



Two-Phase Flow Simulations Of Scour Around Vertical And Horizontal Cylinders

Tim NAGEL , Julien CHAUCHAT, Cyrille BONAMY, Antoine MATHIEU, Xiaofeng LIU, Zhen CHENG & Tian-Jian HSU

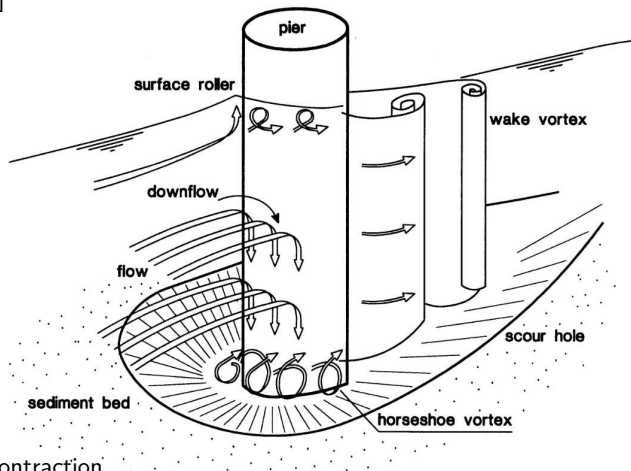


Center for Applied Coastal Research



Vertical circular pile exposed to a steady current

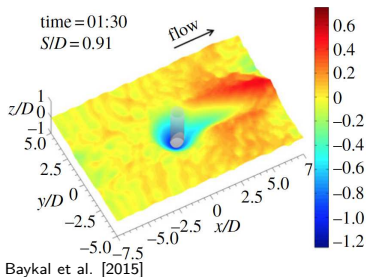
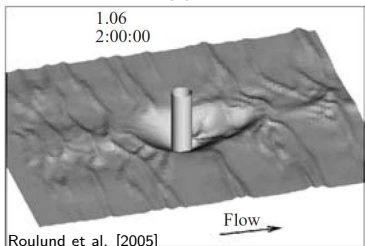
Melville [1988]



- Streamlines contraction.
- Lee-wake vortices (Vortex shedding).
- Downflow in front of the pile / Horseshoe vortex.
- Increase of the sediment transport \Rightarrow **Scour (O. Link)** \Rightarrow Potential structure failure.

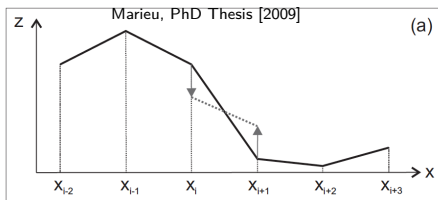
3D Numerical modeling of scour

"Classical" approach



- Critical Shields parameter modification:

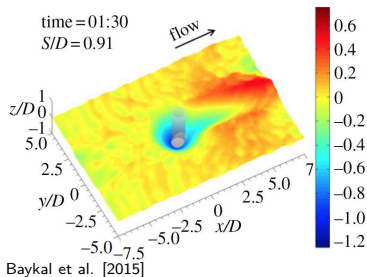
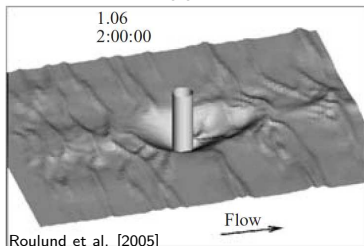
$$\theta_{cS} = \theta_c \left(\cos\beta \sqrt{1 - \frac{\sin^2\alpha \tan^2\beta}{\mu_s^2}} - \frac{\cos\alpha \sin\beta}{\mu_s} \right)$$
- Avalanching model (slope correction above $\beta = 32^\circ$).



- **sediment transport rate \Rightarrow bed shear stress**

3D Numerical modeling of scour

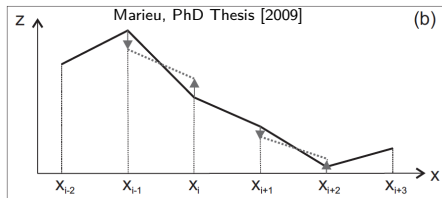
"Classical" approach



- Critical Shields parameter modification:

$$\theta_{cS} = \theta_c \left(\cos\beta \sqrt{1 - \frac{\sin^2\alpha \tan^2\beta}{\mu_s^2}} - \frac{\cos\alpha \sin\beta}{\mu_s} \right)$$

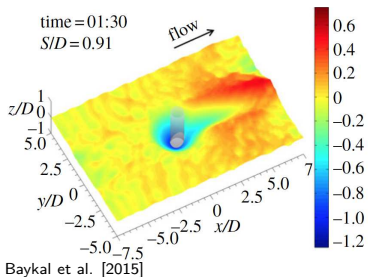
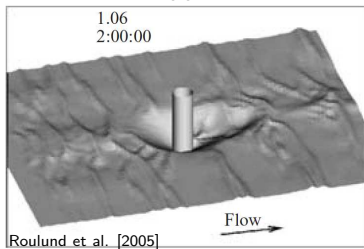
- Avalanching model (slope correction above $\beta = 32^\circ$).



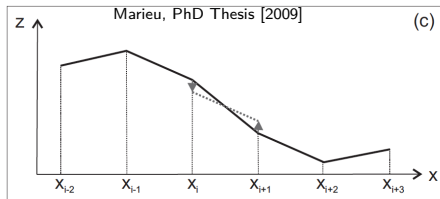
- sediment transport rate \Rightarrow bed shear stress

3D Numerical modeling of scour

"Classical" approach



- Critical Shields parameter modification:
$$\theta_{cS} = \theta_c \left(\cos\beta \sqrt{1 - \frac{\sin^2\alpha \tan^2\beta}{\mu_s^2}} - \frac{\cos\alpha \sin\beta}{\mu_s} \right)$$
- Avalanching model (slope correction above $\beta = 32^\circ$).



- **sediment transport rate \Rightarrow bed shear stress**

Eulerian-Eulerian two-phase flow

- **Does a Eulerian-Eulerian two-phase flow model reproduce the scour phenomenon?**
- **Can we get more insight into the fine scale sediment transport mechanisms involved in the scour process?**
- **Is it possible to parametrize them in "classical" morphodynamics models (upscaling)?**

3D Two-phase flow model equations

$$\text{Fluid: } \frac{\partial(1-\phi)}{\partial t} + \vec{\nabla} \cdot \left((1-\phi) \vec{u}^f \right) = 0$$

$$\underbrace{\rho^f (1-\phi) \frac{D\vec{u}^f}{Dt}}_{\text{unsteady+inertia}} = \underbrace{- (1-\phi) \vec{\nabla} p}_{\text{fluid pressure}} + \underbrace{\vec{\nabla} \cdot \vec{\tau}^f}_{\text{Fluid st.}} + (1-\phi) \vec{f} + \rho^f (1-\phi) \vec{g} \underbrace{- M_D}_{\text{drag}}$$

$$\text{Solid: } \frac{\partial\phi}{\partial t} + \vec{\nabla} \cdot \left(\phi \vec{u}^s \right) = 0$$

$$\rho^s \phi \frac{D\vec{u}^s}{Dt} = -\phi \vec{\nabla} p - \underbrace{\vec{\nabla} \cdot \vec{p}^s + \vec{\nabla} \cdot \vec{\tau}^s}_{\text{granular st.}} + \phi \vec{f} + \rho^s \phi \vec{g} + M_D$$

- $(1-\phi), \phi$: fluid and particles volume fractions
- \vec{u}^f : Average fluid velocity
- \vec{u}^s : Average particle velocity
- p : Fluid pressure
- \vec{p}^s : Particulate pressure
- S_c : Schmidt Number

Drag law:

$$K = 0.75 \frac{24}{Re_p} \left(1 + 0.15 Re_p^{0.687} \right) \frac{\rho^f}{d} \|\vec{u}^f - \vec{u}^s\| (1-\phi)^{-2.65}$$

$$Re_p = (1-\phi) \|\vec{u}^f - \vec{u}^s\| d / \nu^f : \text{particulate Reynolds number}$$

Schiller & Naumann [1933] + Richardson & Zaki [1954]

3D Two-phase flow model equations

$$\text{Fluid: } \frac{\partial(1-\phi)}{\partial t} + \vec{\nabla} \cdot ((1-\phi)\vec{u}^f) = 0$$

$$\underbrace{\rho^f(1-\phi)\frac{D\vec{u}^f}{Dt}}_{\text{unsteady+inertia}} = \underbrace{-(1-\phi)\vec{\nabla}p}_{\text{fluid pressure}} + \underbrace{\vec{\nabla} \cdot \vec{\tau}^f}_{\text{Fluid st.}} + (1-\phi)\vec{f} + \rho^f(1-\phi)\vec{g} \underbrace{- M_D}_{\text{drag}}$$

$$\text{Solid: } \frac{\partial\phi}{\partial t} + \vec{\nabla} \cdot (\phi\vec{u}^s) = 0$$

$$\rho^s\phi\frac{D\vec{u}^s}{Dt} = -\phi\vec{\nabla}p - \underbrace{\vec{\nabla}\tilde{p}^s + \vec{\nabla} \cdot \vec{\tau}^s}_{\text{granular st.}} + \phi\vec{f} + \rho^s\phi\vec{g} + M_D$$

Momentum coupling through drag force:

$$M_D = \phi(1-\phi)K(\vec{u}^f - \vec{u}^s) - \frac{1}{S_c}(1-\phi)K\nu_t^f\vec{\nabla}\phi$$

Drag law:

$$K = 0.75 \frac{24}{Re_p} \left(1 + 0.15 Re_p^{0.687}\right) \frac{\rho^f}{d} \|\vec{u}^f - \vec{u}^s\| (1-\phi)^{-2.65}$$

$$Re_p = (1-\phi) \|\vec{u}^f - \vec{u}^s\| d / \nu^f: \text{ particulate Reynolds number}$$

Schiller & Naumann [1933] + Richardson & Zaki [1954]

- $(1-\phi), \phi$: fluid and particles volume fractions
- \vec{u}^f : Average fluid velocity
- \vec{u}^s : Average particle velocity
- p : Fluid pressure
- \tilde{p}^s : Particulate pressure
- S_c : Schmidt Number

3D Two-phase flow model equations

$$\text{Fluid: } \frac{\partial(1-\phi)}{\partial t} + \vec{\nabla} \cdot ((1-\phi)\vec{u}^f) = 0$$

$$\underbrace{\rho^f(1-\phi)\frac{D\vec{u}^f}{Dt}}_{\text{unsteady+inertia}} = \underbrace{-(1-\phi)\vec{\nabla} p}_{\text{fluid pressure}} + \underbrace{\vec{\nabla} \cdot \overline{\overline{\tau^f}}}_{\text{Fluid st.}} + (1-\phi)\vec{f} + \rho^f(1-\phi)\vec{g} - \underbrace{M_D}_{\text{drag}}$$

$$\text{Solid: } \frac{\partial\phi}{\partial t} + \vec{\nabla} \cdot (\phi\vec{u}^s) = 0$$

$$\rho^s\phi\frac{D\vec{u}^s}{Dt} = -\phi\vec{\nabla} p - \underbrace{\vec{\nabla} \tilde{p}^s + \vec{\nabla} \cdot \overline{\overline{\tau^s}}}_{\text{granular st.}} + \phi\vec{f} + \rho^s\phi\vec{g} + M_D$$

Momentum coupling through drag force:

$$M_D = \phi(1-\phi)K(\vec{u}^f - \vec{u}^s) - \frac{1}{S_c}(1-\phi)K\nu_t^f\vec{\nabla}\phi$$

Drag law:

$$K = 0.75 \frac{24}{Re_p} \left(1 + 0.15 Re_p^{0.687}\right) \frac{\rho^f}{d} \|\vec{u}^f - \vec{u}^s\| (1-\phi)^{-2.65}$$

$$Re_p = (1-\phi) \|\vec{u}^f - \vec{u}^s\| d / \nu^f: \text{ particulate Reynolds number}$$

Schiller & Naumann [1933] + Richardson & Zaki [1954]

- $(1-\phi), \phi$: fluid and particles volume fractions
- \vec{u}^f : Average fluid velocity
- \vec{u}^s : Average particle velocity
- p : Fluid pressure
- \tilde{p}^s : Particulate pressure
- S_c : Schmidt Number

Dense granular flow rheology: $\mu(I)$

Particulate pressure: $\tilde{p}^s = p^{ff} + p^s$

$$\phi_{min}^{fric} = 0.57; \phi_{max} = 0.635 \text{ (spheres).}$$

Permanent Contact: $p^{ff} = Fr \frac{(\phi - \phi_{min}^{fric})^\eta}{(\phi_{max} - \phi)^p}$

Johnson & Jackson [1987]

Shear induced: $p^s = \left(\frac{B_\phi \phi}{\phi_{max} - \phi} \right)^2 \rho^s d^2 \|\overline{\overline{S^s}}\|^2$ $\overline{\overline{S^s}}$: solid phase velocity shear rate tensor

Dense granular flow rheology: $\mu(I)$

Particulate pressure: $\tilde{p}^s = p^{ff} + p^s$

$$\phi_{min}^{fric} = 0.57; \phi_{max} = 0.635 \text{ (spheres).}$$

Permanent Contact: $p^{ff} = Fr \frac{(\phi - \phi_{min}^{fric})^\eta}{(\phi_{max} - \phi)^p}$

Johnson & Jackson [1987]

Shear induced: $p^s = \left(\frac{B_\phi \phi}{\phi_{max} - \phi} \right)^2 \rho^s d^2 \|\overline{\overline{S^s}}\|^2$ $\overline{\overline{S^s}}$: solid phase velocity shear rate tensor

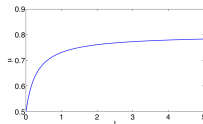
Particulate shear stress: $\overline{\overline{\tau^s}} = \nu_{Fr}^s \overline{\overline{S^s}}$

Frictional shear viscosity:

$$\nu_{Fr}^s = \min \left(\frac{\mu(I) \tilde{p}^s}{\rho^s \left(\|\overline{\overline{S^s}}\|^2 + D_{small}^2 \right)^{1/2}}, \nu_{max} \right)$$

Relates particle shear str. & total part. pressure

$$\mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{I_0/I + 1}, I: \text{Inertial number}$$



GDR Midi [2004], Jop et al. [2006], Forterre & Pouliquen [2008]

Two phase k- ω 2006 fluid turbulence model

Turbulent eddy viscosity: $\nu_t^f = \frac{k}{\max\left(\omega, \frac{1}{8} \sqrt{\frac{2S_{ij}^f S_{ij}^f}{C_\mu}}\right)}$, Wilcox [2006].

$$P = R_{ij}^f \frac{\partial u^f}{\partial x_j}, \nu_G = \nu^f + \sigma \nu_t^f$$

Turbulent Kinetic Energy

$$\frac{Dk}{Dt} = P - C_\mu k \omega + \frac{\partial}{\partial x_j} \left[\nu_G \frac{\partial k}{\partial x_j} \right] - \underbrace{\frac{2K\phi(1 - e^{-BS_t})k}{\rho^f (1 - \phi)}}_{\text{Turbulent drag modulation}} - \underbrace{\frac{\nu_t^f (s - 1)}{S_c (1 - \phi)} \frac{\partial \phi}{\partial x_j} g_j}_{\text{turb. suspension}}$$

Specific Dissipation Rate

$$\frac{D\omega}{Dt} = A_1 \frac{\omega}{k} P - B_1 \omega^2 + \frac{\partial}{\partial x_j} \left[\nu_G \frac{\partial \omega}{\partial x_j} \right] + C_{k\omega} - C_{3\omega} \frac{2K\phi(1 - e^{-BS_t})\omega}{\rho^f (1 - \phi)} - C_{4\omega} \frac{\omega(s - 1)\nu_t^f}{S_c (1 - \phi)k} \frac{\partial \phi}{\partial x_j} g_j$$

$C_{k\omega}$: Cross-diffusion term (Wilcox [2006]).

With S_t the Stokes Number: $S_t = \frac{t_p}{t_l} = \frac{\rho^a / K}{k / 6\varepsilon} = \frac{\text{particle response time}}{\text{time scale of en. eddies}}$

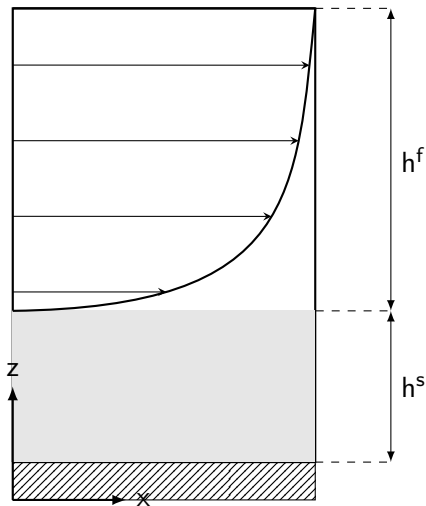
Adapted from two-phase flow $k - \varepsilon$ model (Hsu et al. [2004]).

sedFoam: 3D two-phase num model for sediment transport

- New sedFoam version (openFOAM 5.0 and v1806):
<https://github.com/SedFoam/sedfoam/releases/tag/3.0>. **OpenFOAM®**
- Turbulence-averaged Eulerian two-phase model for sediment transport based on openFOAM (Cheng et al. [2017], Chauchat et al. [2017])
- *fluidfoam* a python package to perform plots with OpenFoam data (<http://bitbucket.org/fluiddyn/fluidfoam/>).
- Latest release improvements:
 - reduced pressure algorithm + stability improvement
 - **two phase k- ω 2006 turbulence model(sedFoamv3.1)**
 - **two phase k- ω / ϵ turbulence model(sedFoamv3.1)**

The screenshot shows the article page for "SedFoam-2.0: a 3-D two-phase flow numerical model for sediment transport" in the journal Geoscientific Model Development. The page includes a navigation menu on the left with options like "About", "Editorial board", "Articles", "Special issues", "Change article", "Subscribe to alerts", "Peer review", "For authors", and "For editors and referees". The main content area displays the article title, authors (Julien Chauchat, Zhen Cheng, The Regal, Cyril Bonamy, and Tian-Jiao He), and the abstract. The abstract describes the paper's focus on sediment transport applications and the implementation of a three-dimensional two-phase flow model. On the right side, there are sections for "Search articles", "Download", "Short summary", "Citation", and "Share". The page also features a "User ID" field, a "Password" field, and a "Follow" button for the journal on Twitter.

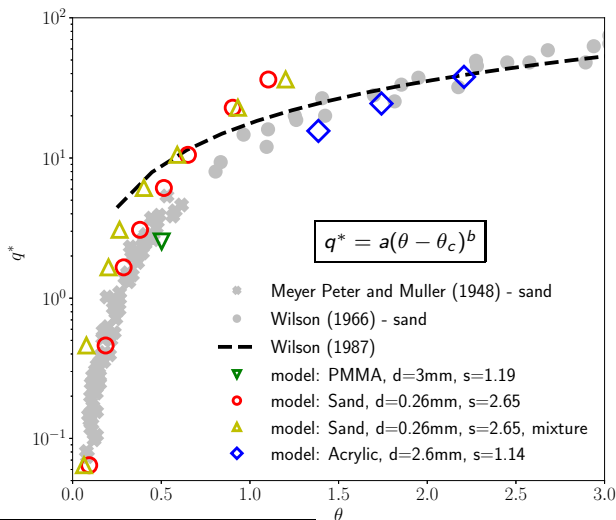
Unidirectional flows (1D2C Computations)



3 experimental reference cases:

- Roulund et al. [2005]:
BedLoad; $d=0.26$ mm ;
Sand; $\rho^s = 2650$ kg/m³.
- Revil-Baudard et al. [2015]:
Sheet-Flow; $d=3$ mm ;
PMMA; $\rho^s = 1190$ kg/m³.
- Sumer et al. [1996]:
Sheet-Flow; $d=2.6$ mm ;
Acrylic; $\rho^s = 1140$ kg/m³.

Unidirectional flows validation



$$q_n^* = \frac{\int \|\vec{u}^s \cdot \vec{t}\| \phi dz}{\sqrt{(s-1)gd^3}}$$

$$\theta = \frac{\max(R_{xz}^f)}{(\rho^s - \rho^f)gd}$$

$$\theta = \frac{\tau_{xz}^f(\phi) + \tau_{xz}^s(\phi)}{(\rho^s - \rho^f)gd}$$

$$q^* = f(\theta) \text{ recovered for a large range of } \theta$$

Bed shear stress	ϕ	a	b
Meyer-Peter and Muller [1948]	-	8	1.50
fluid	-	32.13	2.18
mixture	0.45	31.06	1.57
mixture	0.08	26.14	2.09

Live-Bed case: geometry

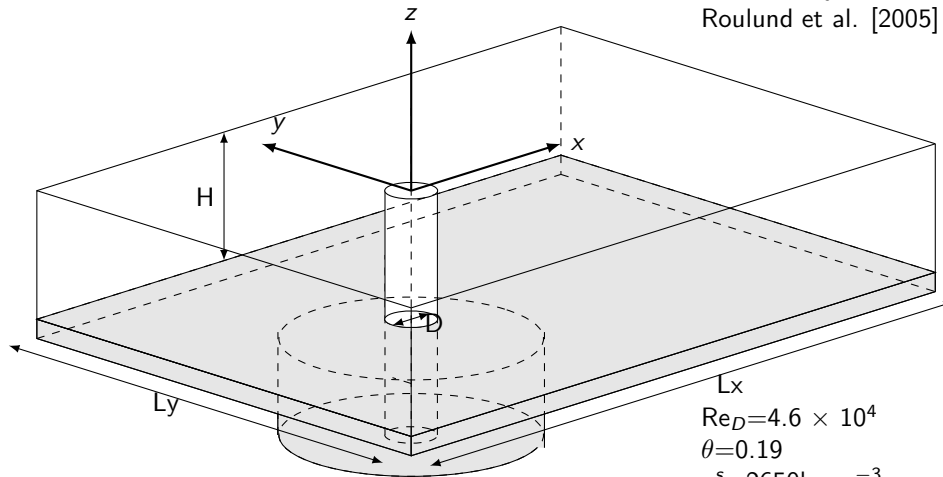
$$D=0.1\text{m}$$

$$H=2D$$

$$L_x=13D, L_y=8D$$

Roulund et al. [2005]

5 308 368 cells (Resolution: $1.2 \cdot 10^{-3}\text{m}$
around cyl perimeter)



$$L_x$$

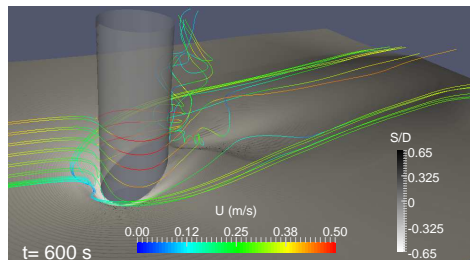
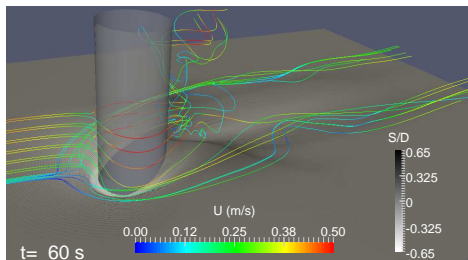
$$Re_D=4.6 \times 10^4$$

$$\theta=0.19$$

$$\rho^s=2650\text{kg}\cdot\text{m}^{-3}$$

$$d=0.26\text{mm}$$

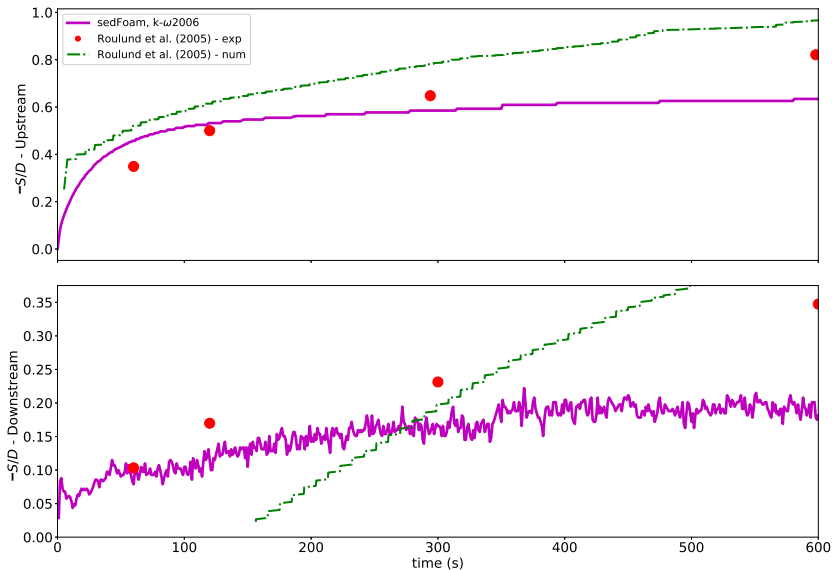
Simulation run up to 600s \Rightarrow 60% of the scour equilibrium



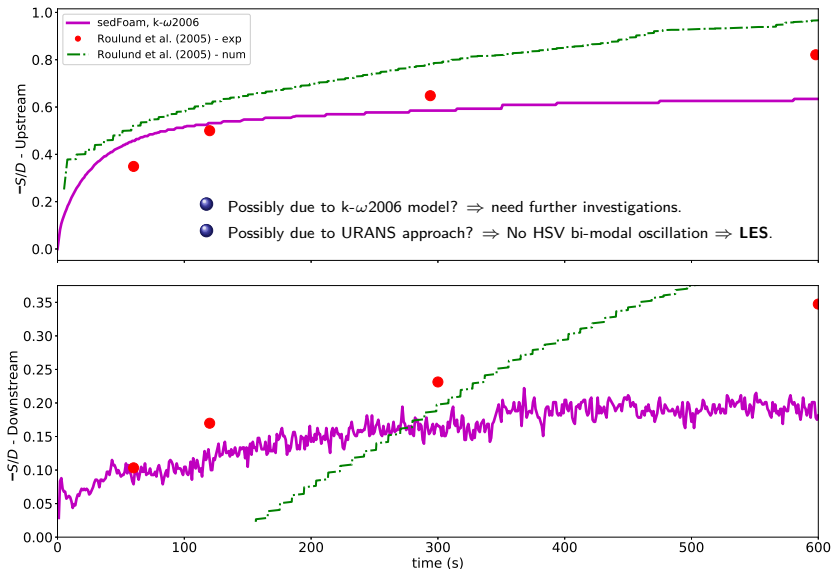
S=Scour depth

- Computational challenge.
- High computational cost \Rightarrow equivalent to 12 years on a single processor.

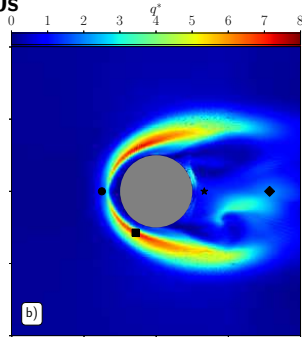
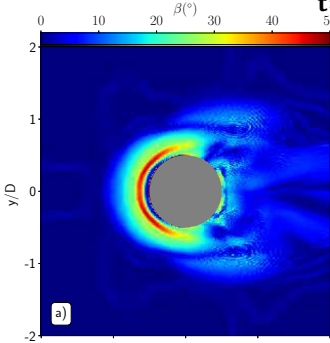
Live-Bed case: Erosion depth evolution in time



Live-Bed case: Erosion depth evolution in time



t=10s

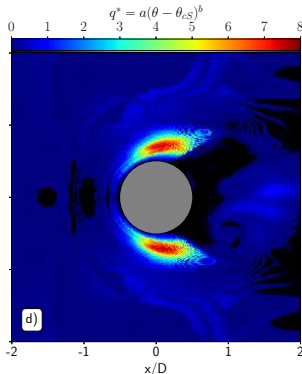
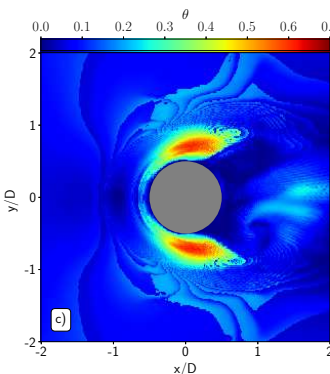


The sediment flux in **b)** is computed using the local sediment velocity and concentration.

In the upstream vicinity of the cylinder the sediment flux is more correlated with the high bed-slope ($0-45^\circ$)

Strong correlation between the maximum bed shear stress and the maximum flux ($45-120^\circ$)

The two legs of flux downstream are correlated with the HSV legs (see Link et al. [2012]).

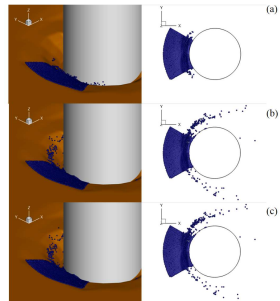
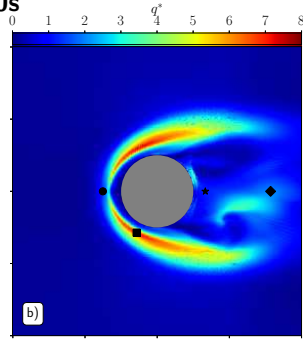
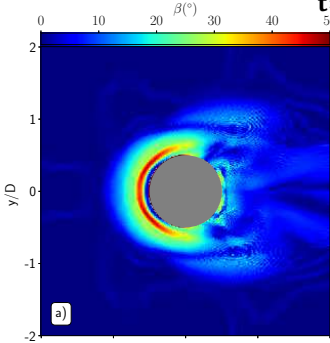


Sediment flux "classical" formula (d):

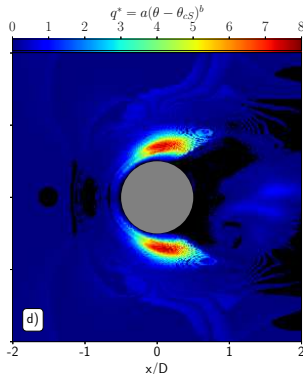
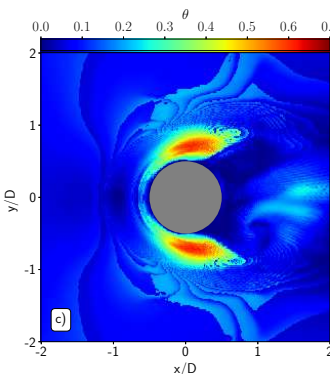
$$q^* = a(\theta - \theta_c(\beta))^b,$$

where $a=26.14$ and $b=2.09$, from 1D simulations.

$t=10s$



Link et al. [2012]

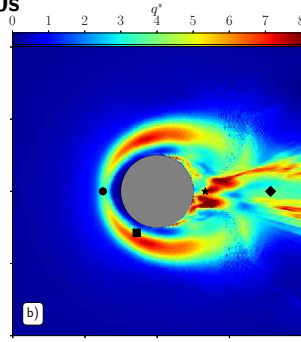
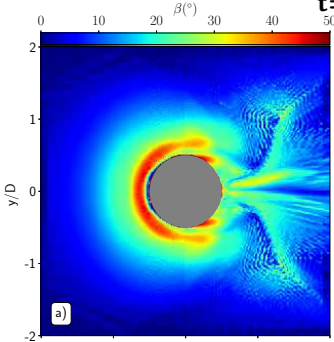


Sediment flux "classical" formula (d):

$$q^* = a(\theta - \theta_c(\beta))^b,$$

where $a=26.14$ and $b=2.09$, from 1D simulations.

t=60s

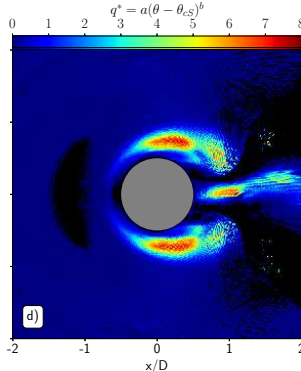
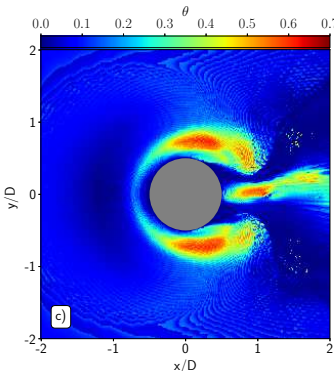


The sediment flux in **b)** is computed using the local sediment velocity and concentration.

In the upstream vicinity of the cylinder the sediment flux is more correlated with the high bed-slope (0-45°)

Strong correlation between the maximum bed shear stress and the maximum flux (45-120°)

The two legs of flux downstream are correlated with the HSV legs (see Link et al. [2012]).

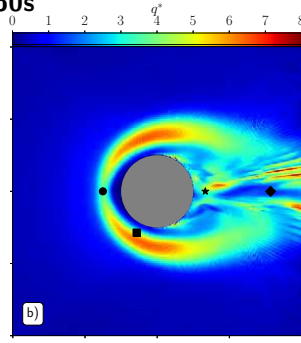
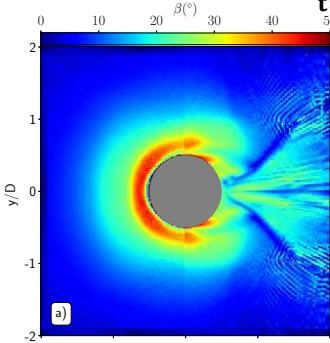


Sediment flux "classical" formula (d):

$$q^* = a(\theta - \theta_c(\beta))^b,$$

where a= 26.14 and b=2.09, from 1D simulations.

t=150s

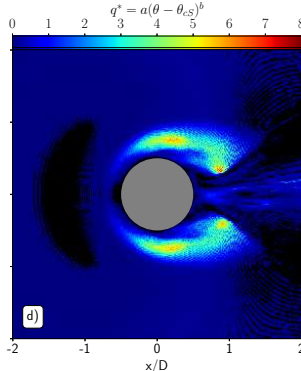
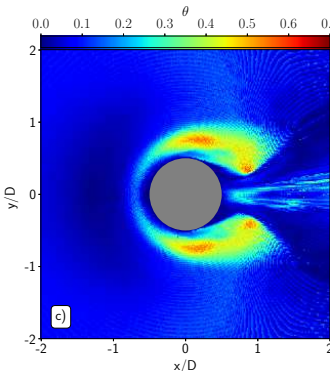


The sediment flux in b) is computed using the local sediment velocity and concentration.

In the upstream vicinity of the cylinder the sediment flux is more correlated with the high bed-slope (0-45°)

Strong correlation between the maximum bed shear stress and the maximum flux (45-120°)

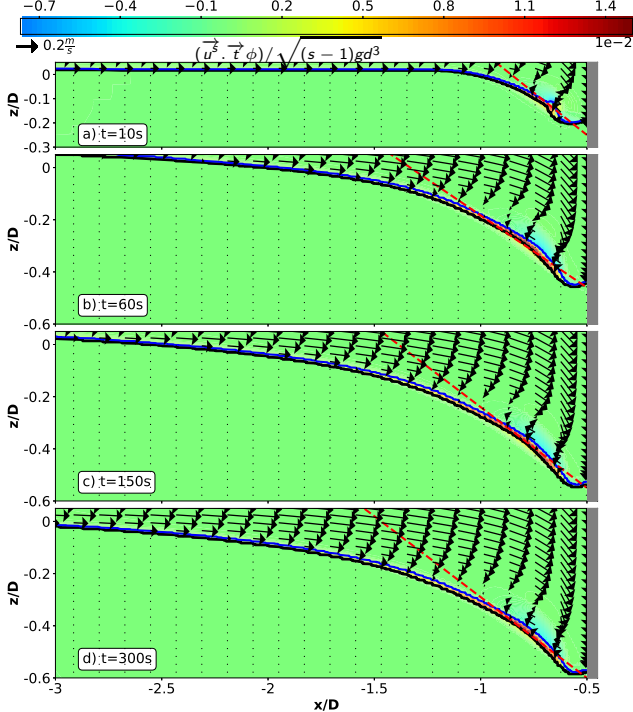
The two legs of flux downstream are correlated with the HSV legs (see Link et al. [2012]).



Sediment flux "classical" formula (d):

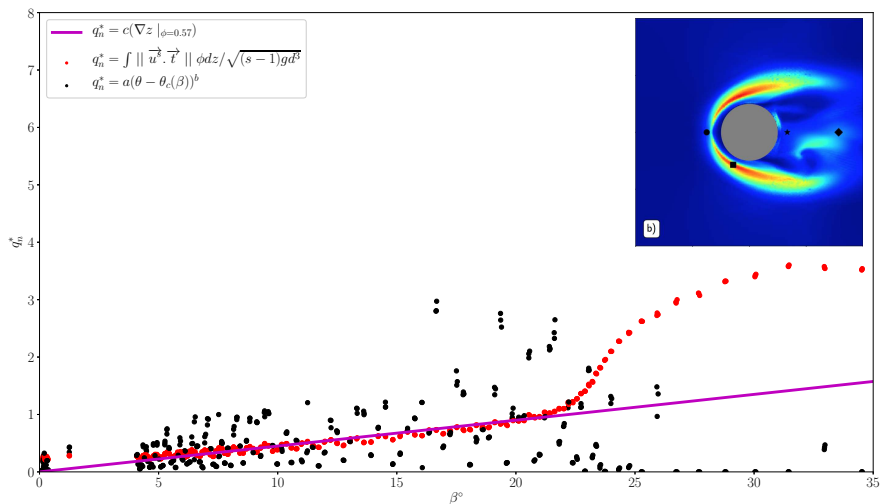
$$q^* = a(\theta - \theta_c(\beta))^b,$$

where a= 26.14 and b=2.09, from 1D simulations.



- Horseshoe vortex position seems stable.
- **A competition between the local bed shear stress (due to fluid flow over the sediments bed) and the gravity is taking place in the scour hole.**
- Bed shear stress is counter acting the gravity in the HSV.
- Avalanche downstream $X/D \approx -0.75$.
- **Importance of avalanches** in the scour process: video courtesy of O. Link, University of Concepcion, Chile.

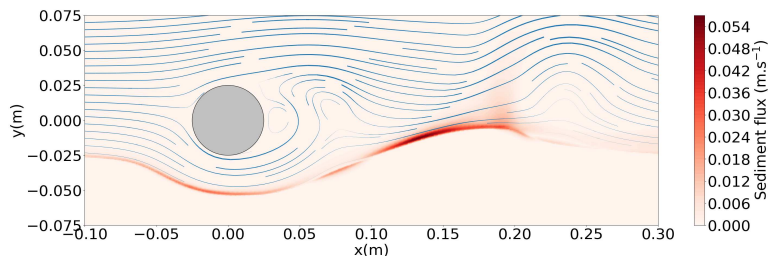
Inspired from large scale morphodynamics models (Dubardier et al. [2017])



- **Proof of concept given:** Eulerian-Eulerian two-phase flow approach is able to simulate 3D scour process (**Pioneering work**).
- $\mu(I)$ rheology is **appropriate**. More work needs to be done on the turbulence models.
- Sediment transport around hydraulic structures **does not depend on the local bed shear stress only**.
- **Gravity is dominant** for sediment transport in the scour mark (**Avalanching**).
- **Parametrization** of the gravity contribution (\approx large scale beaches models) \Rightarrow Ongoing work.

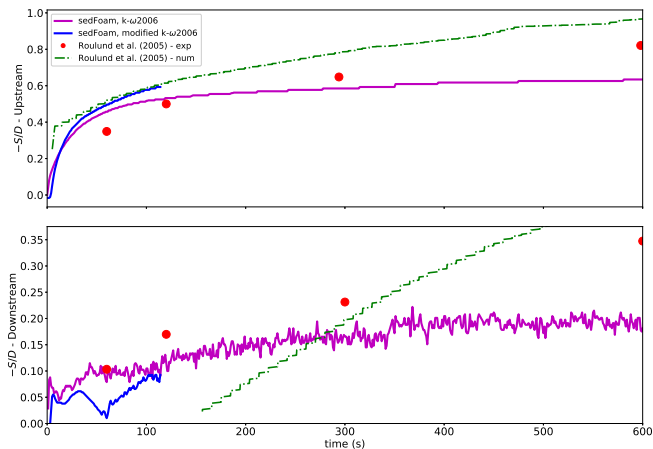
Two-phase flow simulation of scour:

- Study two-dimensional configuration of scour under a pipeline - tunneling and lee-wake erosion stages.
- Simplified configuration for turbulence model improvement \Rightarrow modified $k-\omega$ 2006.



Mathieu, Chauchat, Bonamy & Nagel [In prep. for Adv. Water Ressources]

- Use the **modified k- ω 2006** from Mathieu et al. [In prep.] to improve the long term agreement with Roulund et al. [2005] exp. results.



- Perform **LES two-phase flow simulations** of the 3D scour around the pile.
- Improve the proposed sediment transport parametrization ($\beta > 25^\circ \dots$).

Modified k- ω 2006 turbulence model

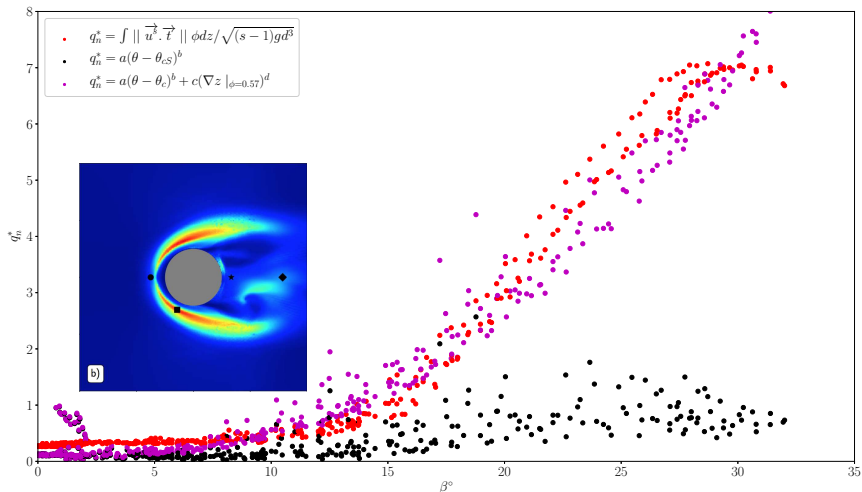
Turbulent eddy viscosity: $\nu_t^f = \frac{k}{\max\left(\omega, C_{lim} \frac{\|s^f\|}{C_\mu} \mathcal{H}(\phi_{cut} - \phi)\right)}$, Mathieu et al [In prep].

Heavyside function \Rightarrow suppress stress limiting term where $\phi > \phi_{cut}$

Coefficiented differently:

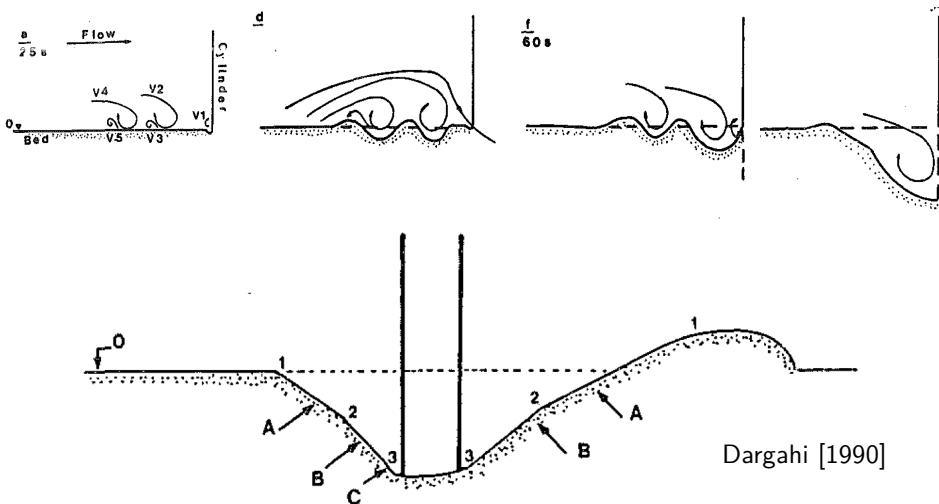
- $\phi > \phi_{cut}$ (dense sediment concentration regions) \Rightarrow k- ϵ behavior
- $\phi < \phi_{cut}$ (dilute sediment concentration regions) \Rightarrow k- ω 2006 behavior

Sediment transport rate parametrization



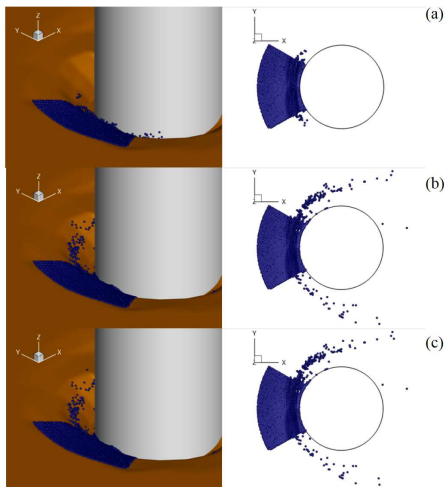
Scour around a cylindrical pile

The Scour Hole formation

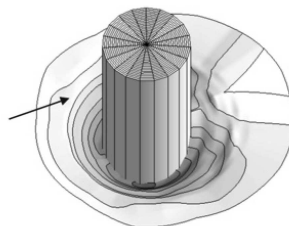


Scour around a cylindrical pile

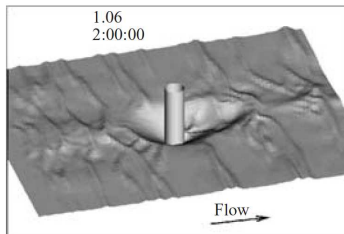
Sediment erosion and equilibrium



Link et al. [2012]

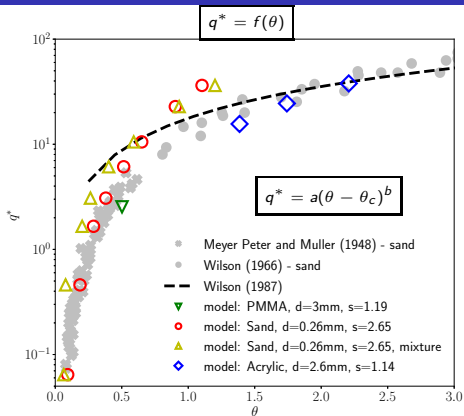


Link et al. [2012]. Clear water Regime.



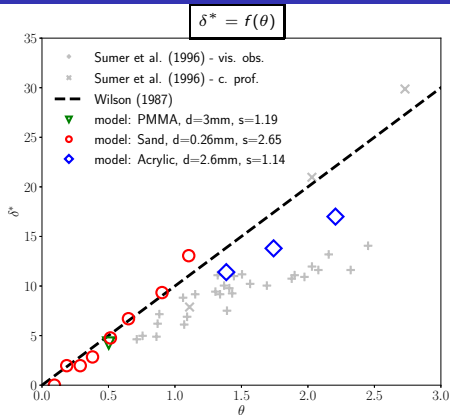
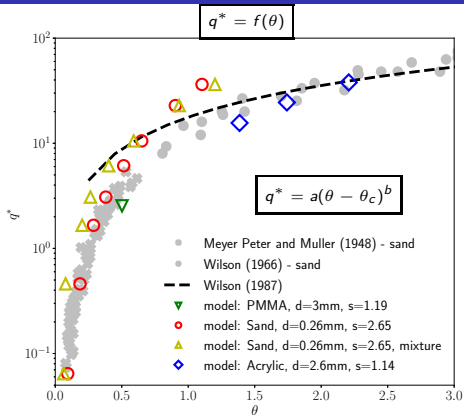
Roulund et al. [2005]. Live-Bed Regime.

Unidirectional flows



Bed shear stress	ϕ	a	b
Meyer-Peter and Muller [1948]	-	8	1.50
Wong and Parker [2006]	-	3.97	1.50
fluid	-	32.13	2.18
mixture	0.45	31.06	1.57
mixture	0.3	28.56	1.59
mixture	0.08	26.14	2.09

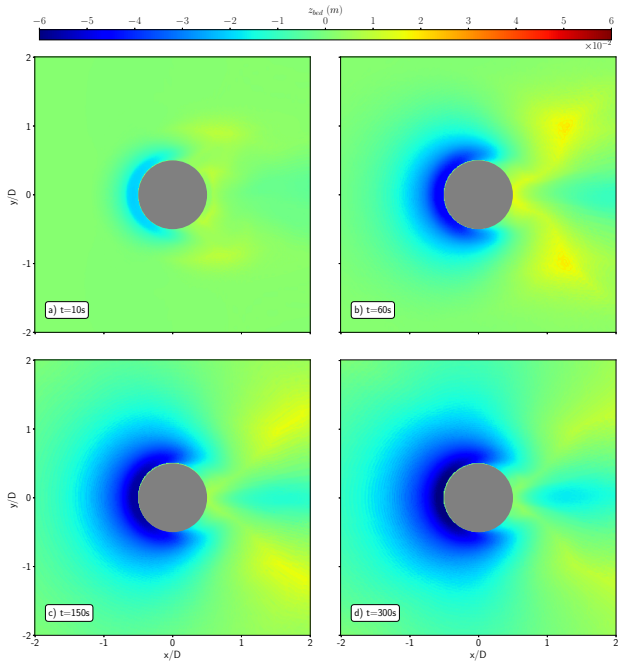
Unidirectional flows



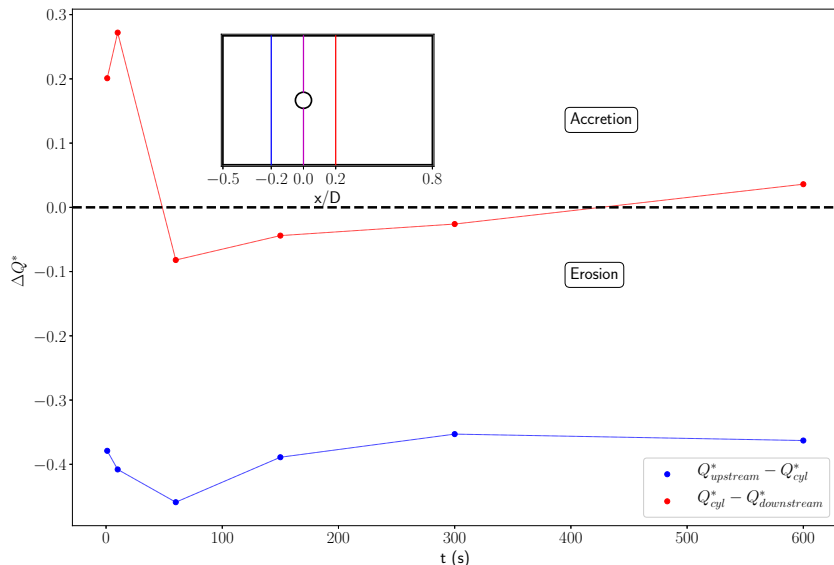
Bed shear stress	ϕ	a	b
Meyer-Peter and Muller [1948]	-	8	1.50
Wong and Parker [2006]	-	3.97	1.50
fluid	-	32.13	2.18
mixture	0.45	31.06	1.57
mixture	0.3	28.56	1.59
mixture	0.08	26.14	2.09

$$\delta^* = \frac{z(\phi|0.08) - z(\phi|0.57)}{d}$$

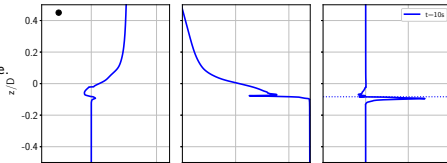
Dimensionless bedload transport layer



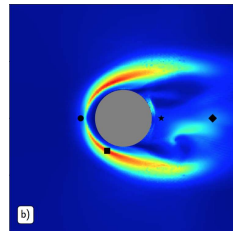
Live-Bed case: Erosion depth evolution in time



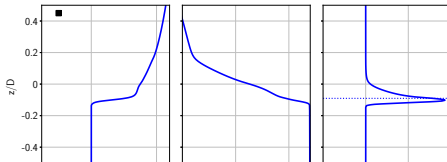
- Strong bedload sediments flux oriented toward the pile.
- Suspended load going upslope.



0.25D upstream



- Bedload (majority) and suspended load.
- Downstream sediments transport.



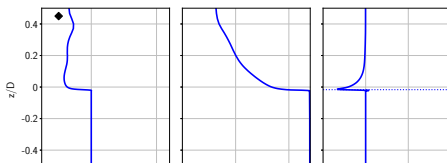
HSV Legs

- Recirculation area.
- Upstream suspended load.



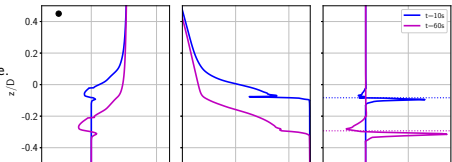
0.2D downstream

- Suspended load mostly.
- Upstream (10s),

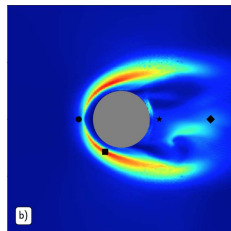


1.7D downstream

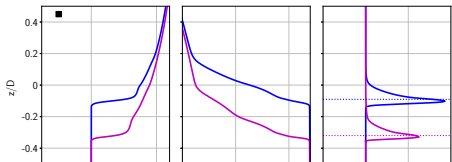
- Strong bedload sediments flux oriented toward the pile.
- Suspended load going upslope.



0.25D upstream

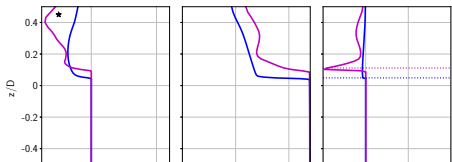


- Bedload (majority) and suspended load.
- Downstream sediments transport.



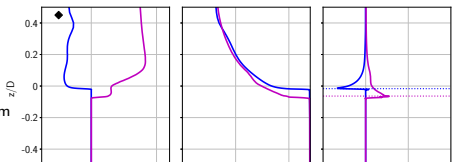
HSV Legs

- Recirculation area.
- Upstream suspended load.



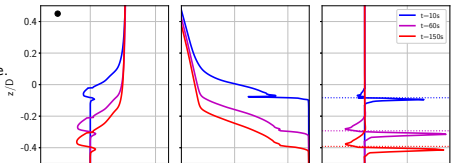
0.2D downstream

- Suspended load mostly.
- Upstream (10s), downstream (60s and 150s).

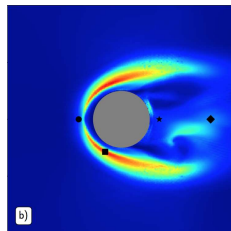


1.7D downstream

- Strong bedload sediments flux oriented toward the pile.
- Suspended load going upslope.

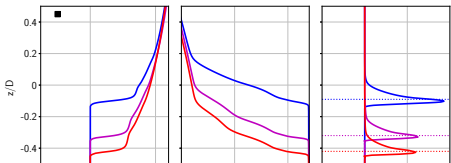


0.25D upstream

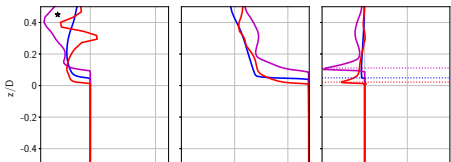


HSV Legs

- Bedload (majority) and suspended load.
- Downstream sediments transport.

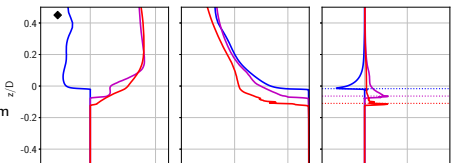


- Recirculation area.
- Upstream suspended load.



0.2D downstream

- Suspended load mostly.
- Upstream (10s), downstream (60s and 150s).



1.7D downstream