A NUMERICAL STUDY OF THE IMPORTANCE OF NONLINEAR EFFECTS FOR FABRY-PEROT RESONANCE OF WATER WAVES

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INTRODUCTION

When a regular wave train propagates over a patch of periodic bottom corrugations on an otherwise flat bottom (with still water depth h), the so called Bragg resonance phenomenon can appear, leading to a significant reflection of the incident waves due to the presence of the ripple patch. This effect is maximum when the wavelength of the surface waves (noted $\lambda = 2\pi/k$) is twice that of the bottom ripples (noted $\lambda_b = 2\pi/k_b$). This phenomenon has been studied both experimentally (e.g. Davies & Heathershaw, 1984) and theoretically within the linear wave theory framework (e.g. Mei, 1985; Dalrymple & Kirby, 1986).

When two patches of sinusoidal bottom ripples (with respective lengths $L_1 = N_1\lambda_b$ and $L_2 = N_2\lambda_b$, and amplitude d) are considered, separated by a distance L called the resonator length (with constant water depth h), the combined effect of Bragg resonance due to the two patches give rise to the so-called Fabry-Perot (F-P) resonance, also observed in other fields of physics. F-P resonance for water waves was recently studied by Couston *et al.* (2015) within the linear theory framework, by applying the asymptotic approach by Mei (1985) and considering non-zero seaward reflection following Yu & Mei (2000). These authors derived a set of results regarding the reflection and transmission of waves over the two patches and the amplification factor of the waves within the resonator. According to this latter expression, large amplification factor of the wave height (i.e. higher than 2) could be obtained in the resonator area, even with a small relative ripple amplitude d/h = O(0.1).

SCOPE OF PRESENT WORK

As is mentioned above, available results of F-P resonance are based on a linear potential theory approach, based on two simplifying assumptions:

(a) the surface waves are of infinitesimal amplitude so that the free surface bottom boundary conditions can be linearized and applied at the undisturbed still water level $z = 0$ instead of $z = \eta(x, t)$;

(b) the bottom boundary condition (BBC) is also simplified considering small bottom gradients (see Mei, 1985). Keeping only first-order terms of the bottom slope, the exact BBC:

$$\frac{\partial\varphi}{\partial z} + \frac{\partial h}{\partial x}\frac{\partial\varphi}{\partial x} = 0 \quad \text{on} \quad z = -h + \delta(x)$$

is approximated by:

$$\frac{\partial\varphi}{\partial z} + \delta\frac{\partial^2\varphi}{\partial z^2} - \frac{\partial\delta}{\partial x}\frac{\partial\varphi}{\partial x} = 0 \quad \text{on} \quad z = -h$$

Here $\varphi(x, z, t)$ denotes the wave potential, and $\delta(x)$ is the elevation of the corrugations relative to the (constant) mean bottom elevation $z = -h$.

In this study, we aim at studying the consequences of both these assumptions though numerical simulations in a nonlinear potential framework. The effect of finite ripple height (i.e. of assumption (b)) and of finite wave height (i.e. of assumption (a)) on the F-P mechanism will thus be assessed and quantified.

NUMERICAL MODELLING APPROACH

The numerical simulations are carried out using the fully nonlinear and dispersive potential wave model whispers-3D, based on Yates & Benoit (2015). In this model, the nonlinear kinematic and dynamic free surface boundary conditions are rewritten as evolution equations of the free surface elevation (η) and the free surface velocity potential ($\psi = \varphi(z = \eta)$). These equations are integrated in time using a third-order Strong Stability Preserving Runge-Kutta scheme (SSP-RK3). At each time step the vertical velocity at the free surface is estimated from η and ψ (a so-called Dirichlet-to-Neumann problem) by solving the Laplace equation for the velocity potential in the fluid domain. The vertical variation of the velocity potential φ is approximated using a spectral approach with an orthogonal basis of Chebyshev polynomials, up to a given order N_T, usually chosen between 5 and 10. This modeling approach has been validated with a series of challenging cases including the propagation of nonlinear regular or irregular waves over various beach profiles (Raoult *et al.*, 2016).

RESULTS AND DISCUSSION

In the first part of the study, we simulate numerically the complete linear problem in order to compare the numerical results with the analytical (asymptotic) predictions of Couston *et al.* (2015). We start by considering low values of the relative ripple amplitude, namely d/h in order to match with the frame of assumption (b) above.

An example of results using the linear version of whispers-3D for the case $d/h = 0.1$ is presented in fig. 1, considering the same parameters and settings as the case studied by Couston *et al.* (2015) in their fig. 6, except for the simulation duration (t/T = 200). Here we plotted the local amplitude of the waves, showing the effects of reflections due to the patches of ripples, and a typical standing wave pattern in the resonator area.

A good agreement is observed with a maximum amplification factor in the resonator area of about 2.25 from the numerical simulation, against about 2.47 from the analytical prediction in this case.

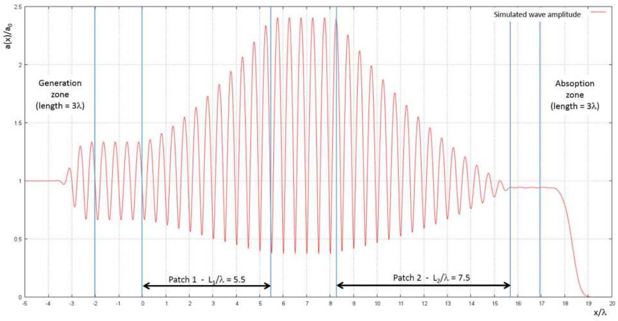

Figure 1 - Horizontal profile of the relative wave amplitude computed with the linear version of whispers-3D for the case $k_b h = 1.64$, $k_b d = 0.164$, $L/\lambda = 2.75$, $N_1 = 11$, $N_2 = 15$.

However when *d/h* is increased (say to 0.2 for instance, or higher values), differences become visible between the asymptotic linear solution (relying on assumptions (a) and (b) above) and the numerical model (which only assumes (a)). In this case, the FP resonance does not develop for incident waves with wave-number computed from the asymptotic theory. Instead, F-P resonance is observed to manifest at a slightly lower wave-number compared to the theoretical prediction. An explanation of this downshift of the resonating wave number is proposed and discussed. Furthermore, we observe that the range of incident wave-numbers prone to F-P resonance is very narrow in the set-up considered here.

In the second part of the study, nonlinear simulations with finite amplitude waves (thus getting rid of assumption (a) above) are also performed, to demonstrate how finite amplitude effects may also affect the F-P resonance mechanism. The whole set of numerical simulations will be presented and discussed during the Conference and in the final paper.

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