

*The State of the Art and Science of Coastal Engineering*

# ANALYTICAL SOLUTION OF BEACH PROFILE RESPONSE TO SEA LEVEL RISE - APPLICATION FOR TECTONIC MOVEMENT IN SENDAI, JAPAN

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# INTRODUCTION

- The sea level is expecting to increase due to the global climate changes. Therefore, researches on the SLR impacts to the coastal morphological changes becomes more important.
- Many researchers (Bruun, 1962; Edelman, 1972; Swart, 1976; Hands, 1984; Kriebel and Dean, 1993, Khang and Tanaka, 2008) have investigated the response of the beach profiles and nearshore to changes in water level. Concern has primarily been with **rising water level** since it is recognized as being a major contributor to the cause of **increased shore erosion**.
- Bruun (1962) employed an equilibrium concept to estimate shoreline retreat to sea level rise (SLR). However, this method can only give the maximum shoreline retreated length; it was not able to discuss the dynamic response process of a beach against SLR.
- Kriebel and Dean (1993) developed a simple approach to quantify the beach profile response to a time-varying sea level during an idealized storm conditions.



# OBJECTIVES

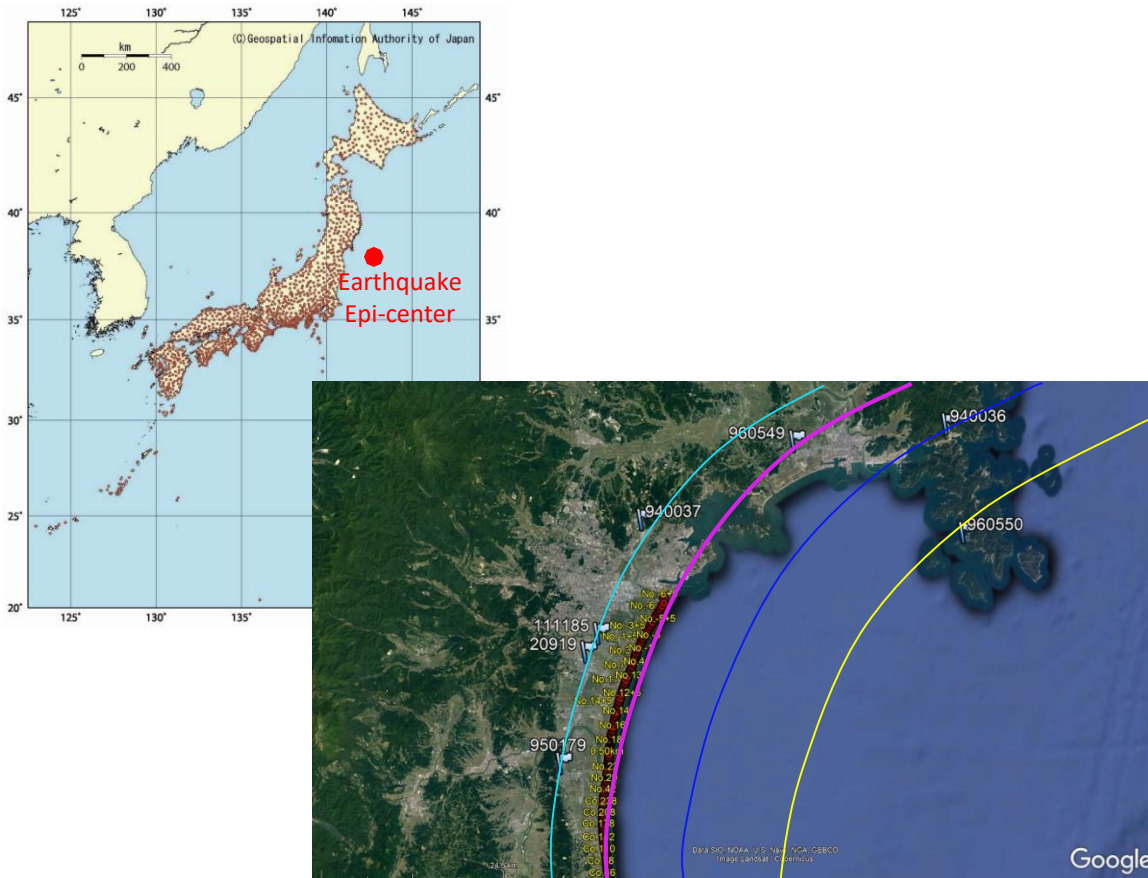
- In the Sendai case with instantaneous bed level subsidence, which caused by the 2011 Tohoku Earthquake and Tsunami, followed by rather rapid recovery of the original bed level. This **bed level change is inversely related to the SLR process** and the data provide a unique opportunity to **evaluate the EBP concept** (*i.e.*, Bruun rule, 1962).
- to develop a new analytical solutions based on the convolution method to describe beach-profile response to sea water level change.



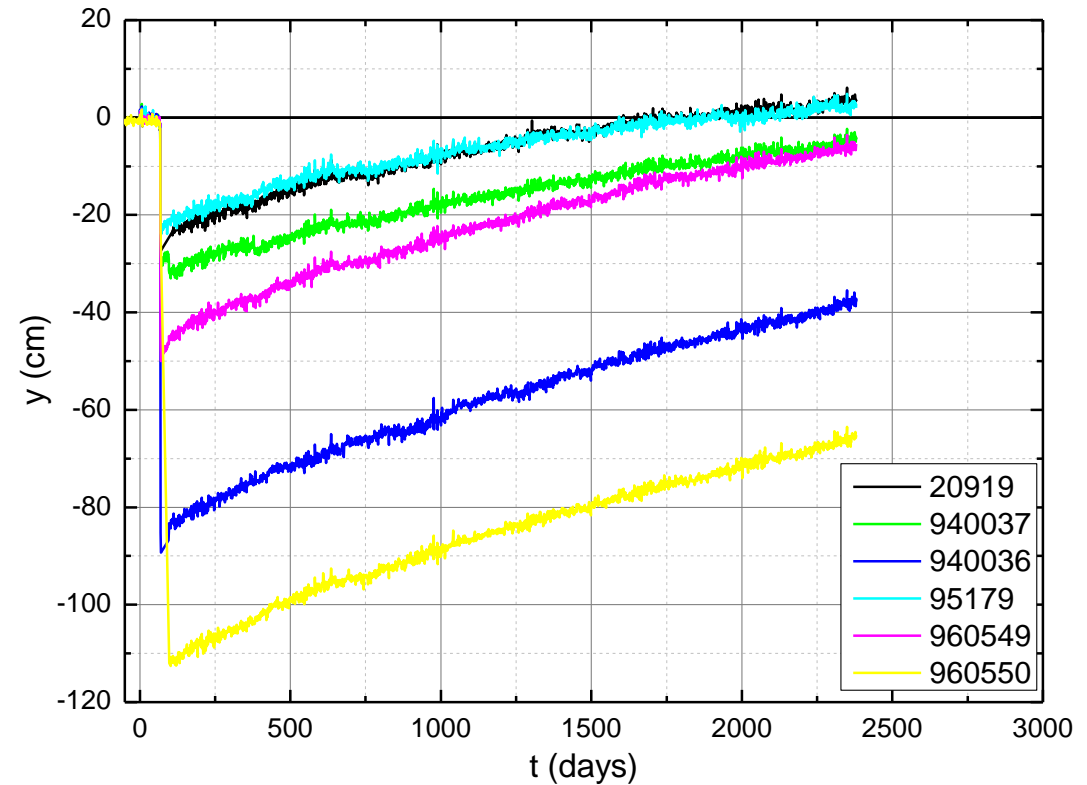
# DATA COLLECTION AND ANALYSIS

- The 2011 Great East Japan great earthquake caused a maximum of land subsidence about 1m in Tohoku area. After that, the land rising is gradually occurring. The land rising is equivalent to the sea level rise process.

GEONET : *GNSS Earth Observation Network System*



GEONET Stations in Miyagi Prefecture

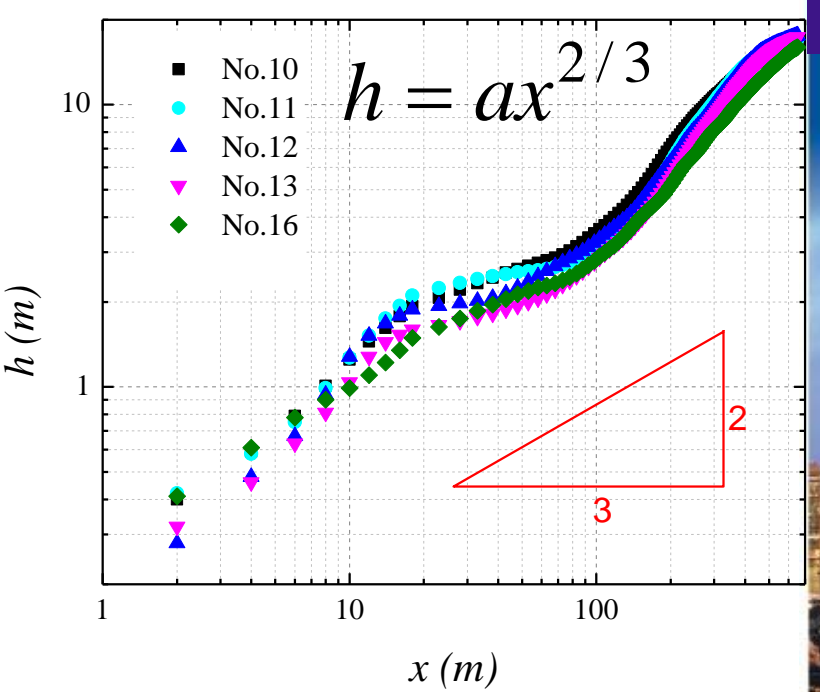
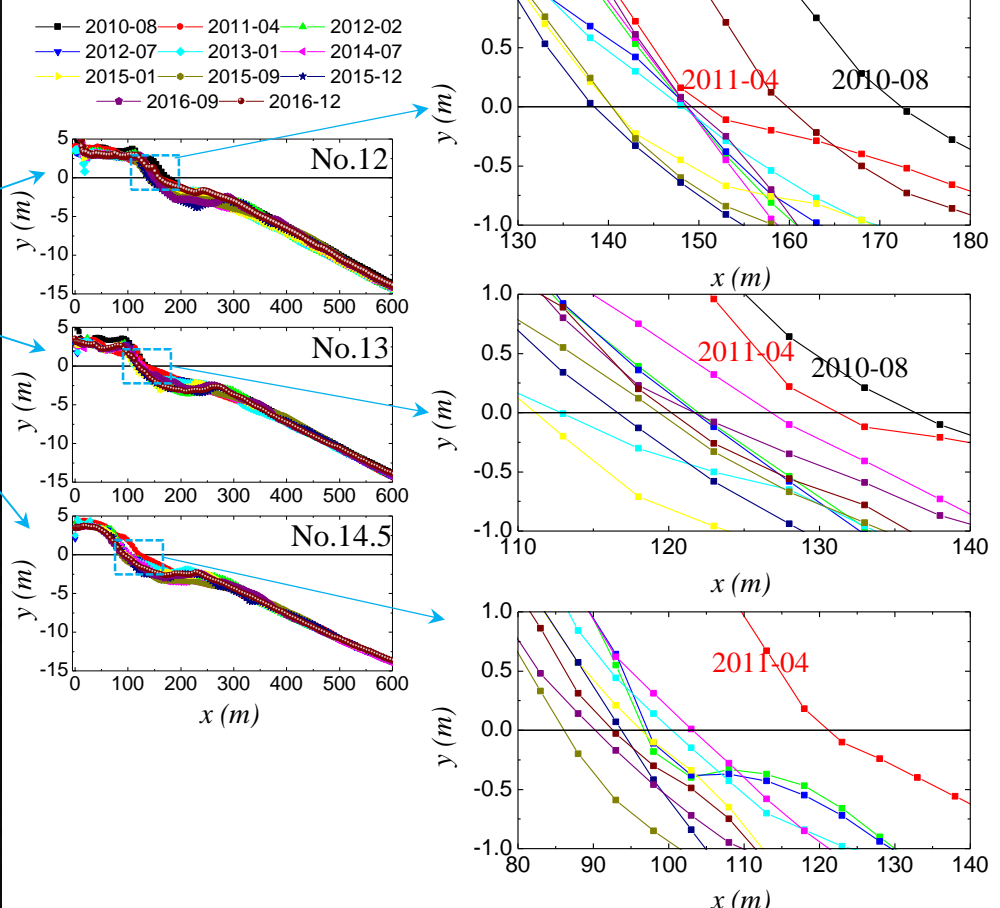
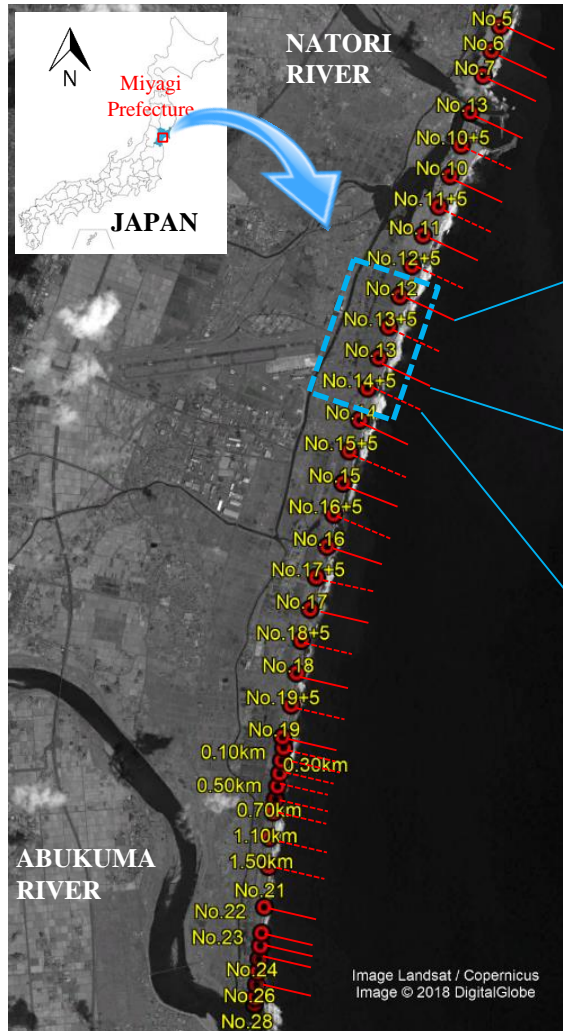


Time series of Land Subsidence and rising measurements



# DATA COLLECTION AND ANALYSIS – Cross-shore profiles

Detailed, high-quality data sets obtained after the 2011 Tohoku Earthquake and Tsunami, including wave climate, beach profiles surveyed every 6 months.



Measured profiles are followed the Bruun Rule

# THEORETICAL MODEL DEVELOPMENT

- The theoretical model proposed by Kriebel and Dean (1993) to describe the beach profile response to a time-varying sea level is,

$$\frac{dy}{dt} = \alpha \left\{ y^{\infty} f(t) - y \right\} \quad (1)$$

$y$  is shoreline position;  $y^{\infty}$ : the maximum response at equilibrium,  $t$  the time, and  $\alpha$  a characteristic rate parameter that may be related to the typical response time scale of the morphological system. Function  $f(t)$  is external force term which describes a varying sea level, and in this case it is measured land rising function.

- The general solution to this equation may be written as a **convolution integral** as

$$y(t) = \alpha y^{\infty} \int_0^t f(\tau) \exp(-\alpha(t - \tau)) d\tau \quad (2)$$

where  $\tau$  is a dummy integration variable.



# ANALYTICAL MODEL DEVELOPMENT

➤ In this study, we assumed a two forcing function.

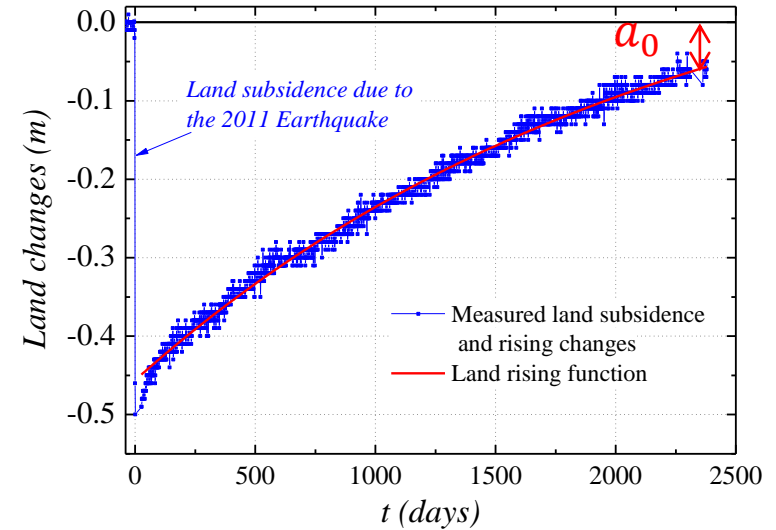
No Land recovering after subsidence :

$$f(t) = -1 \quad (3)$$

With Land recovering after subsidence:

$$f(t) = -(1-a_0)\exp(-\beta t) - a_0 \quad (4)$$

$a_0$  is a remained distance to the original ground



Land subsidence and rising

## 1. SOLUTION FOR NO LAND RECOVERING CASE

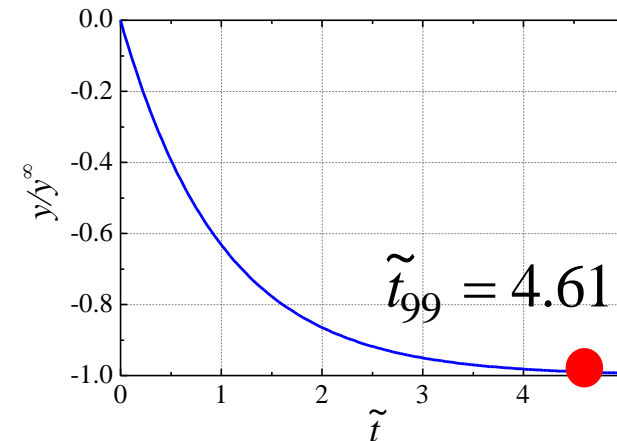
➤ Replacing Eq.(3) into Eq.(2) and solving the convolution integral we obtain

$$y^* = e^{-\tilde{t}} - 1 \quad (5)$$

Where  $y^* = y/y^\infty$  non-dimensional quantity

$\tilde{t} = \alpha t$  non-dimensional parameter

When  $y^\infty = 99\%$  then  $\tilde{t}_{99} = 4.61$



# ANALYTICAL MODEL DEVELOPMENT

## 2. SOLUTION FOR GENERAL LAND RECOVERING CASE

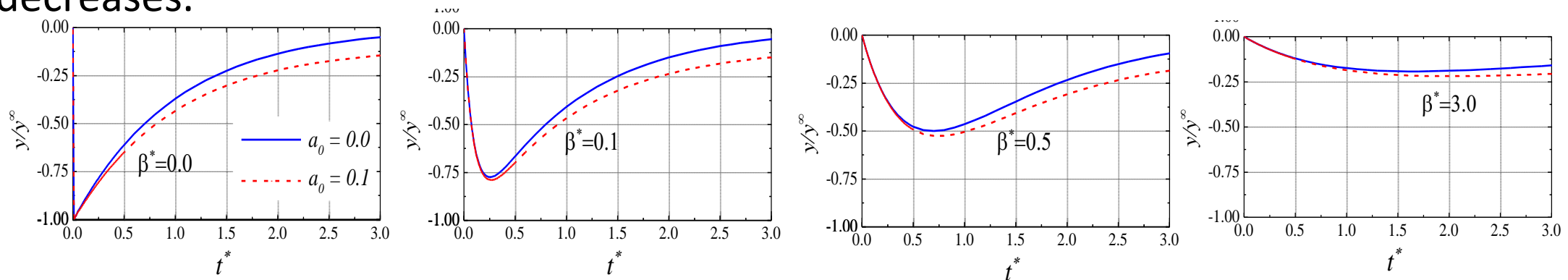
- Replacing Eq.(4) into Eq.(2) and solving the convolution integral we obtain

$$y^* = \left( \frac{1-a_0}{1-\beta^*} \right) e^{\frac{-t^*}{\beta^*}} \left\{ 1 - e^{\left(\frac{1}{\beta^*}-1\right)t^*} \right\} - a_0 \left( 1 - e^{\frac{-t^*}{\beta^*}} \right) \quad (6)$$

Where:  $t^* = \beta t$

$\beta^* = \beta / \alpha$  is non-dimensional parameter expresses the ratio between the morphological and bed recovery time scales

- $\beta^* = 0$  indicates that the time constant of the beach response is zero, so a shoreline response occurs instantly in response to a change in external force (Land rising function).
- As  $\beta^*$  increases, the shoreline response occurs late and the height of the peak decreases.





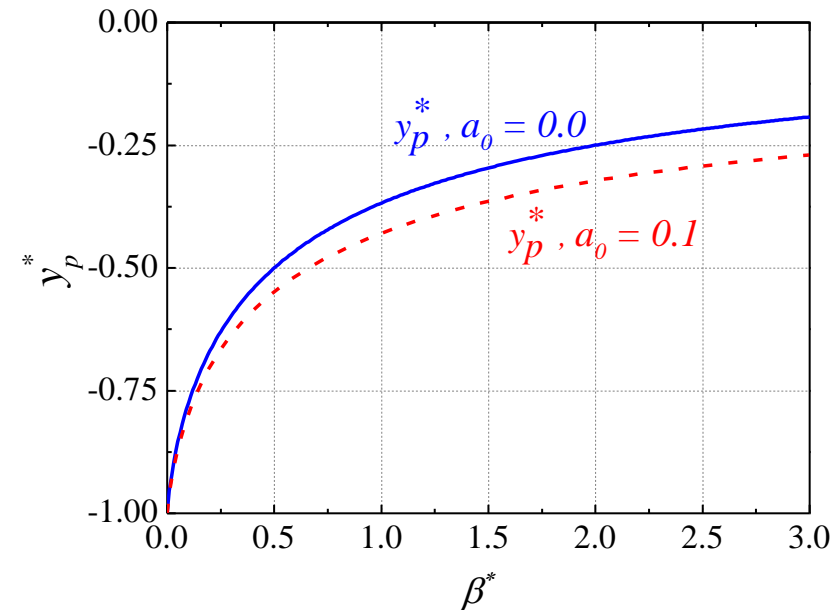
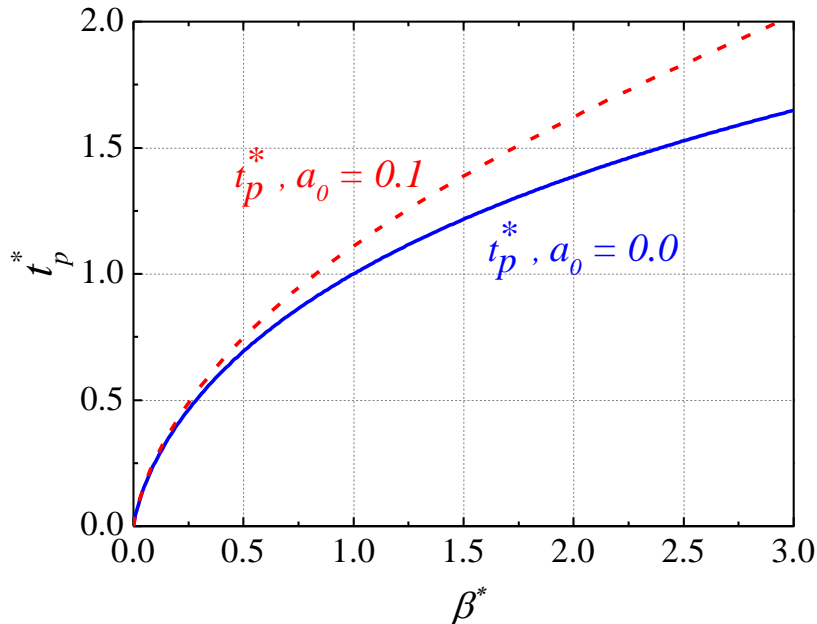
# ANALYTICAL MODEL DEVELOPMENT

## 2. SOLUTION FOR GENERAL LAND RECOVERING CASE

The peak occurrence time  $t_p^*$  and peak shoreline position  $y_p^*$  are obtained from the first derivative with respect to  $t^*$  in Eq. (6) = 0 as follows.

$$t_p^* = \frac{\beta^* \ln(a^*)}{\beta^* - 1} \quad (7) \quad y_p^* = \frac{(a_0 \beta^* - 1)}{\beta^*} a^{*\frac{1}{1-\beta^*}} - a_0 \quad (8)$$

$$a^* = \frac{\beta^* (1 - a_0)}{1 - a_0 \beta^*} \quad (9)$$



# ANALYTICAL SOLUTION

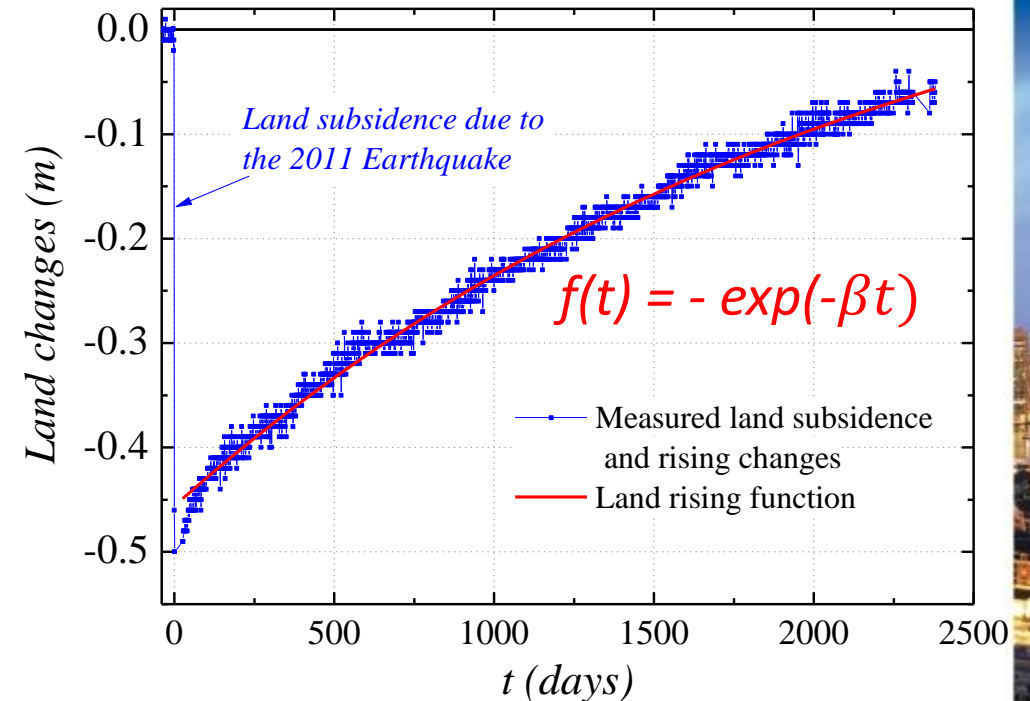
## 2. SOLUTION FOR LAND RECOVERING CASE when $a_0 = 0.0$

$$y^* = \left( \frac{1}{1-\beta^*} \right) e^{\frac{-t^*}{\beta^*}} \left\{ 1 - e^{\left( \frac{1}{\beta^*} - 1 \right) t^*} \right\} \quad (11)$$

$$t_p^* = \frac{\beta^* \ln \beta^*}{\beta^* - 1} \quad (12)$$

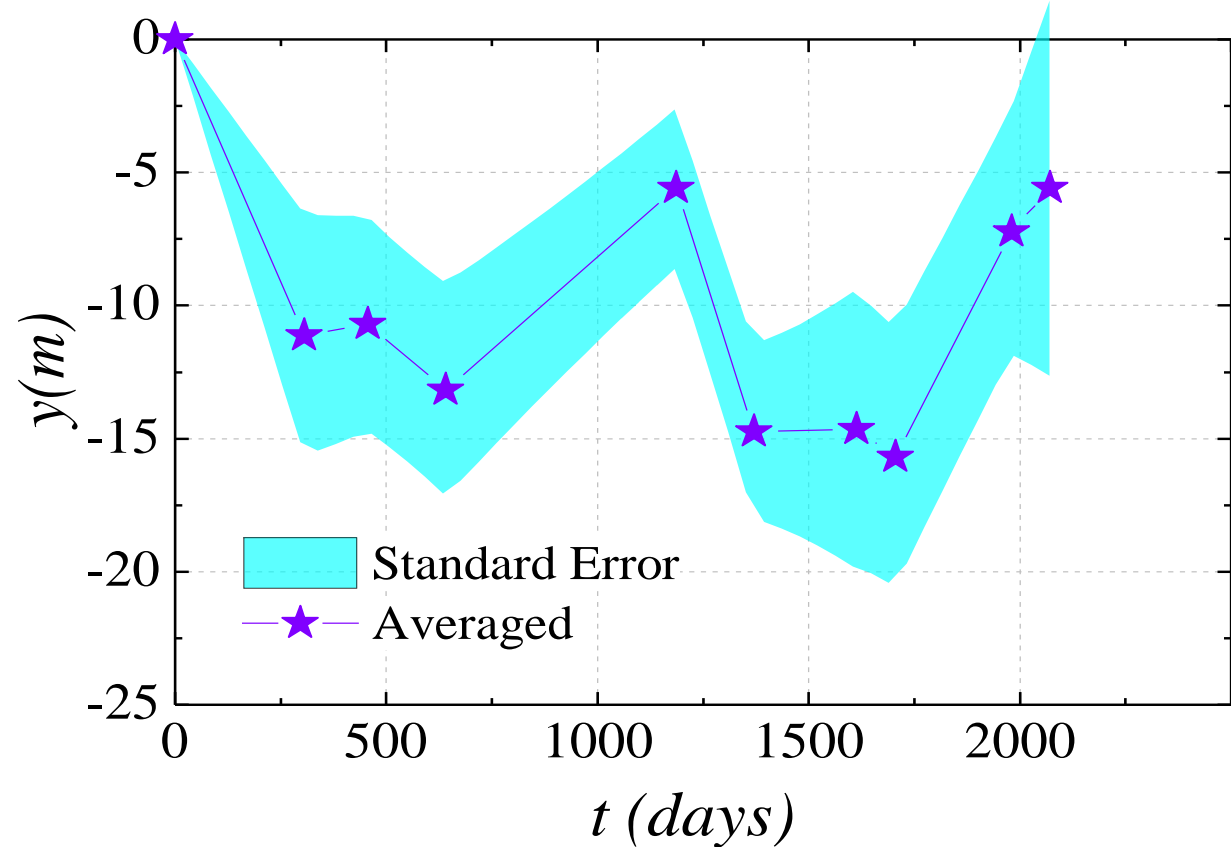
$$y_p^* = \beta^* \frac{\beta^*}{(1-\beta^*)} \quad (13)$$

- $\beta^*$  is a calibrate parameter depend on two unknown  $\beta$  and  $\alpha$ .
- Based on the current land rising data, the best fit was found when  $\beta = 7.5 \times 10^{-4}$  (1/day)



# ANALYTICAL MODEL CALIBRATION

- The time constant of the beach deformation,  $\alpha$ , was calibrated by fixing the above  $\beta$  value and adjusting the maximum shoreline retreat  $y_{\infty}$  in order to obtain the best agreement to the measured shoreline positions.
- Even though the measured data shows a fluctuation shoreline. However, the all data indicates that the shorelines were retreated after the earthquake and then start recovering.



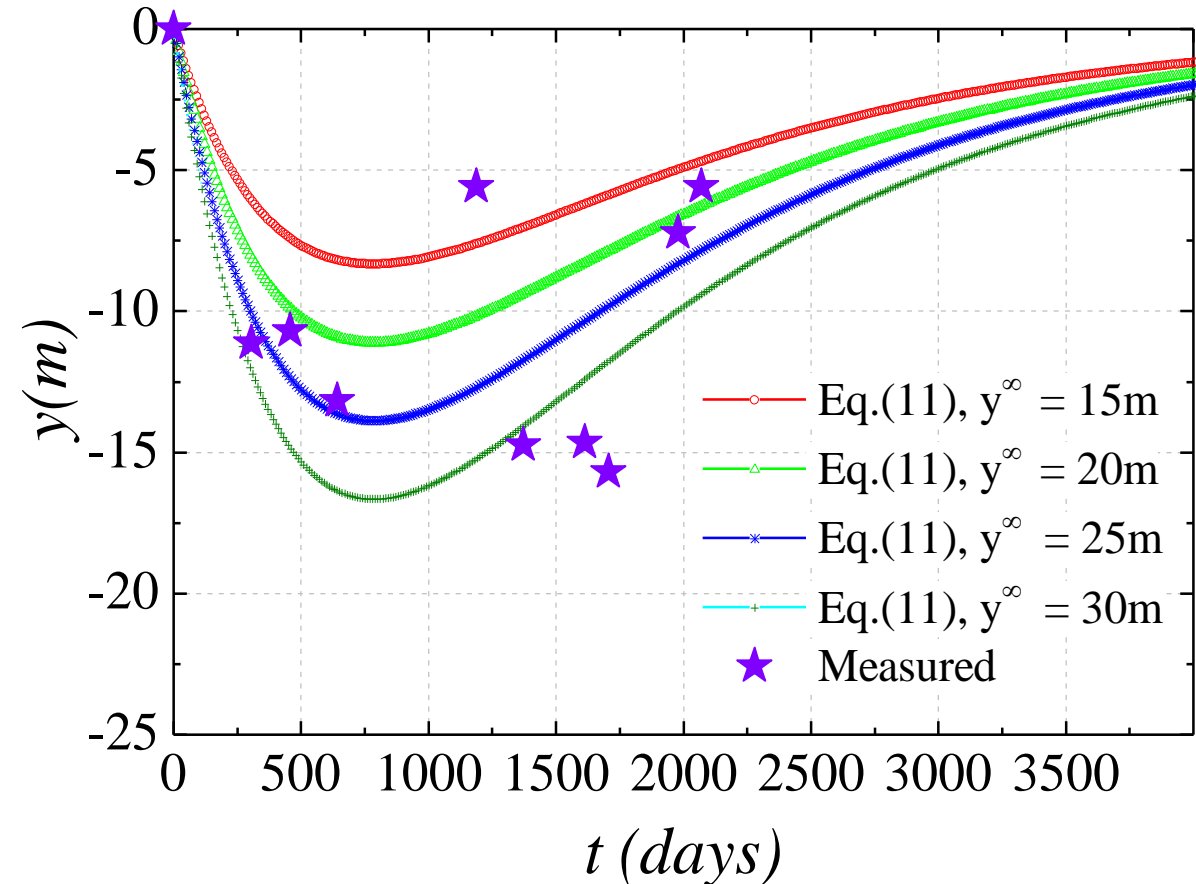
Variation of calibrated shoreline positions



# ANALYTICAL MODEL CALIBRATION

➤ it was found that the value of  $\alpha = 2.0 \times 10^{-3}$  (1 /day) and equivalent to the time constant  $T_{SL} = 500$  days . In addition,  $y_{\infty} = 20$  m was obtained from the consistency of the theoretical value and measured value.

➤ On the other hand, according to Bruun's rule,  $y_{\infty} = 21.4$  m, which is good agreement with the above calibrated value for this coast.



Comparison of measured values of shoreline fluctuation and analytical results



# CONCLUSIONS

- In this study, new analytical solutions are developed based on the linear response theory to describe shoreline position changes due to the sea level rise.
- Detailed, high-quality data sets obtained after the 2011 Tohoku Earthquake and Tsunami, including wave climate, beach profiles and bed level change due to tectonic movement were used to validate the model.
- As a result, the time dependent beach profile responses due to the SLR forcing is predicted. The maximum shoreline retreat amount due to land recovering by new method is in good agreement with the value estimated from the Bruun's rule.
- In addition, the obtained time constant of the beach deformation under the sea water level rise was much larger than beach response by the storm condition.



**THANKS FOR YOUR ATTENTION!**

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