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The State of the Art and Science of Coastal Engineering

INFLUENCE OF BEACH LENGTH ON THE DEVELOPMENT OF ESTUARINE DELTAS

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1 Background

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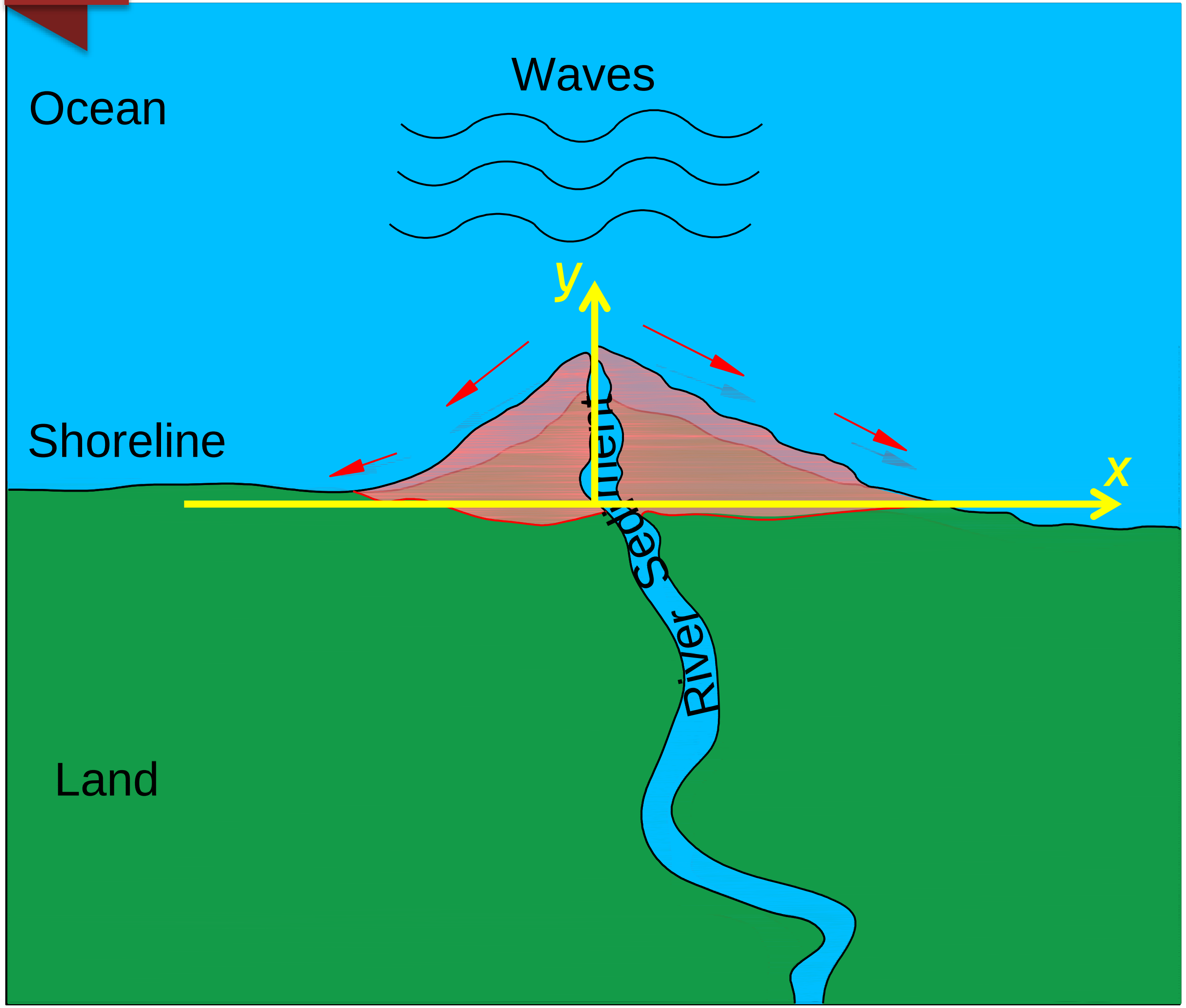
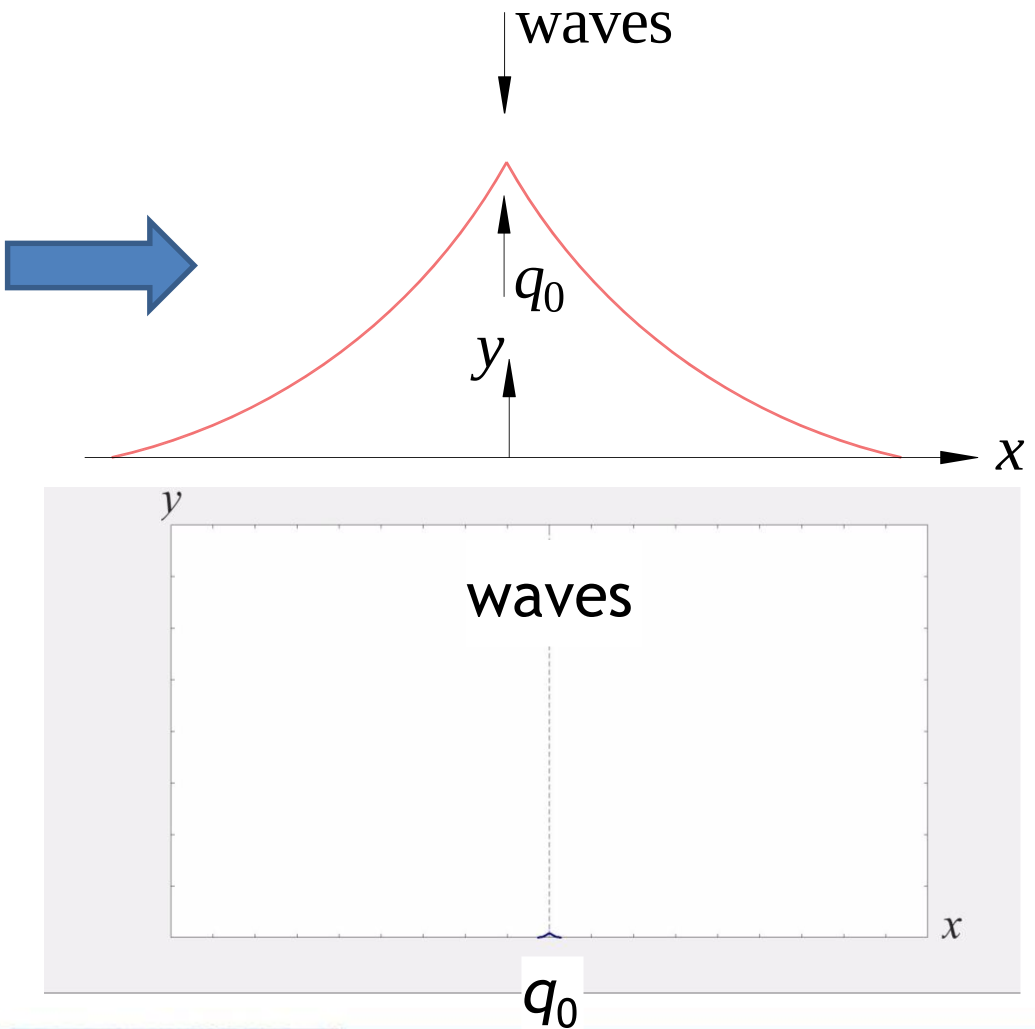


Larson et al. (1987):



1 Background

- **Larson et al. (1987):**

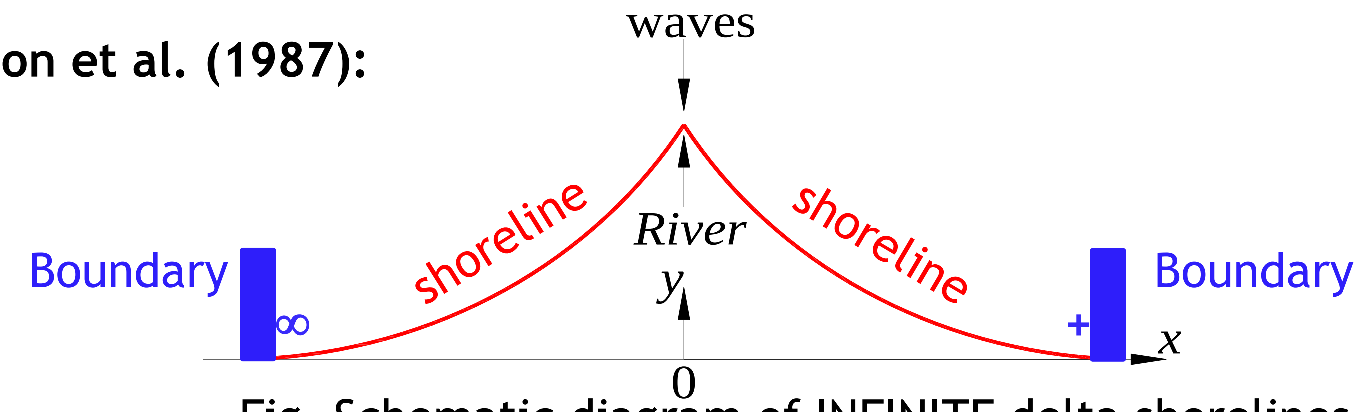


Fig. Schematic diagram of INFINITE delta shorelines

A new analytical solution to study the evolution of FINITE delta shorelines would be useful.

- **In reality:**

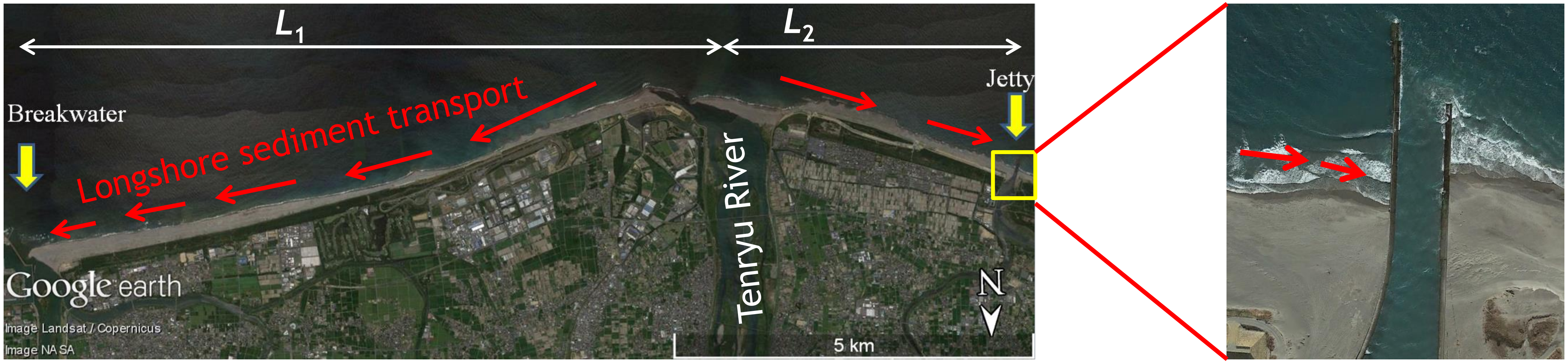


Fig. An example of FINITE delta shorelines in Tenryu River delta, Japan

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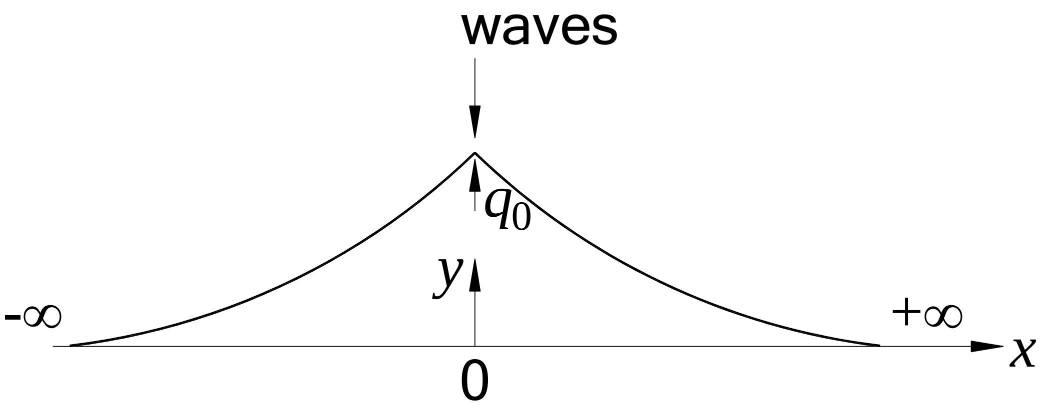
New Analytical solution

2 Method

Governing equation: $$\frac{\partial y}{\partial t} = \varepsilon \frac{\partial^2 y}{\partial x^2} \quad (1)$$

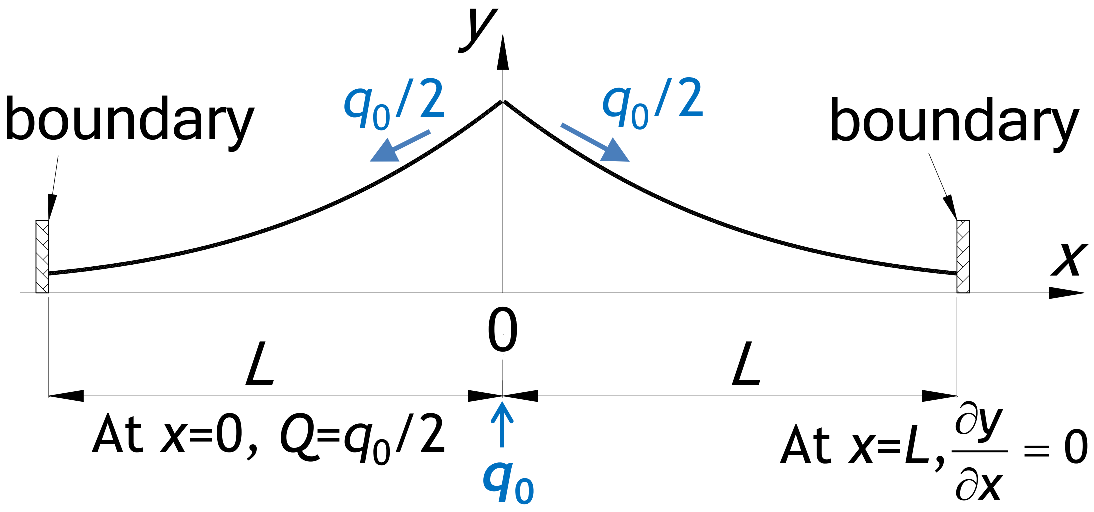
- **Without** rigid boundary (Larson et al., 1987):



$$y = \frac{q_0}{D}\sqrt{\frac{t}{\pi\varepsilon}}e^{-x^2/(4\varepsilon t)} - \frac{q_0}{D}\frac{|x|}{2\varepsilon}erfc\left(\frac{|x|}{2\sqrt{\varepsilon t}}\right) \quad (2)$$

for: **-∞<*x*<+∞**

- **With** rigid boundaries (present study):



$$y = \frac{q0}{2\varepsilon DL}\left[\frac{x^2}{2} - Lx + \frac{L^2}{3} + \varepsilon t - \frac{2L^2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}e^{-n^2\pi^2\frac{\varepsilon t}{L^2}}\cos\left(\frac{n\pi x}{L}\right)\right]$$

for: ***-L<x<+L*** (3)

3

Results

3 Results – Theoretical discussion

Dimensionless form:

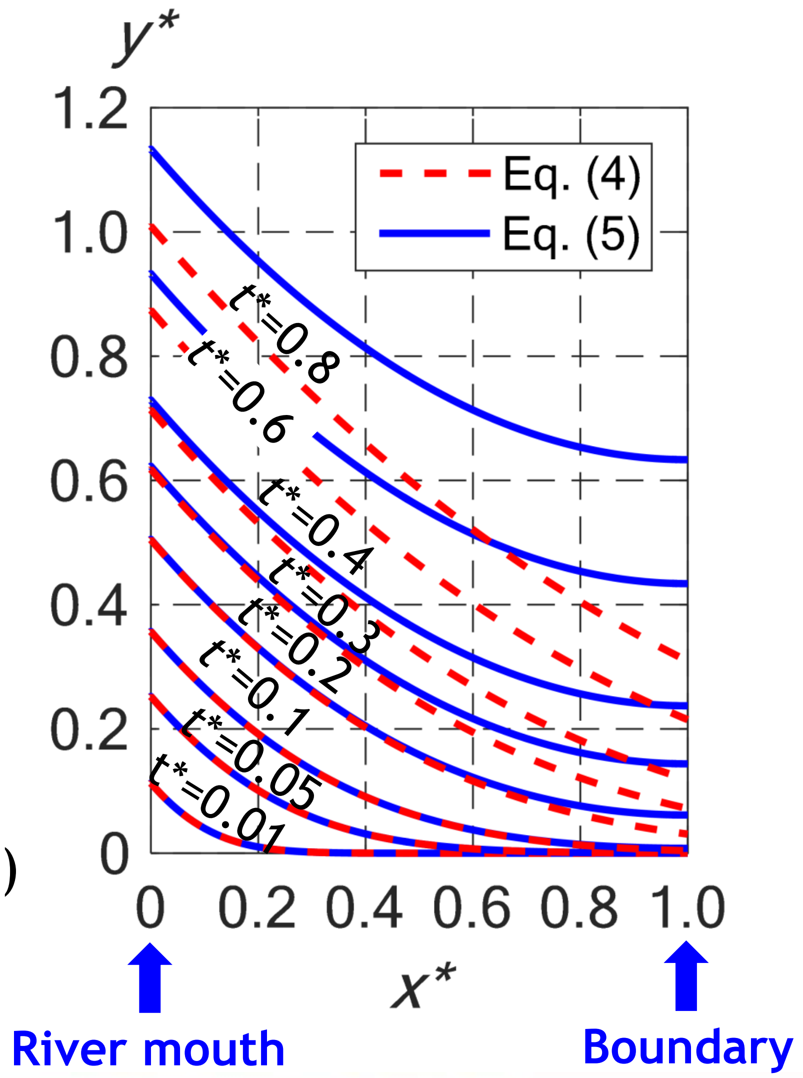
Let: $y^* = y\frac{2\varepsilon D}{q_0 L}; \quad x^* = \frac{x}{L}; \quad t^* = \frac{\varepsilon t}{L^2};$

Without rigid boundary (Larson et al., 1987):

$$y^* = 2\sqrt{\frac{t^*}{\pi}}\, e^{-\left(x^{*2}/4t^*\right)} - |x^*|\, erfc\left(\frac{|x^*|}{2\sqrt{t^*}}\right) \qquad (4)$$

With rigid boundaries:

$$y^* = \frac{x^{*2}}{2} - |x^*| + \frac{1}{3} + t^* - \boxed{\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{e^{-n^2\pi^2 t^*}}{n^2}\cos(n\pi x^*)} \qquad (5)$$



- At small t^*: no effect of boundary to shoreline change at the delta tip: Eq. (4).
- At large t^*: **fifth term in Eq. (5)** cancels, shoreline with parabolic shape advances at constant speed.

3 Results – theoretical discussion

Shoreline evolution at $x^*=0$ (y_0^*):

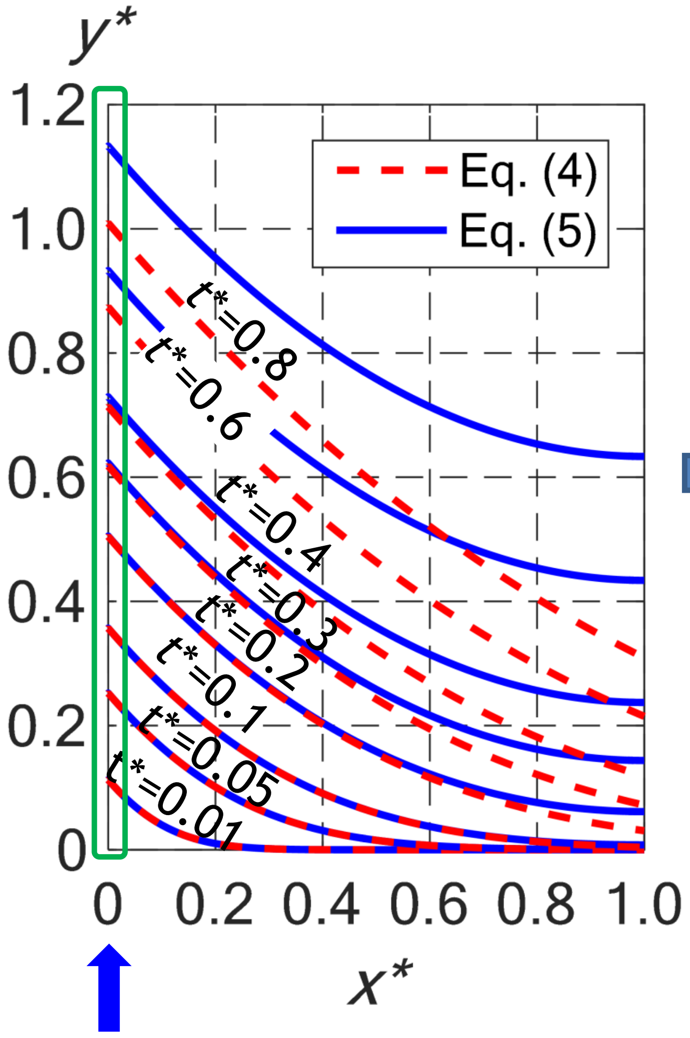
$$y_0^* = t^* + \frac{1}{3} - \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{e^{-n^2\pi^2 t^*}}{n^2} \tag{6}$$

At small t^*:

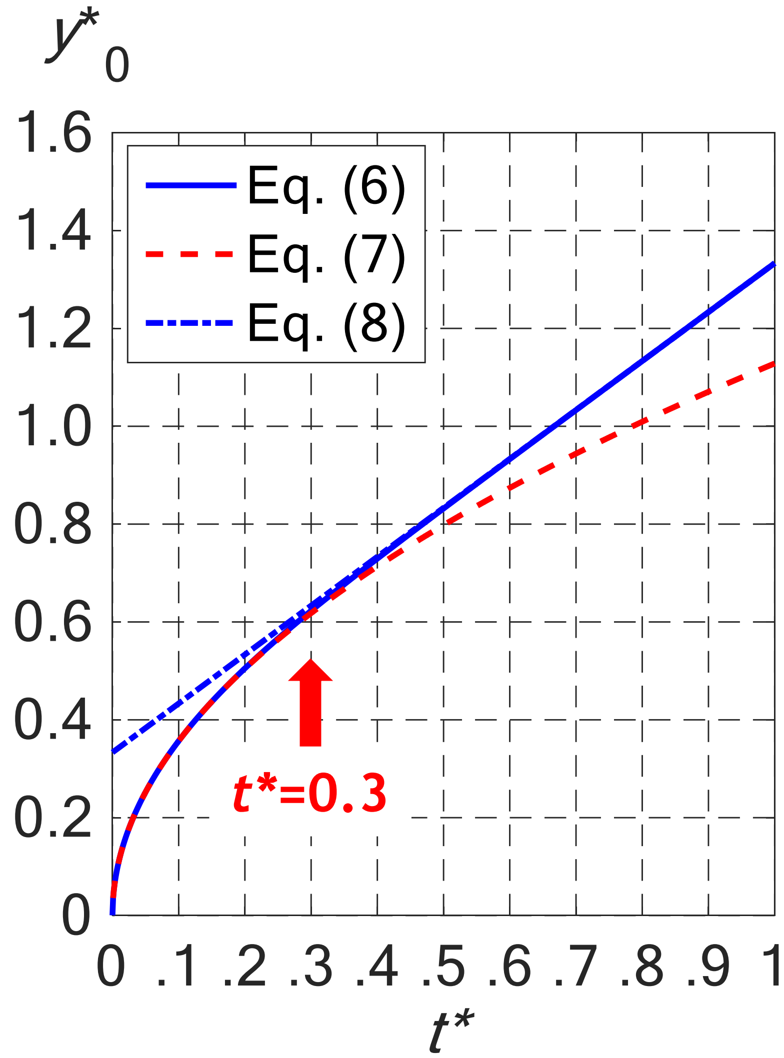
$$y_0^* = 2\sqrt{\frac{t^*}{\pi}} \tag{7}$$

At large t^*:

$$y_0^* = t^* + \frac{1}{3} \tag{8}$$



River mouth



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Results – Theoretical discussion

Shoreline evolution at $x^*=1$ (y_1^*):

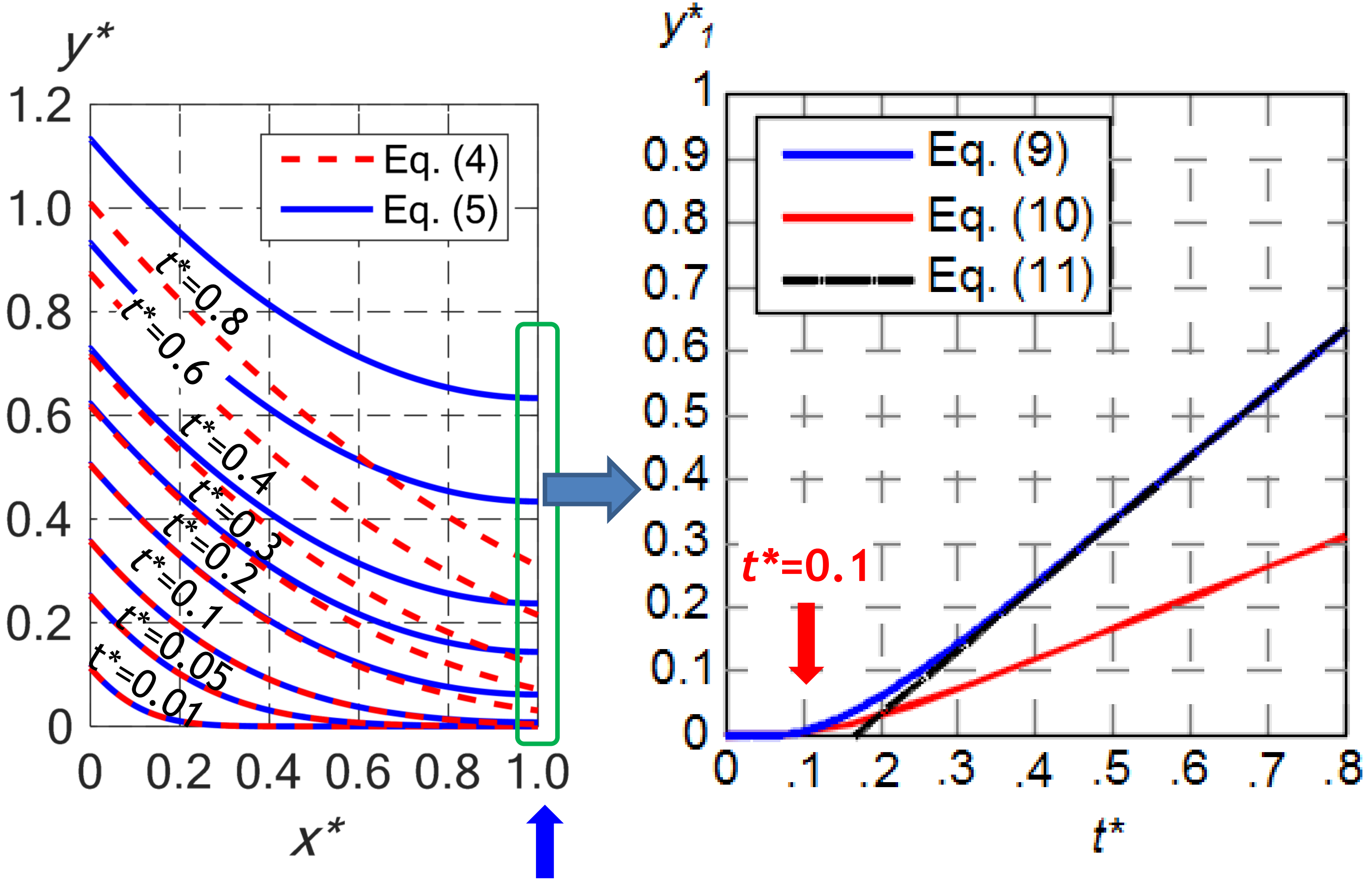
$$y_1^* = t^* - \frac{1}{6} - \frac{2}{\pi^2}\sum_{n=1}^{\infty}(-1)^n \frac{e^n}{n^2} \qquad (9)$$

At small t^*:

$$y_1^* = 2\sqrt{\frac{t^*}{\pi}}\, e^{-\left(\frac{1}{4t^*}\right)} - erfc\left(\frac{1}{2\sqrt{t^*}}\right) \qquad (10)$$

At large t^*:

$$y_1^* = t^* - \frac{1}{6} \qquad (11)$$



Boundary

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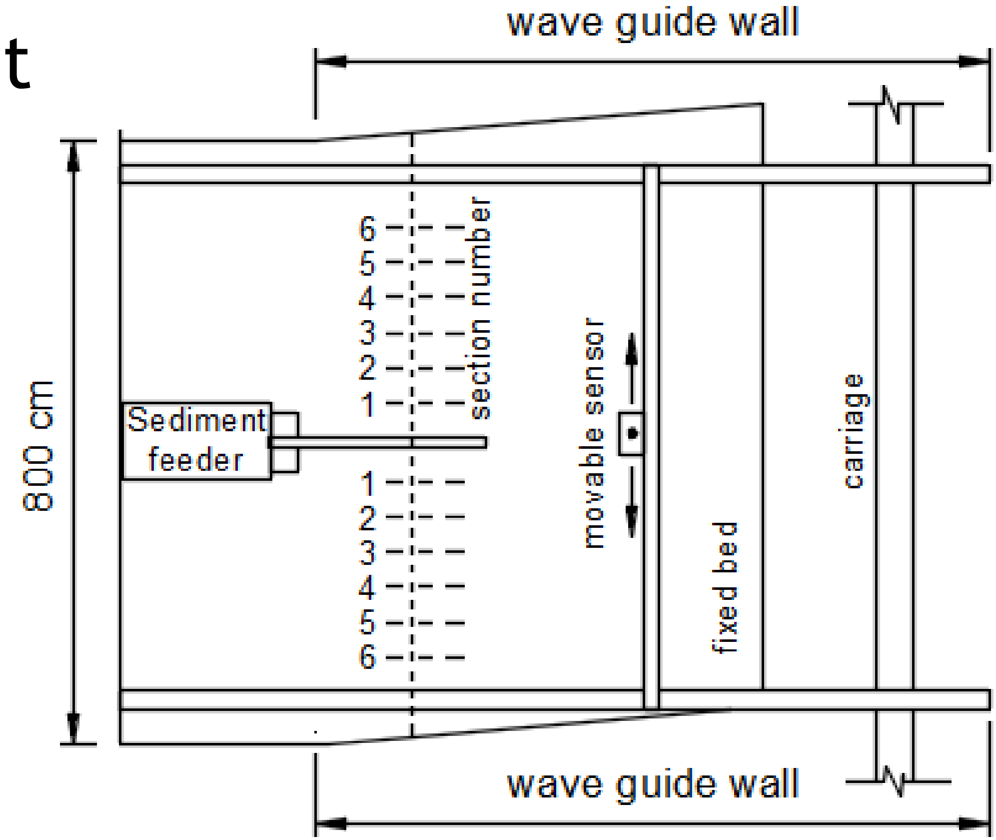
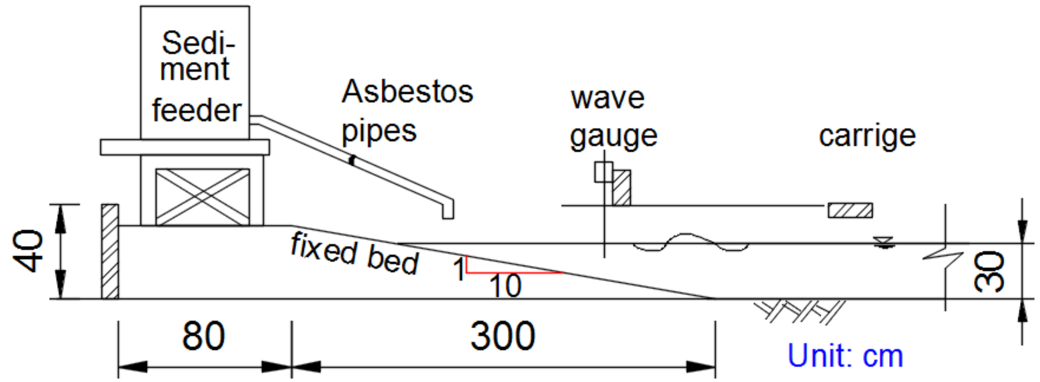
Results – Compare with Experiment

Experiment (Refaat, 1990)

- Experiment was performed in a wave basin 35x10m;
- Fixed bed with slope 1:10;
- Sediment discharge as a point source.
- Wave conditions: constant water depth of 30 cm; wave height of 2 cm and period of 0.8 sec; angle of breaking wave to initial shoreline a_0=0 deg.;
- Sediment supply rate Q=7.06 cm^3/sec;
- Run time: 80 min = 4,800 sec;
- Shoreline positions were measured at every 10 min and 50 cm interval.





Unit: cm

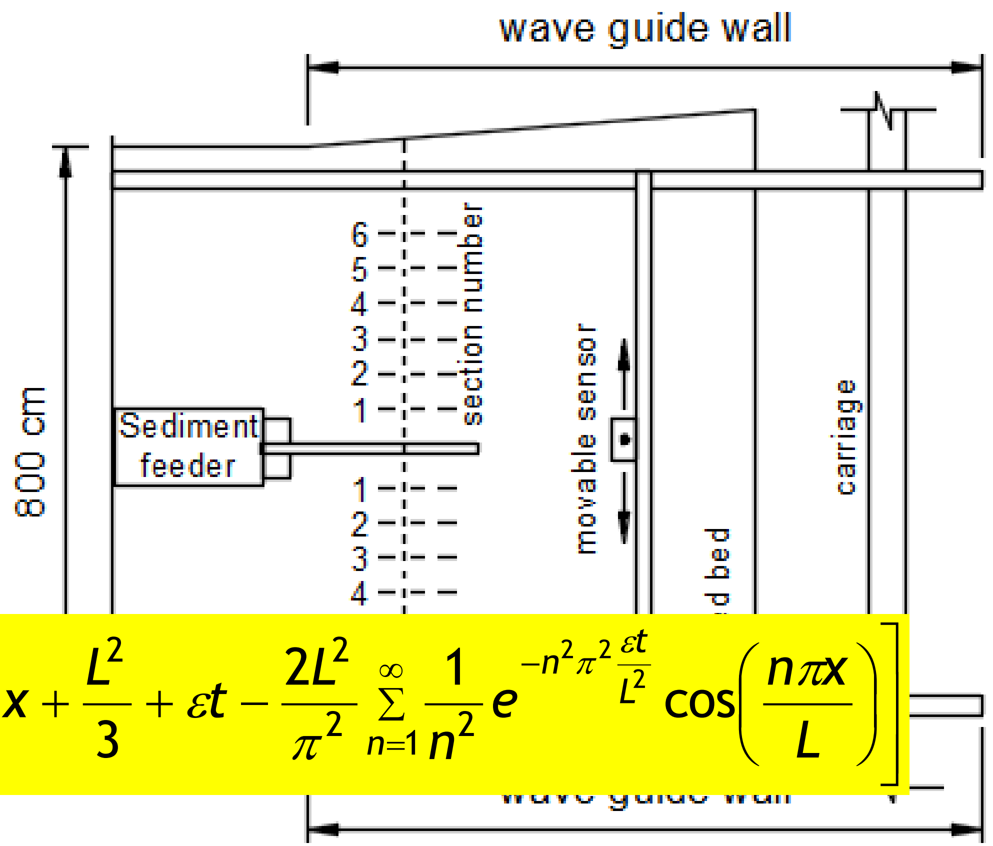
3 Results – Compare with Experiment

Comparison with experiment (Refaat, 1990):

Q_0 – sediment supply (cm^3/sec)

a_0 – wave crest angle to the straight initial shoreline (degree)

$L = 800/2 = 400$ cm; $\varepsilon = ?$



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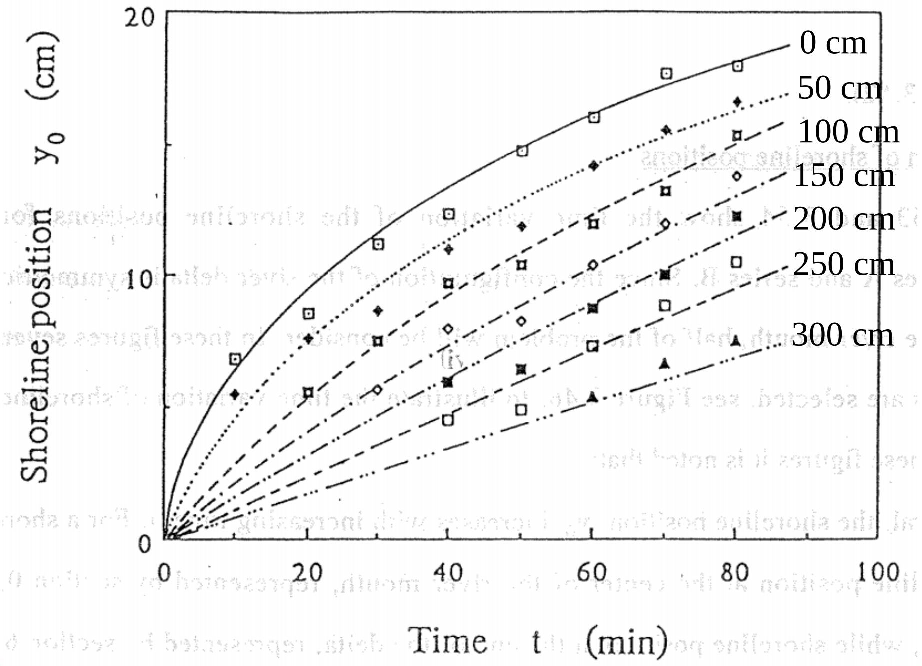


Exp. No.	Q_0 (cm^3/sec)	Run time (min)	a_0 (deg.)
Series A A-1	7.06	80	0

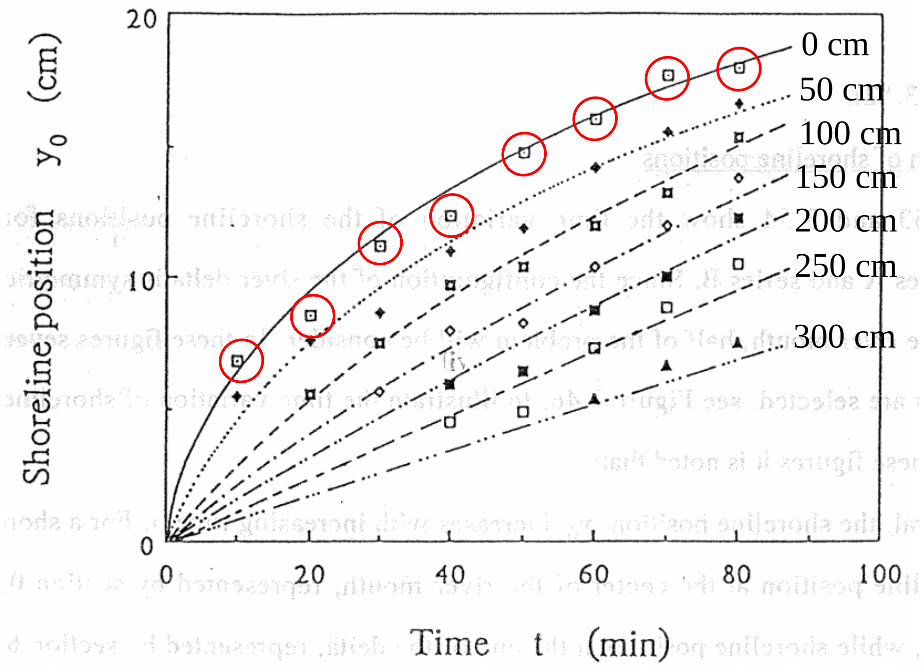
Results – Compare with Experiment

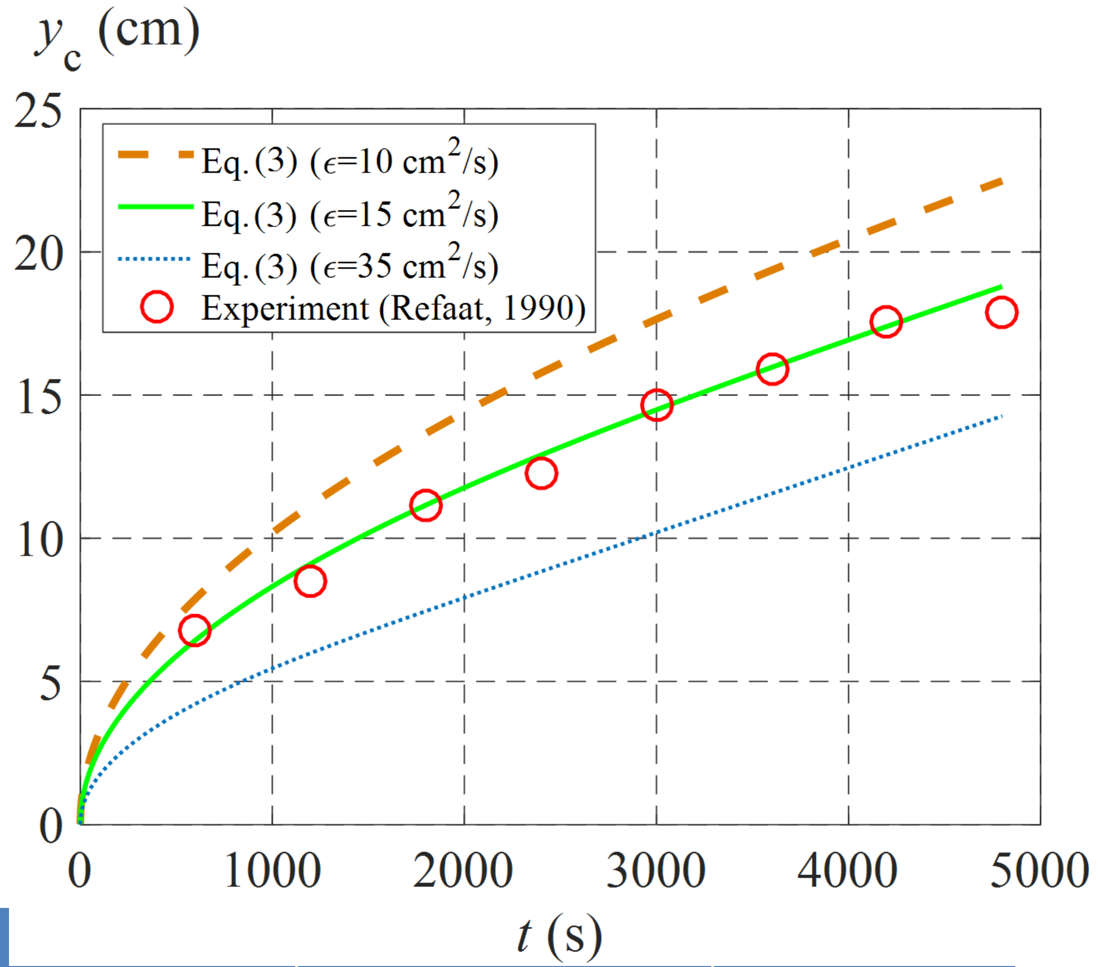
Comparison with experiment (Refaat, 1990):

Q_0 - sediment supply (cm^3/sec)

a_0 - wave crest angle to the straight initial shoreline (degree)

L = 800/2=400 cm; ε= 15cm^2/s





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Series A A-1	7.06	80	0

3 Results – Compare with Experiment

Comparison with experiment (Refaat, 1990):

Q_0 – sediment supply = 7.06 (cm^3/sec); a_0 = 0^0

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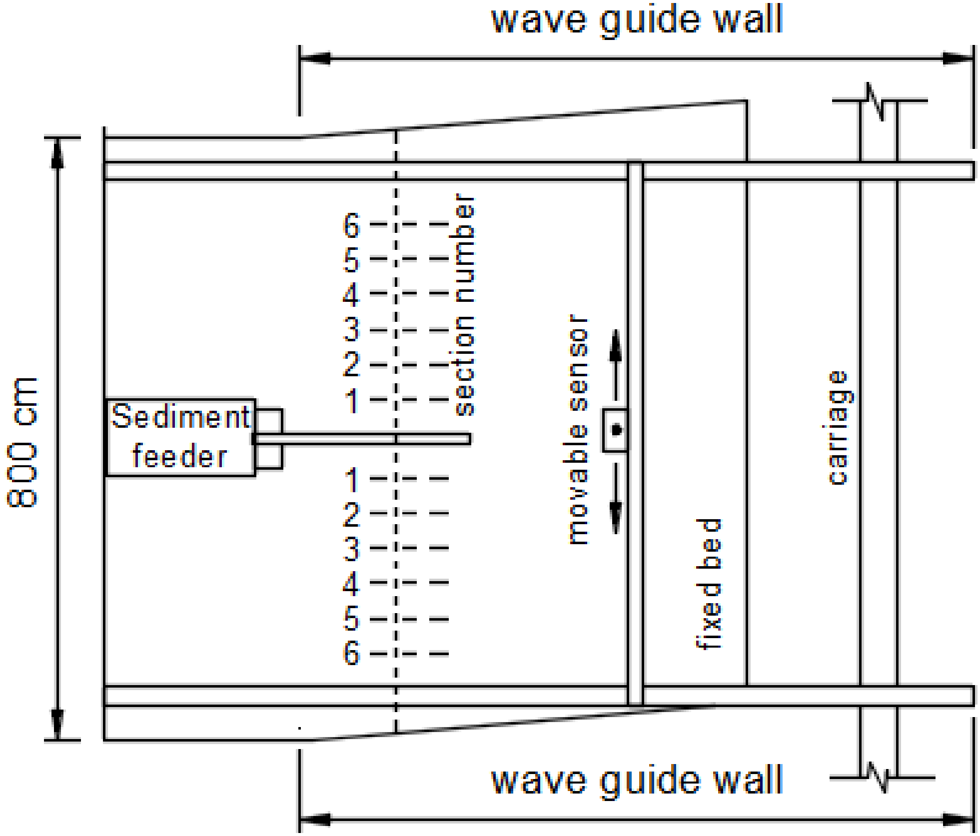

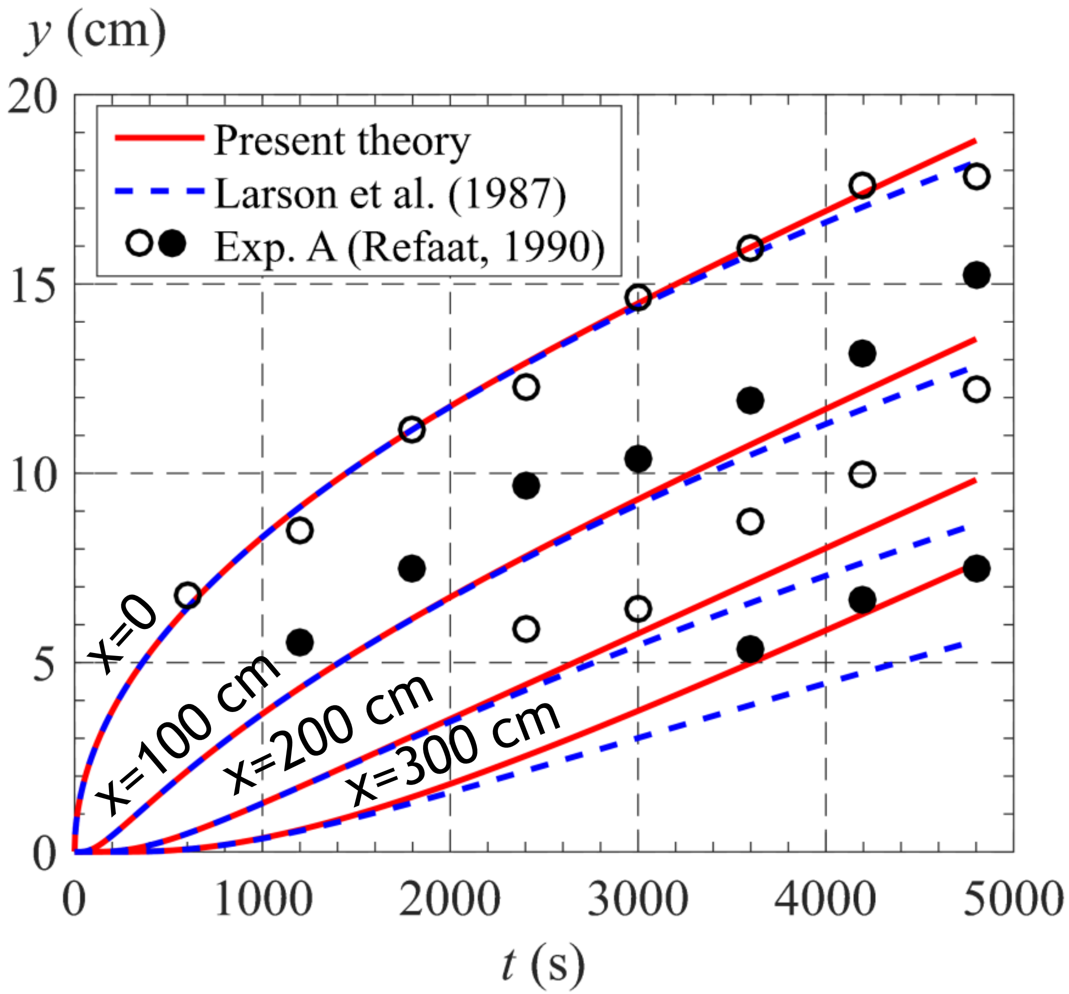

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Conclusions

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Conclusions

- A solution for studying the development of delta coastline with effect of the lateral boundaries has been obtained.

- The critical times when the lateral boundary has effect on the shoreline evolution are:
  - t^* = 0.1 at the boundary and
  - t^* = 0.3 at the river mouth;

- Comparison with experiment data shows the usefulness of present theory for simulating river delta coastlines with lateral boundary.

THANK YOU FOR YOUR KIND ATTENTION!