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3-D Numerical Modeling of Sediment Transport Near Coastal Inlets

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3-D Shallow Water Flow Equations

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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$$\frac{\partial u}{\partial t}+\frac{\partial (uu)}{\partial x}+\frac{\partial (vu)}{\partial y}+\frac{\partial (wu)}{\partial z}=-\frac{1}{\rho}\frac{\partial p_a}{\partial x}-\frac{1}{\rho}\left(\rho_0 g\frac{\partial \eta}{\partial x}+g\int_z^\eta \frac{\partial \rho}{\partial x}dz\right)$$

$$+\frac{\partial}{\partial x}\left(\nu_{tH}\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu_{tH}\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(\nu_{tV}\frac{\partial u}{\partial z}\right)-\frac{1}{\rho}\frac{\partial S_{xx}}{\partial x}-\frac{1}{\rho}\frac{\partial S_{xy}}{\partial y}+f_c v$$

$$\frac{\partial v}{\partial t}+\frac{\partial (uv)}{\partial x}+\frac{\partial (vv)}{\partial y}+\frac{\partial (wv)}{\partial z}=-\frac{1}{\rho}\frac{\partial p_a}{\partial y}-\frac{1}{\rho}\left(\rho_0 g\frac{\partial \eta}{\partial y}+g\int_z^\eta \frac{\partial \rho}{\partial y}dz\right)$$

$$+\frac{\partial}{\partial x}\left(\nu_{tH}\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu_{tH}\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(\nu_{tV}\frac{\partial v}{\partial z}\right)-\frac{1}{\rho}\frac{\partial S_{yx}}{\partial x}-\frac{1}{\rho}\frac{\partial S_{yy}}{\partial y}-f_c u$$

Eddy Viscosity

Vertical

$$\nu_{tV} = \sqrt{\left(l_{mV}^2 \left|\bar{S}_V\right|\right)^2 + \left(l_{mH}^2 \left|\bar{S}_H\right|\right)^2}$$

where $|S_V|$ and $|S_H|$ are shear strains in vertical and horizontal directions:

$$\left|\bar{S}\right|_V = [(\partial u/\partial z)^2 + (\partial v/\partial z)^2]^{1/2}$$

$$\left|\bar{S}\right|_H = [2(\partial u/\partial x)^2 + 2(\partial v/\partial y)^2 + (\partial u/\partial y + \partial v/\partial x)^2]^{1/2}$$

l_{mV} and l_{mH} are vertical and horizontal mixing lengths:

$$l_{mV} = \kappa z\sqrt{1 - z/h} \qquad l_{mH} = \kappa \min(l, c_m h)$$

$$l_{mV} = \sqrt{X l_{mc}^2 + (1 - X) l_{mw}^2} \qquad X = U_c^2 \Big/ \left(U_c^2 + 0.5 U_{wm}^2\right)$$

where z is the vertical coordinate above the bed, l is the horizontal distance to the nearest solid wall, and h is the flow depth.

Horizontal

$$\nu_{tH} = \nu_{tV} + c_{wf} U_{wm} H + c_{br} h \left(\frac{D_{br}}{\rho}\right)^{1/3}$$

Boundary Conditions

Free surface kinematic condition

$$\frac{\partial \eta}{\partial t} + u_h \frac{\partial \eta}{\partial x} + v_h \frac{\partial \eta}{\partial y} = w_h$$

Surface shear stress due to wind

$$\tau_{si} = \rho_a C_D W W_i$$

where ρ_a is air density, C_D is the wind drag coefficient, and W is the wind velocity. The drag coefficient is calculated using the formula of Hsu (1988) and modified for high wind speeds based on field data by Powell et al. (2003).

Bed shear stress

$$\tau_{bx} = \rho c_f u_b \sqrt{u_b^2 + v_b^2 + 0.5 U_{wm}^2}, \quad \tau_{by} = \rho c_f v_b \sqrt{u_b^2 + v_b^2 + 0.5 U_{wm}^2}$$

where u_b and v_b are the x- and y-velocities near the bed; c_f is the bed friction coefficient; and U_{wm} is the maximum orbital bottom velocity of wave.

$$c_f = \left[\frac{\kappa}{\ln\left(30 z_P / k_s\right)} \right]^2$$

CMS-Wave

- Spectral wave-action balance equation (Mase, 2001)

$$\frac{\partial (c_x N)}{\partial x} + \frac{\partial (c_y N)}{\partial y} + \frac{\partial (c_\theta N)}{\partial \theta} = \frac{K}{2\sigma}\left[\frac{\partial}{\partial y}\left(cc_g \cos^2\theta \frac{\partial N}{\partial y}\right) - \frac{cc_g}{2}\cos^2\theta \frac{\partial^2 N}{\partial y^2}\right] - \varepsilon_b N - S$$

$$N = \frac{E(f,\theta)}{\sigma}$$

- Characteristic velocities

$$c_x = c_g \cos\theta + U_x \qquad c_y = c_g \sin\theta + U_y$$

$$c_\theta = \frac{\sigma}{\sinh 2kh}\left(\sin\theta \frac{\partial h}{\partial x} - \cos\theta \frac{\partial h}{\partial y}\right) + \cos\theta\left(\sin\theta \frac{\partial U_x}{\partial x} - \cos\theta \frac{\partial U_x}{\partial y}\right) + \sin\theta\left(\sin\theta \frac{\partial U_y}{\partial x} - \cos\theta \frac{\partial U_y}{\partial y}\right)$$

- Dispersion relation

$$\sigma^2 = gk\tanh(kh) \qquad \sigma = \omega - \vec{k}\cdot\vec{U}$$

Wave Radiation Stress

Formula of Mellor (2008)

$$S_{ij} = \int_0^\infty \int_{-\pi}^{\pi} \left\{ k(f)E(f,\theta) \left[\frac{k_i(f)k_j(f)}{k(f)^2} \frac{\cosh^2 k(h+z')}{\sinh kD \cosh kD} - \delta_{ij} \frac{\sinh^2 k(h+z')}{\sinh kD \cosh kD} \right] + \delta_{ij} E_D(f,\theta) \right\} d\theta df$$

where E is the wave energy, k is the wave number, θ is the angle of wave propagation to the onshore direction, f is the wave frequency, h is the still water depth, D is the total water depth, z' is the vertical coordinate referred to the still water level, and E_D is a modified Dirac delta function which is 0 if $z \neq \eta$ and has the following quantity:

$$\int_{-h}^{\eta+} E_D dz = E/2$$

Surface Roller

- As wave breaks, part of the energy goes into the aerated region known as surface roller as momentum and later transferred to the flow below.
- Roller energy balance (Stive and de Vriend, 1994)

$$\frac{\partial(2E_{sr}c_j)}{\partial x_j} = -D_{sr} + f_e D_{br}$$

$E_{sr} \rightarrow$ Rolle energy density

$D_{br} \rightarrow$ Wave breaking dissipation

$D_{sr} \rightarrow$ Surface roller dissipation

$f_e \rightarrow$ Efficiency factor

- Assumptions
  - Roller direction in same direction as waves
- Roller dissipation $D_{sr} = \frac{g 2E_{sr}\beta_D}{c}$

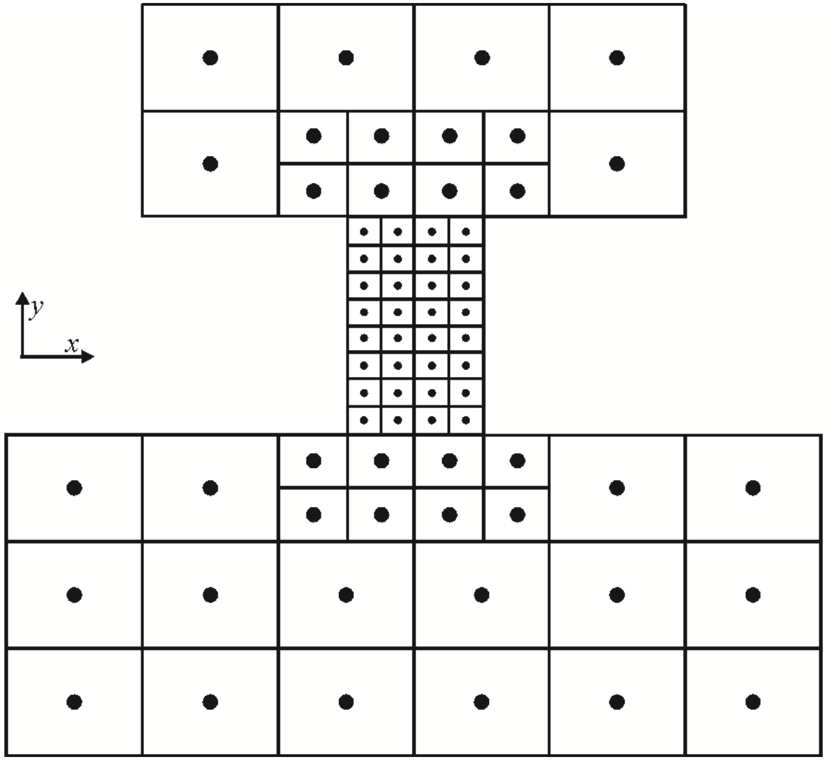
$\beta_D \rightarrow$ Roller dissipation coefficient

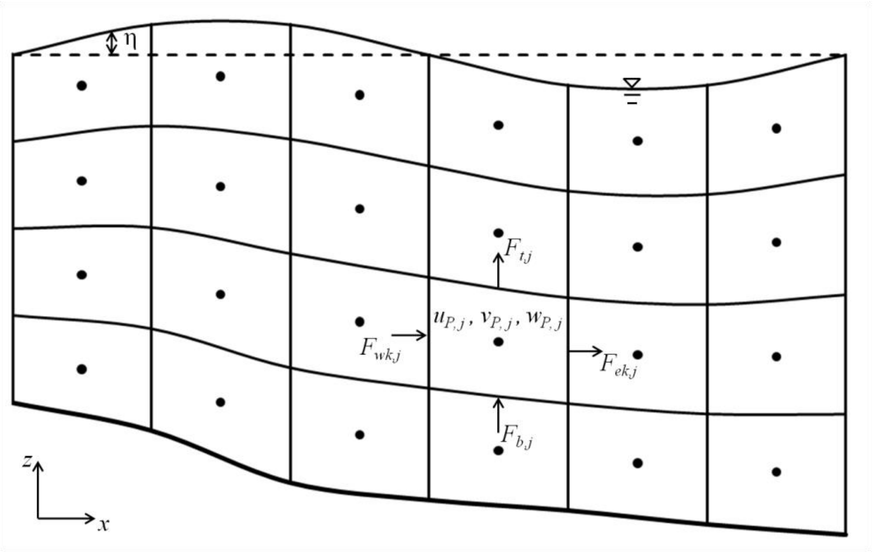
- Roller stress $R_{ij} = 2E_{sr} R_z w_i w_j$

$$R_z = 1 - \tanh\left[\frac{2(z-\eta)}{H_s}\right]^4$$

(Warner et al., 2008)

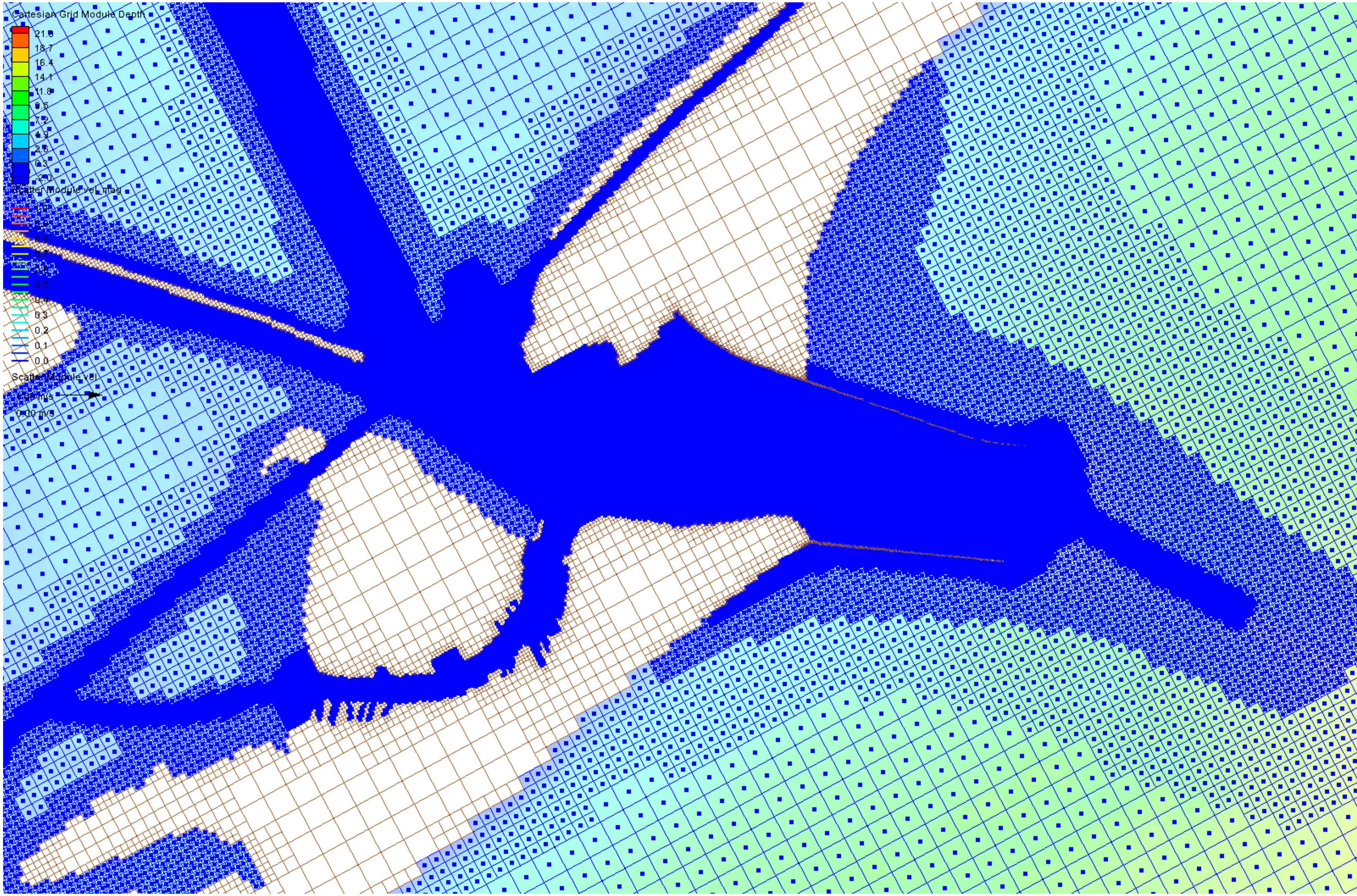
3D Mesh System





Quadtree rectangular in horizontal, and σ coordinate in vertical

Galveston Entrance Channel, TX



Numerical Solution Methods

- Finite volume method;
- Fully implicit;
- Non-staggered (collocated) grid;
- SIMPLEC, with under-relaxation;
- Rhie and Chow's (1983) momentum interpolation for interface fluxes;
- Upwind schemes:
  - Hybrid, Exponential, HLPA
- Solvers:
  - GMRES
- Drying and wetting: "Freezing" dry nodes.

3-D Sediment Transport Model

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Suspended Load Transport

$$\frac{\partial c_k}{\partial t}+\frac{\partial\left[\left(u_j-\omega_{s,k}\delta_{j3}\right)c_k\right]}{\partial x_j}=\frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_c}\frac{\partial c_k}{\partial x_j}\right)$$

Bed Load Transport

(k=1, 2, …, N)

$$\frac{\partial\left(q_{bk}/u_{bk}\right)}{\partial t}+\frac{\partial\left(\alpha_{bxk}q_{bk}\right)}{\partial x}+\frac{\partial\left(\alpha_{byk}q_{bk}\right)}{\partial y}+\frac{1}{L}\left(q_{bk}-q_{b*k}\right)=0$$

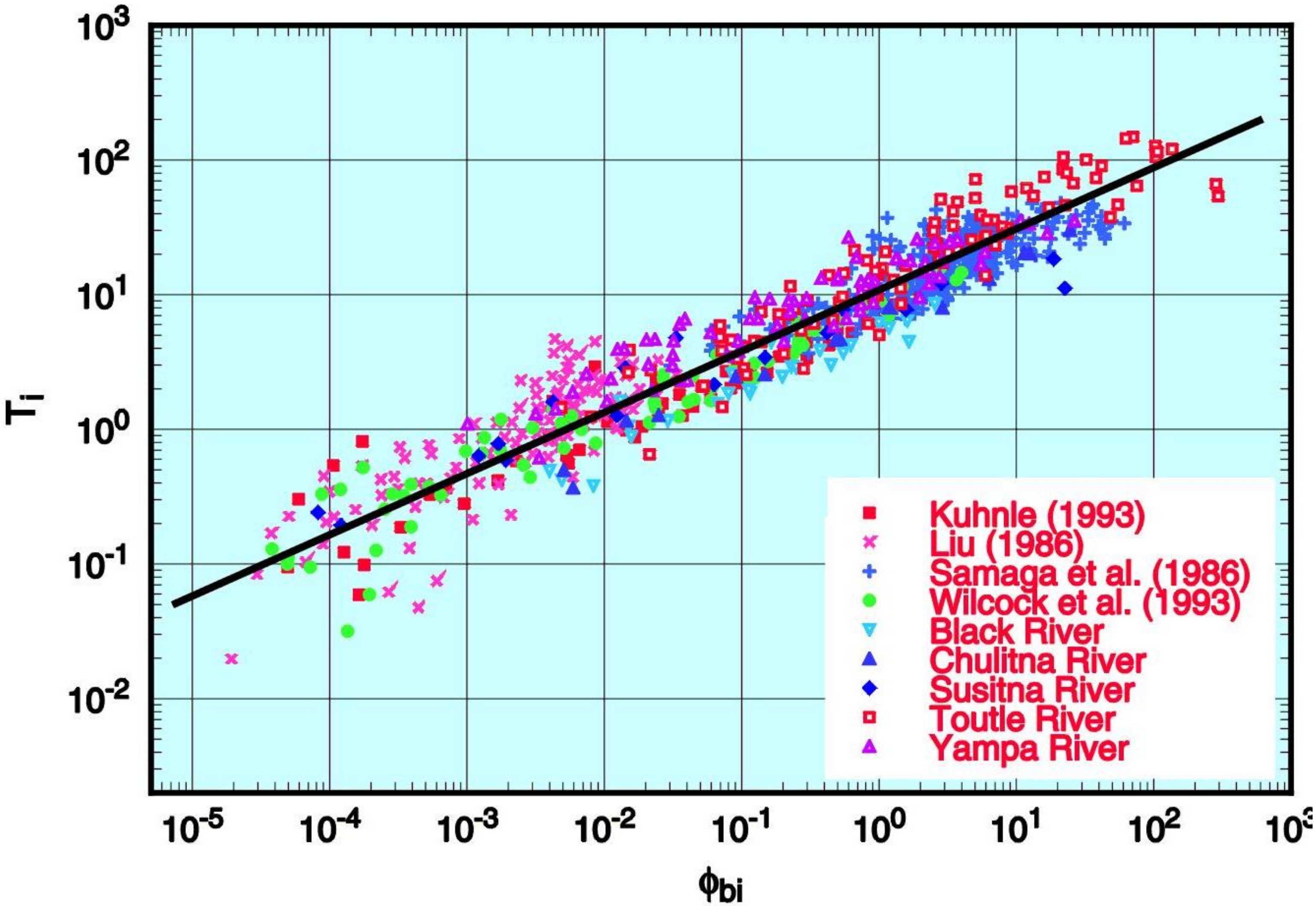
Bed Change

$$\left(1-p'_m\right)\frac{\partial z_{bk}}{\partial t}=D_{bk}-E_{bk}+\frac{1}{L}\left(q_{bk}-q_{b*k}\right)$$

Bed Material Mixing

$$\frac{\partial(\delta_m p_{bk})}{\partial t}=\frac{\partial z_{bk}}{\partial t}+p^*_{bk}\left(\frac{\partial\delta_m}{\partial t}-\frac{\partial z_b}{\partial t}\right)$$

Wu et al. (2000) Bed Load Formula

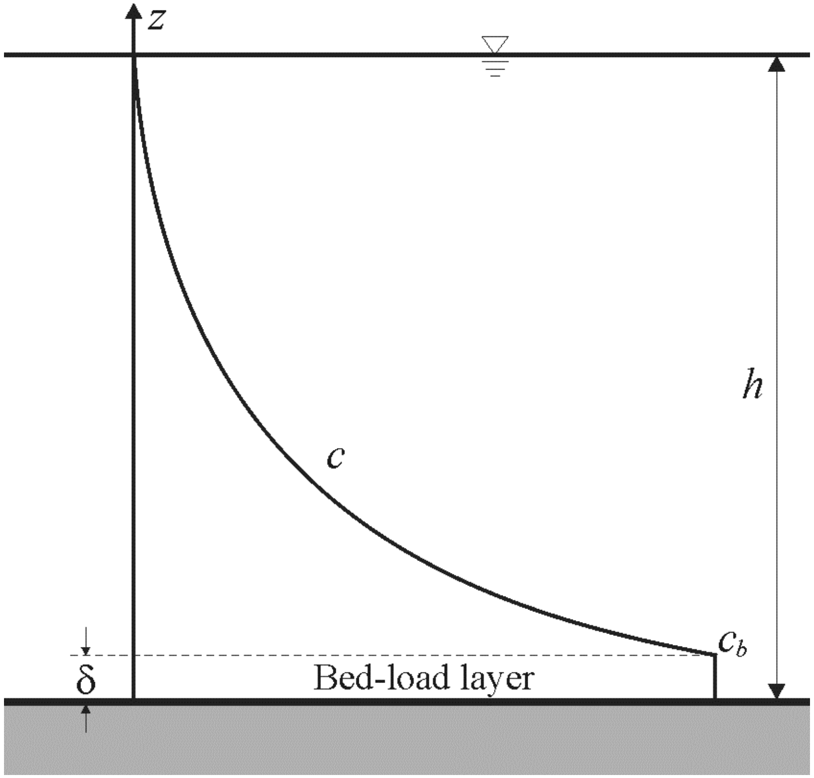


Extended to Coastal Sedimentation by Wu and Lin (2014, Coastal Engineering)

Near-Bed Suspended-load Concentration

Near-bed suspended-load concentration is related to bed-load transport rate:

$$c_{*bk} = \frac{q_{*bk}}{\delta u_{bk}}$$



Bed-load layer thickness:

$$\delta = \max\left(2.0 d_{50}, 0.5\Delta_r, 0.01h\right)$$

Bed-load velocity:

$$\frac{u_{bk}}{\sqrt{\left(\rho_s/\rho - 1\right) g d_k}} = 1.64\left(\frac{\tau_b'}{\tau_{cri,k}} - 1\right)^{0.5}$$

$$\Rightarrow \quad c_{*bk} = \frac{0.0032}{\delta} p_{bk} d_k \left(\frac{\tau'_b}{\tau_{cri,k}} - 1\right)^{1.7}$$

(Wu and Lin, 2014)

Mean Bottom Shear Stress

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- Wave-current bottom friction

$$\tau'_b = \sqrt{\tau'^2_{b,c} + \tau'^2_{b,wm} + 2\tau'_{b,c}\tau'_{b,wm}\cos\varphi}$$

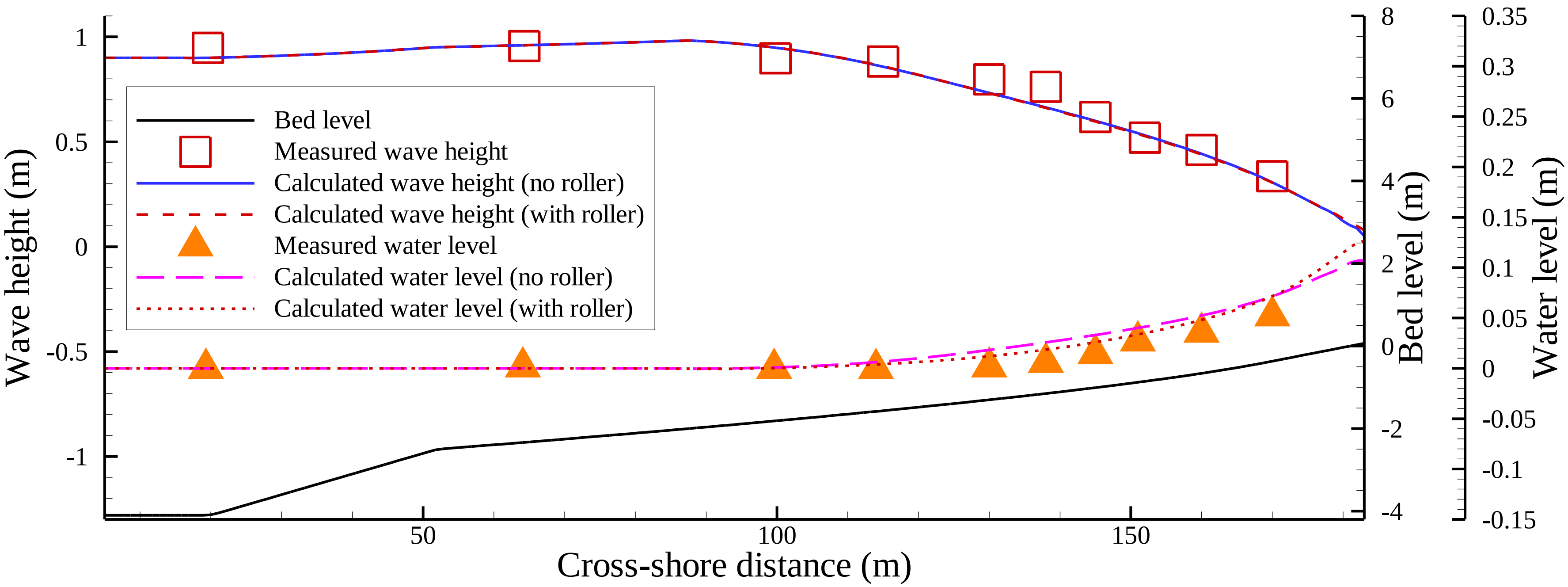
$$\tau'_{b,wm} = \frac{1}{4}\rho f'_w U^2_{wm} \qquad f'_w = 0.237\left(A_w / k'_s\right)^{-0.52}$$ (Soulsby, 1997)

$$A_w = U_{wm} T_w / 2\pi$$

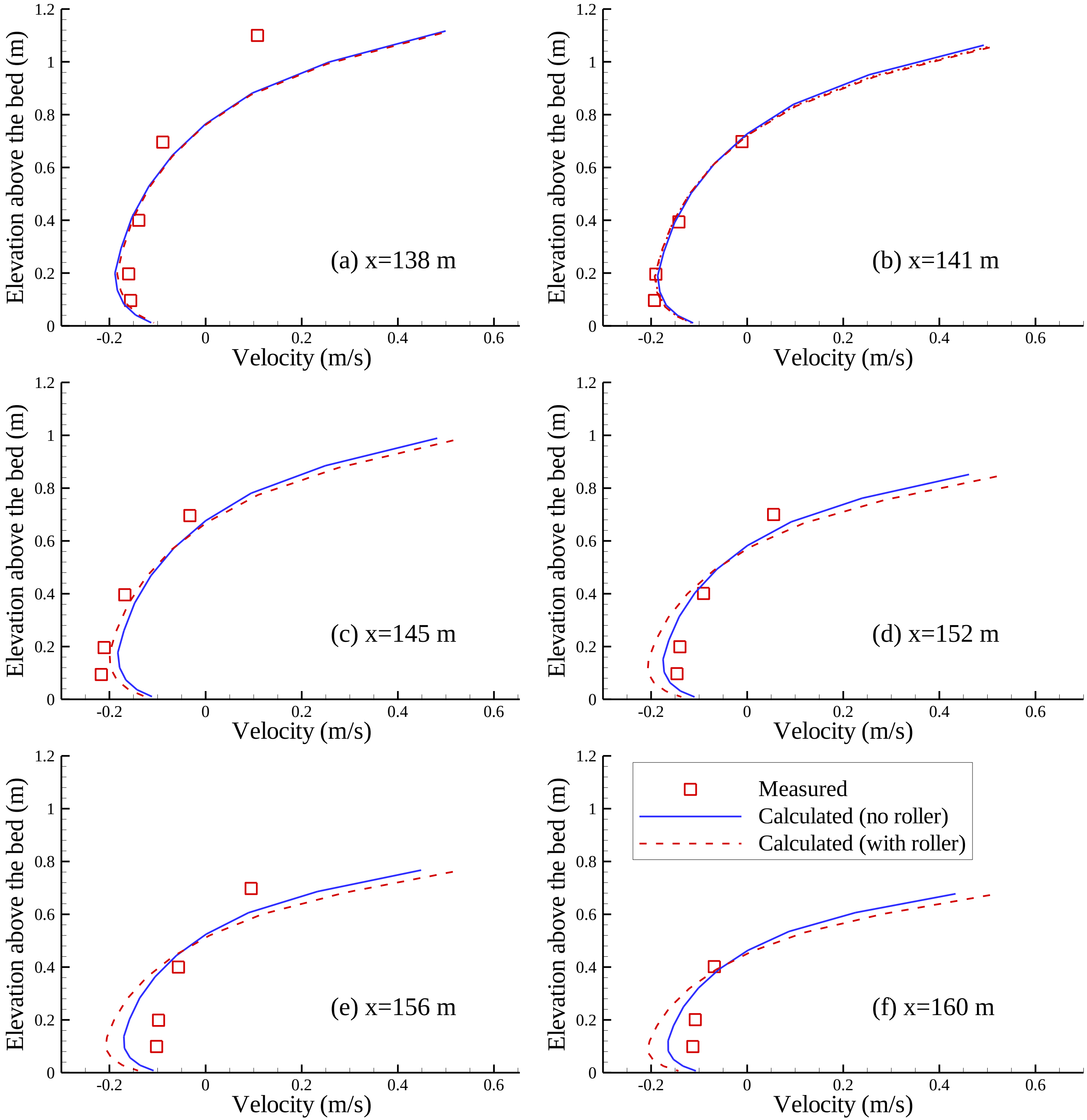
- Bottom wave orbital velocity

$$U_{wm} = \frac{\pi H_s}{T_p \sinh(kh)}$$

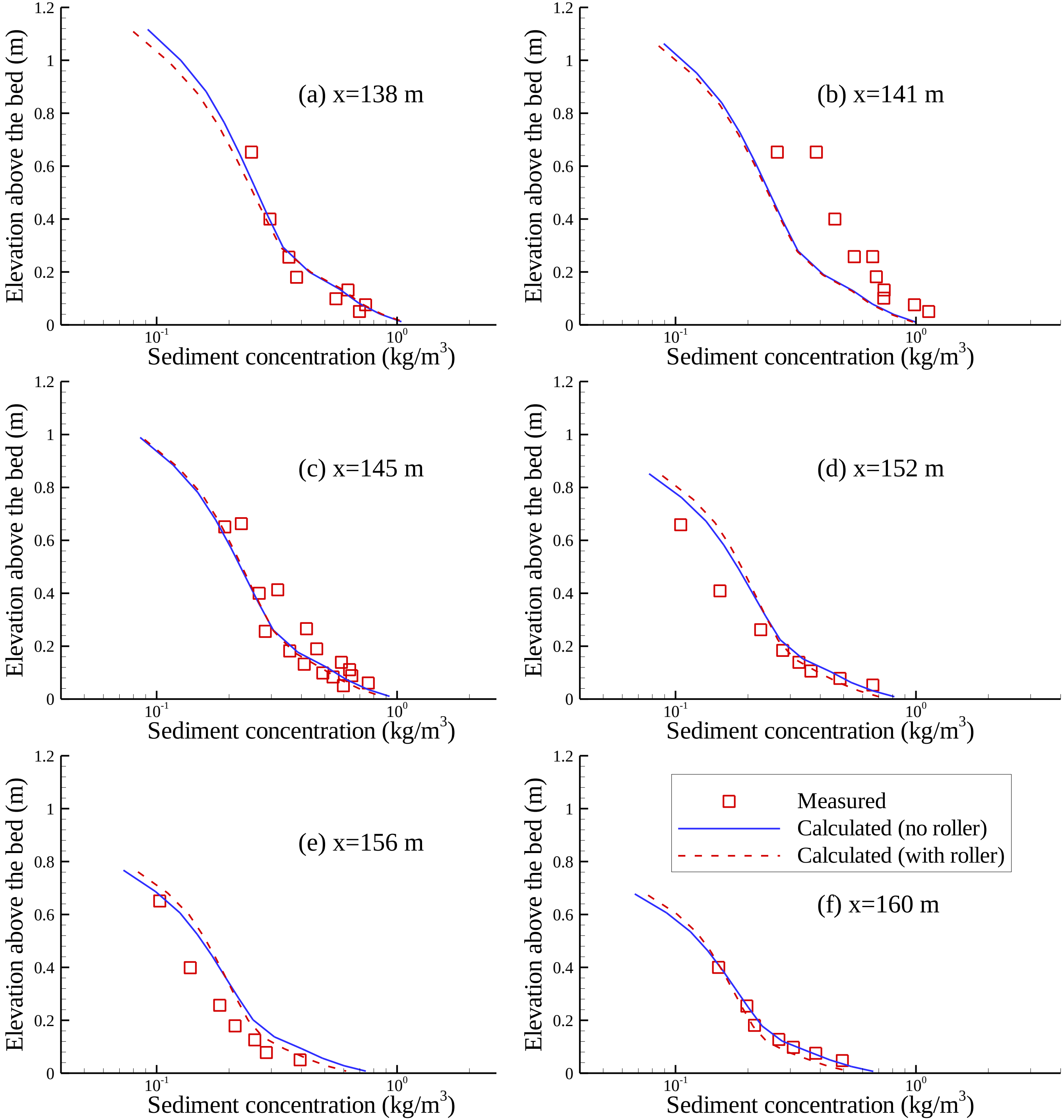
Undertow Current and Sediment Transport in Roelvink and Reniers's (1995) Experiment



Experiment LIP11D 1A conducted in a 233 m long, 5 m wide and 7 m deep wave tank (Roelvink and Reniers 1995). D_{50}=0.22 mm. h=4.1 m, H_{s0}=0.9 m, and T_p =5 s. The bed friction coefficient c_f =0.012. The suspended-load Schmidt number is calibrated as 0.23. Bed change is not simulated in this case. a uniform cross-shore grid spacing of 1.0 m and 13 layers in the vertical direction with variable relative thickness (layer thickness over local flow depth) of 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.06, 0.05, 0.04, 0.03, and 0.02 from the water surface to the bottom



(a) x=138 m
(b) x=141 m
(c) x=145 m
(d) x=152 m
(e) x=156 m
(f) x=160 m
Elevation above the bed (m)
Velocity (m/s)
Measured
Calculated (no roller)
Calculated (with roller)



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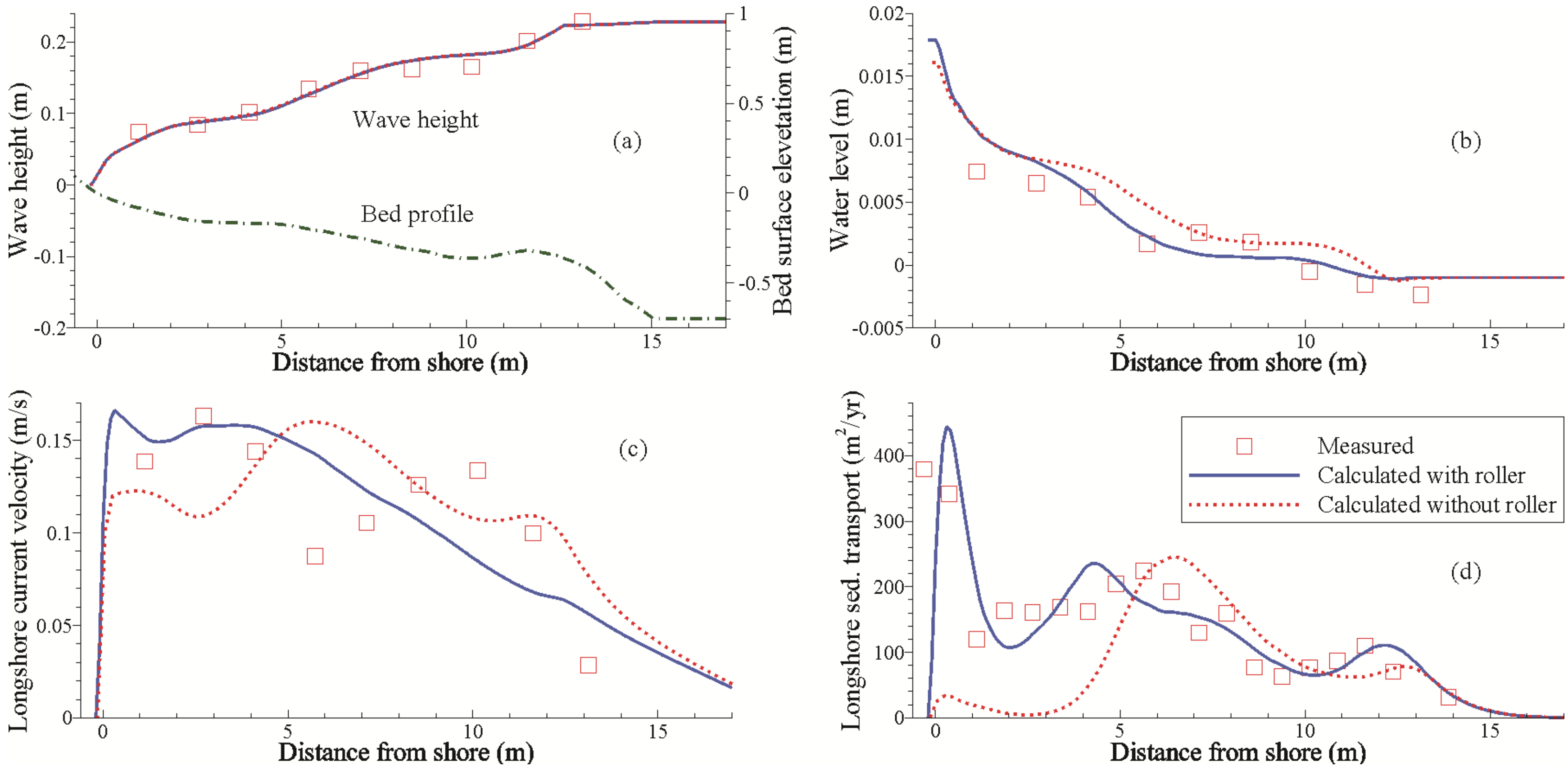
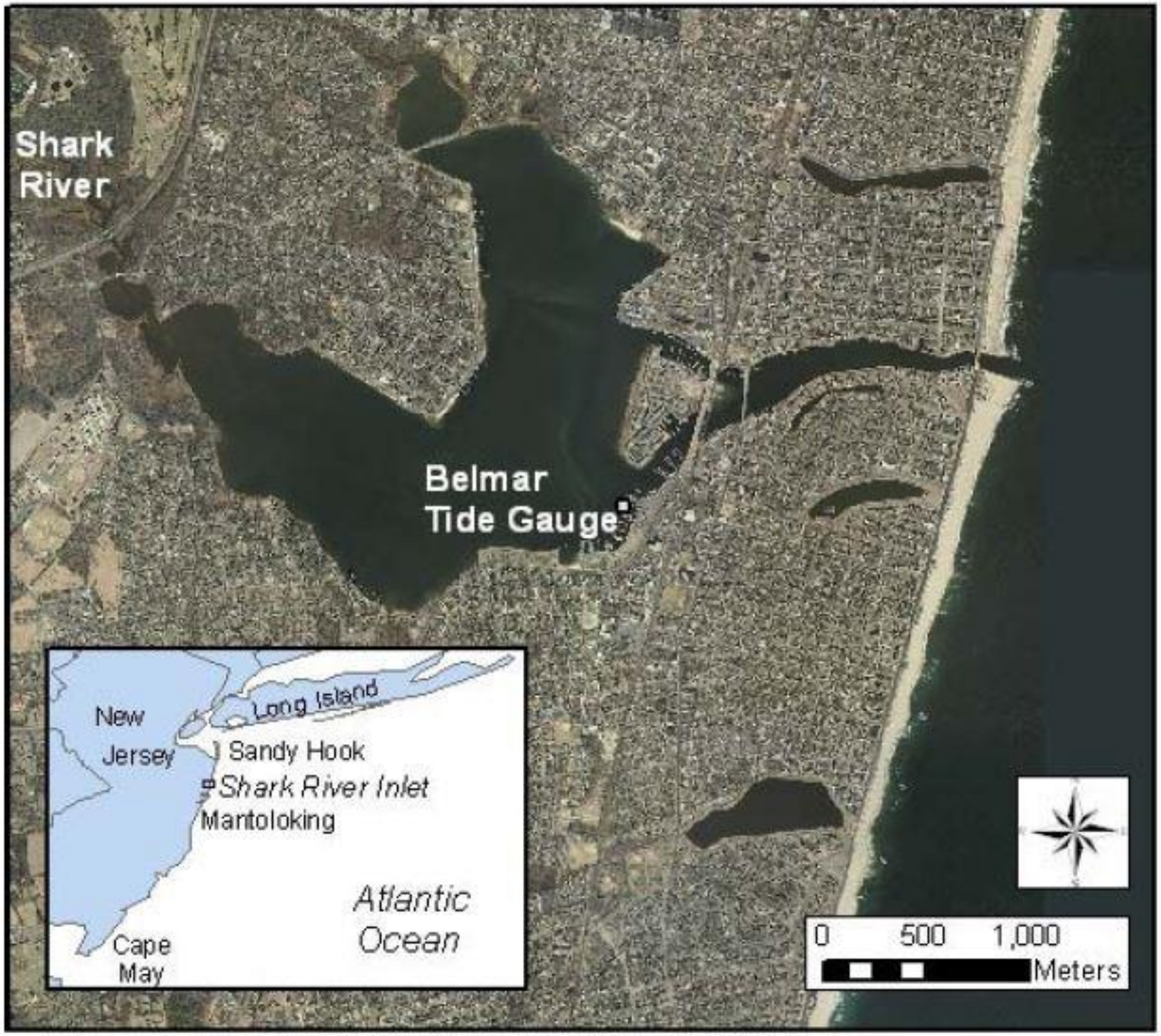
Longshore Sediment Transport – LSTF case 1

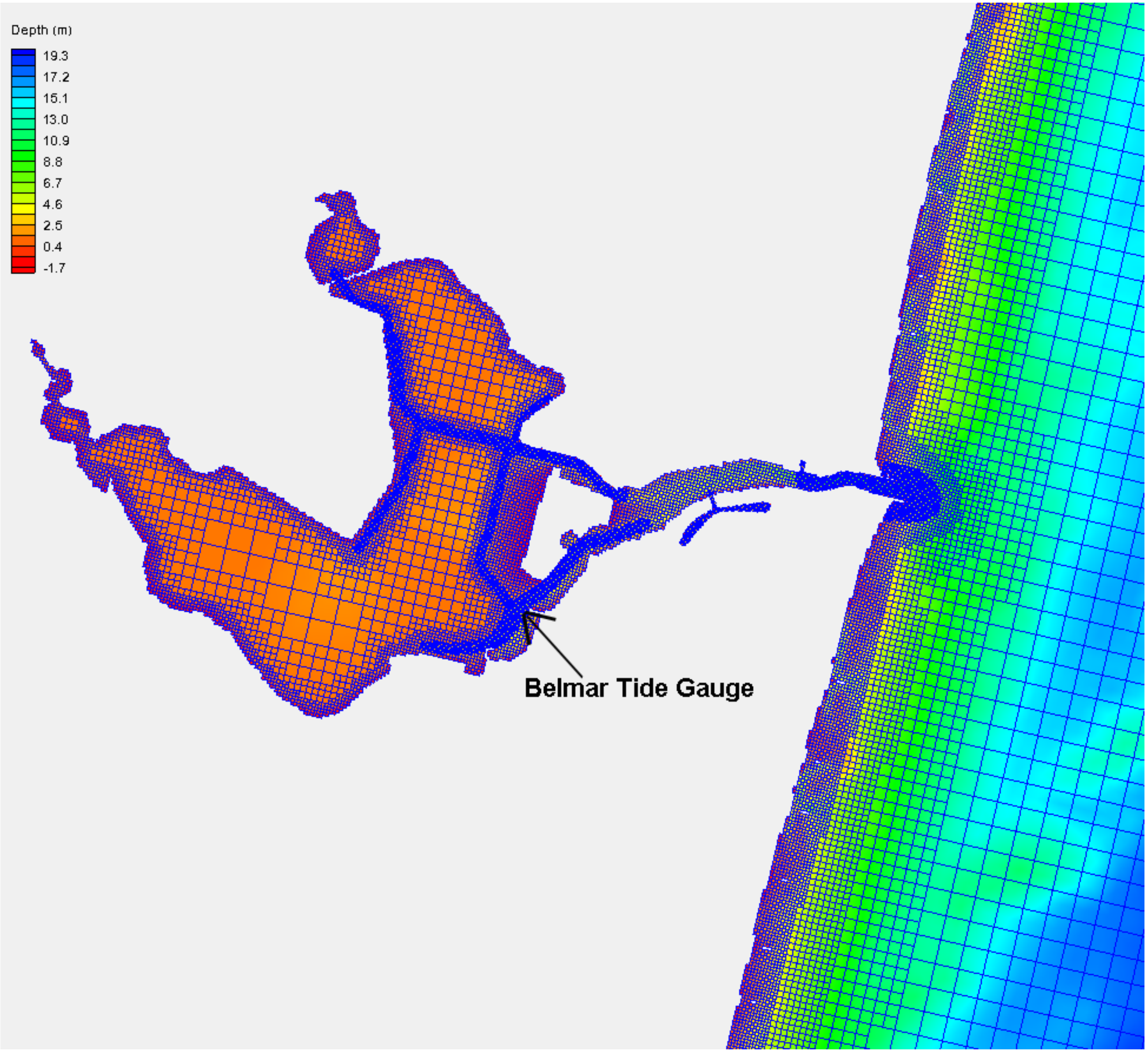


LSTF case 1: (a) Significant wave height (with bed profile), (b) Water level, (c) Longitudinal current, and (d) Longitudinal sediment transport. H_s=0.228 m, T_p=1.465 s, and incident wave angle was 6.5°. Sediment size was 0.15 mm. 16 uniform layers in vertical.

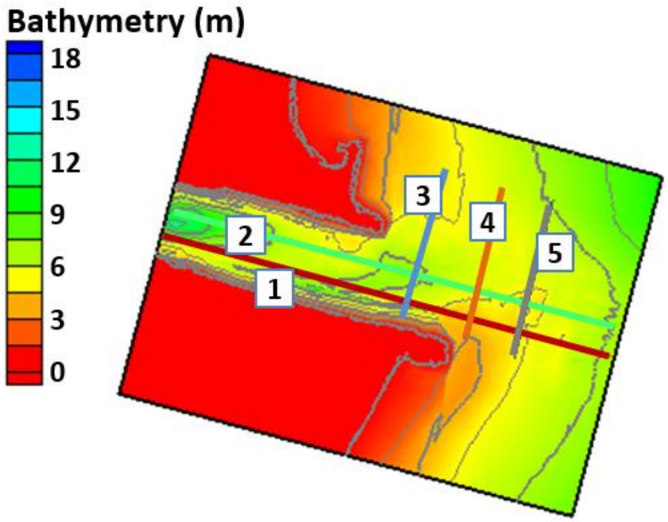
Shark River Inlet, USA

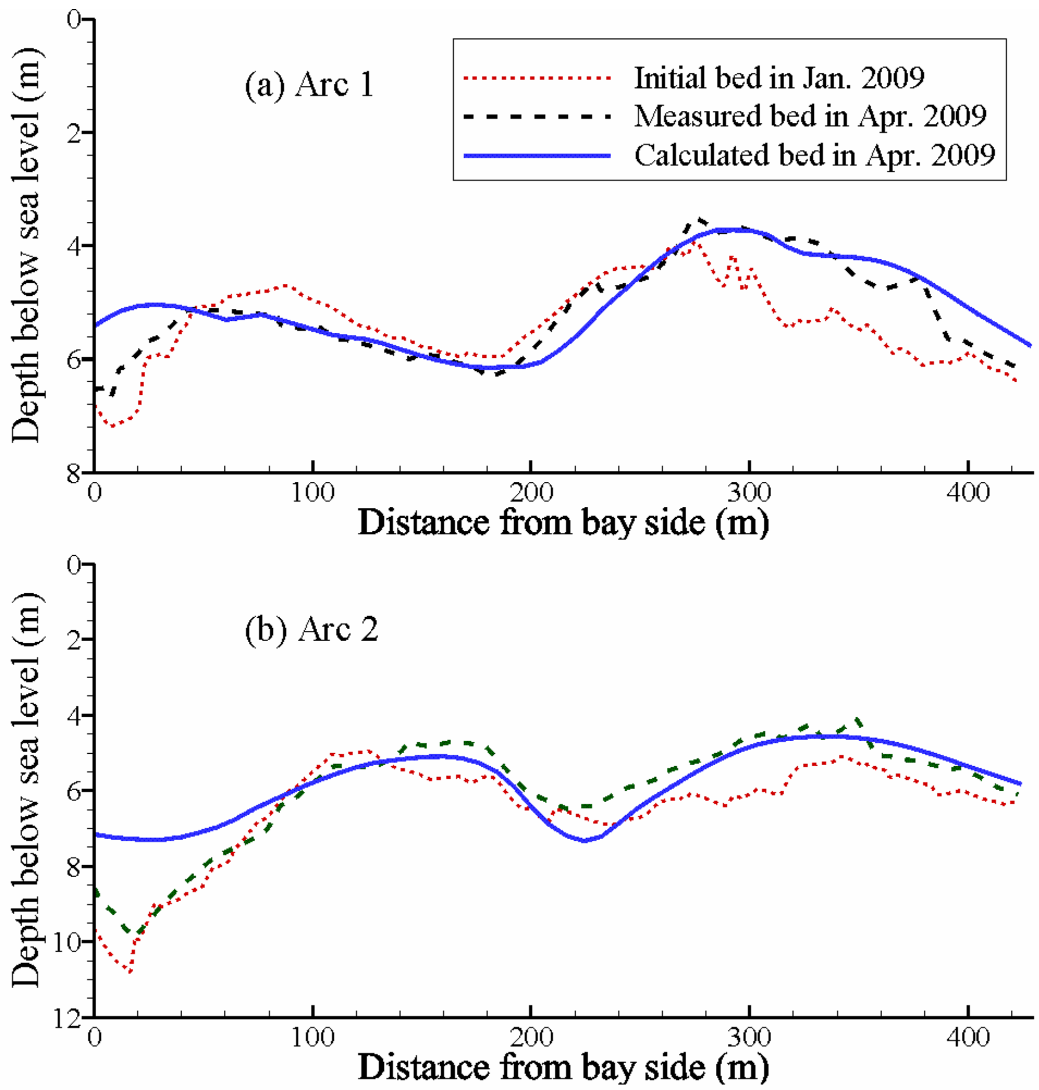


Shark River Inlet, USA



Shark River Inlet, USA





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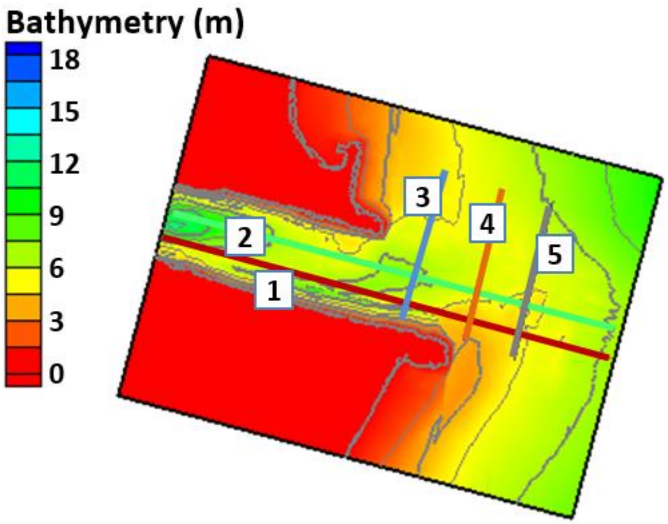

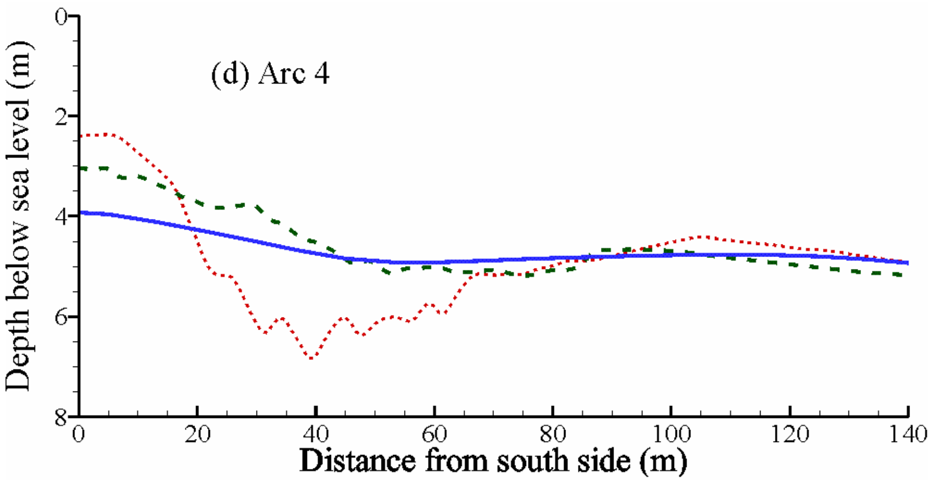

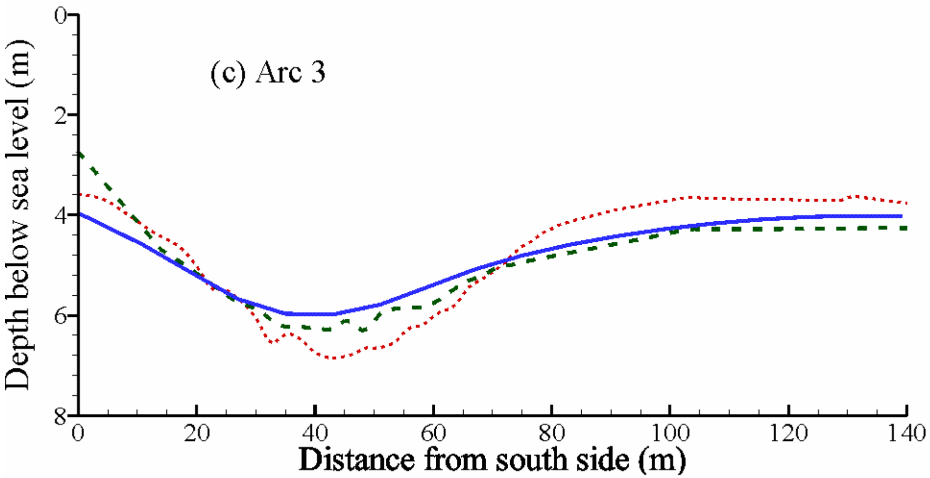

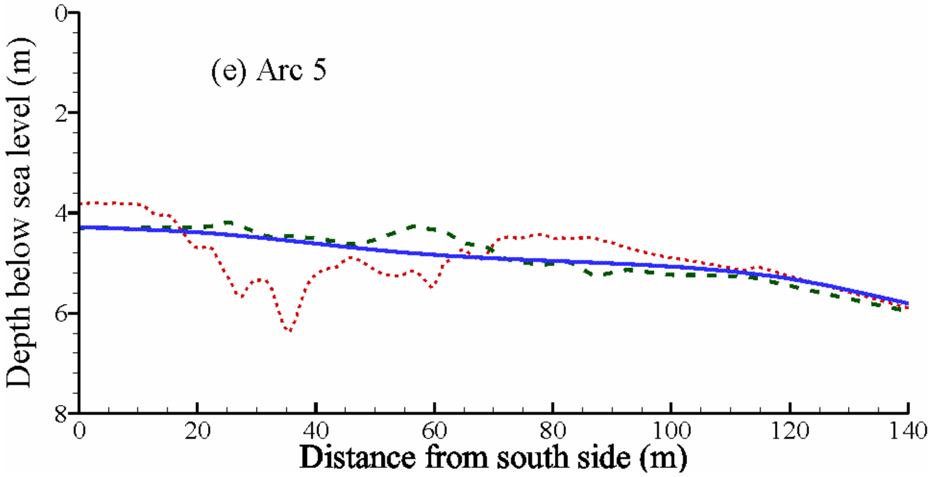

Summary

- 3-D shallow water flow model has been developed for coastal sedimentation.
- A modified mixing length model is used for turbulence closure.
- The flow model is coupled with CMS-Wave model.
- The model equations are solved with implicit finite-volume method.
- The sediment transport model considers multiple-sized, total-load transport.
- The model has been tested using laboratory and field measurements.

Comments and Suggestions?

Thanks!