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*The State of the Art and Science of Coastal Engineering*

## Statistics of extreme waves in coastal waters:

Large scale experiments and advanced numerical simulations

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# Statistics of extreme waves in coastal waters:

Large scale experiments and advanced numerical simulations

## Outline of this presentation:

1. Background and motivations for the study
2. Introduction of the large scale experiments in Tainan Hydraulics Laboratory (THL)
3. Brief introduction of two highly accurate numerical models
4. Data processing and comparison between the measured/simulated results
5. Conclusion and Outlook

# 1. Background and motivation

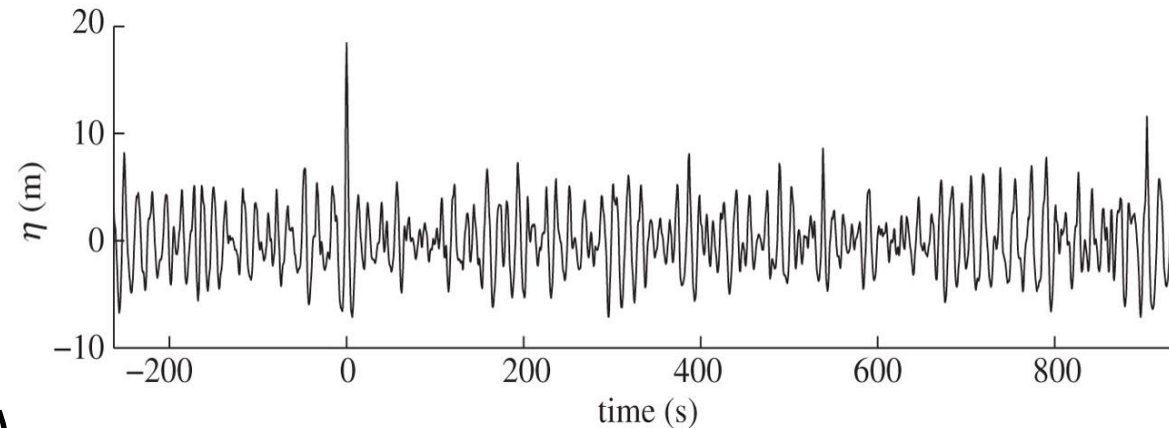
## Definition

$$\frac{H}{H_s} > 2 \text{ or } \frac{\eta_{crest}}{H_s} > 1.25 \text{ (Dysthe } et al. \text{ 2008)}$$

## Motivation

- Vital importance for the coastal structures design and for the ship navigation safety;
- Theoretically unclear effect of a bottom gradient on the statistics of freak waves;
- Limited experimental results with irregular waves propagating over variable bottom;

New year wave (1<sup>st</sup> January 1995) in Draupner platform

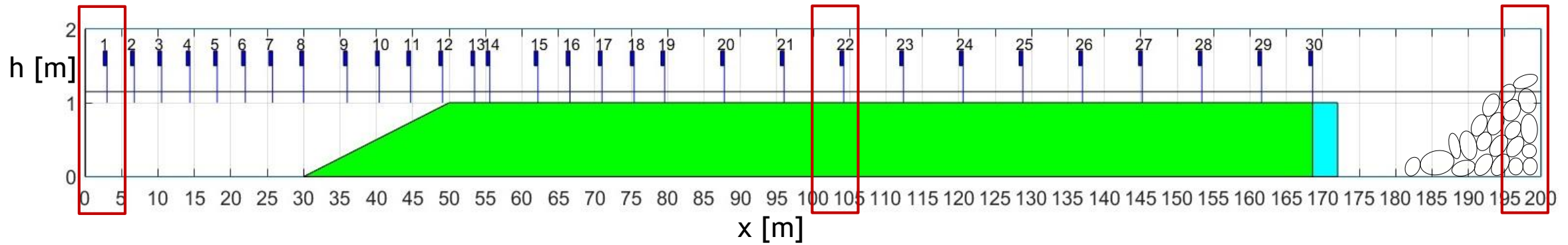


## Some existing results

- Trulsen *et al.* (2012) showed that the presence of a bottom slope will give rise to local maximum of skewness and kurtosis, and also, the freak wave probability;
- Kashima *et al.* (2014) studied different bottom shapes.



## 2. Large scale experiments in THL



Piston-type absorbing wave maker



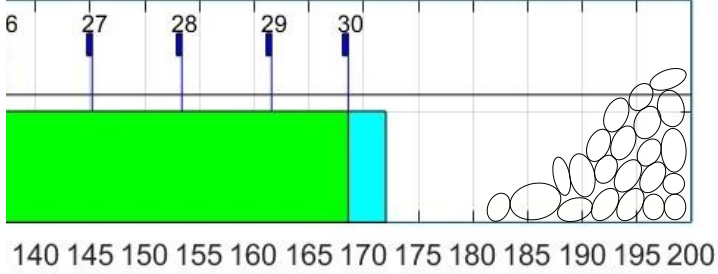
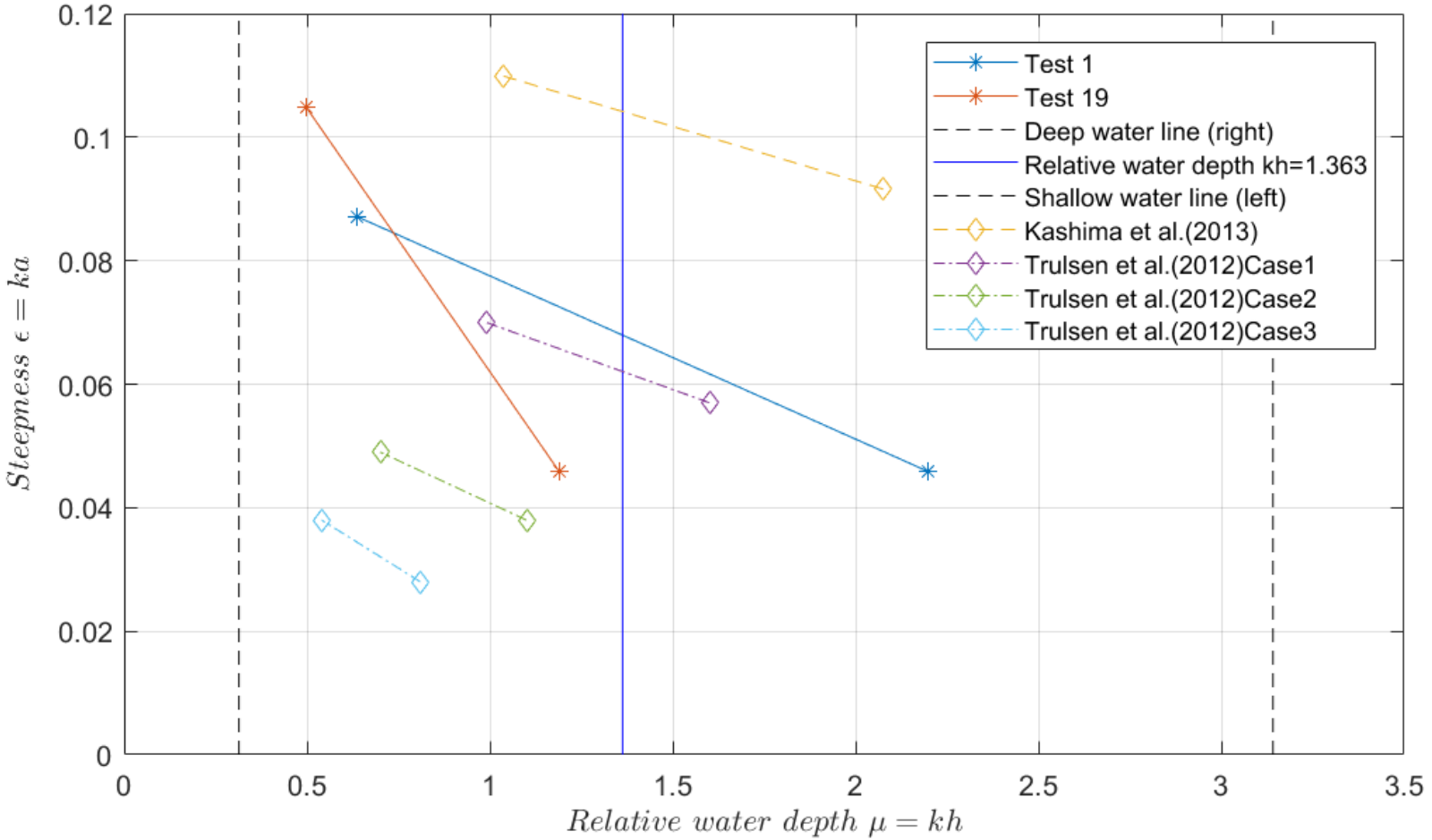
30 wave gauges  
( $F_s = 100 \text{ Hz}$ )



Wave absorption  
zone



# 2. Large scale experiments in THL



Shallower zone			breaking
$h'$	$k_p H_s'$	$Ur'$	
0.64	0.087	26.73	no
0.50	0.105	67.62	little

Note: here list 2 of the 25 experiments in total

Where:  $Ur = H_s L_p^2 / h^3$

$$H_s' = H_s \sqrt{C_g^{deep}} / \sqrt{C_g^{shallow}}$$

$$h' = h - 1 \text{ m}$$

$$C_g = \frac{1}{2} \left[ 1 + \frac{2k_p h}{\sinh(2k_p h)} \right] \frac{\omega}{k_p}$$



# 3. Highly accurate numerical models

## Some key points of the numerical models:

1. Potential formulation: Boussinesq-type model (Bingham et al., 2009) and a recently developed model Whispers3D (based on Yates and Benoit 2015), both are formulated in terms of velocity potential variable;
 

Free surface boundary condition (BC)

$$\phi_t + \nabla\eta\nabla\phi + \eta_t = 0$$

on  $z = \eta(x)$

$$\phi_t + \frac{1}{2}(\nabla\phi + \phi_z)^2 + g\eta = 0$$

2. Basic assumptions: (1) *inviscid* and *irrotational flow* with constant density  $\rho$  (*incompressible*) fluid; (2) *non-overturning waves*;
 



3. The behavior of the Boussinesq model has been tested for both slowly varying bottom shape (linear shoaling) and rapidly varying bottom (Bragg resonance), see more details in Bingham et al., (2009);
 

Lateral BC

Laplace equation in the bulk

$$\Delta\phi = 0$$

Lateral BC

4. The behavior of the Whispers3D modeling approach has been validated for both its fully nonlinear version (Raoult et al., 2016) and its linearized version (Benoit et al., 2017).
 

Bottom BC

$$\phi_z + \nabla h \nabla \phi = 0, \text{ on } z = -h(x)$$



# 3. Highly accurate numerical model: Boussinesq

Following Bingham *et al.* (2009):

- The velocity potential is approximated by a infinite number of Taylor series around an arbitrary vertical position  $z = \hat{z}(x)$ : Normally  $\hat{z} = 0.5h(x)$  is chosen

$$\phi(x, z, t) = \hat{\phi} + (z - \hat{z})\hat{\phi}^{(1)} + \frac{1}{2} (z - \hat{z})^2\hat{\phi}^{(2)} + \dots + \frac{1}{n!} (z - \hat{z})^n\hat{\phi}^{(n)} + \dots \quad (1)$$

- Then the potential could be rewritten as follows:  $\hat{z} = \hat{z}(\delta x)$  is assumed here!

$$\phi(x, z, t) = (J_{01} + \delta J_{11}\nabla\hat{z})\hat{\phi}^* + (J_{02} + \delta J_{12}\nabla\hat{z})\hat{w}^* \quad (2)$$

- The system is solvable together with bottom boundary condition: Finite difference scheme for x

$$w + \nabla h \nabla \phi = 0, \text{ on } z = -h(x) \quad (3)$$

- Finally the model can be integrated in time (**4<sup>th</sup> order R-K method**) using the Zakharov equation:

$$\eta_t + \nabla\eta\nabla\tilde{\phi} - \tilde{w}^2(1 + (\nabla\eta)^2) = 2v\eta_{xx} \quad (4)$$

$$\tilde{\phi}_t + g\eta + \frac{1}{2}(\nabla\tilde{\phi})^2 - \frac{1}{2}\tilde{w}^2(1 + (\nabla\eta)^2) = -2v\tilde{\phi}_{zz} \quad (5)$$

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Note: See the analytical expression of  $L_0(\hat{z}\nabla), J_{01}, J_{11}, J_{02}, J_{12} \dots$  in the work of Bingham *et al.* (2009), they are not given here for the simplicity.



# 3. Highly accurate numerical model: Whispers3D

Following Tian and Sato (2008):

- First the variable bottom is mapped into a rectangular domain by using a new vertical variable  $s(x, z, t)$ :

$$s(x, z, t) = \frac{2z + h(x) - \eta(x, t)}{h(x) + \eta(x, t)}, \quad (1)$$

- Then the system with potential variable  $\varphi(x, s(x, z, t), t) \equiv \phi(x, z, t)$  in the transformed space  $(x, s)$  :

$$\varphi_{xx} + 2s_x \varphi_{xs} + (s_x^2 + s_z^2) \varphi_{ss} + s_{xx} \varphi_s = 0 \quad \text{for } s \in [-1, 1], \quad (2)$$

$$\varphi(x, 1) = \tilde{\Phi}(x) \quad \text{on } s = +1, \quad (3)$$

$$(h + \eta)h_x \varphi_x + 2(1 + h_x^2) \varphi_s = 0 \quad \text{on } s = -1. \quad (4)$$

- The spectral approximation is adopted in the vertical:  $T_n(s)$  is the Chebyshev polynomial

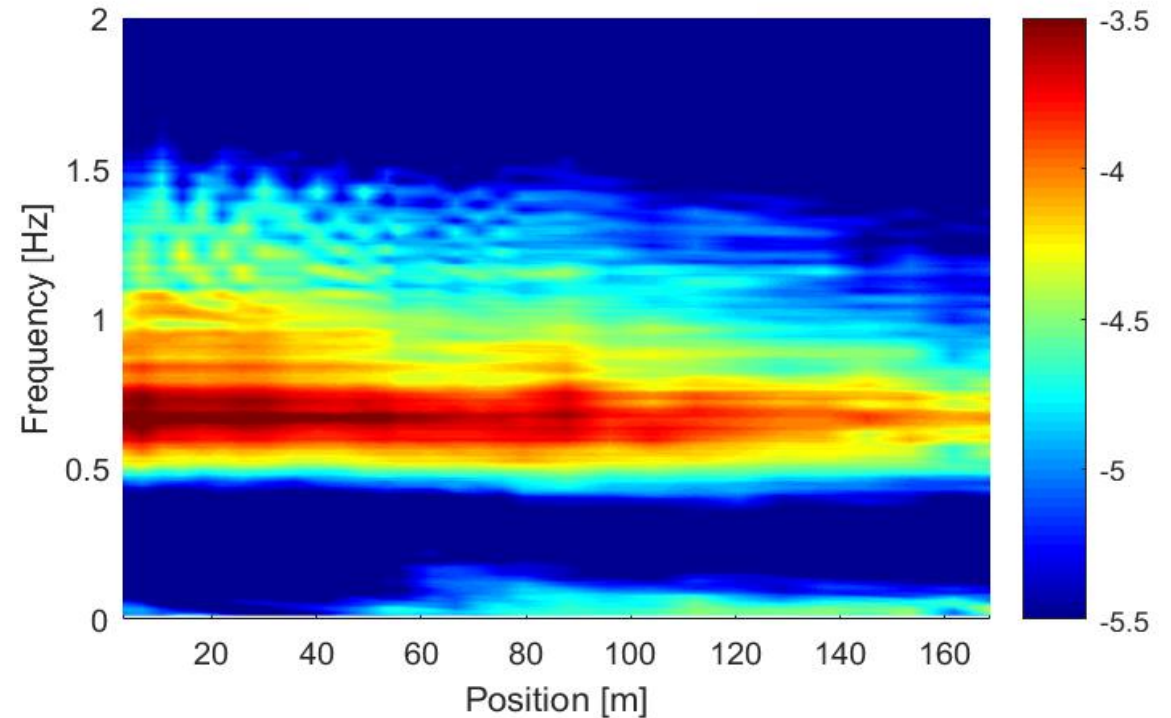
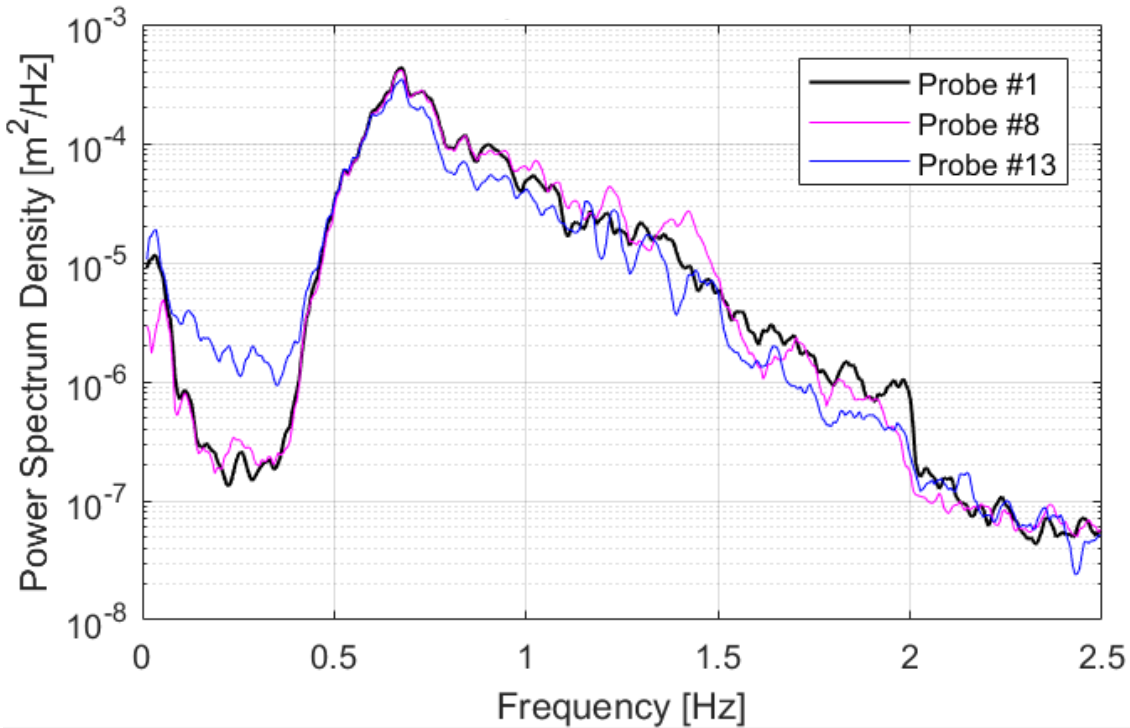
$$\varphi(x, s) \approx \sum_{n=0}^{N_T} a_n(x) T_n(s), \quad (5)$$

- The unknowns are the  $N_T + 1$  coefficients  $a_n(x), n = 0, 1, \dots, N_T$ , using Galerkin method to eq.(2), we get  $N_T - 1$  equations, together with eqs.(3) and (4) is the linear system that we solve at each time step. Finally the system is stepped forward in time by using third order SSP-RK3 scheme.





# 4. Data processing and comparison



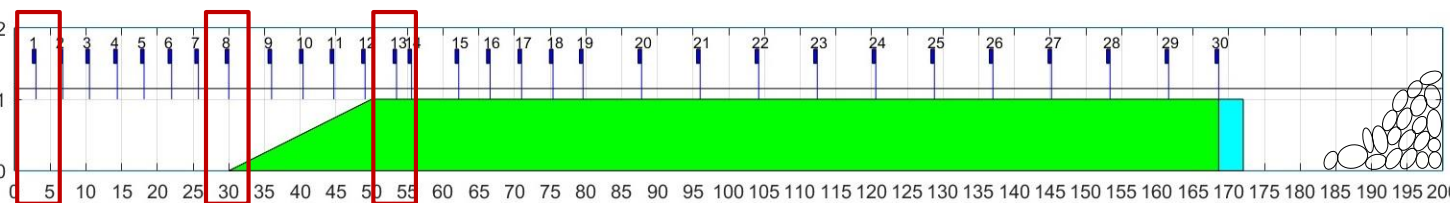
(PS: The numbers on color map represent the power of 10)

**Case 1 :  $T_p = 1.5$  s,  $H_s = 0.05$  m,  $h = 1.2$  m,  $\gamma = 3.3$**

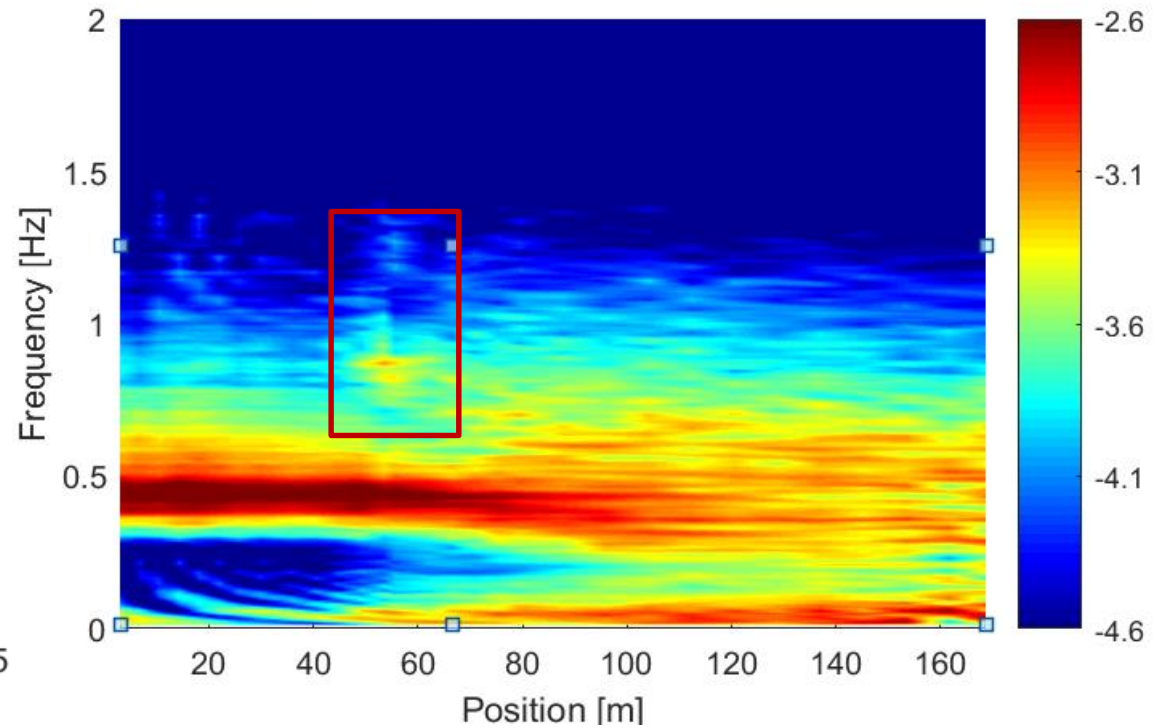
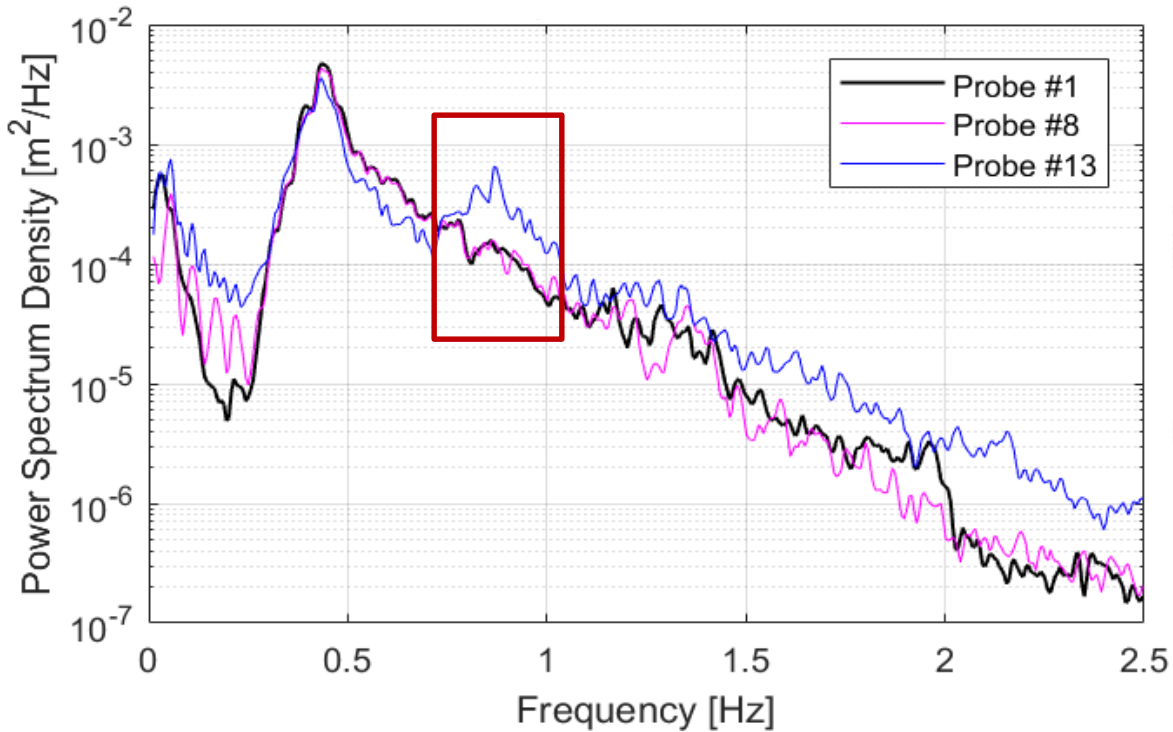


**Fourier analysis**

- Sampling frequency 100Hz
- $2^{13}$ -Points of FFT ( $\Delta f = 0.0061$  Hz)
- Welch method with Hanning window
- 50% overlap of the signal

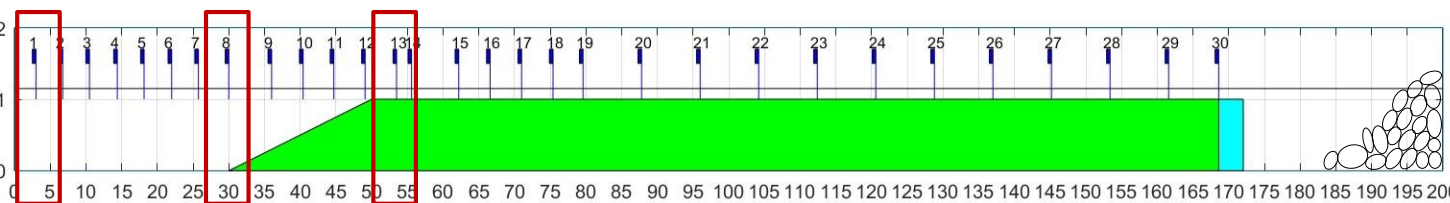


# 4. Data processing and comparison



(PS: The numbers on color map represent the power of 10)

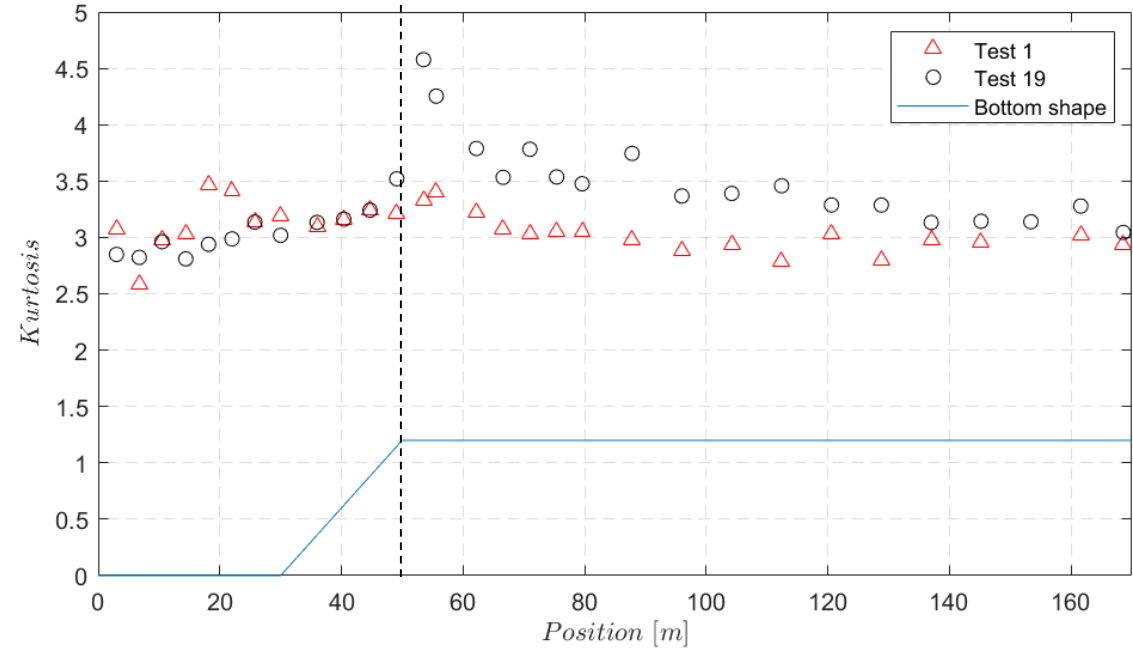
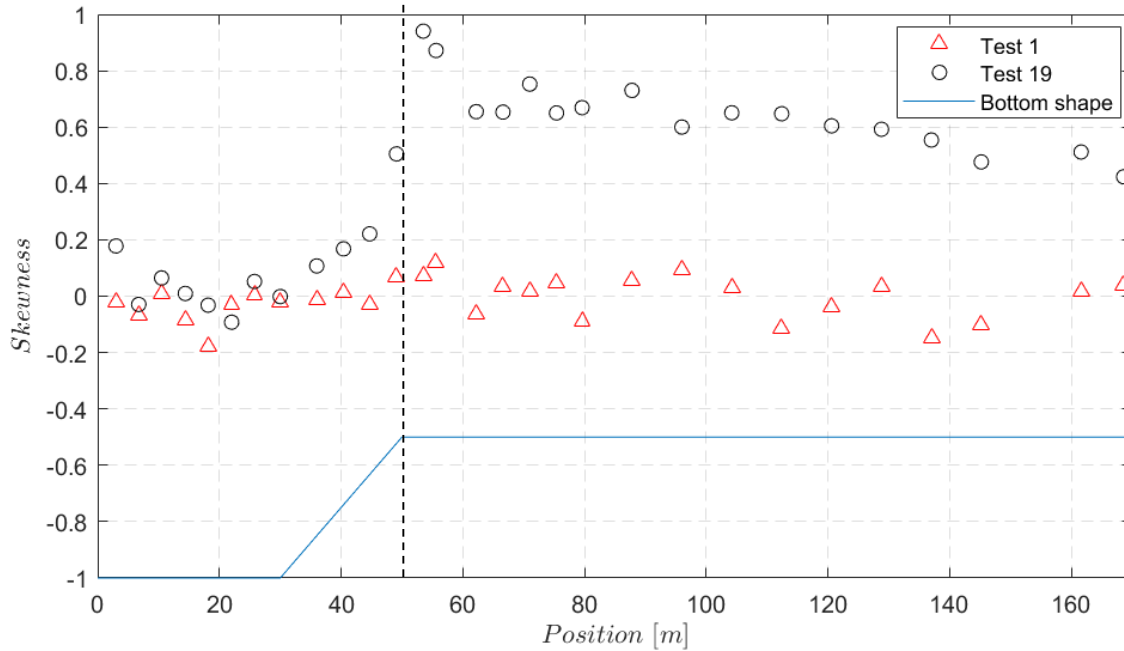
**Case 19:  $T_p = 2.3$  s,  $H_s = 0.10$  m,  $h = 1.3$  m,  $\gamma = 3.3$**



## Fourier analysis

- Sampling frequency 100Hz
- $2^{13}$ -Points of FFT ( $\Delta f = 0.0061$  Hz)
- Welch method with Hanning window
- 50% overlap of the signal

# 4. Data processing and comparison



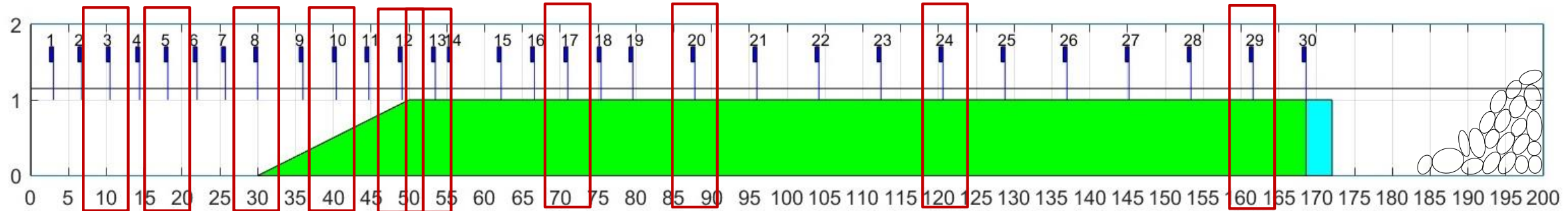
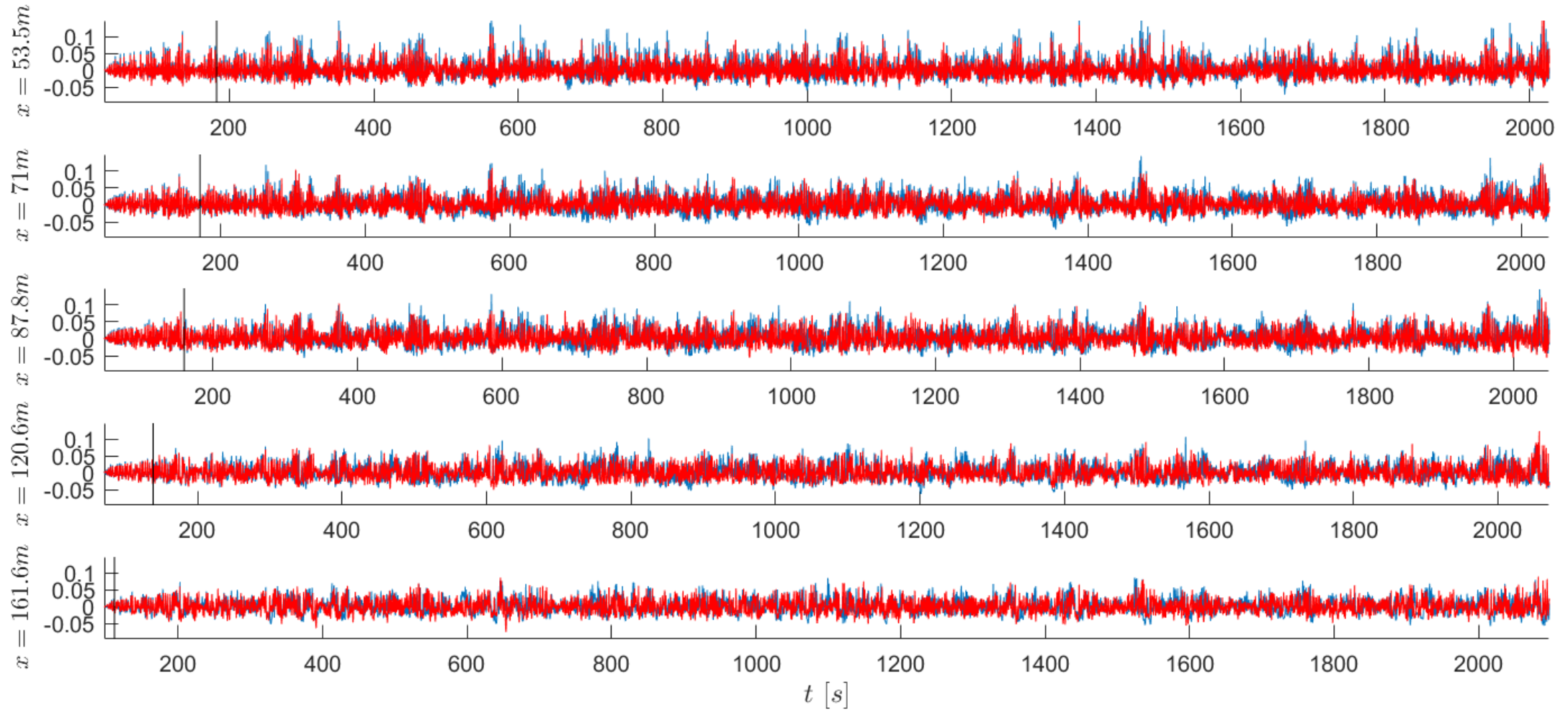
$$Skewness = \frac{\langle (\eta - \langle \eta \rangle)^3 \rangle}{m_0^{3/2}}$$

$$Kurtosis = \frac{\langle (\eta - \langle \eta \rangle)^4 \rangle}{m_0^2}$$

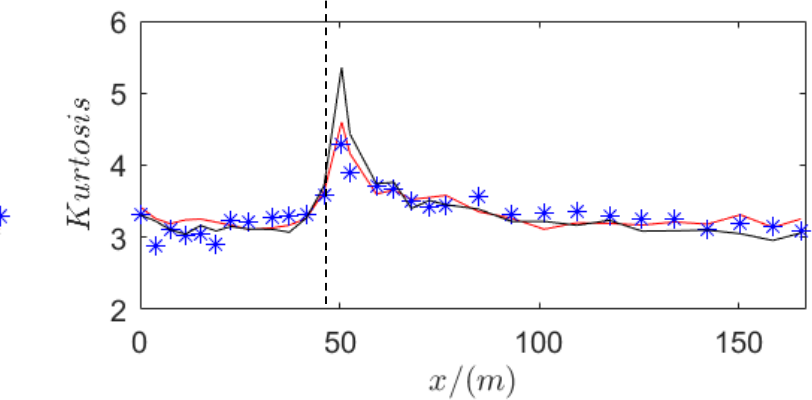
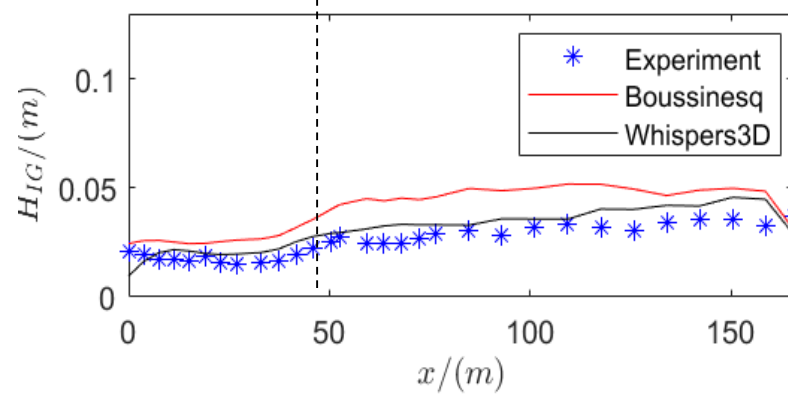
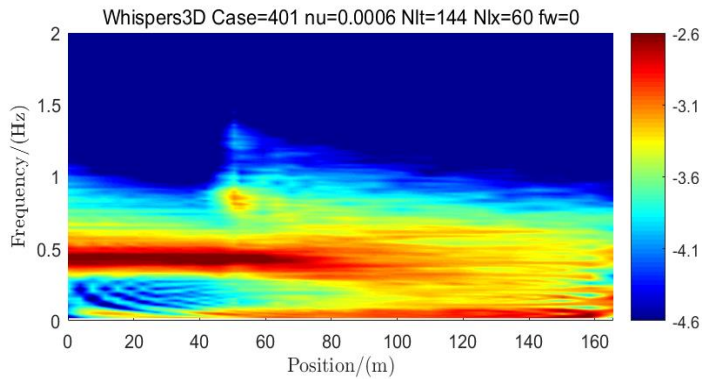
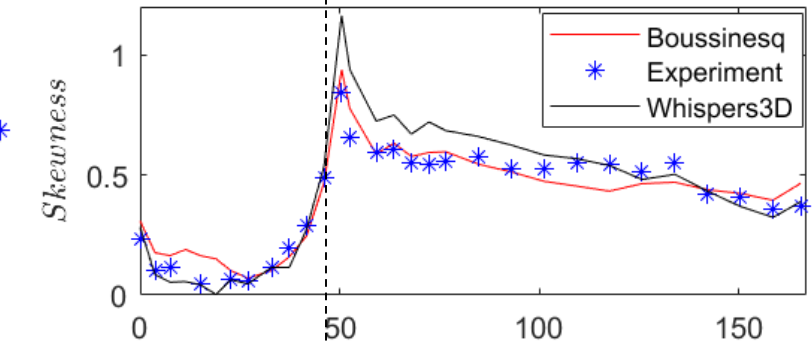
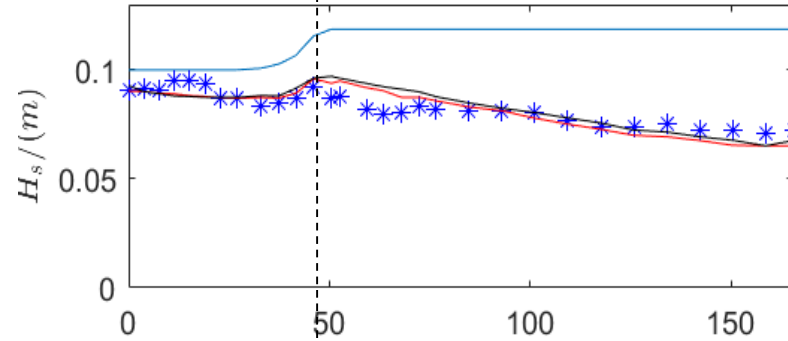
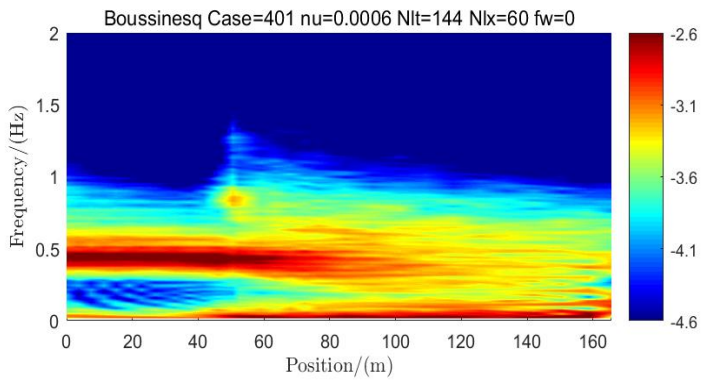
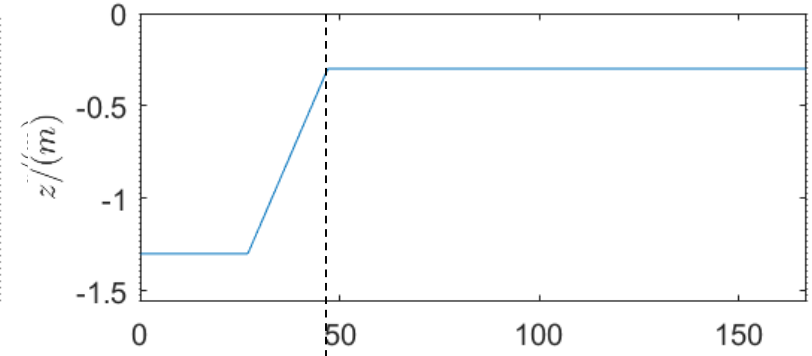
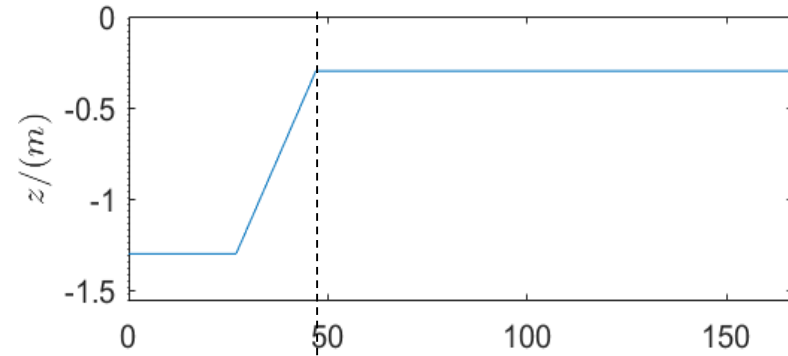
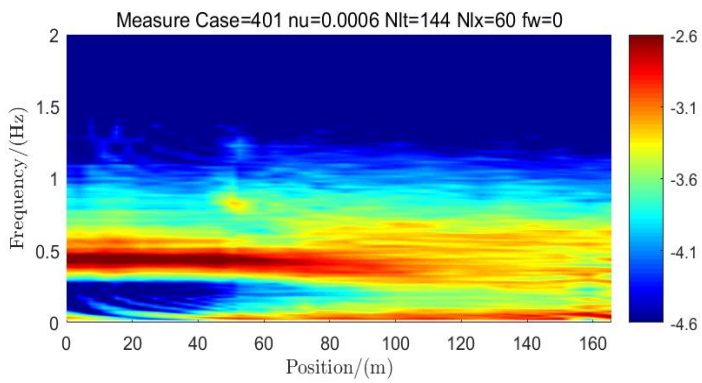
JONSWAP	$h$	$H_s$	$T_p$	$\gamma$	$k_p h$	Deeper zone			Shallower zone			breaking
						$k_p H_s$	$Ur$	$k_p h'$	$k_p H_s'$	$Ur'$		
Test 1	1.2	0.05	1.5	3.3	2.20	0.046	0.340	0.64	0.087	26.73	no	
Test 19	1.3	0.10	2.3	3.3	1.19	0.046	2.143	0.50	0.105	67.62	little	



# 4. Data processing and comparison



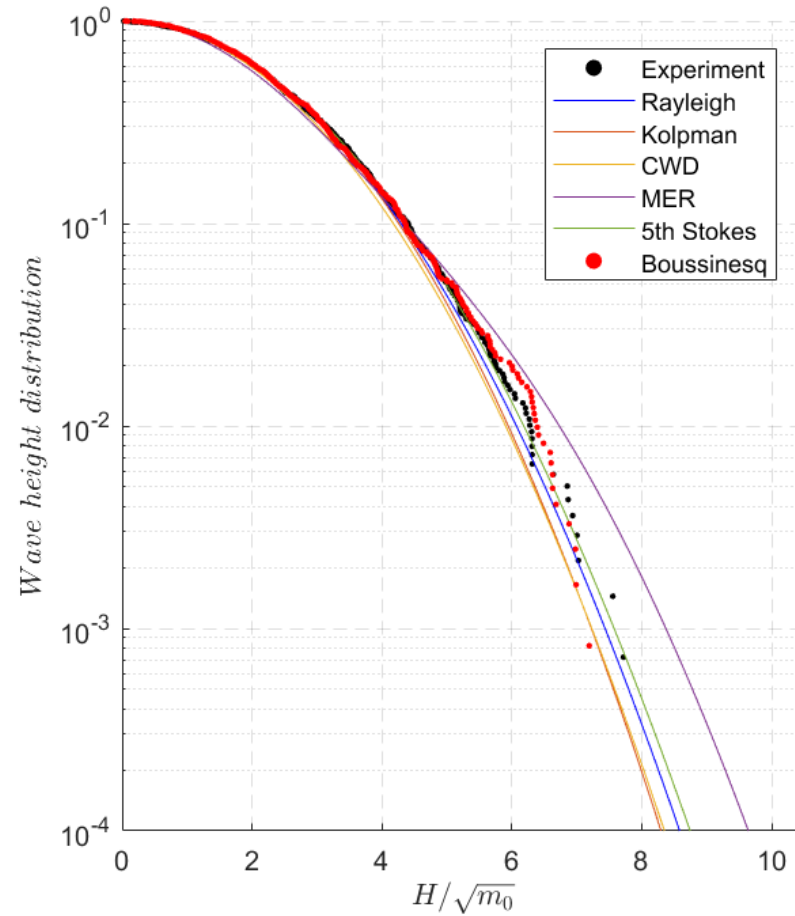
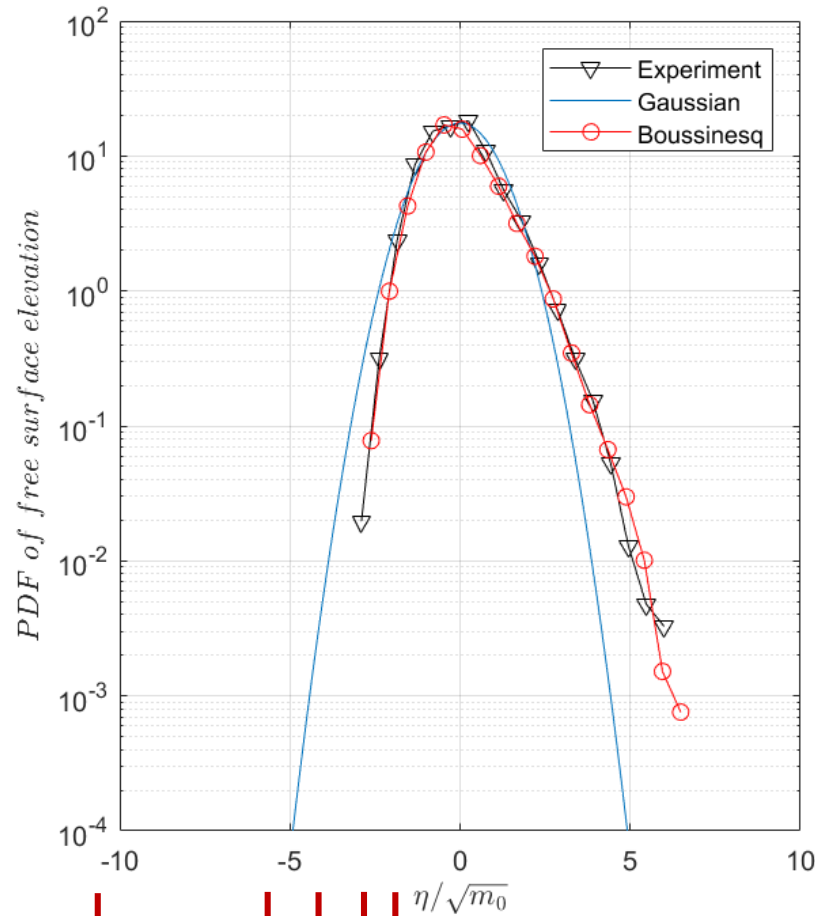
# 4. Data processing and comparison



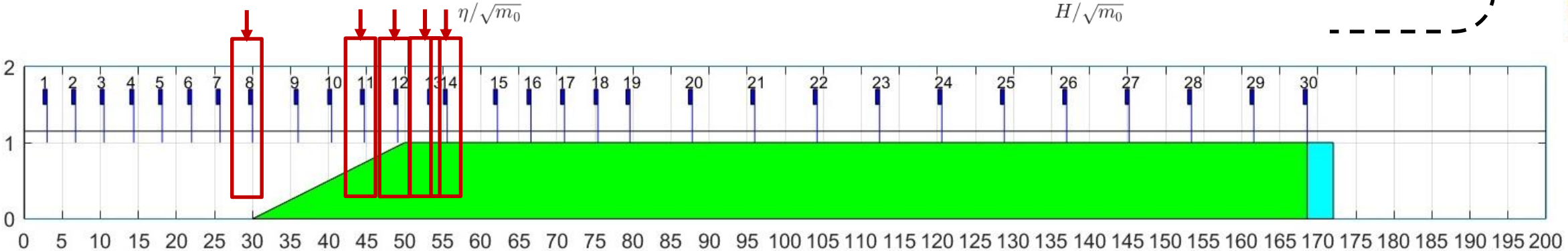
# 4. Data processing and comparison

Case=401 nu=0.0006 NIt=144 Nlx=60 fw=0 Probe #14

1  
A  
h  
s  
2  
A  
v  
3



jk, 2000),



# 5. Conclusion and outlook

- Experiments with large Ursell number:
  - Second-order nonlinear effect is significant around the end of the slope;
  - Local maximum values for skewness and kurtosis are found around the end of the slope;
  - No distribution can predict the wave height distribution equally well for all probes;
  - The wave energy is significantly redistributed over a wide range of frequencies for long-distance propagation.

- Simulations with numerical models:
  - The agreement with experiment data is generally good for both models;
  - The Boussinesq model can describe the spatial variation of low frequency waves but overestimates the energy, whereas Whispers3D behaves better;
  - The Whispers3D overestimates the statistical parameters at some location whereas the Boussinesq model has a better agreement with the experiments;



# 5. Conclusion and outlook

## What's next:

1. Improve and calibrate the simulation with two models (mainly dissipation);
2. Include wave breaking in both two models;
3. Conduct more simulations to do statistical analysis of freak waves;
4. Bispectral analysis;
5. The distribution of the envelop of the free surface.



Thank you for your attention  
Merci pour votre attention  
谢谢，再见。