REEF3D::NSEWAVE, A Three-Dimensional Non-Hydrostatic Wave Model on a Fixed Grid

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Motivation for Wave Modeling

Coastal Engineering

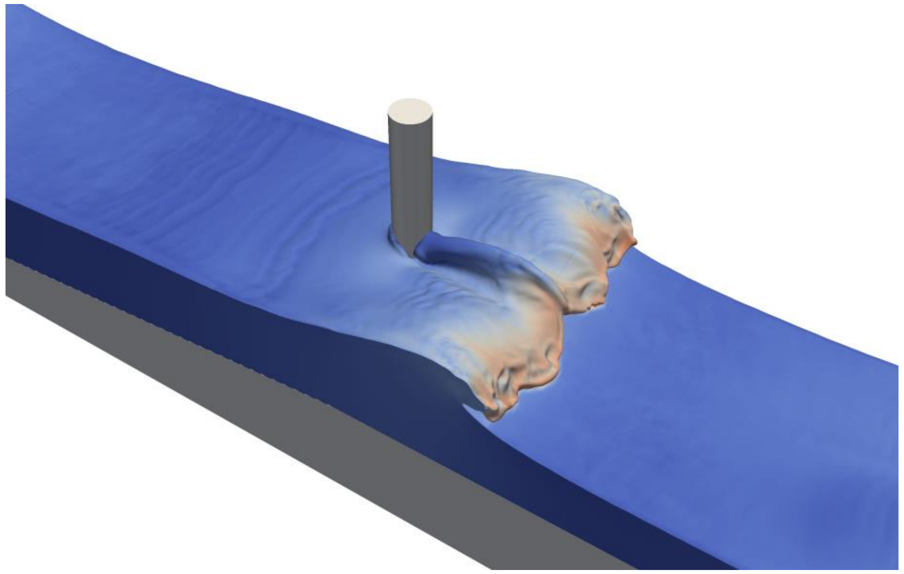
Aquaculture

Offshore Structures: Floating and Mooring

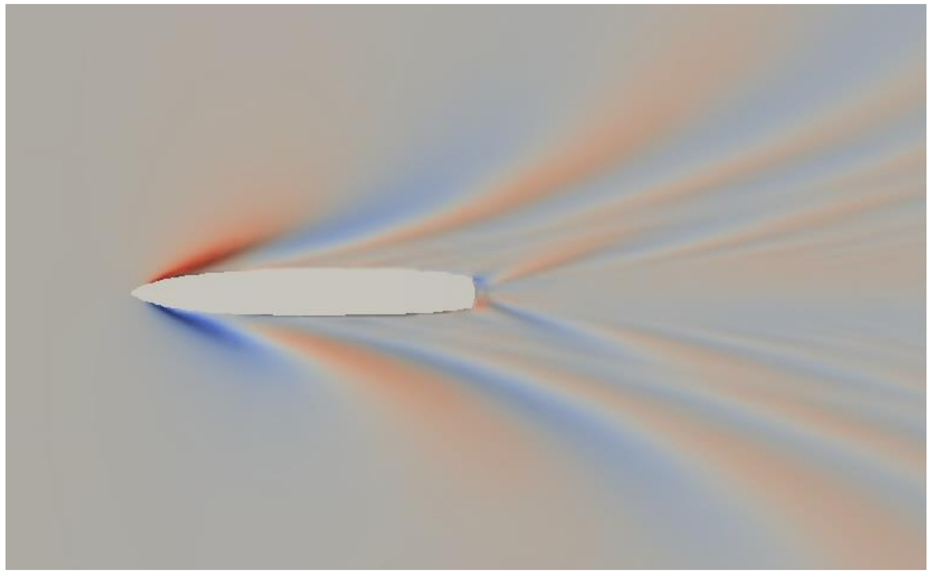
Structures in Waves / Ocean Energy

REEF3D::CFD

- **Solves:**
  - Full 3D Navier-Stokes Equations
  - Free Surface: Two-Phase Flow - Water & Air
  - Turbulence



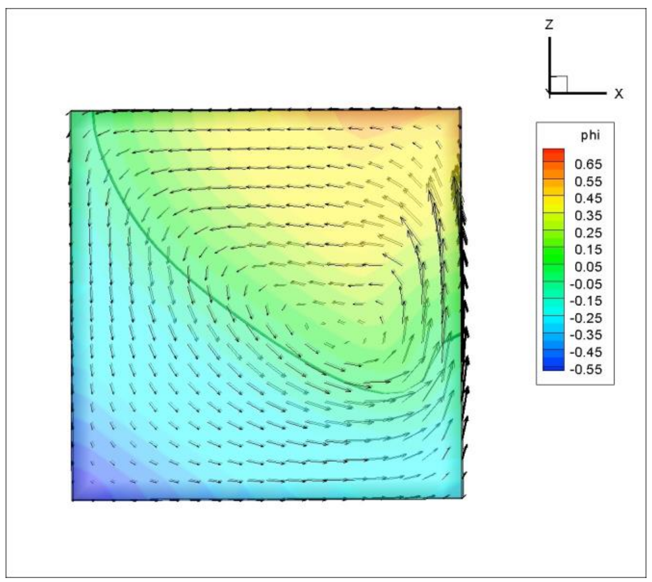
- **Focus on:**
  - Free Surface Flows
  - Wave Hydrodynamics
  - Wave Structure Interaction
  - Floating Structures
  - Open Channel Flow
  - Sediment Transport



- **The Code:**
  - C++ (modular & extensible)
  - Parallel Computing / HPC
  - Open-Source
  - Developed at the Department of Civil and Environmental Engineering, NTNU Trondheim

Free Surface Modeling

Interface Capturing
REEF3D::CFD



$$\phi(\vec{x},t)\begin{cases} > 0 \; if \;\; \vec{x} \in phase\ 1 \\ = 0 \; if \;\; \vec{x} \in \Gamma \\ < 0 \; if \;\; \vec{x} \in phase\ 2 \end{cases} \qquad , |\nabla \phi| = 1$$

$$\phi_t + \vec{u} \cdot \nabla \phi = 0$$

Interface Tracking
REEF3D::NSEWAVE

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}\int_{-d}^{\zeta} u dz + \frac{\partial}{\partial y}\int_{-d}^{\zeta} v dz = 0$$

- single-valued free surface
- based on depth-integrated continuity
  - (less interface resolution depended)
- Eulerian mesh: level set function implicitly defines free surface for full NSE-solver
- larger CFL number and dx possible (see results)

Governing Equations

Incompressible RANS Equations:

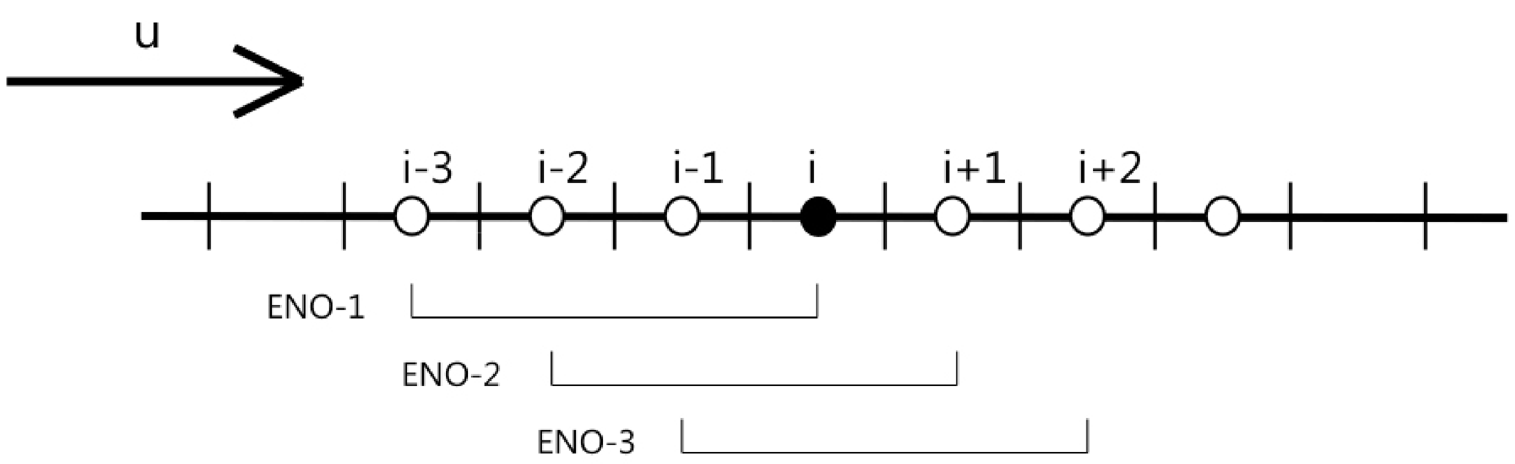
$$\frac{\partial U_i}{\partial x_i} = 0$$

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left[(\nu + \nu_t)\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\right] + g_i$$

- Temporal Discretization
- Spatial Discretization
- Pressure Solution
- Turbulence Modeling

Spatial Discretization

Convection Discretization: Conservative 5th-order WENO



$$U\frac{\partial U}{\partial x} \approx \frac{1}{\Delta x}\left(\tilde{U}_{i+1/2}U_{i+1/2} - \tilde{U}_{i-1/2}U_{i-1/2}\right)$$

$$U^{\pm}_{i+1/2} = \omega^{\pm}_1 U^{1\pm}_{i+1/2} + \omega^{\pm}_2 U^{2\pm}_{i+1/2} + \omega^{\pm}_3 U^{3\pm}_{i+1/2}$$

- can handle large gradient
- high accuracy
- maintains the sharpness of the extrema

Time Discretization

3rd-order TVD Runge-Kutta:

$$\phi^{(1)} = \phi^n + \Delta t L\left(\phi^n\right)$$

$$\phi^{(2)} = \frac{3}{4}\phi^n + \frac{1}{4}\phi^{(1)} + \frac{1}{4}\Delta L\left(\phi^{(1)}\right)$$

$$\phi^{n+1} = \frac{1}{3}\phi^n + \frac{2}{3}\phi^{(2)} + \frac{2}{3}\Delta L\left(\phi^{(2)}\right)$$

Adaptive Time-Stepping:

$$\delta t \leq 2\left(\left(\frac{|u|_{max}}{\delta x} + V\right) + \sqrt{\left(\frac{|u|_{max}}{\delta x} + V\right)^2 + \frac{4|g|_{g1}}{\delta x}}\right)^{-1}$$

with

$$V = max\left(\nu + \nu_t\right) \cdot \left(\frac{2}{(\delta x)^2} + \frac{2}{(\delta y)^2} + \frac{2}{(\delta z)^2}\right)$$

R
constraint:
- implicit diffusion treatment

Pressure

Projection Method:

1. solve NS-equation without pressure gradient

2. Poisson equation for Pressure

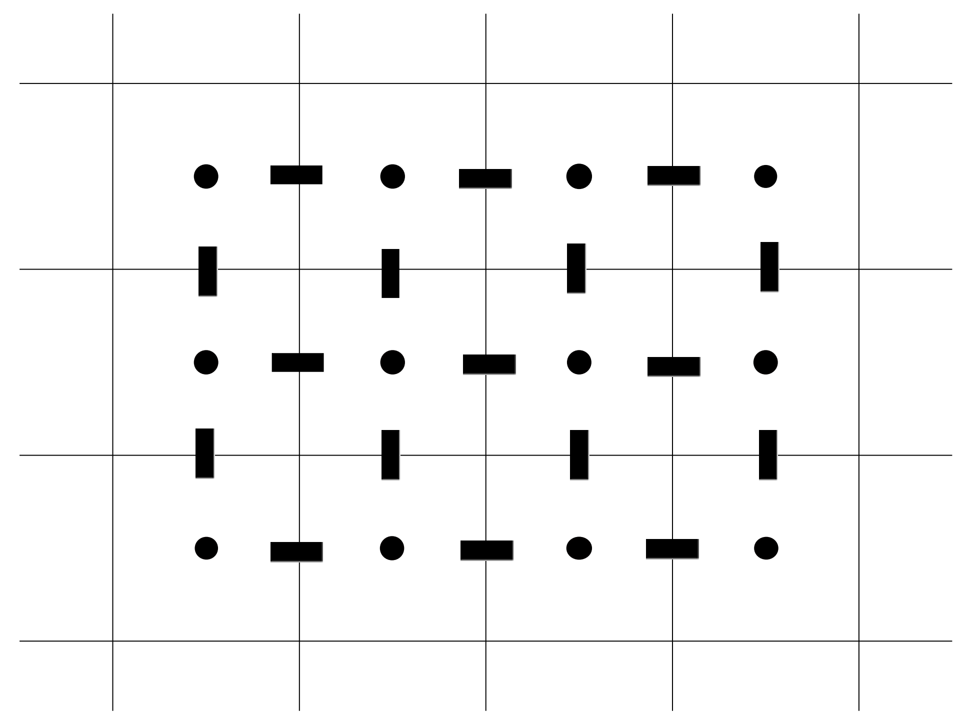
$$-\frac{\partial}{\partial x_i}\left(\frac{1}{\rho\left(\phi^n\right)}\frac{\partial p}{\partial x_i}\right) = -\frac{1}{\Delta t}\frac{\partial u_i^*}{\partial x_i}$$

3. Iterative Solution of Poisson equation
   hypre: BiCGStab + geometric
multigrid

4. Correct the intermediate velocity field

$$u_i^{n+1} = u_i^* - \frac{\Delta t}{\rho\left(\phi^n\right)}\frac{\partial p}{\partial x_i}$$

Staggered Grid:
- tight pressure-velocity coupling
- important for 2-phase flow:
density jump across the interface

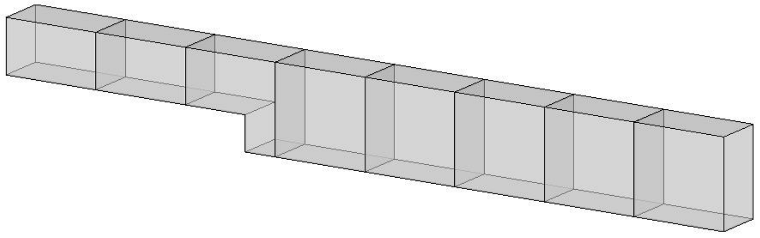


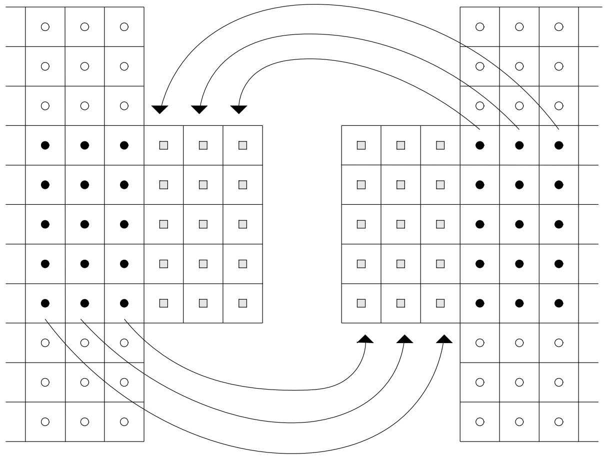
Fc
- Gravity included in RANS-Equations
- P includes hydrostatic and dynamic
parts

$$F = \int_\Omega (-\mathbf{n}P + \mathbf{n}.\tau)d\Omega$$

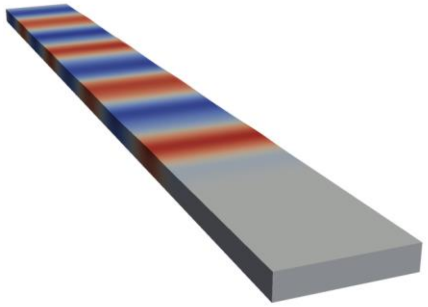
Parallelization

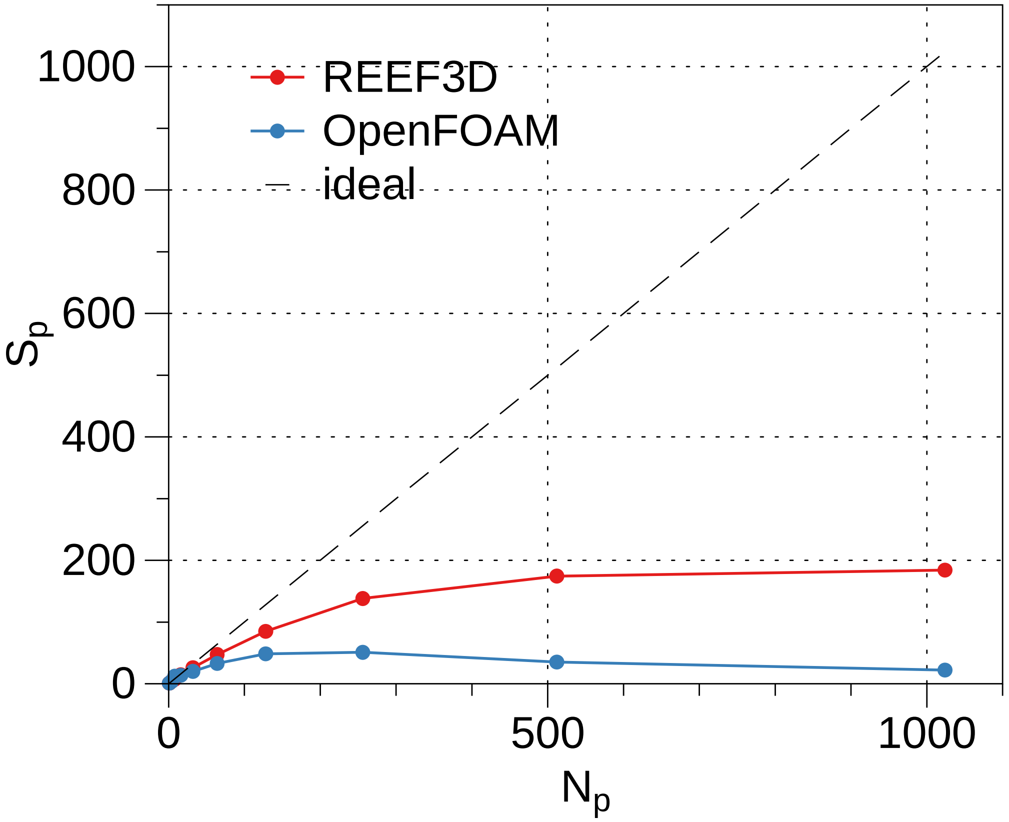
- **Parallelization**
- MPI
- domain decomposition



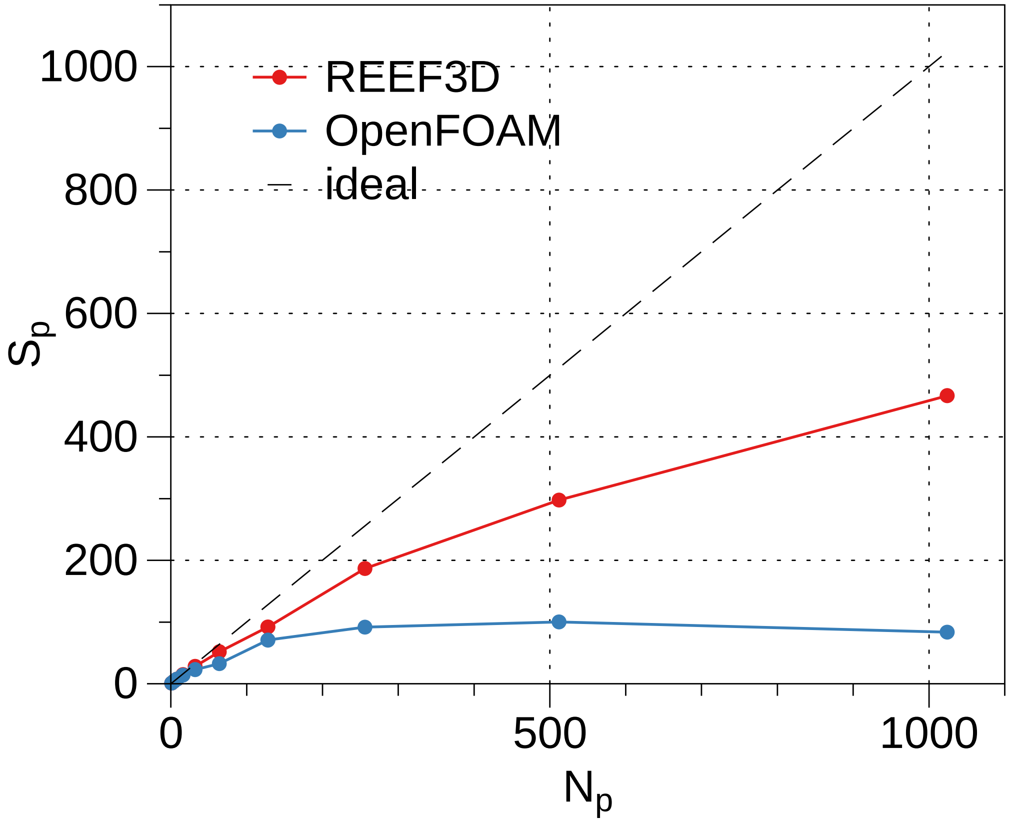


Scaling test for 3D wave basin





2 mil. cells

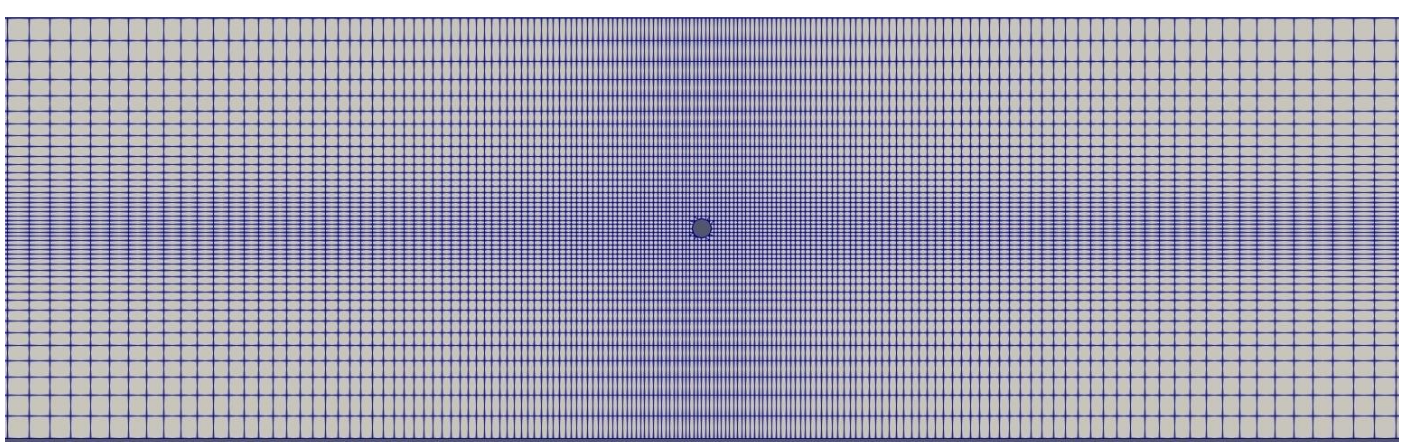


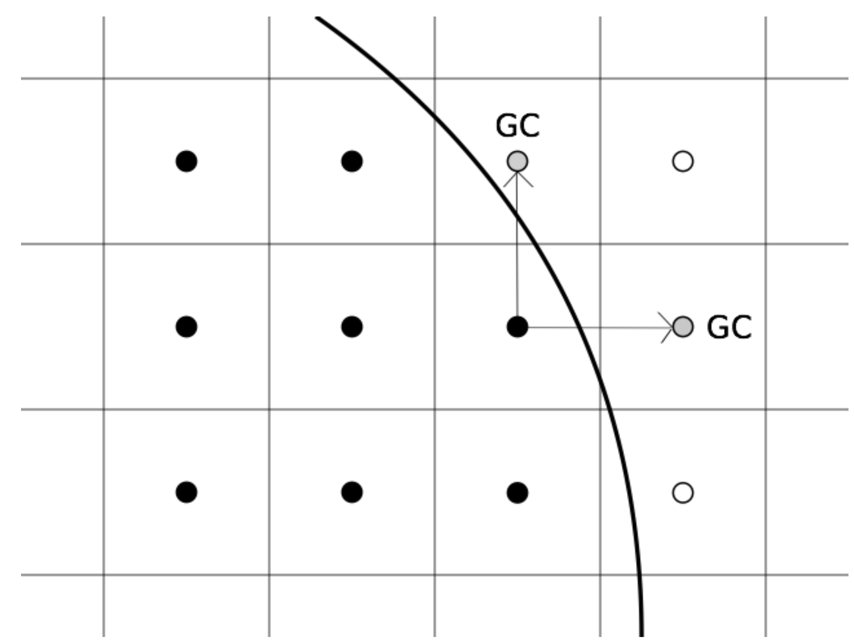
12 mil. cells

Rectilinear Grid

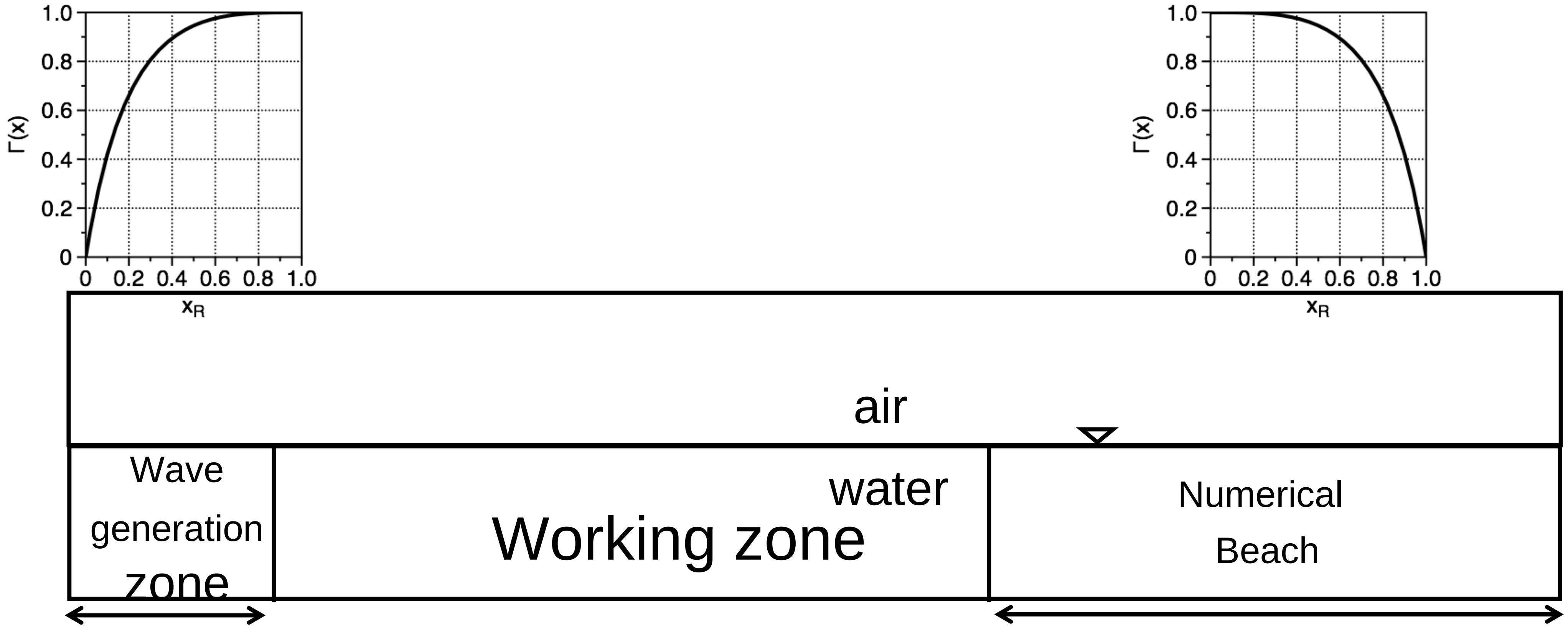
Rectilinear Grid:
- Implementation of numerical algorithms is straightforward
- level set method is Eulerian
- complex geometries are possible with immersed boundary

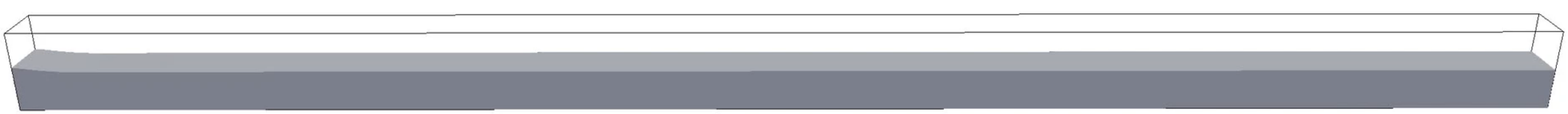
Ghostcell Immersed Boundary:
- no extra boundary treatment
- works well with Parallelization with MPI
- works well with immersed boundary





Numerical Wave Tank





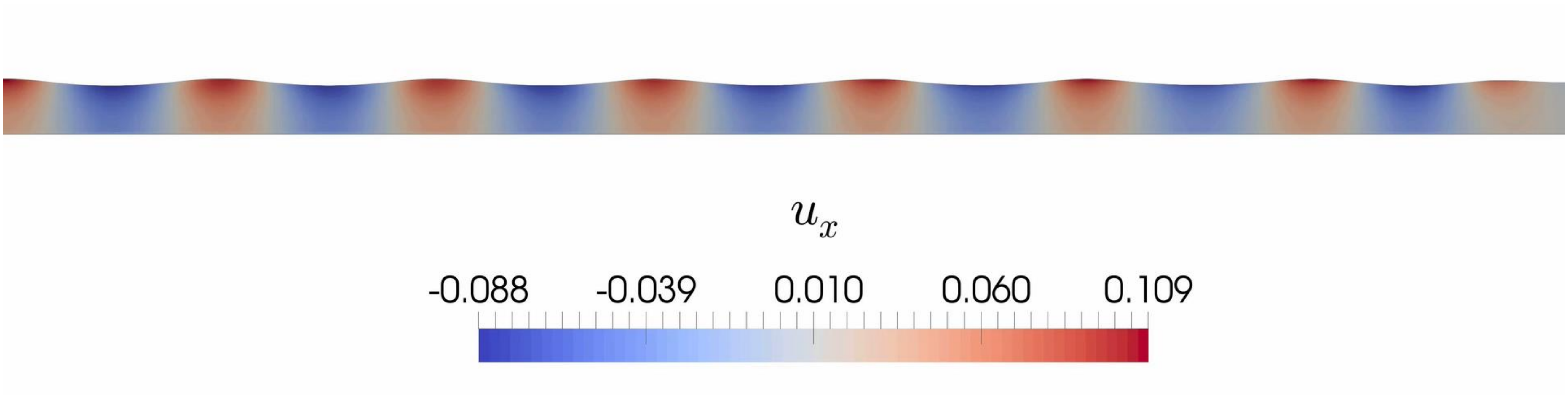
other available methods

- active wave absorption (AWA)
- Dirichlet wave generation

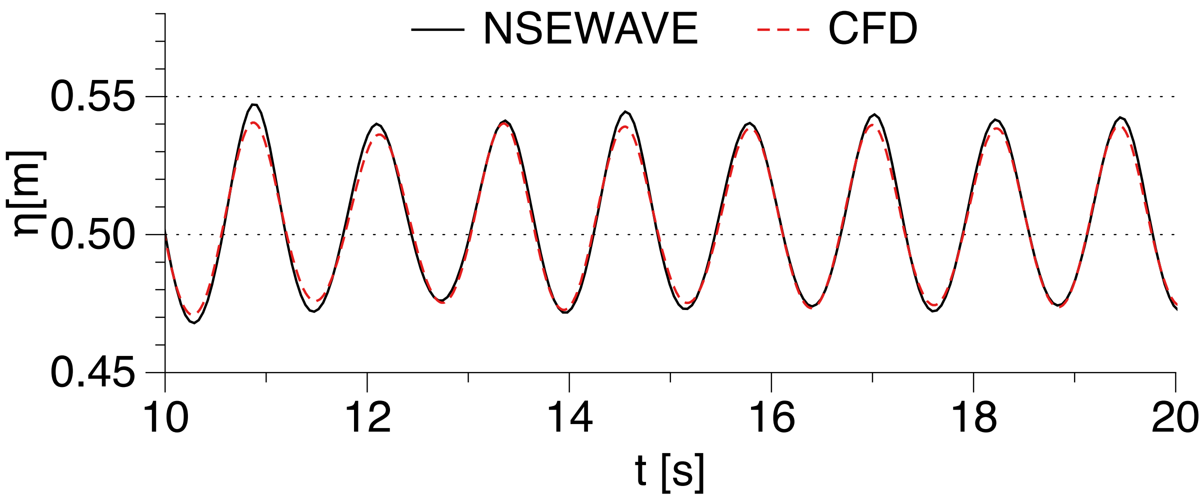
2D Wave Tank Tests

Setup:
- 18 m long
- 1 m high
- d=0.505 m
- H = 0.07m
- L = 2 m

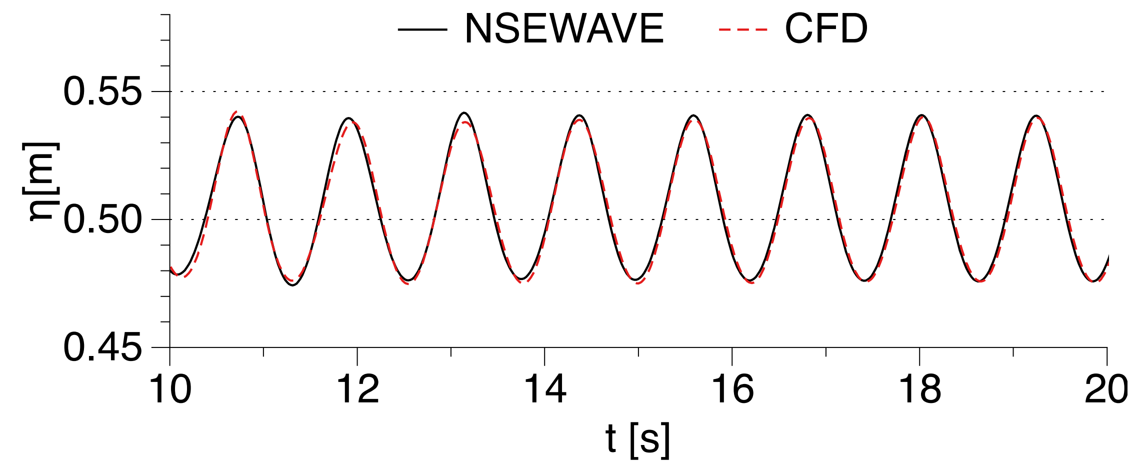
Grid:
- dx
    - 0.1 m
    - 0.05 m
    - 0.025 m
    - 0.0125 m



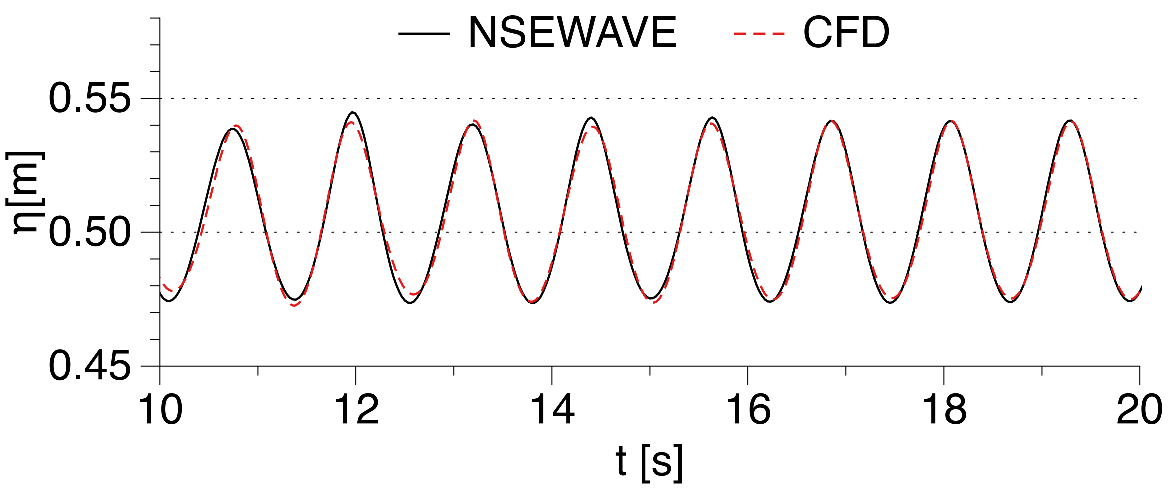
2D Wave Tank Tests



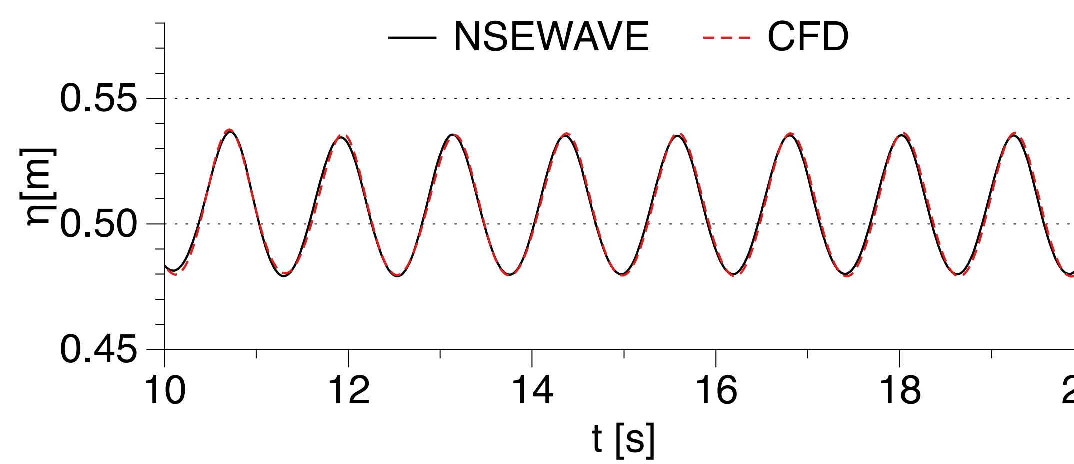
(a) $dx = 0.1$ m at $x = 9$ m



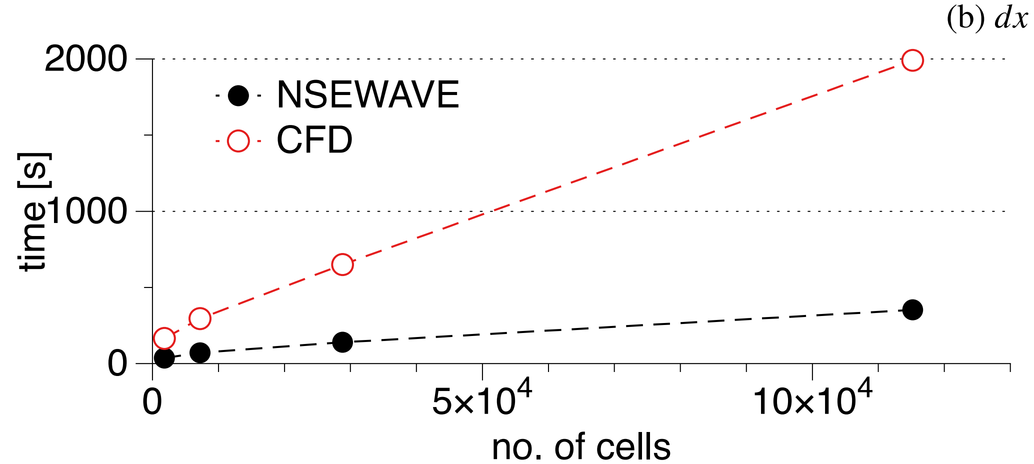
(b) $dx = 0.025$ m at $x = 9$ m



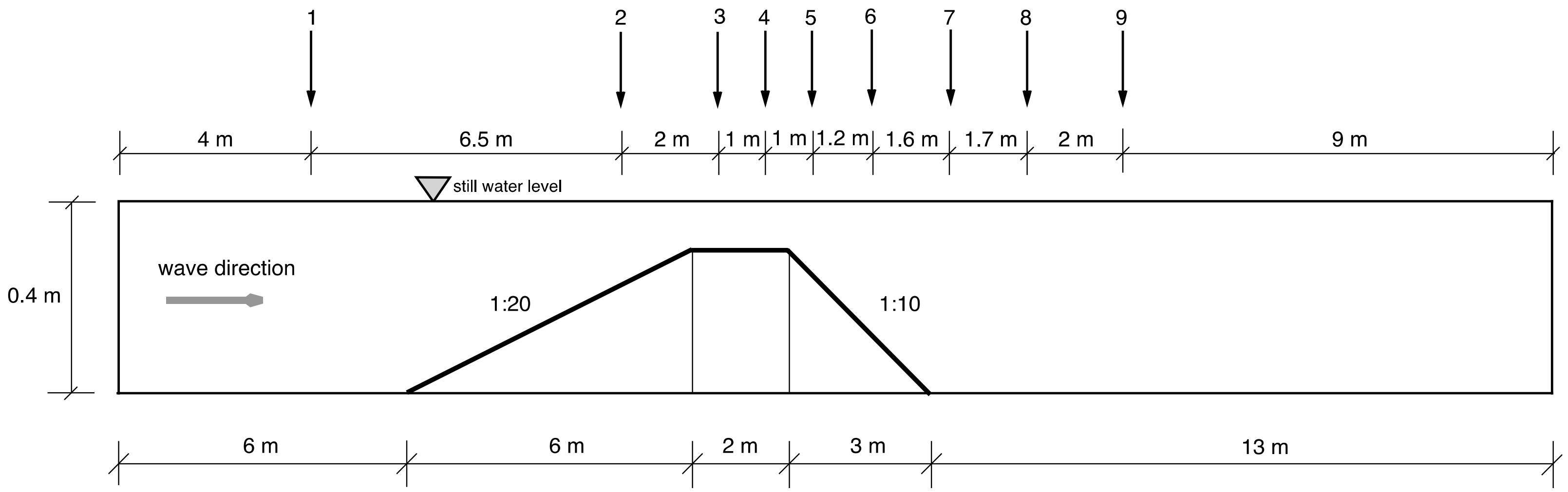
(b) $dx = 0.05$ m at $x = 9$ m

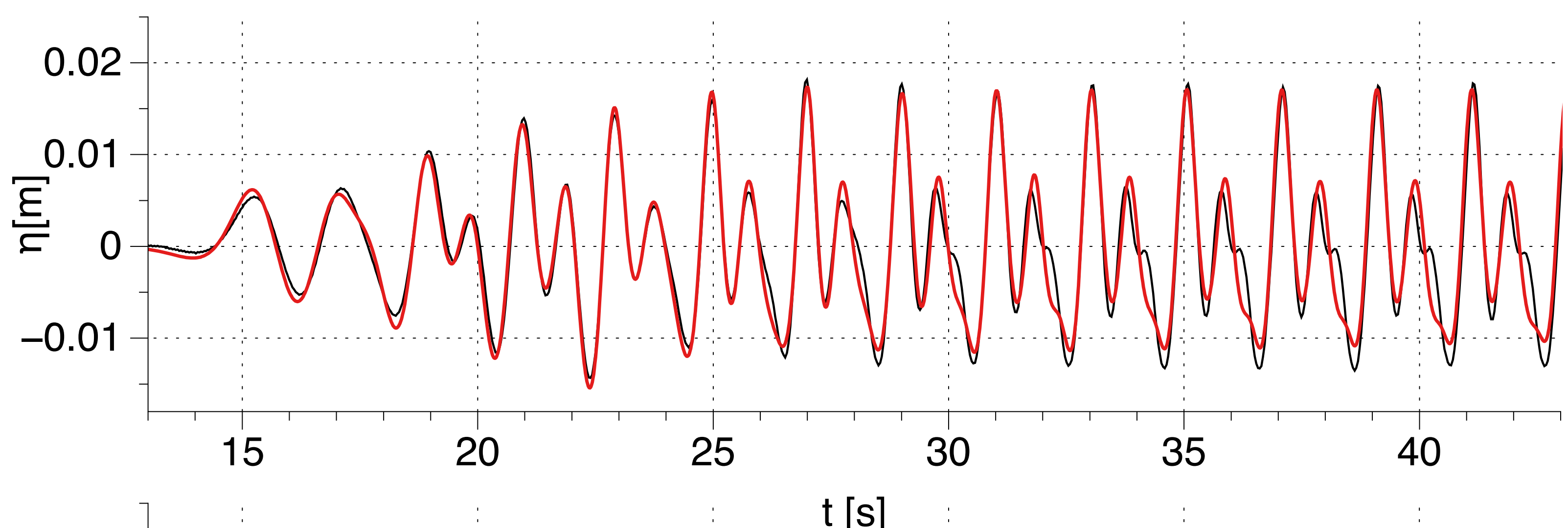


(b) $dx = 0.0125$ m at $x = 9$ m

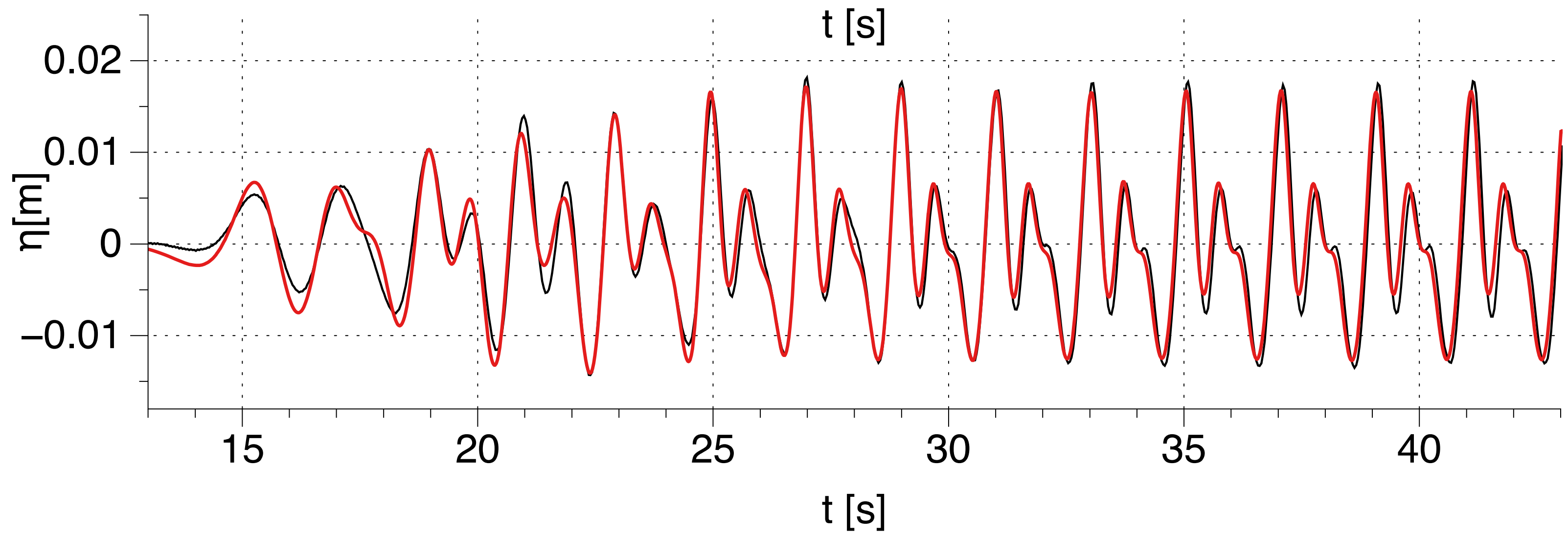


NSEWAVE vs CFD : Beji & Battjes at x = 21m





NSEWAVE
dx=0.0125m

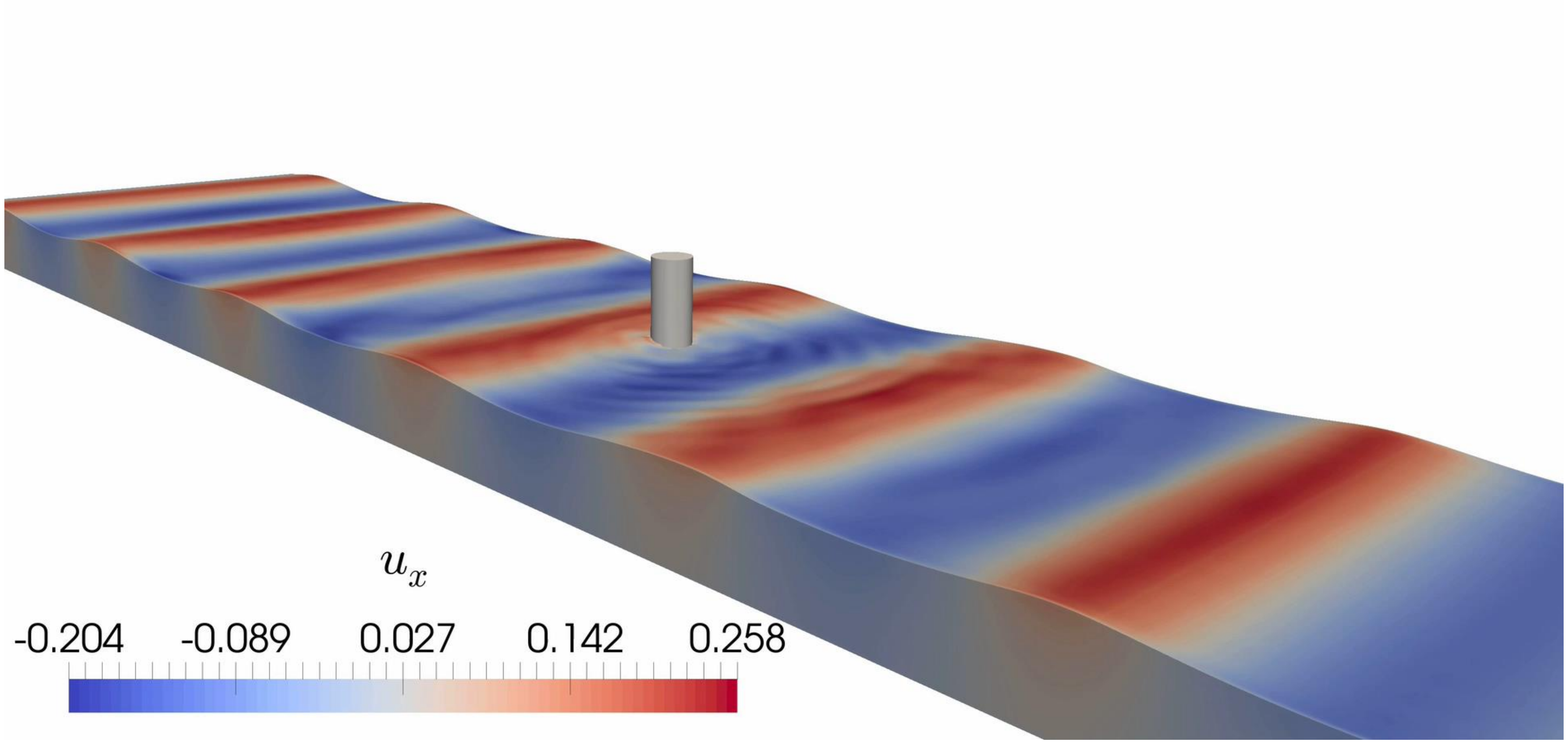


CFD
dx = 0.005m

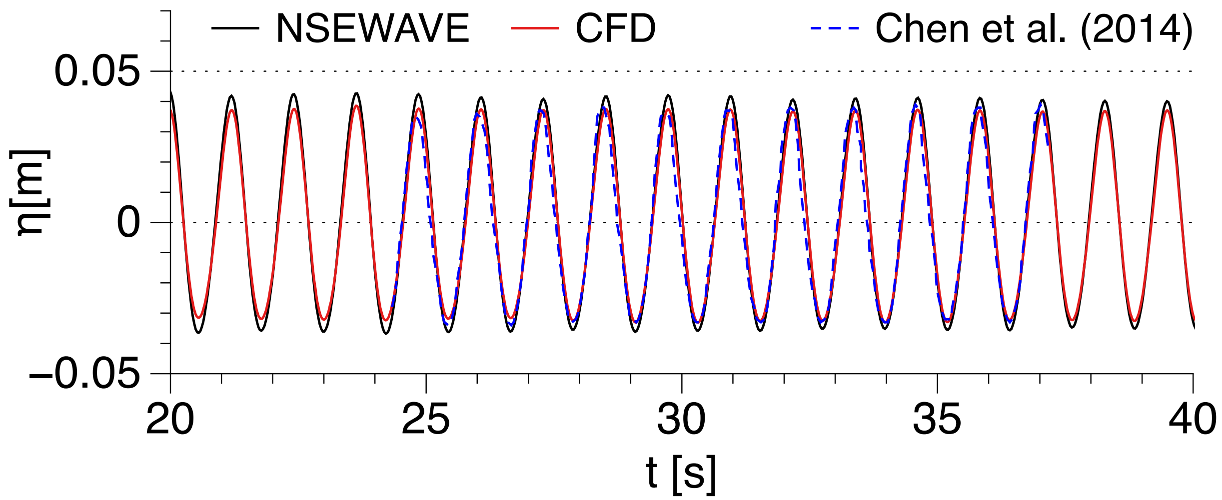
Non-Breaking Wave Forces

Setup:

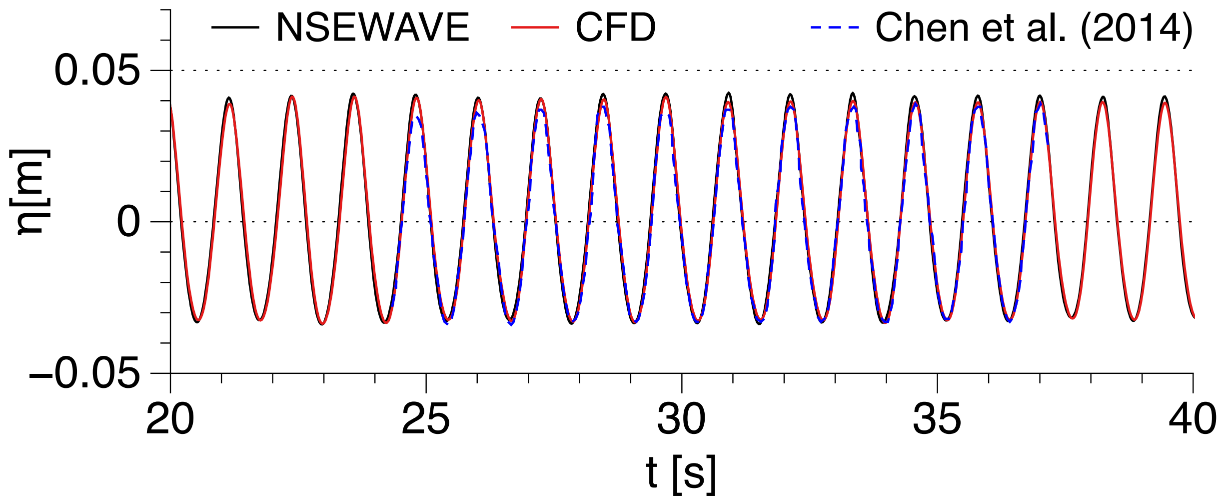
- Chen et al., CE 2014
- 18 m long
- 3 m wide
- 1 m high
- d=0.505 m
- H = 0.07m
- L = 2 m
- dx
    - 0.05 m
    - 0.025 m
    - 0.0125 m



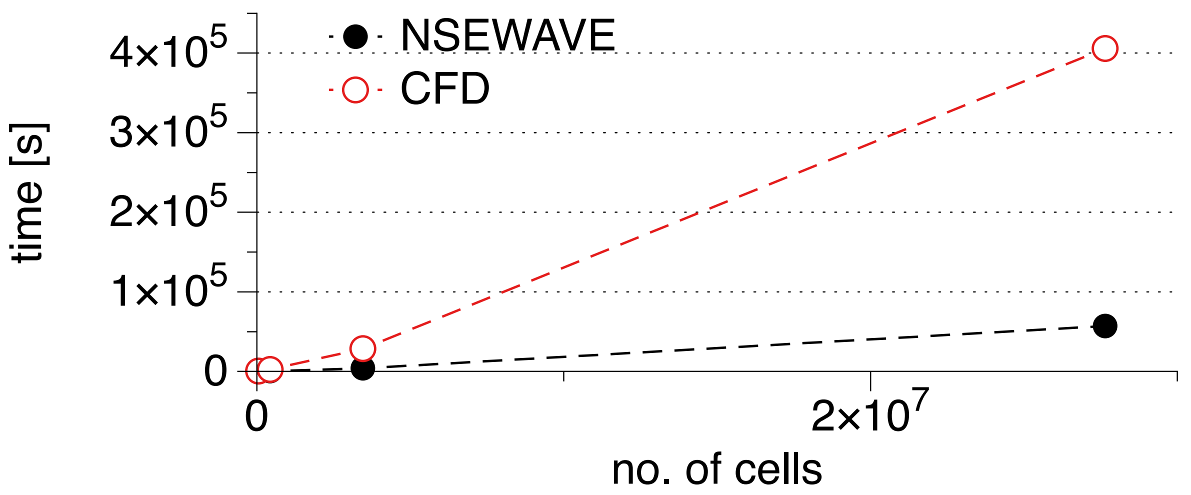
Free Surface and Speed-Up



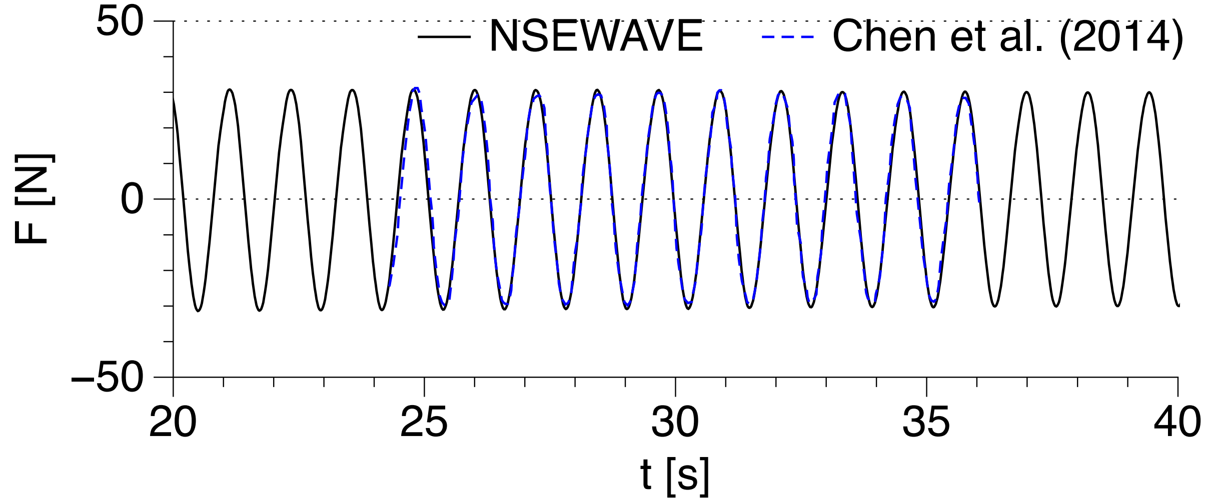
(a) $dx = 0.050$ m



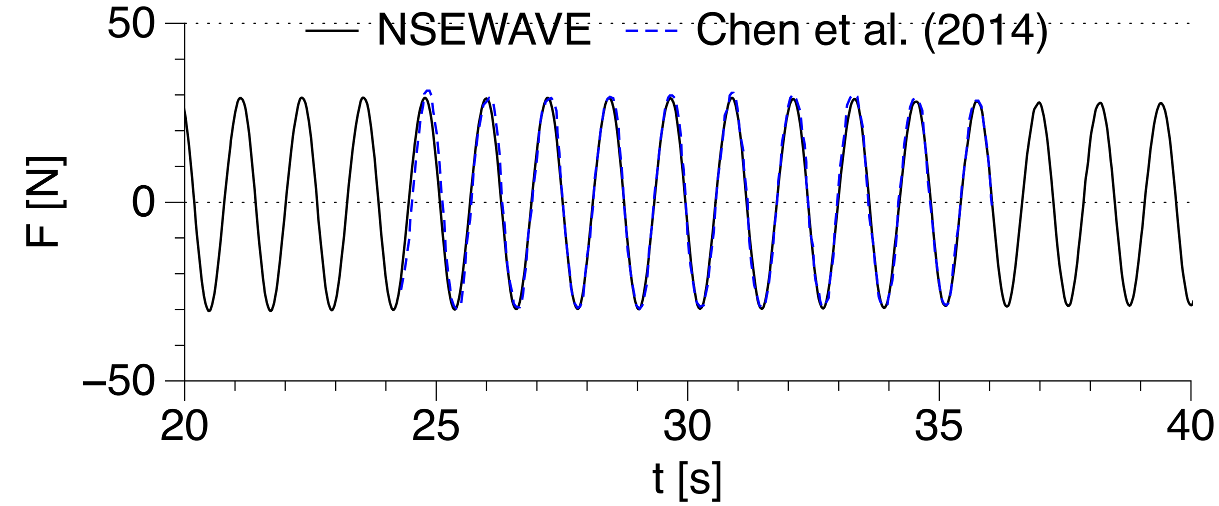
(b) $dx = 0.025$ m



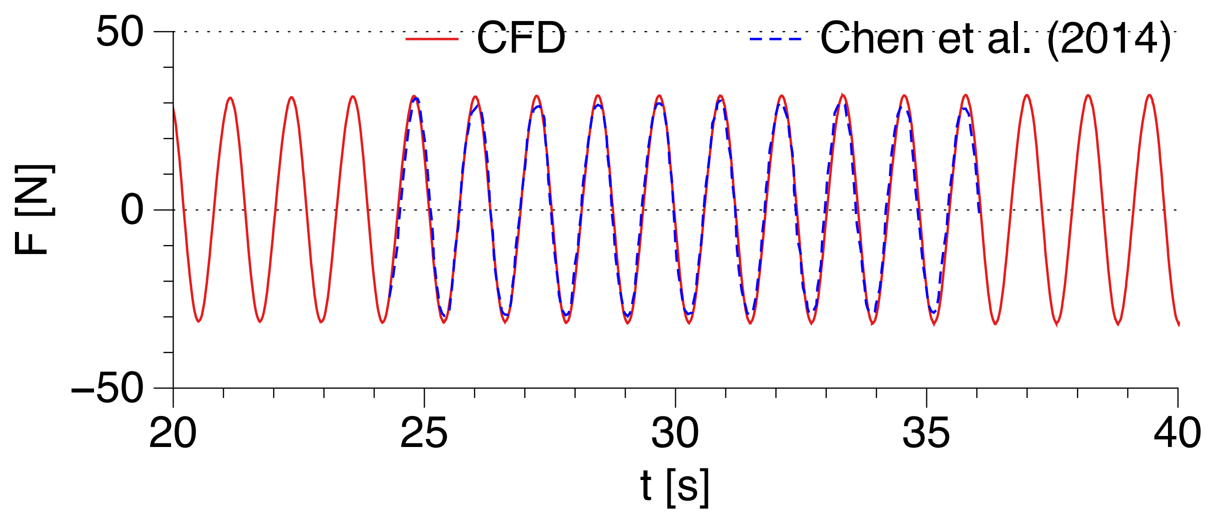
Non-Breaking Wave Forces



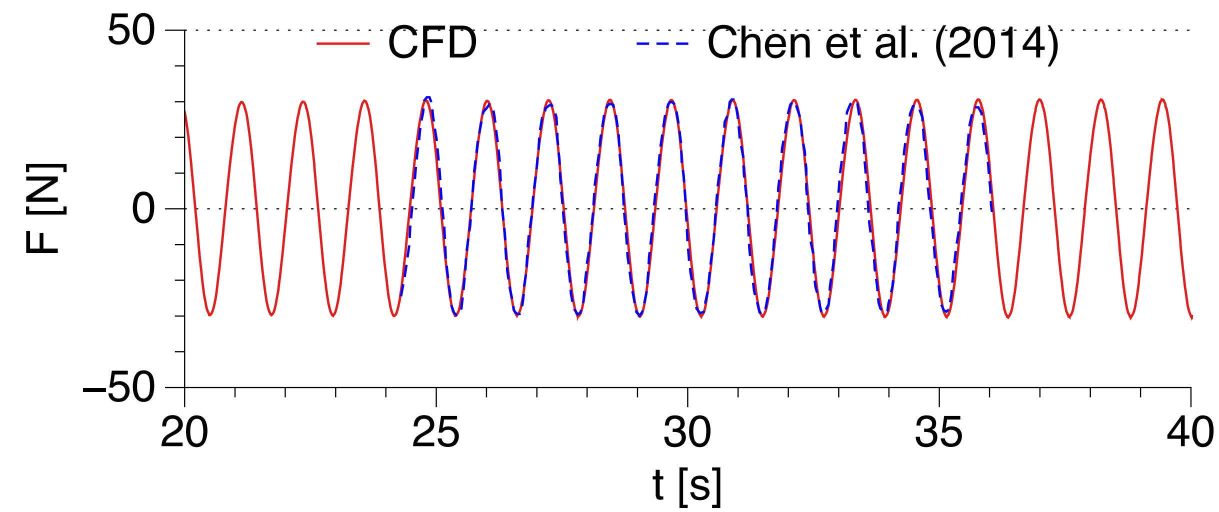
(a) $dx = 0.025$ m using NSEWAVE



(a) $dx = 0.0125$ m using NSEWAVE



(b) $dx = 0.025$ m using CFD



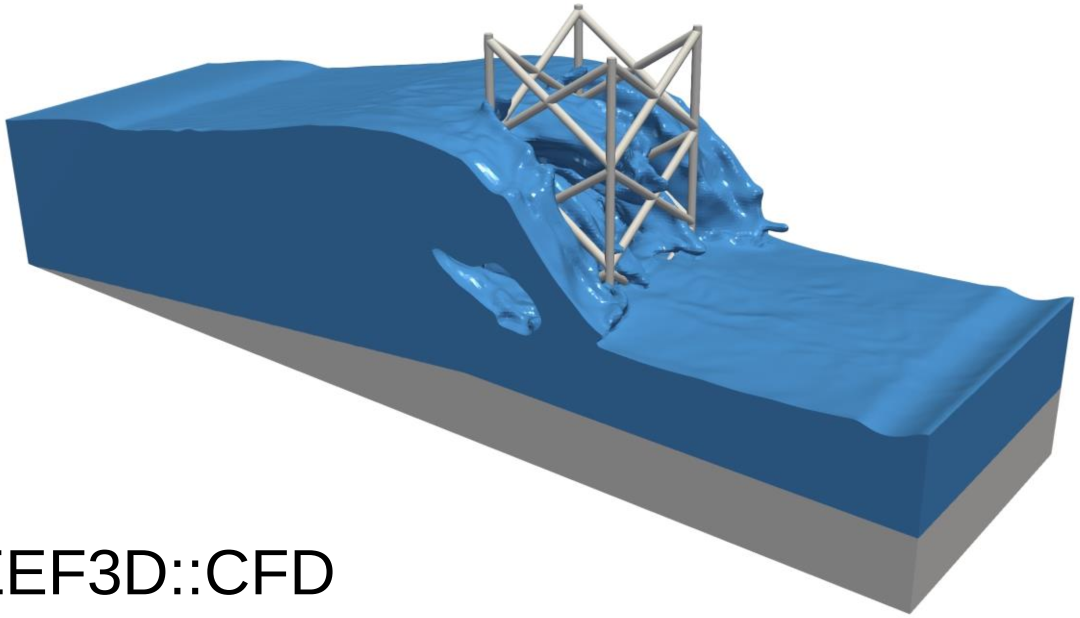
(b) $dx = 0.0125$ m using CFD

Conclusions

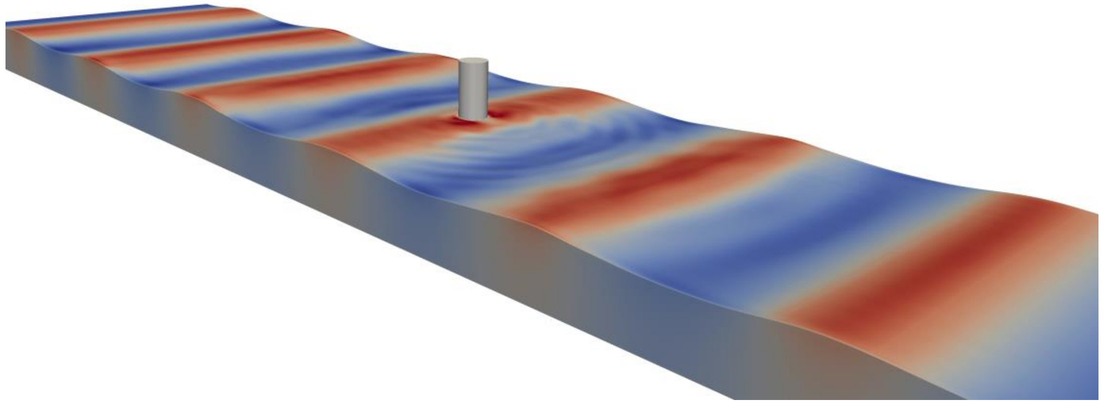
- REEF3D::NSEWAVE
  - single-values free surface
  - straightforward addition to existing code
  - reduced computational speed
    - large CFL number
    - larger dx

- Good Agreement between REEF3D and experimental data

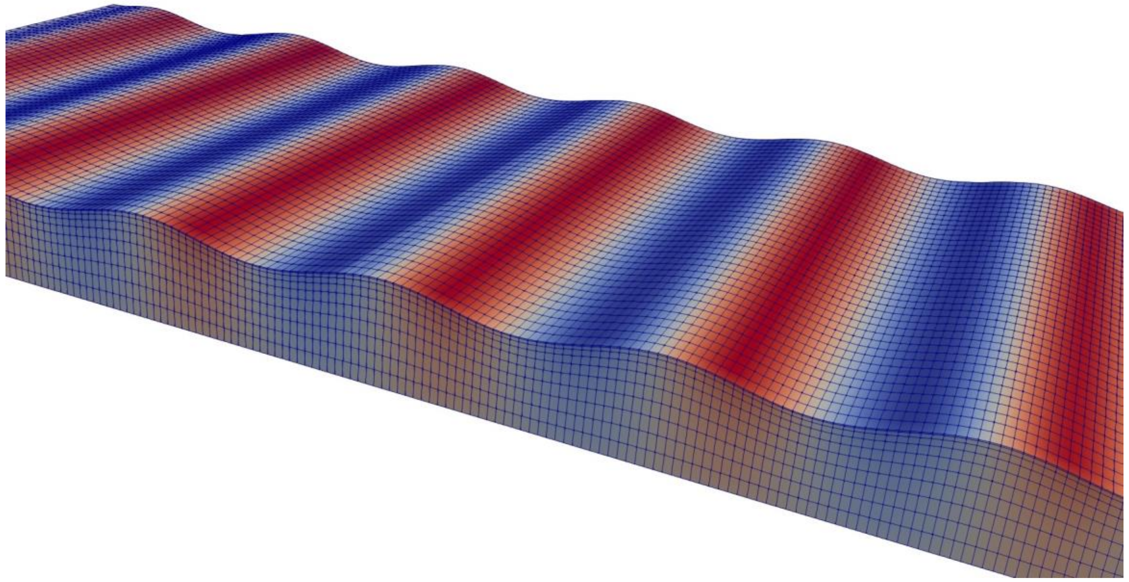
REEF3D : Open-Source Hydrodynamics



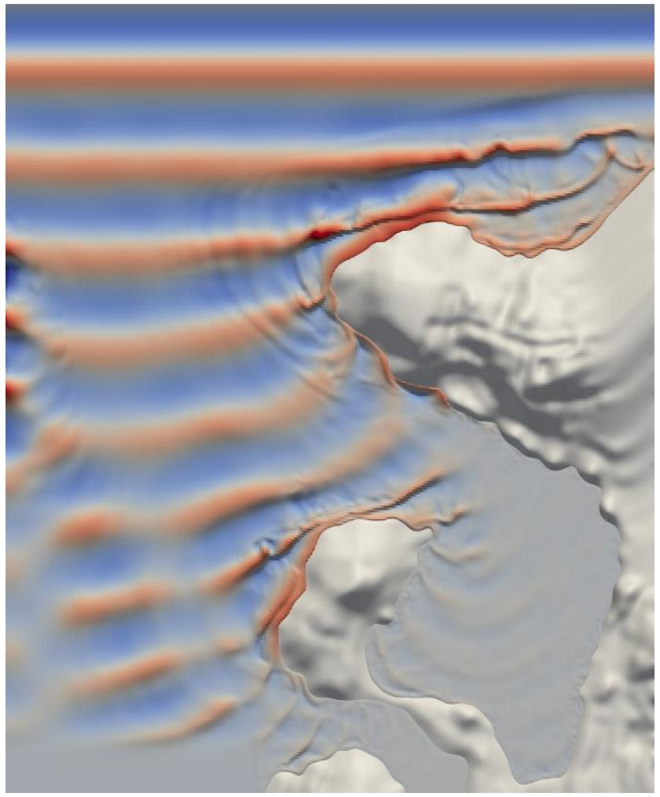
REEF3D::CFD



REEF3D::NSEWAVE



REEF3D::FNPF



REEF3D::SFLOW