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NUS National University of Singapore

A Unified Analytical Solution For Wave Scattering By Rectangular-Shaped Objects

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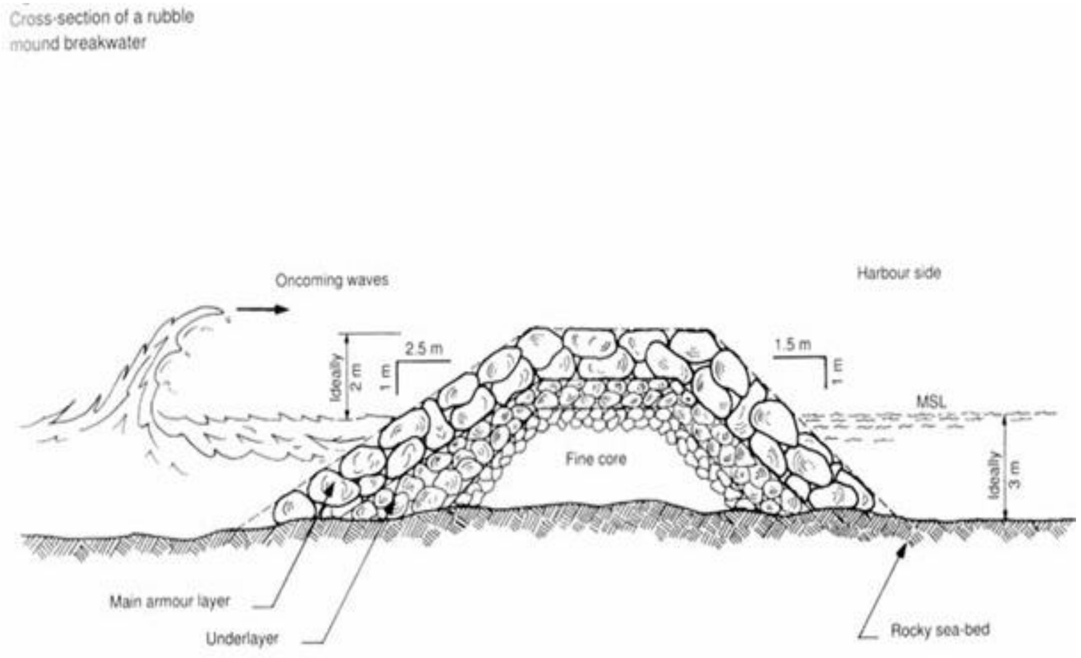
Outline

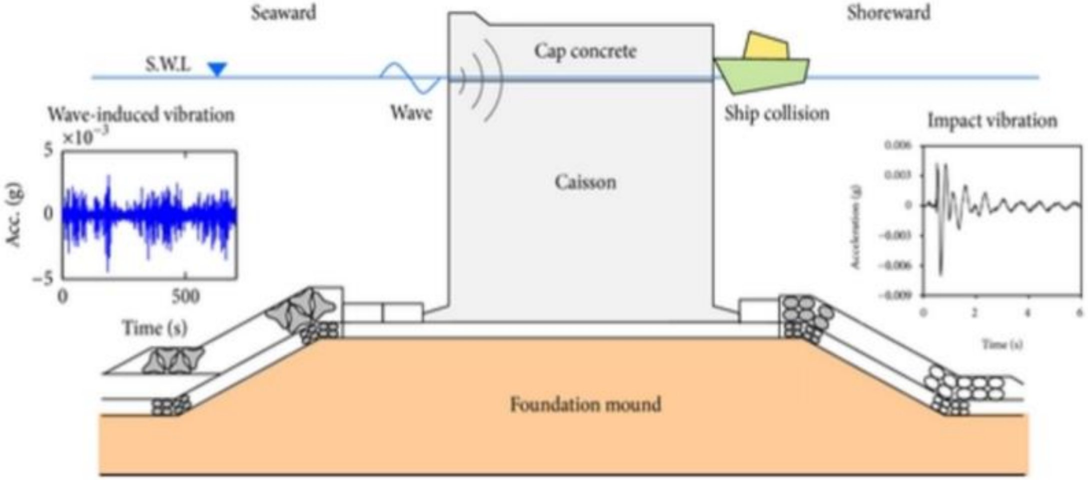
- Introduction and theoretical background
- Model description
  - ➢ A boundary value problem
  - ➢ Solving method
  - ➢ Model validation
  - ➢ Some results from the model
- Conclusion

Breakwaters

- Protecting shorelines
- Providing sheltered area for sea vessels



Rubble Mound breakwater



Caisson breakwater

A Classical Mathematical Model

Sollitt and Cross (1972)

Effects of permeable objects can be described by three parameters.

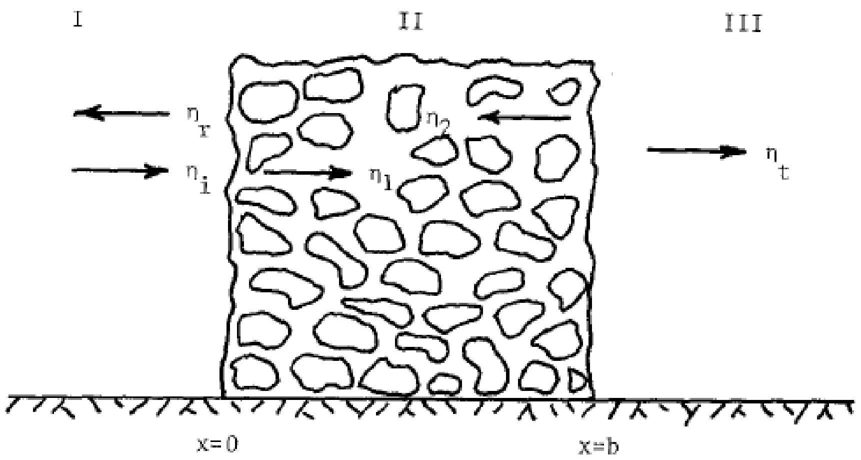


Figure 1. Crib Style Breakwater

- Porosity: ε
- Inertial coefficient: s
- Linearized friction: f

$$\frac{\partial \vec{u}}{\partial t} = -\boldsymbol{\nabla}\left(gz + \frac{P}{\rho}\right) - \left(\alpha\vec{u} + \beta\vec{u}|\vec{u}|\right) - \Upsilon\frac{\partial \vec{u}}{\partial t}$$

dissipative term

Inertial term

- Define an inertial coefficient, $s = 1 + \Upsilon$
- Linearize the dissipative term by an equivalent linear resistance term through a friction coefficient f, $\alpha\vec{u} + \beta\vec{u}|\vec{u}| \rightarrow f\omega\vec{u}$

$$s\frac{\partial \vec{u}}{\partial t} = -\boldsymbol{\nabla}\left(gz + \frac{P}{\rho}\right) - f\omega\vec{u}$$

- Perform the curl operation $\rightarrow \boldsymbol{\nabla} \times \vec{u} = 0$
- A velocity potential ϕ can be defined $\vec{u} = \boldsymbol{\nabla}\phi$
- Mass equation becomes $\boldsymbol{\nabla}^2\phi = 0$

Development of Study

Based on Sollitt and Cross (1972)'s model

- Dalrymple et al (1991) studied reflection and transmission from fully extended surface-piercing porous structures under wave attack
- Rojanakamthorn et al (1989) developed a model for wave transformation on submerged breakwater
- Losada et al (1996) extended Rojanakamthorn et al (1989)'s model to 3-D
- Yu and Chwang (1994) studied water wave over submerged porous plate
- etc…

Objective of Study

This paper presents a unified solution for wave scattering by stationary objects, which consist of a submerged rectangular plate and a floating rectangular dock (Fig. 1).

Characteristics of these objects:

- Same width and aligned centerlines
- Either permeable or solid

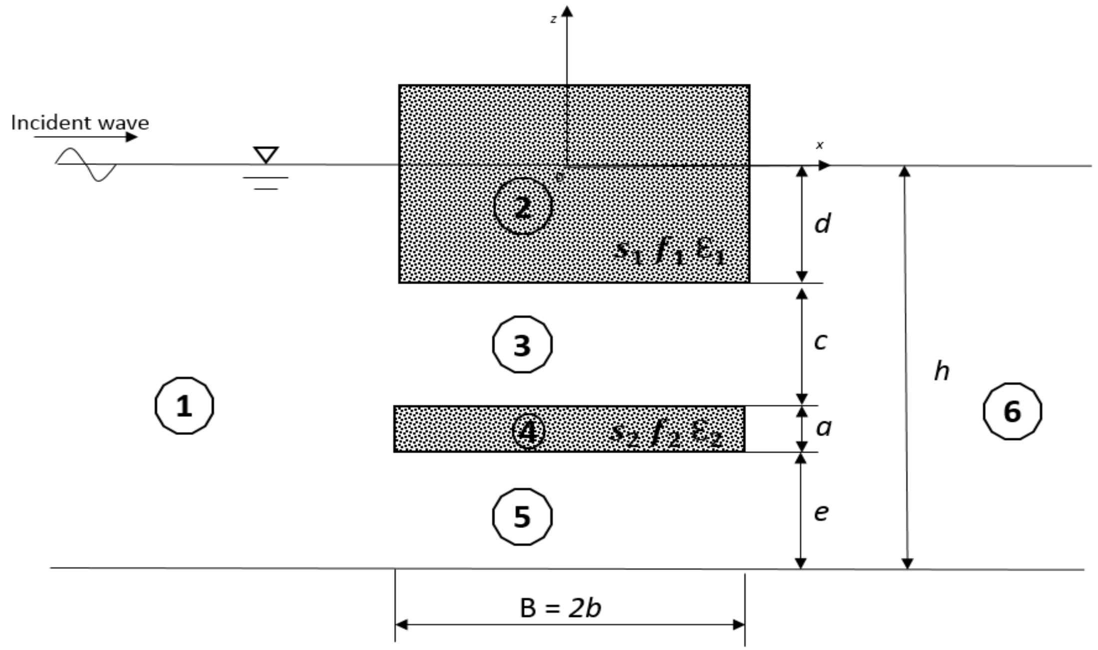
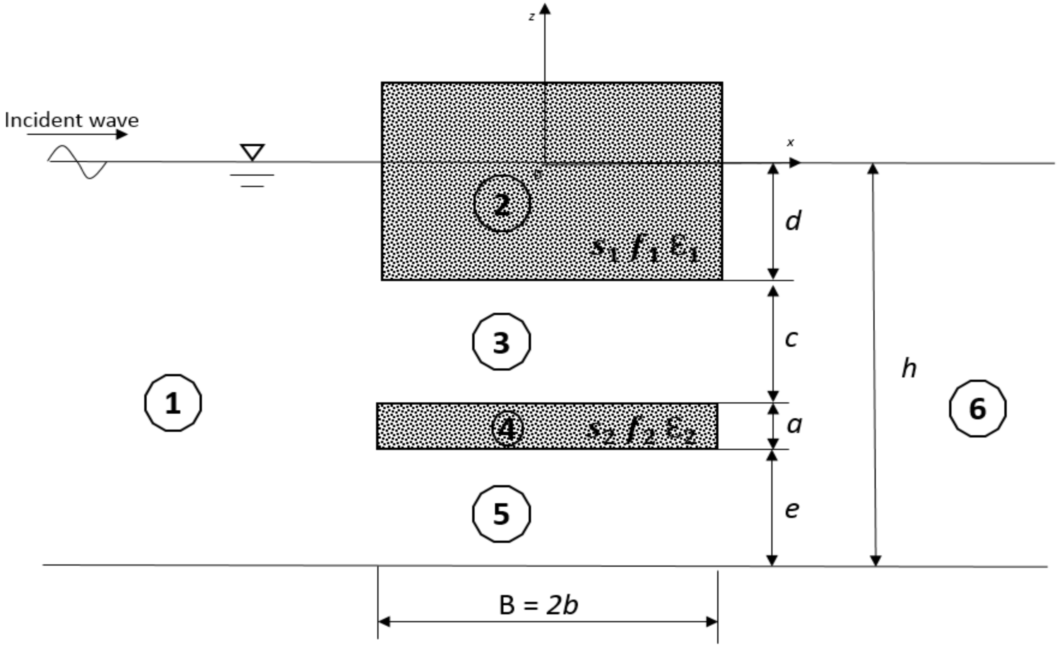


Fig. 1. Sketch of wave scattering by a combination of a floating permeable breakwater and a submerged horizontal permeable plate with finite thickness. Both objects are stationary.

Formulation of Problem



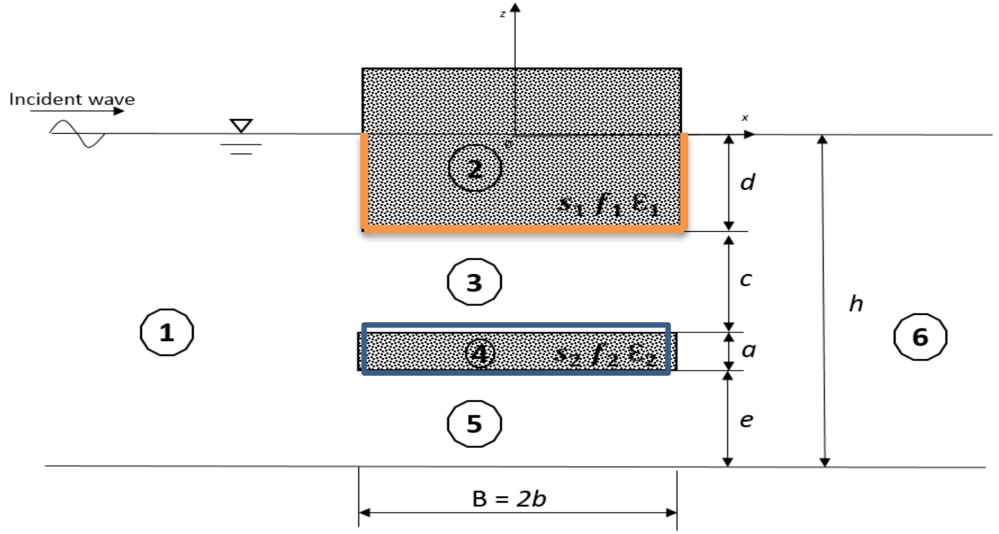
- Within the framework of linear potential flow theory
- Following Sollitt and Cross (1972)'s model

Fluid motions can be described by a potential function $\Phi_j(x, z, t)$, (j = 1-6)

For harmonic incident waves with angular frequency ω,
$\Phi_j(x, z, t)$ = Re[$\phi_j(x, z)e^{-i\omega t}$],

where Re[.] represents the real part of the argument; $i = \sqrt{-1}$; t is the time and $\phi_j(x, z)$ represents the spatial distribution of the potential function.

Formulation of Problem



- Governing equation

$$\frac{\partial^2 \phi_j}{\partial x^2} + \frac{\partial^2 \phi_j}{\partial z^2} = 0, \qquad (j = 1 - 6)$$

- Boundary conditions

$$\frac{\partial \phi_j}{\partial z} = K\phi_j, \qquad (j = 1 \text{ and } \ 6) \qquad \text{on } \ z = 0,$$

$$\frac{\partial \phi_2}{\partial z} = K(s_1 + if_1)\phi_2, \quad \text{on } \ z = 0,$$

$$\frac{\partial \phi_j}{\partial z} = 0, \qquad (j = 1, 5, 6) \qquad \text{on } \ z = -h,$$

$$(s_{1(2)} + if_{1(2)})\phi^- = \phi^+ \quad \text{on } \boldsymbol{\Gamma}$$

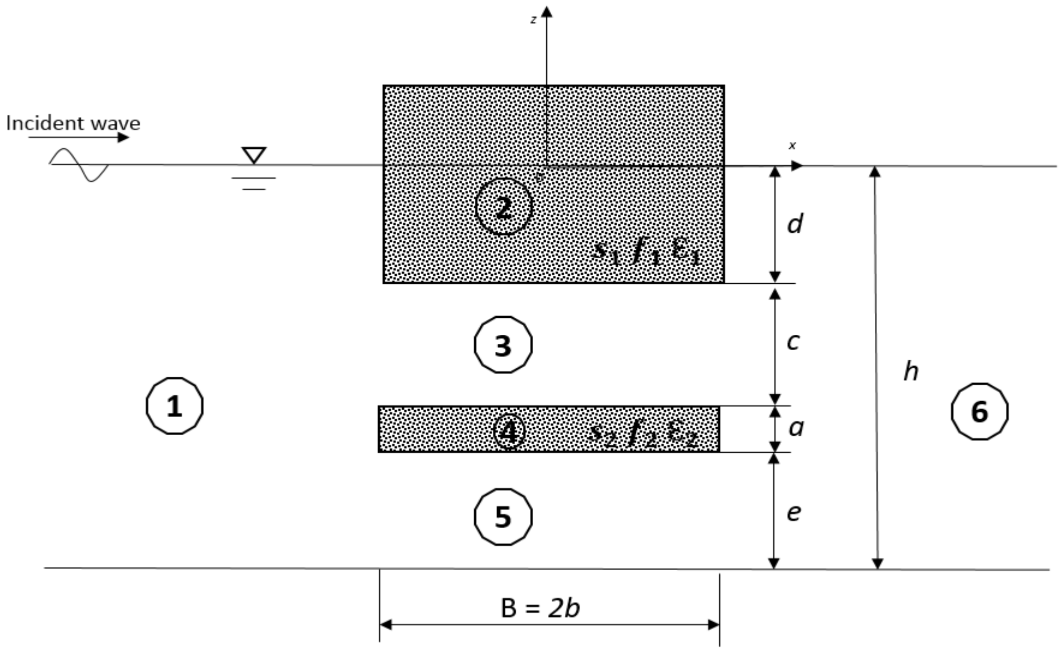
$$\varepsilon_{1(2)} \frac{\partial \phi^-}{\partial n} = \frac{\partial \phi^+}{\partial n} \quad \text{on } \boldsymbol{\Gamma}$$

Boundary Value Problem

where $K = \omega^2/g$, and $s_{1(2)}$ and $f_{1(2)}$ are, respectively, the inertial coefficient and linearized resistance coefficient of the permeable breakwaters (Sollitt and Cross, 1972). **Γ** denotes common boundary between the permeable objects and surrounding fluid.

Difficulties in solving BVP

- A non-trivial same wavenumber (inside breakwater and in the water region above/beneath) assumption is necessary for obtaining the solution
- A complex dispersion relation occurs and there exists difficulties of finding the complex roots



Same wavenumber for region 2, 3, 4 and 5.
And wavenumber is complex.

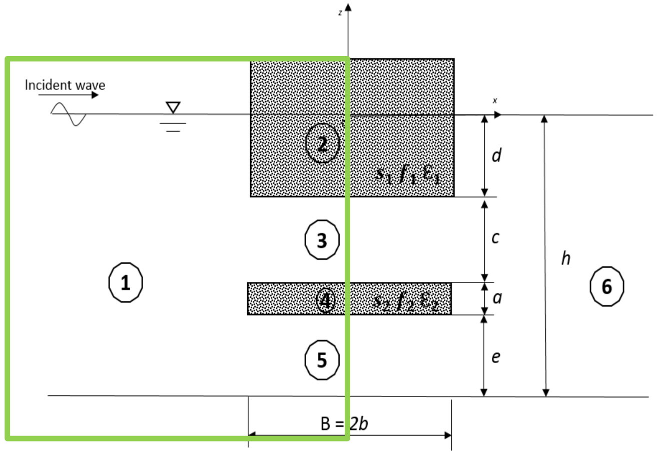
Potential Decomposition Method

- For the purpose of simplicity Mei and Black (1969)

$$\phi(x,z) = \frac{1}{2}[\phi^S(x,z) + \phi^A(x,z)],$$

Symmetric

Anti-symmetric



- Further decompose $\phi^S(x,z)/\phi^A(x,z)$ into two auxiliary potentials

$$\phi_j^{S(A)} = \phi_{j,v}^{S(A)} + \phi_{j,u}^{S(A)},$$

$j = 2,3,4,5$ ← Middle region

Vertical

Horizontal

- Seek appropriate boundary conditions in order to construct the solutions for these auxiliary potentials in the middle region

Ref: Lee and Liu (1995); Liu et al. (2012); Liu and Li, (2013)

Potential Decomposition Method

The appropriate boundary conditions include:

Due to symmetric and anti-symmetric requirement at the centerline Mei and Black (1969):

$$\frac{\partial \phi^{S}_{j,v(u)}}{\partial x} = 0, \quad x = 0, \quad j = 2,3,4,5$$

$$\phi^{A}_{j,v(u)} = 0, \quad x = 0, \quad j = 2,3,4,5.$$

Auxiliary Potential functions $\phi^{S(A)}_{v(h)}$ are constructed, and all the eigenvalues are simple real numbers

Other homogeneous boundary conditions:

- On the edge of rectangular bodies:

$$\phi^{S}_{j,u} = 0, \quad \frac{\partial \phi^{A}_{j,u}}{\partial x} = 0, \quad x = -b, \quad j = 2,3,4,5$$

- On the free surface:

$$\frac{\partial \phi^{S(A)}_{2,v}}{\partial z} = 0, \quad z = 0$$

- Along the interface:

$$\frac{\partial \phi^{S(A)}_{j,v}}{\partial z} = 0, \quad j = 2,3,4,5$$

- On the bottom

$$\frac{\partial \phi^{S(A)}_{5,v(u)}}{\partial z} = 0, \quad z = -h$$

Potential Decomposition Method

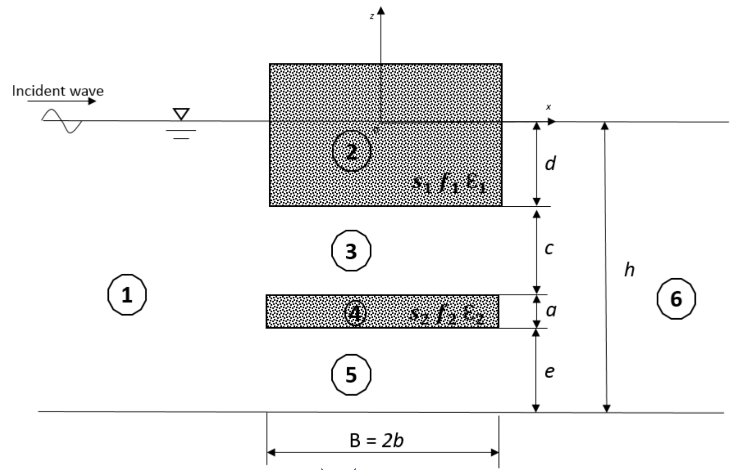
Physcially, the matching boundary conditions are required to ensure the continuity of pressure and flux between the fluids and objects:

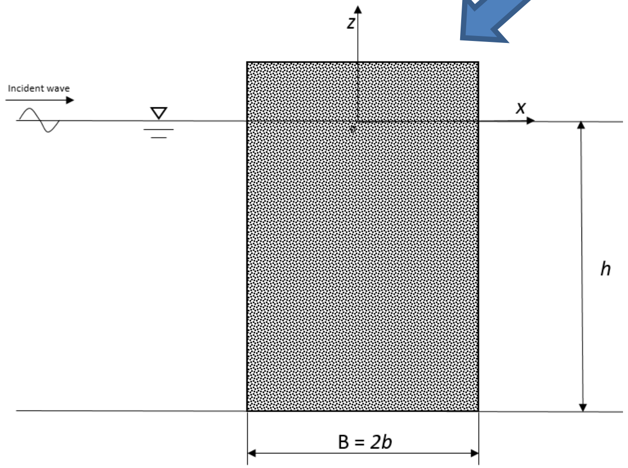
$$\phi_i^+ = \begin{cases} \phi_j^+ \\ (s_{1(2)} + if_{1(2)})\phi^- \end{cases} \quad \& \quad \frac{\partial \phi_i^+}{\partial n} = \begin{cases} \dfrac{\partial \phi_j^+}{\partial n} \\ \varepsilon_{1(2)} \dfrac{\partial \phi^-}{\partial n} \end{cases} \quad \text{on } \boldsymbol{\Gamma}$$

where the superscripts + and – distinguish the pure water region and breakwater region, *i*, *j* denote different region, respectively.

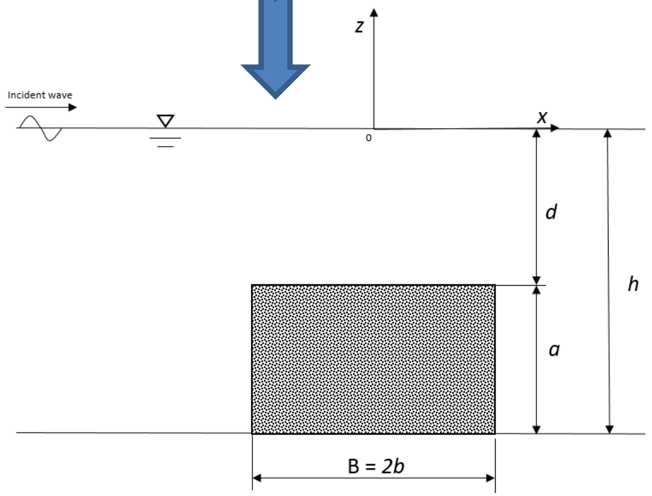
The Boundary Value Problem is complete and the unknown expression coefficients of auxiliary potential functions $\phi_{v(h)}^{S(A)}$ can be determined.

Model Validations with Different Breakwater Configurations

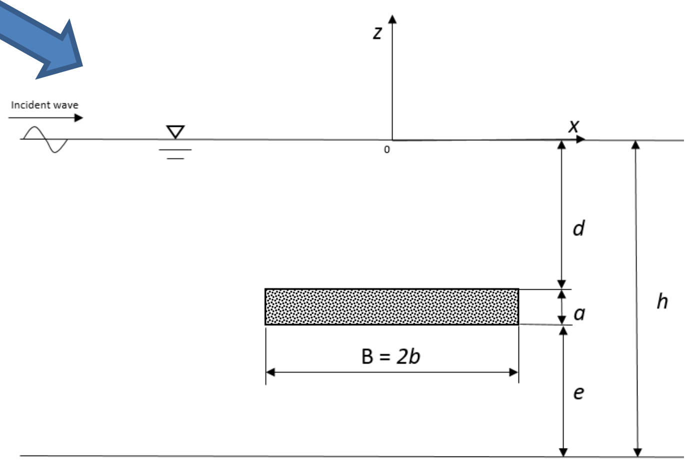




Liu and Li (2013)



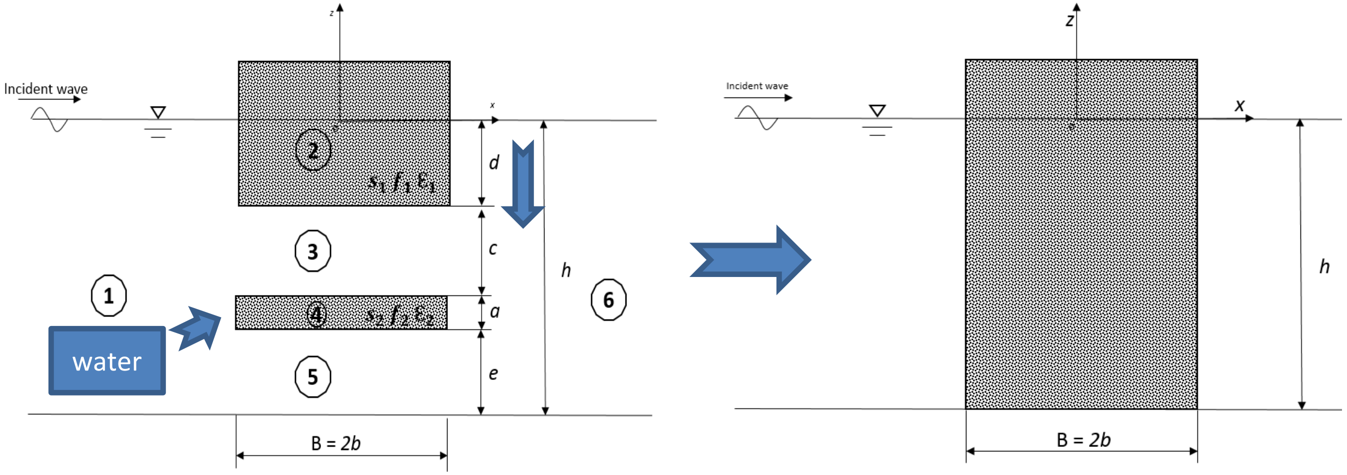
Lee and Liu (1995)



Liu et al. (2012)

Model Validations with Different Breakwater Configurations

Validation with Fully Extended Surface-Piercing Breakwater Liu and Li (2013)



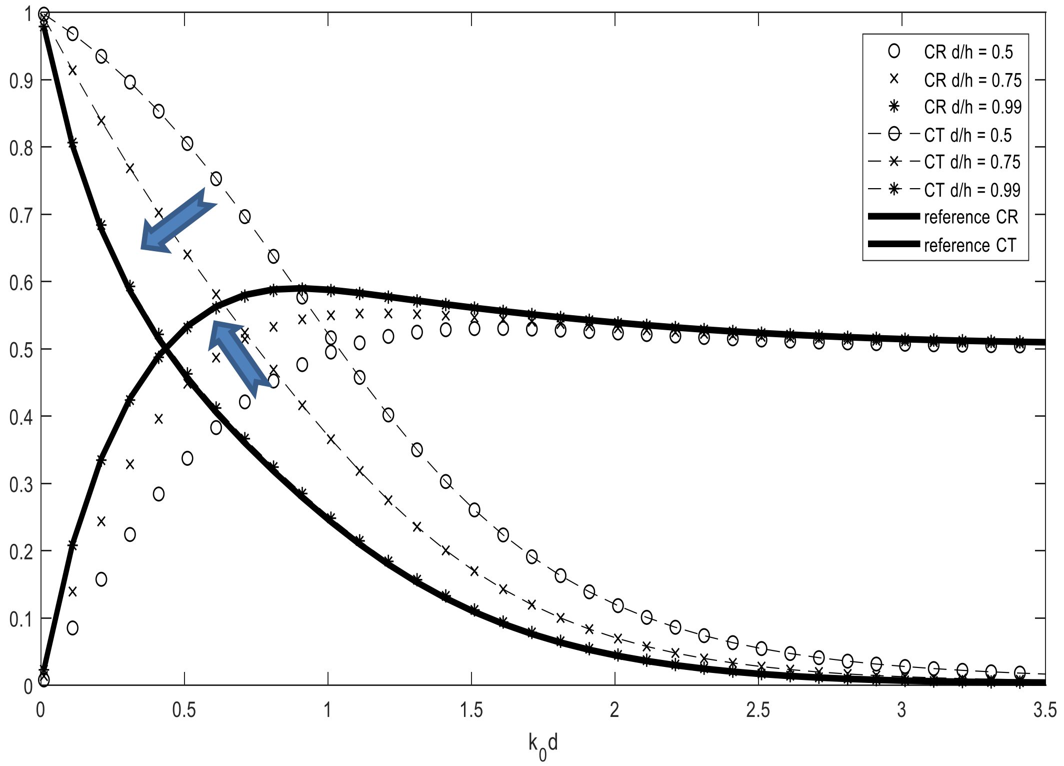


Fig. 2 Comparisons of reflection coefficient, C_R, transmission coefficient, C_T, between the general solution and the fully extended surface piercing solution at B/h = 1.0, ε_1 = 0.45, f_1=2.0, s_1=1.0, ε_2 = 1.0, f_2=0.0, s_2=1.0, $c = a = e = (h\text{-}d)/3$, d/h = ratio (ratio=0.5, 0.75, 0.99). The bold line named 'reference C_R (C_T)' is from fig 4 of Liu and Li, (2013) served as reference. The symbols of circle, cross, asterisk represent the C_R (C_T) under different d/h

Model Validations with Different Breakwater Configurations

Validation with Submerged bottom-mounted Breakwater Lee and Liu (1995)

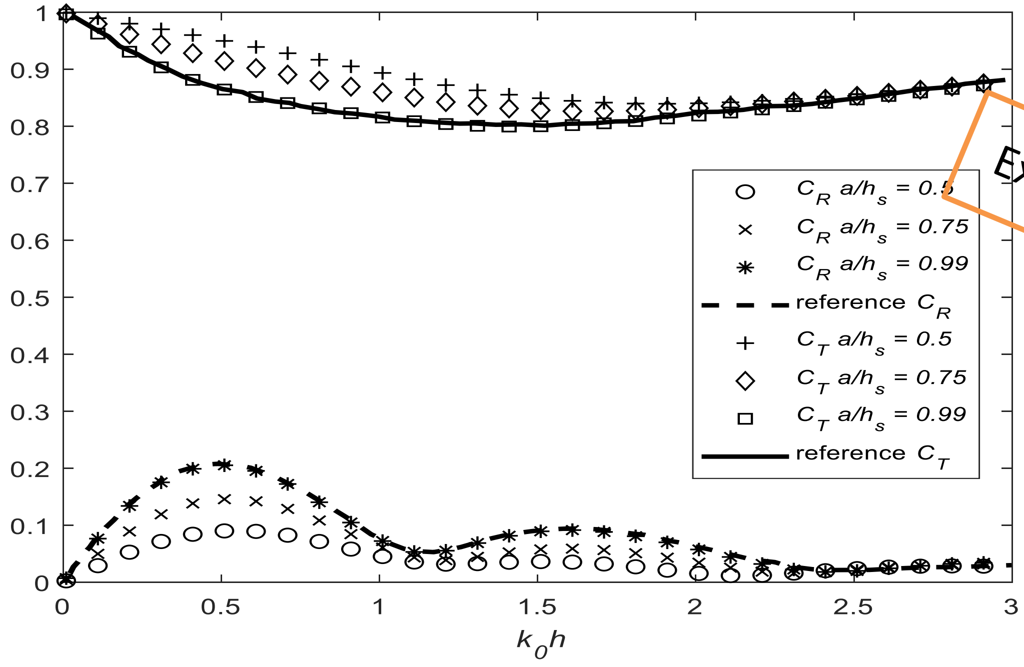


Fig. 3. Variation of reflection coefficient, C_R, and transmission coefficient, C_T, versus relative water depth k_0h, for a submerged permeable plate with downward increasing thickness.

Excellent agreement

Validation with Suspended Plate Liu et al. (2012)

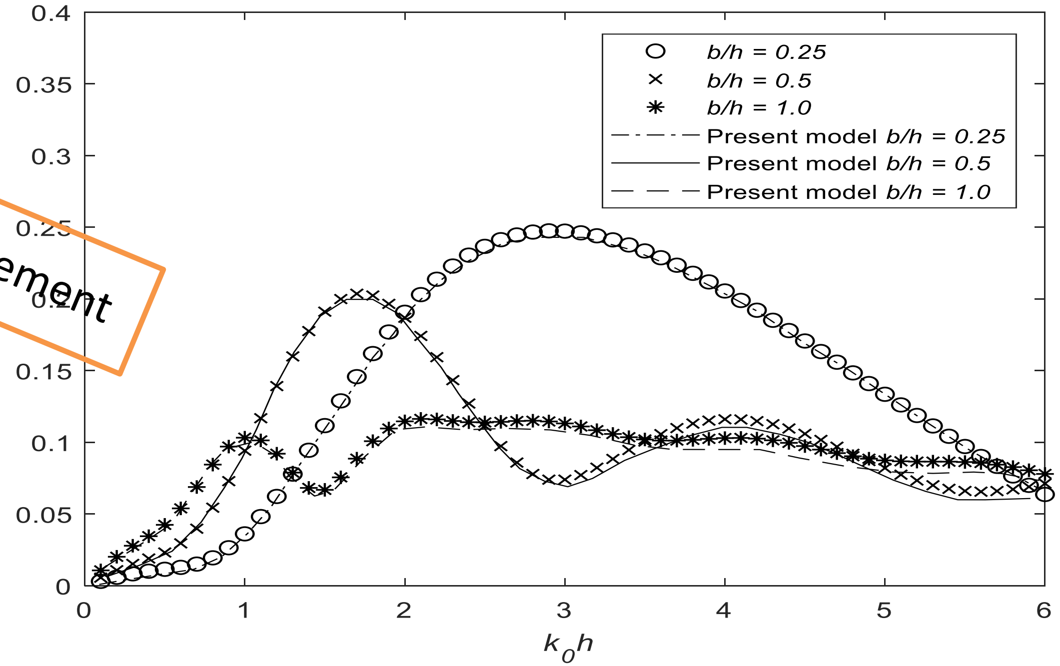
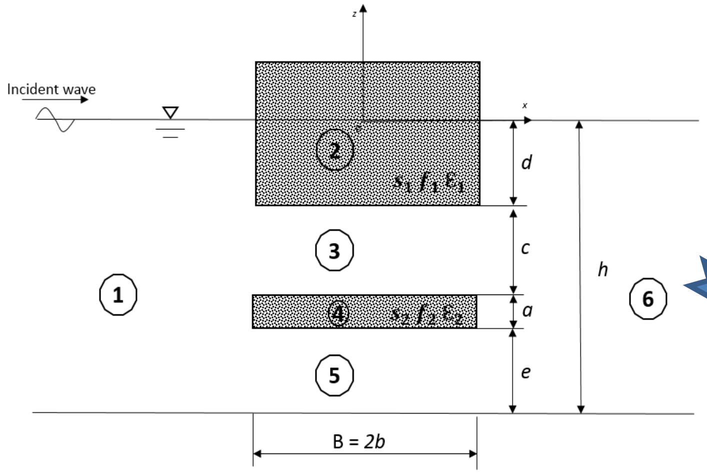


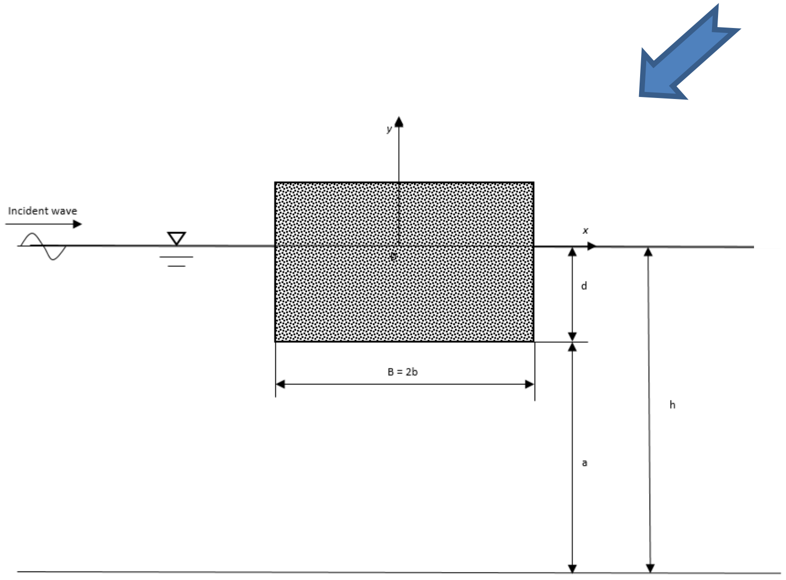
Fig. 4. Variations of reflection coefficient C_R, for a submerged permeable plate versus relative water depth k_0h for three different relative width of plate b/h = (0.25, 0.5, 1.0).

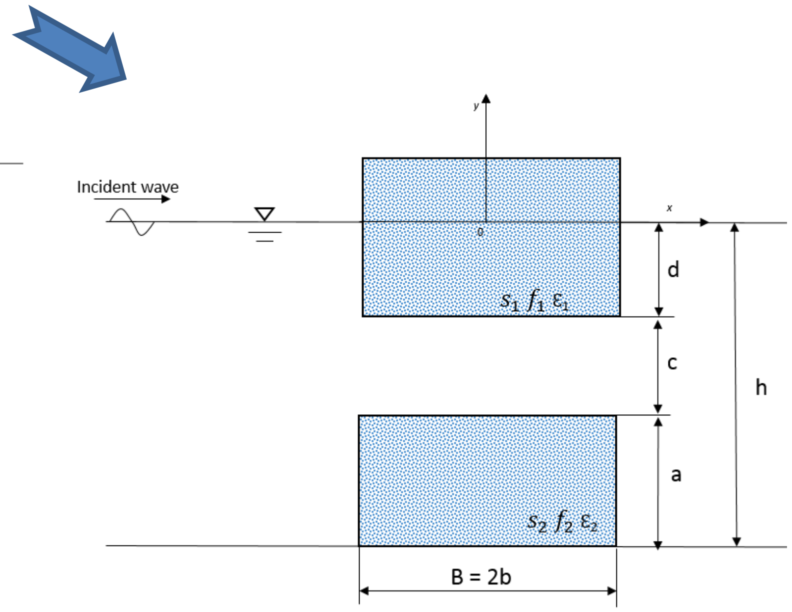
Other types of breakwater can be produced

Other New Solutions Produced from the General Model are:

- A stationary floating breakwater (left Fig)
- A combination of a floating and a bottom mounded submerged permeable breakwater (right Fig)







Conclusion

- A unified analytical solution is presented for wave scattering by stationary breakwaters
- The general model successfully produces the existing solutions for different types of single object and it can also produce new solutions for a single floating breakwater and for the case with a combined floating and bottom-mounted breakwater.

Thank you

References

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Annex

Sollitt and Cross (1972)

Combining the steady state damping law proposed by Ward with the additional inertial damping law proposed by the authors yields the appropriate replacement for the resistance forces in Eq. (1).

$$\frac{\partial q}{\partial t} = -\frac{1}{\rho}\nabla(p+\gamma z) - \frac{\nu}{K_p}\varepsilon q - \frac{C_f}{\sqrt{K_p}}\varepsilon^2 q|q| - \frac{1-\varepsilon}{\varepsilon}C_M\frac{\partial q}{\partial t}$$

$$\nabla \cdot q = 0$$

Linear resistance term | square resistance term | Inertial resistance term

where ν is the kinematic viscosity, K_p is the intrinsic permeability, C_f is a dimensionless turbulent resistance coefficient and ε is the porosity of the medium. The linear term governs low Reynolds number flow and the square law term dominates high Reynolds number flow.

The non dissipative inertial resistance term may be transposed to the left hand side of the equation and an inertial coefficient, S, defined as

$$S = 1 + \frac{1-\varepsilon}{\varepsilon}C_M$$

The dissipative stress term in momentum equation is replaced by an equivalent linear resistance term by the Lorentz's hypothesis of equivalent work

$$\frac{\nu\varepsilon q}{K_p} + \frac{C_f\varepsilon^2}{\sqrt{K_p}} q|q| \rightarrow f\sigma q$$

where σ is the angular frequency of the periodic motion and f is a dimensionless friction or damping coefficient.

So the linearized momentum equation is:

$$S\frac{\partial q}{\partial t} = -\frac{1}{\rho}\nabla(p+\gamma z) - f\sigma q$$

Performing the curl operation on this equation demonstrates the irrotationality of the seepage velocity field, that is

$$\sigma(iS+f)\nabla \times q = -\frac{1}{\rho}\nabla\times\nabla(p+\gamma z) = 0$$

Thus, $\nabla \times q = 0$, the flow field is irrotational and a velocity potential, Φ, may be defined wherein

$$q = \nabla\Phi$$

Annex

In the boundary value problem, there are several boundary conditions:

- the homogeneous Robin condition at the free surface
- the homogeneous Neumann condition at the seabed
- the nonhomgenous Dirichlet/Neumann conditions at the interface.

To reconstruct the BVP

1. We consider two auxiliary potentials, for (anti)symmetric part respectively, with either a homogeneous Neumann condition or a homogeneous Dirichlet condition at the boundaries of each region.
2. A series of eigenfunctions with corresponding eigenvalues can be constructed in each region.
3. As a result, the expression of potential function in each region is obtained.
4. Finally, the matching conditions between neighbouring potentials are used to determine the unknown coefficients in the reconstructed potentials.