A NONLINEAR AND DISPERSIVE 3D MODEL FOR COASTAL WAVES USING RADIAL BASIS FUNCTIONS

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INTRODUCTION

Accurate wave propagation models are required for the design of coastal structures and the evaluation of coastal risks. Nonlinear and dispersive effects are particularly important in the nearshore environment. Two-dimensional cross-shore (2DV) wave models can be used as a preliminary step in coastal studies, but 3D models are needed to capture fully the effects of alongshore bathymetric variations, variable wave incidence, the presence of coastal or harbor structures, etc.

MODEL

Yates and Benoit (2015) developed a numerical model based on fully nonlinear potential flow theory. By assuming non-overturning waves, the kinematic and dynamic free surface boundary conditions are expressed as evolution equations of the free surface elevation and velocity potential, following Zakharov (1968). At each time step, the free surface vertical velocity is estimated by solving the Laplace equation for the velocity potential in the domain. Following Tian and Sato (2008), a spectral approach is used to expand the velocity potential in the vertical as a linear combination of Chebyshev polynomials. The accuracy of the 2DV model was validated with several non-breaking experimental test cases (Benoit et al., 2014; Raoult et al., 2016).

Here the model is extended to 3D using scattered nodes (for flexibility) to discretize the horizontal domain. Spatial derivatives are estimated at each node using a linear combination of the function values at neighboring points using Radial Basis Functions (RBF) (Wright and Fornberg, 2006). The accuracy of the method depends on the number of neighboring nodes (*Nsten*) and the chosen RBF type (e.g. multiquadric, Gaussian, polyharmonic spline (PHS), thin plate spline, etc.), with associated shape factor *C* for some of them.

RESULTS

The simulation results are compared to theoretical solutions and laboratory experiments for three test cases. The first case simulates the propagation of a regular wave (uniform in the transversal direction) over a flat bottom. The results are compared to those obtained with the 2DV model to test the accuracy of the 3D model. A series of sensitivity tests were completed as a function of the chosen RBF (including the parameter C for some of them), the stencil's size *Nsten* and an added polynomial. The second test case simulates the propagation of regular waves over a semi-circular step that acts like a focusing lens, based on the experiments of Whalin (1971) (Figure 1). Comparison between the experimental and simulated spatial evolution of the first three harmonic amplitudes shows the model's ability to reproduce well wave shoaling and nonlinear wave interactions (Figure 2). Finally, the third test case simulates the experiments of Vincent & Briggs (1989), which evaluate regular and irregular wave transformation over an elliptical bump.

CONCLUSION

These test cases show the nonlinear and dispersive capabilities of the 3D model, as well as the sensitivity of the model to the parameters of the RBF-FD method. Based on these tests, recommendations can be drawn regarding the choice of a RBF type (here a PHS RBF proved to be more accurate and robust), the degree of the added polynomial and a range of sizes for the stencil of nodes.

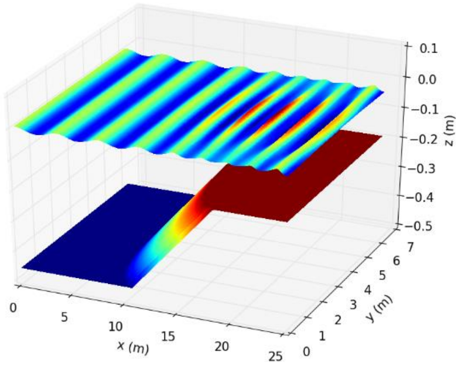

Figure 1 - Bathymetry of the experiments of Whalin (1971) and snapshot of the simulated free surface elevation.

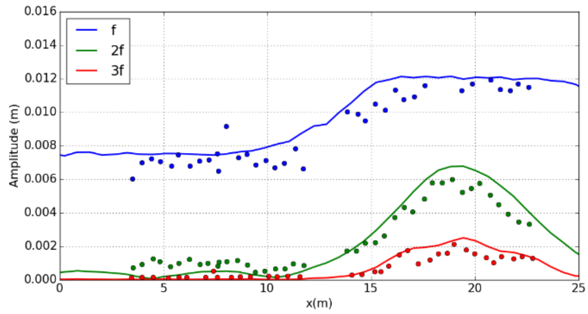

Figure 2 - Simulated (solid line) and measured (dots) harmonic amplitudes along the center axis of the basin for an experiment of Whalin (1971) with T = 2 s and A = 7.5 mm.

REFERENCES

Benoit, Raoult, Yates (2014): Fully nonlinear and dispersive modelling of surf zone waves: non-breaking tests, Coastal Engineering Proceedings, vol. 1(34), waves.15.

Raoult, Benoit, Yates (2016): Validation of a fully nonlinear and dispersive wave model with laboratory non-breaking experiments, Coastal Eng., vol. 114, pp. 194–207.

Tian, Sato (2008): A numerical model on the interaction between nearshore nonlinear waves and strong currents. Coast. Eng. J., vol. 50(4), pp. 369-395.

Vincent, Briggs (1989): Refraction-Diffraction of Irregular Wave over a Mound, J. Wat. Port Coast. Ocean Eng., vol. 115, pp. 269-284.

Whalin (1971): The limit of applicability of linear wave refraction theory in a convergence zone. Technical report, DTIC Document.

Wright, Fornberg (2006): Scattered node compact finite difference-type formulas generated from radial basis functions, J. Comp. Phys., vol. 212, pp. 99-123.

Yates, Benoit (2015): Accuracy and efficiency of two numerical methods of solving the potential flow problem for highly nonlinear and dispersive water waves, Int. J. Num. Meth. Fluids, vol. 77, pp. 616-640.

Zakharov (1968): Stability of periodic waves of finite amplitude on the surface of a deep fluid. J. Appl. Mech. Tech. Phys., vol. 9, pp. 190-194.