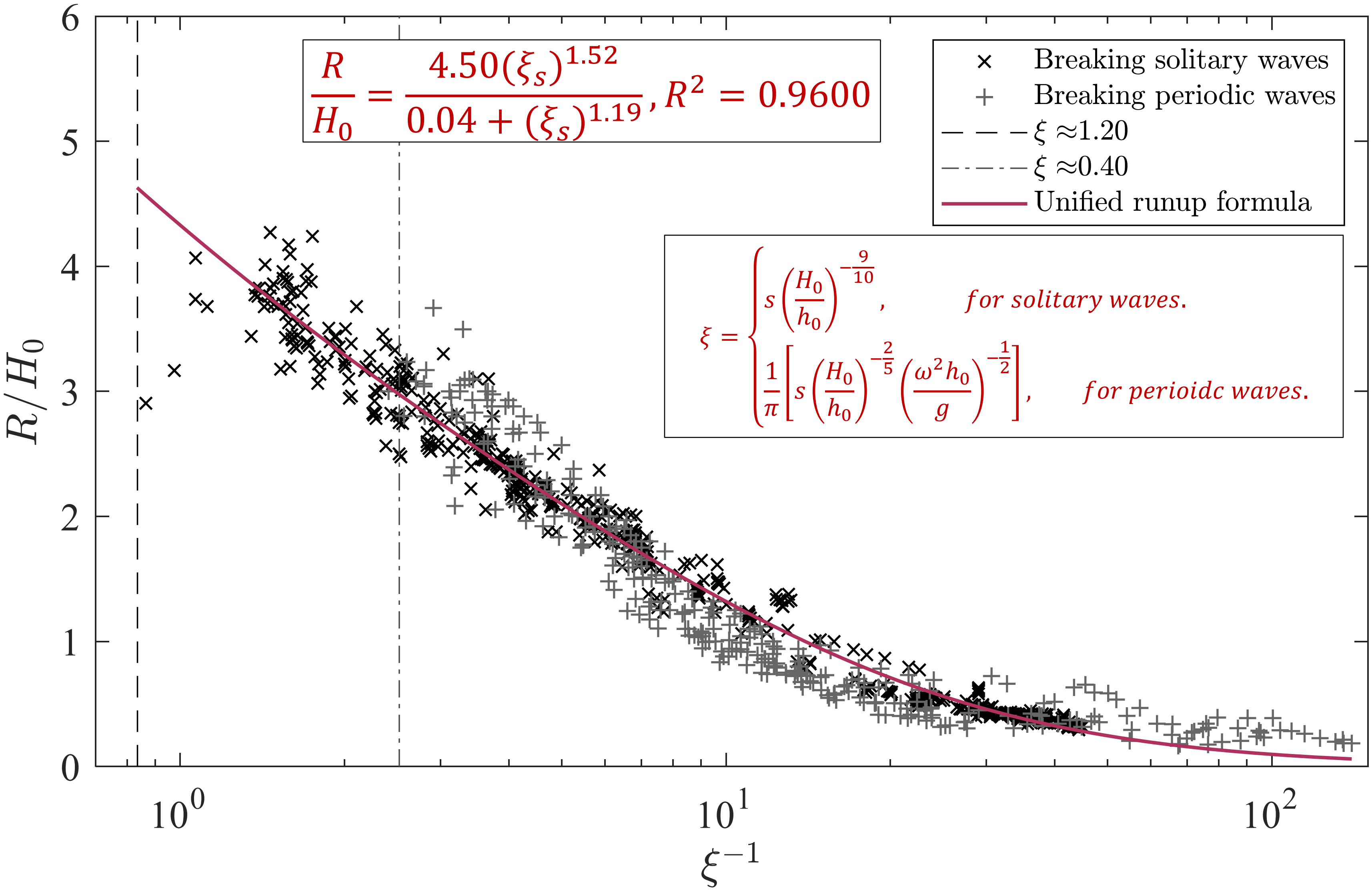
	Periodic waves
Iribarren number	$\xi_{IR} = s(H_0/h_0)^{-\frac{1}{2}}((\omega^2 h_0)/g)^{-\frac{1}{2}}(2\pi)^{\frac{1}{2}}$
Present study	$\xi_p = s(H_0/h_0)^{-\frac{2}{5}}((\omega^2 h_0)/g)^{-\frac{1}{2}}$



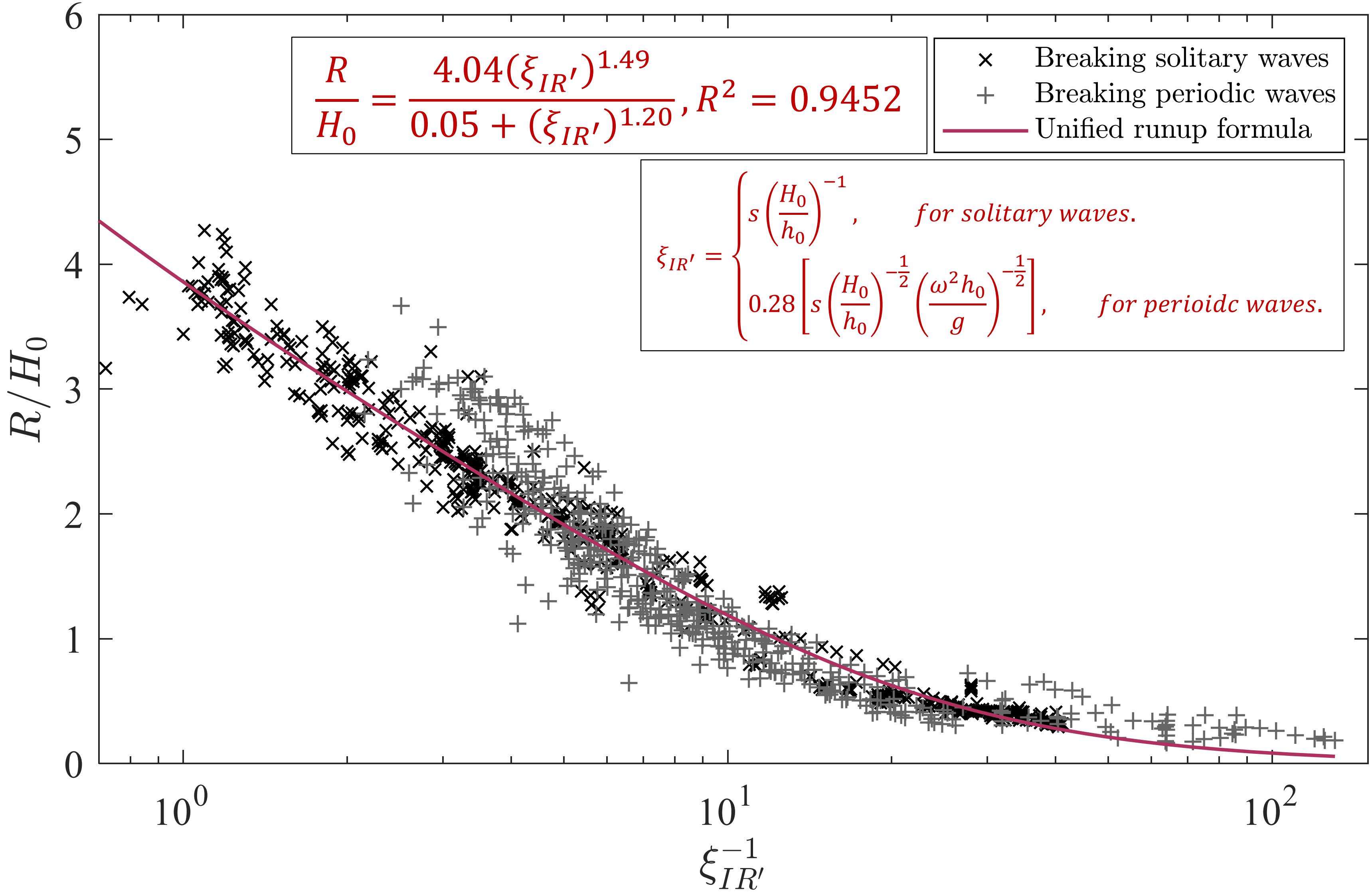
Periodic wave runup – available experiments



Unified runup formula – solitary and periodic waves



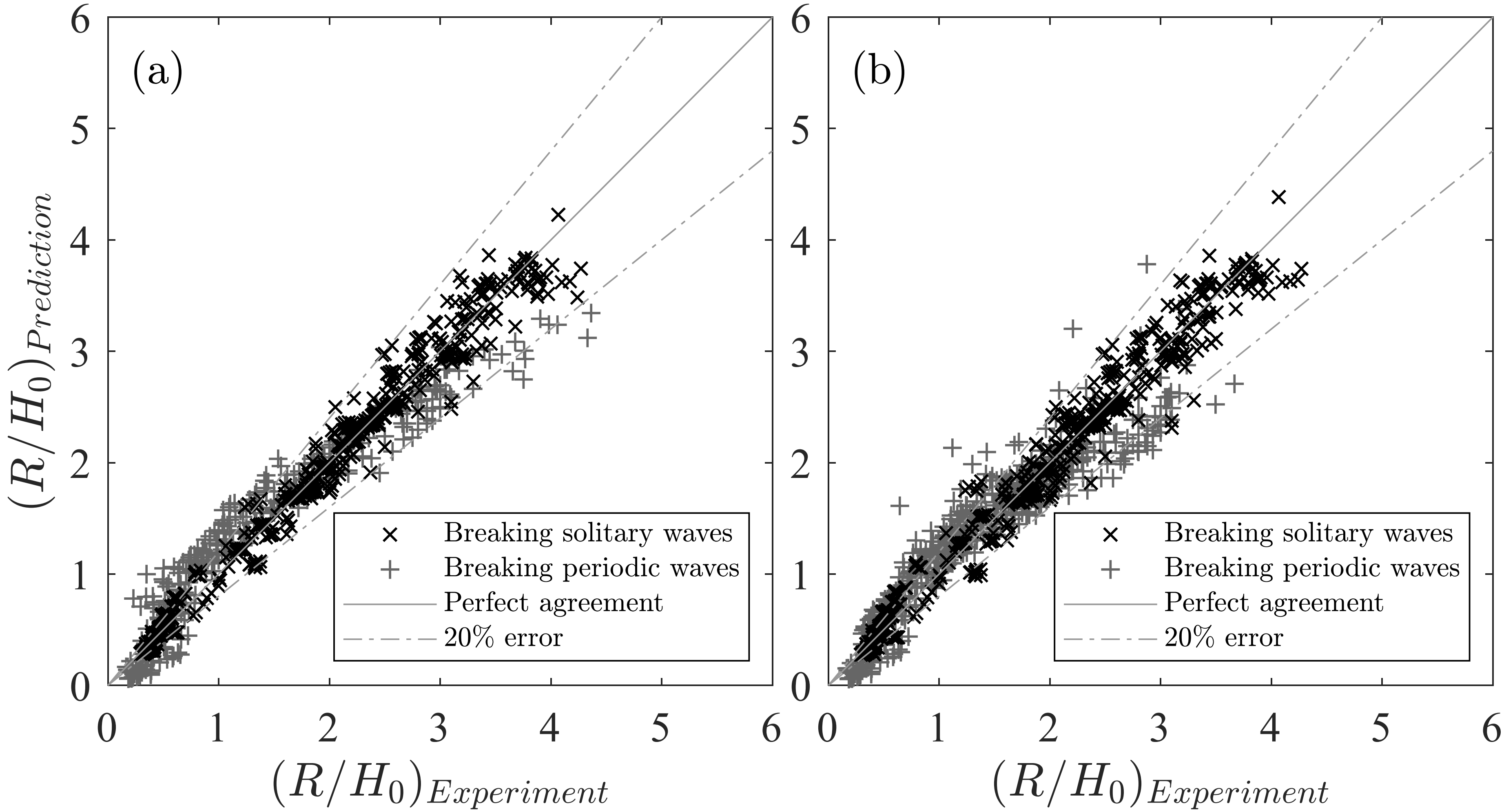
Unified runup formula – solitary and periodic waves



Unified runup formula – solitary and periodic waves

$$\frac{R}{H_0} = \frac{4.50(\xi_s)^{1.52}}{0.04 + (\xi_s)^{1.19}}, R^2 = 0.9600$$

$$\frac{R}{H_0} = \frac{4.04(\xi_{IR'})^{1.49}}{0.05 + (\xi_{IR'})^{1.20}}, R^2 = 0.9452$$



Conclusions

- Using laboratory data, the normalized runup heights of solitary and periodic waves can be characterized by a single surf parameter, which is based on theoretical breaking criteria.
- Additional laboratory experiments in a large-scale wave flume were performed for breaking solitary waves on a 1/100 slope. The new experimental data extend the range of surf parameters for solitary waves.
- Although surf parameters are defined separately for solitary and periodic waves, an empirical formula for the normalized runup height can be used for estimating the runup heights of breaking solitary and periodic waves on uniform slopes.
- The proposed empirical formula is applicable for different breakers from surging, plunging to spilling breakers.
- Since the formula is deduced from laboratory data, where the beach material and scale effects appear to be minor, the applications of this formula for field conditions should be exercised cautiously.

❖ Wu, Y.-T. , Liu, P.L.-F., Hwang, K.-S., Hwung, H.-H. (2018) "On the runup of laboratory-generated breaking solitary and periodic waves on a uniform

Thank you for your attention~!!!

NUS
National University of Singapore

經濟部水利署
國立成功大學
水工試驗所
Tainan Hydraulics Laboratory
National Cheng Kung University, since 1950

IWDRC,NCKU
國立成功大學國際波動力學研究中心
INTERNATIONAL WAVE DYNAMICS RESEARCH CENTER