

The State of the Art and Science of Coastal Engineering

Numerical Simulation Of Interactions Between Water Waves And Moored-Floating Breakwater

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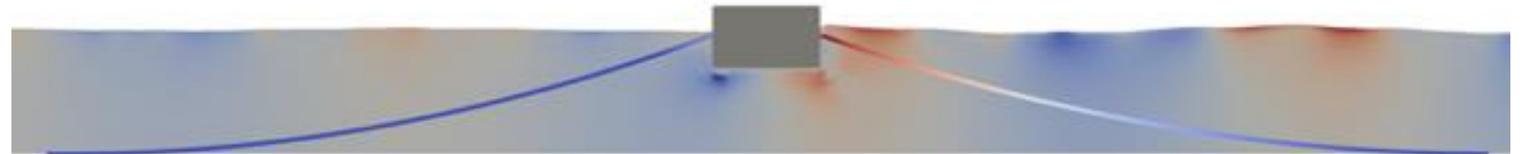
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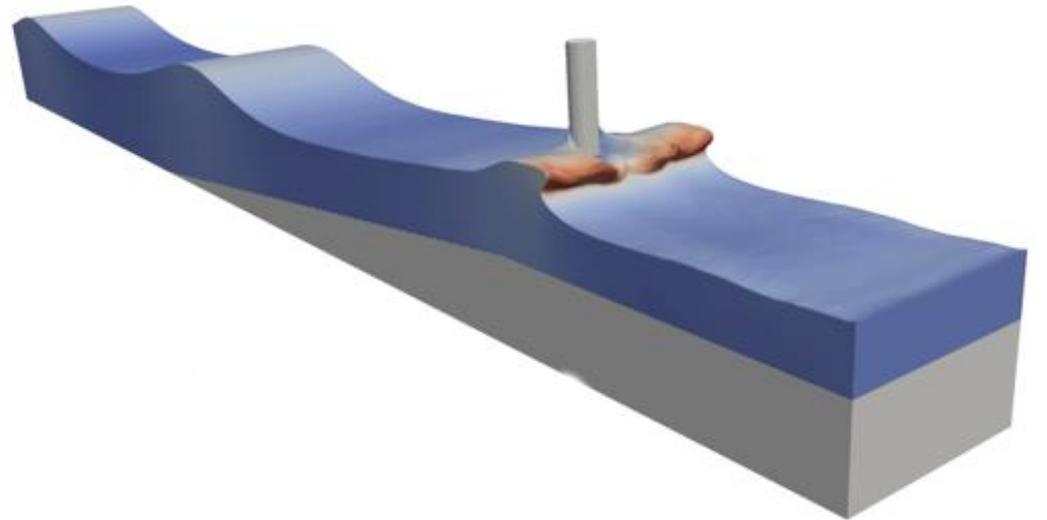
Objectives

- Modelling the influence of mooring systems on the motion of breakwaters
 - Complex FSI including flexible structures
- CFD
 - Accurate wave modelling
 - Non-linear wave-structure interaction
 - Viscous effects around the body
- Implementation of 6DOF algorithm
- Implementation of mooring system



REEF3D: Open Source CFD Solver

- Developed at the Marine Civil Engineering Group, Department of Civil and Transport Engineering, NTNU Trondheim, Norway
- 3D numerical wave tank
 - Free Surface Flows
 - Wave Hydrodynamics
 - Wave Forces
 - Fluid-Structure Interaction
- MPI parallelized C++ code
- Published under GNU GPL v3



Governing Equations

- Incompressible RANS and continuity equations in non-conservative form

$$\frac{\partial u_i}{\partial x_i} = 0,$$
$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\nu \cdot \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right) + g_i,$$

- Level-Set Method for capturing the free surface

$$\Phi(\mathbf{x}, t) \begin{cases} > 0 & \text{if } \mathbf{x} \in \text{phase 1} \\ = 0 & \text{if } \mathbf{x} \in \Gamma \\ < 0 & \text{if } \mathbf{x} \in \text{phase 2} \end{cases}, \quad |\nabla \Phi| = 1$$

$$\frac{\partial \Phi}{\partial t} + u_j \frac{\partial \Phi}{\partial x_j} = 0.$$

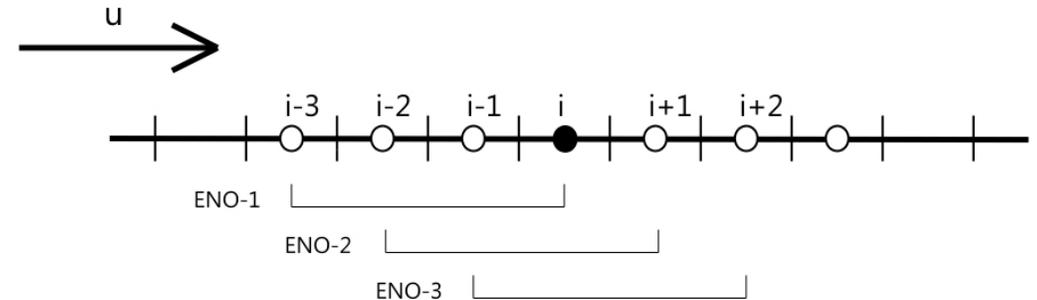
- Reinitialisation after each step to keep Φ a signed distance function

Numerical Discretisation

- Finite Difference Method on Cartesian grid
- Convection terms: 5th-order accurate WENO scheme
 - Non-oscillatory behaviour near large gradients
 - Keeps high-order accuracy in comparison to TVD schemes

$$U \frac{\partial U}{\partial x} \approx \frac{1}{\Delta x} \left(\tilde{U}_{i+1/2} U_{i+1/2} - \tilde{U}_{i-1/2} U_{i-1/2} \right)$$

$$U_{i+1/2}^{\pm} = \omega_1^{\pm} U_{i+1/2}^{1\pm} + \omega_2^{\pm} U_{i+1/2}^{2\pm} + \omega_3^{\pm} U_{i+1/2}^{3\pm}$$



- Diffusion terms: Implicit discretisation for stability and efficiency reasons
- Temporal terms in momentum equation: 3rd-order TVD Runge-Kutta scheme
 - Adaptive time steps based on CFL criterion

Fluid Solution Algorithm

- Staggered grid
 - Tight velocity-pressure coupling
 - Avoiding parasitic currents above the free surface
- Chorin's projection method for incompressible flows
 - Poisson equation for pressure
 - Velocity satisfies continuity equation after correction
- Implicit boundary treatment
 - Ghost cell immersed boundary method
 - Extrapolation of solution in solid regions
 - High stability through numerical simplicity

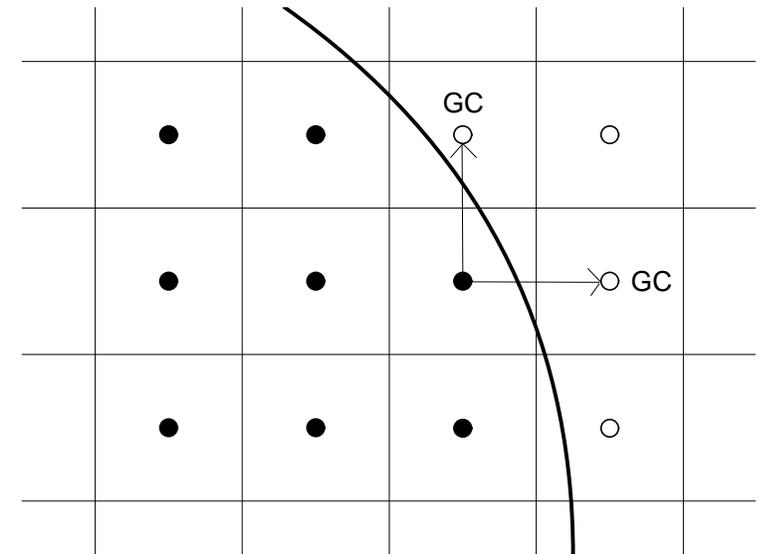
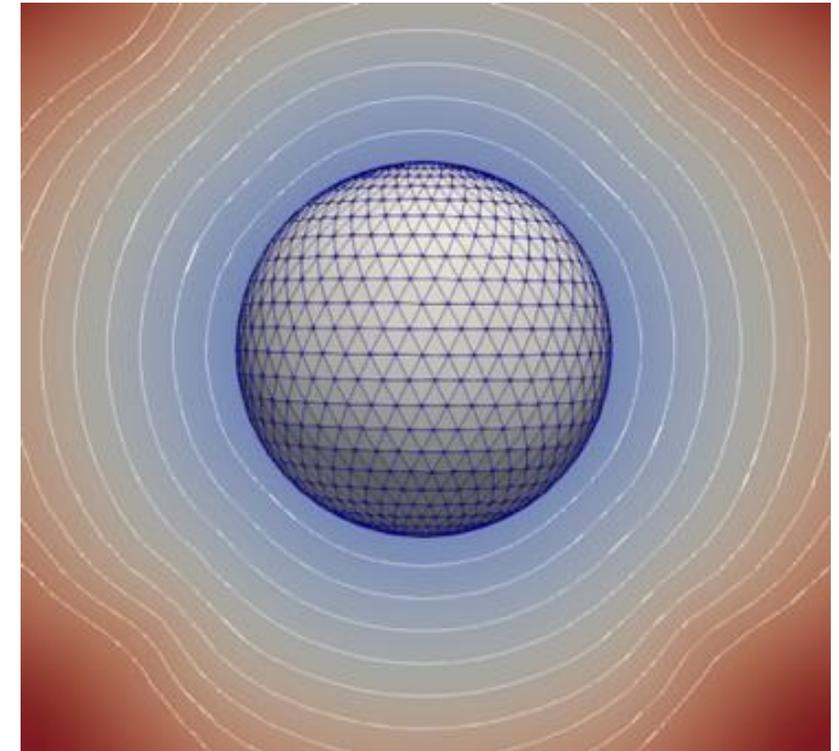


Illustration of GC-IBM

6DOF Algorithm

- Geometry described by triangular mesh
- Implicit description of rigid bodies
 - Ray-tracing algorithm for closest distance information
 - Signed distance function around body using reinitialisation algorithm
- Advantages:
 - No moving or overset meshes
 - High numerical stability
 - Fast and parallelised process



Contour of level set function
around moving body

6DOF Algorithm: Rigid Body Dynamics

- Body moves as rigid body in 6DOF
 - Translational motion from Newton's law

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{pmatrix} = \frac{1}{m} \cdot \begin{pmatrix} F_{x_1, \mathbf{x}} \\ F_{x_2, \mathbf{x}} \\ F_{x_3, \mathbf{x}} \end{pmatrix}$$

- Rotational motion in principal, body-fixed coordinate system

$$\begin{aligned} I_x \ddot{\xi}_1 + \dot{\xi}_2 \dot{\xi}_3 \cdot (I_z - I_y) &= M_{1, \xi}, \\ I_y \ddot{\xi}_2 + \dot{\xi}_1 \dot{\xi}_3 \cdot (I_x - I_z) &= M_{2, \xi}, \\ I_z \ddot{\xi}_3 + \dot{\xi}_1 \dot{\xi}_2 \cdot (I_y - I_x) &= M_{3, \xi}. \end{aligned}$$

$$\mathbf{I} = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} = \begin{bmatrix} mr_x^2 & 0 & 0 \\ 0 & mr_y^2 & 0 \\ 0 & 0 & mr_z^2 \end{bmatrix} \quad \mathbf{M}_\xi = (M_{1, \xi}, M_{2, \xi}, M_{3, \xi})^T = \mathbf{J}_1^{-1} \cdot \mathbf{M}_x$$

$$\mathbf{J}_1 = \begin{bmatrix} c\mathbf{x}_6 c\mathbf{x}_5 - s\mathbf{x}_6 c\mathbf{x}_4 + c\mathbf{x}_6 s\mathbf{x}_5 s\mathbf{x}_4 & s\mathbf{x}_6 s\mathbf{x}_4 + c\mathbf{x}_6 c\mathbf{x}_4 s\mathbf{x}_5 \\ s\mathbf{x}_6 c\mathbf{x}_5 & c\mathbf{x}_6 c\mathbf{x}_4 + s\mathbf{x}_4 s\mathbf{x}_5 s\mathbf{x}_6 & -c\mathbf{x}_6 s\mathbf{x}_4 + s\mathbf{x}_5 s\mathbf{x}_6 c\mathbf{x}_4 \\ -s\mathbf{x}_5 & c\mathbf{x}_5 s\mathbf{x}_4 & c\mathbf{x}_5 c\mathbf{x}_4 \end{bmatrix}$$

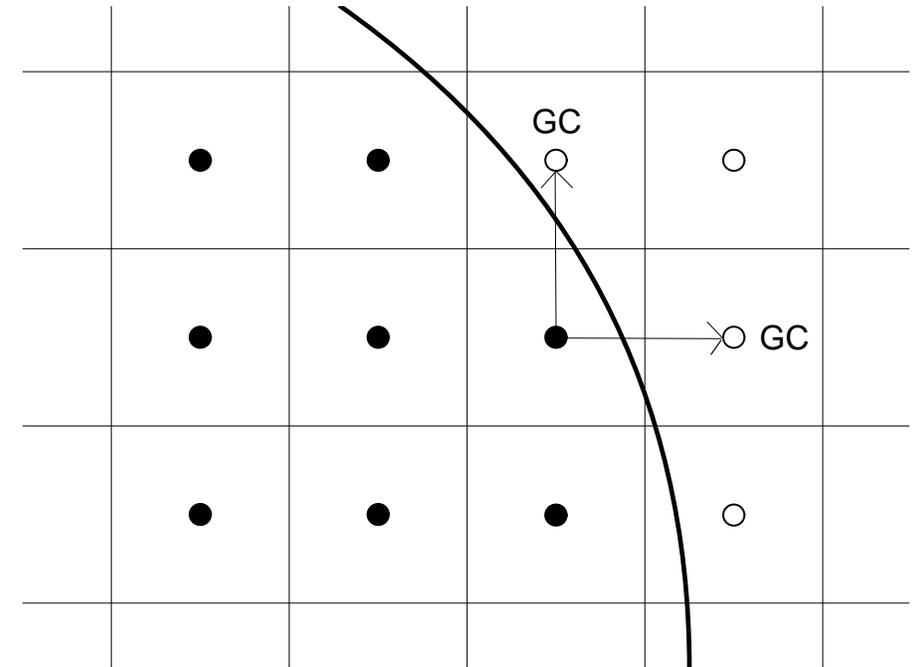
- System is solved explicitly using an AB-scheme

6DOF Algorithm: Coupling

- Weak coupling without sub-iterations
- Ghost cell immersed boundary method
 - Interface velocity from body dynamics
 - Interpolation of staggered velocity components
 - Pressure gradient from velocities

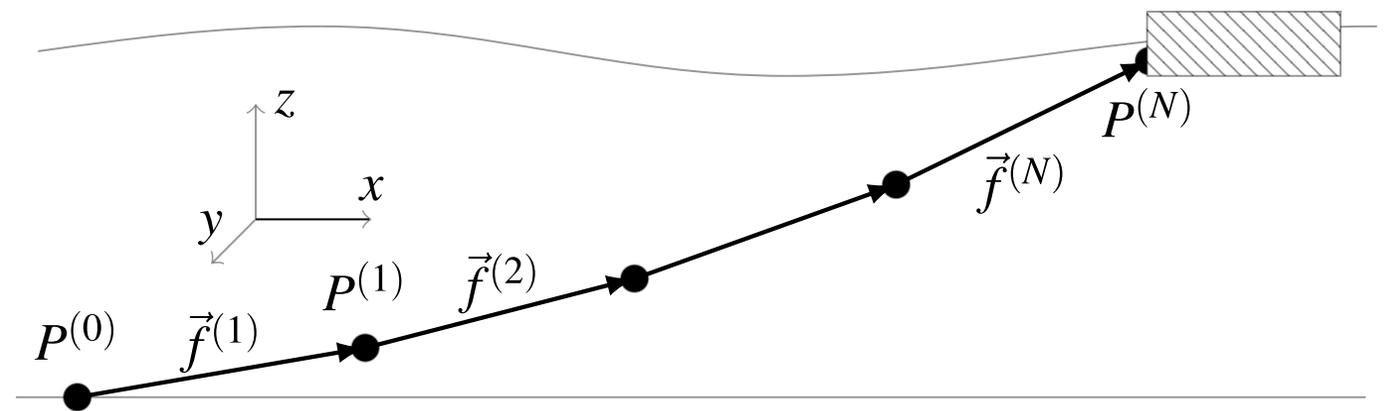
$$\mathbf{u} = \dot{\mathbf{x}} + \mathbf{r} \times \dot{\boldsymbol{\xi}}$$
$$\frac{\partial p}{\partial x_i} = -\frac{1}{\rho} \frac{du_i}{dt}$$

- Preventing pressure oscillations during solid/fluid change
 - Extrapolation of fluid values to solid region



Quasi-Static Mooring Model

- Analytical models lack flexibility for general application
- Simple numerical model in order to keep efficiency
 - Quasi-static motion
 - Explicit coupling as additional forces and moments
- Tension Element Method
 - Neglecting bending stiffness
 - Elastic material
 - Discretisation in mass points
 - Here: just gravity forces



Quasi-Static Mooring Model

- Static force equilibrium at each point

$$\vec{f}^{(v+1)} F_T^{(v+1)} - \vec{f}^{(v)} F_T^{(v)} = -\vec{F}_G^{(v)} \quad \vec{F}_G^{(v)} = q\vec{g} \cdot \left(\frac{a^{(v)} + a^{(v+1)}}{2} \right), \quad v = 1, \dots, N-1$$

- System of equations is solved for unit normal vectors of bars

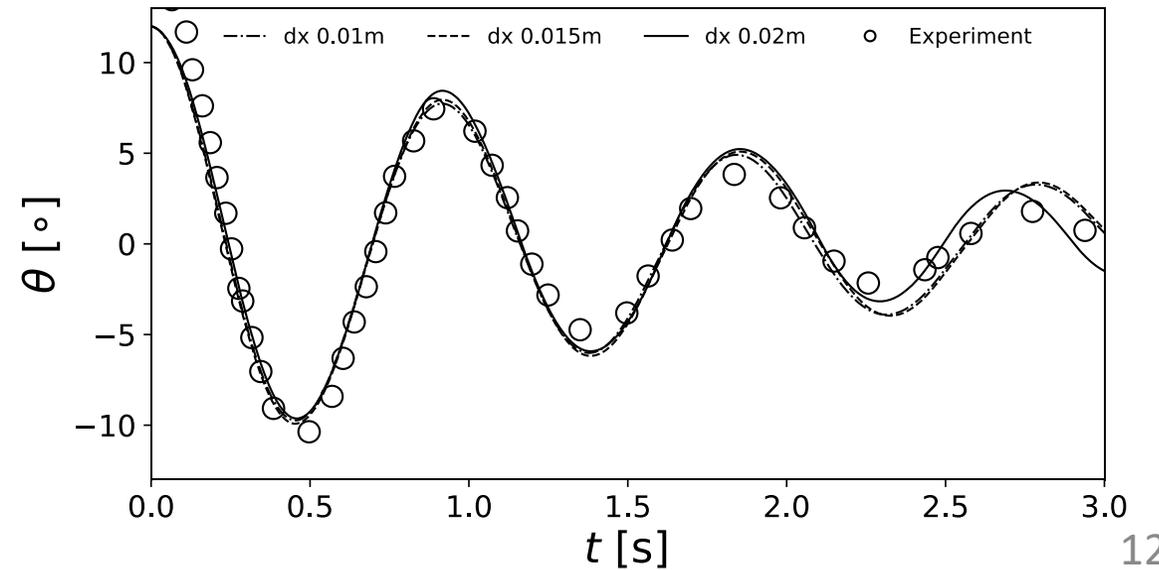
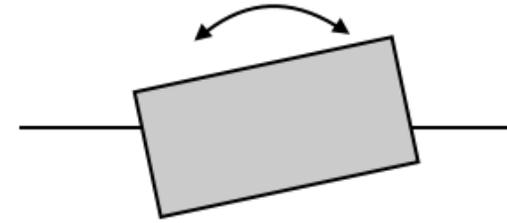
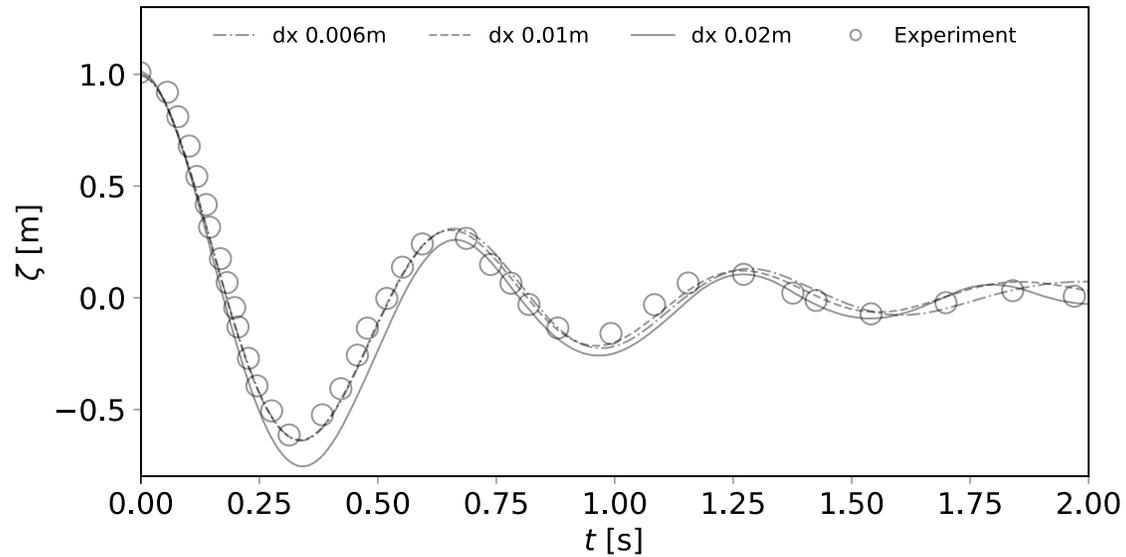
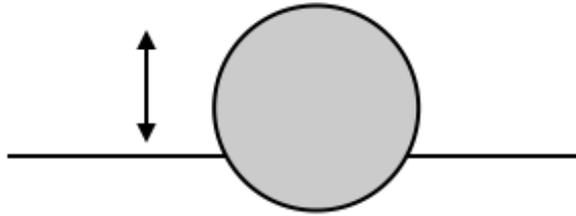
$$\mathbf{A} \cdot \mathbf{F} = \mathbf{B}, \quad \mathbf{A} = \begin{pmatrix} \mathbf{T} \\ \mathbf{L} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -\mathbf{G} \\ \vec{c} \end{pmatrix}$$

- Geometrical constraint to close system and control physics
 - Bar lengths a as function of stiffness
- System solved using successive approximation

$$\sum_{v=1}^{N+1} \vec{f}^{(v)} a^{(v)} = \vec{c}$$

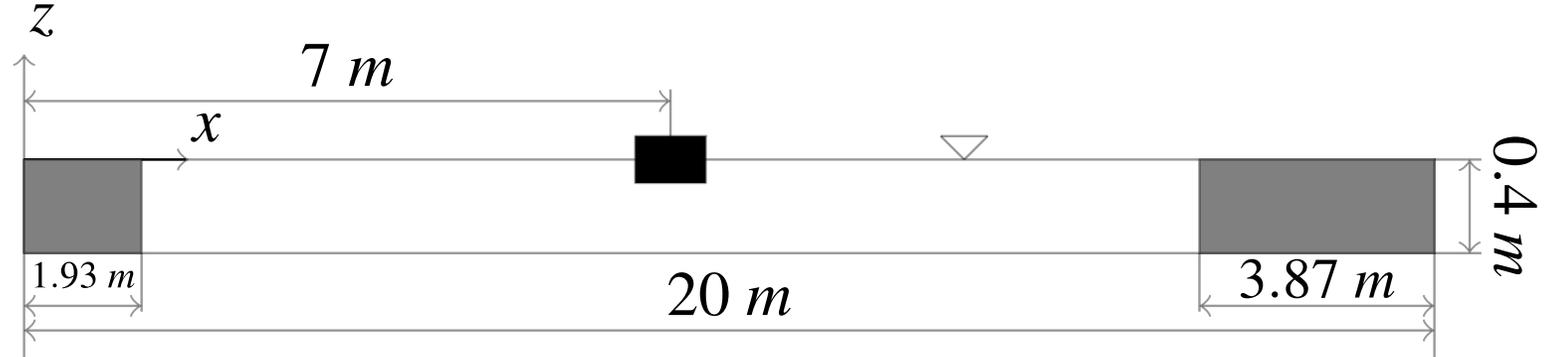
2D Decay Tests

- Single motion calculations
- Good convergence to experimental data

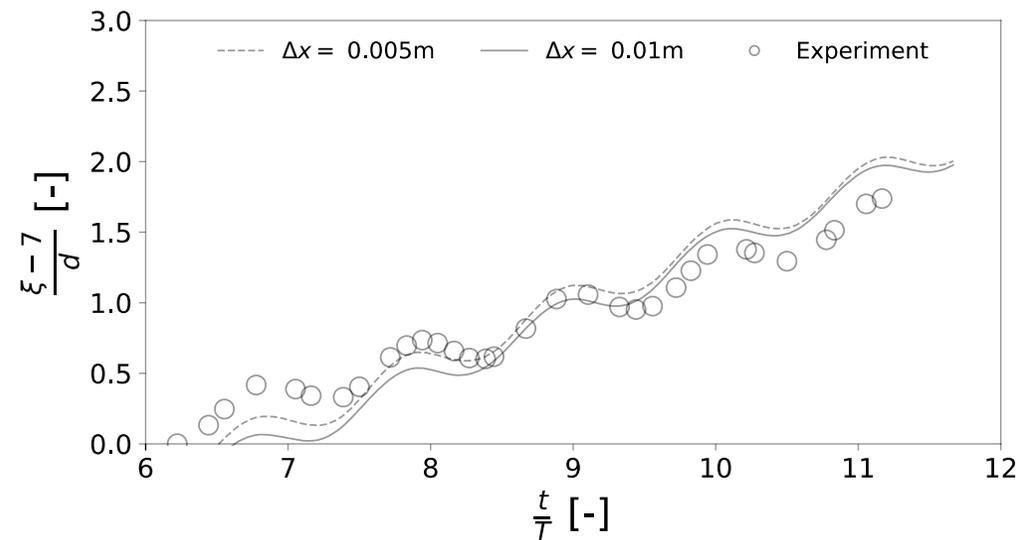
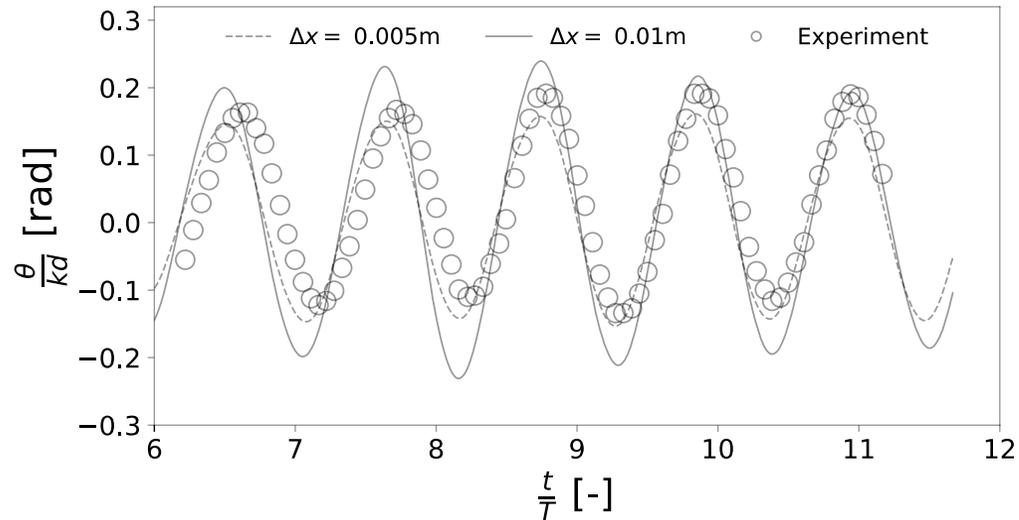
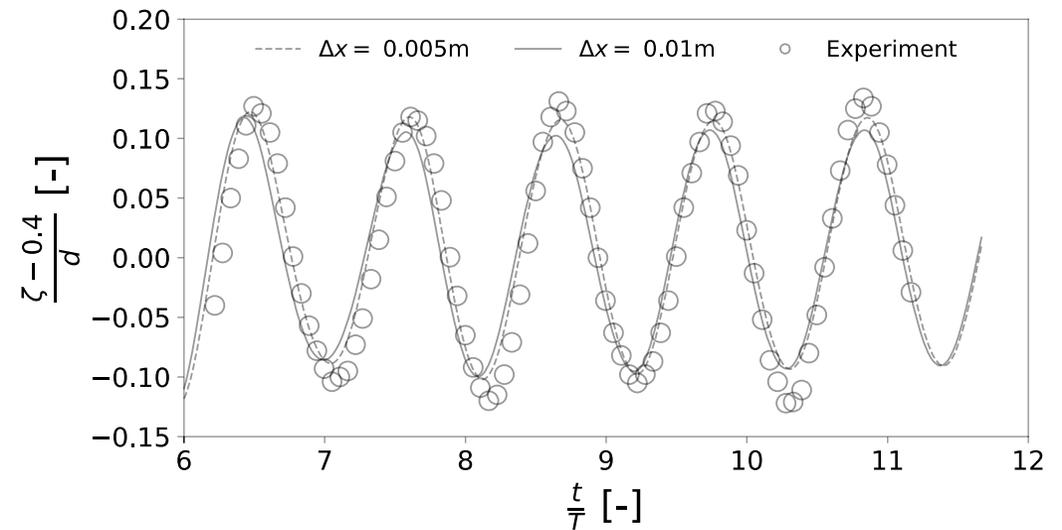
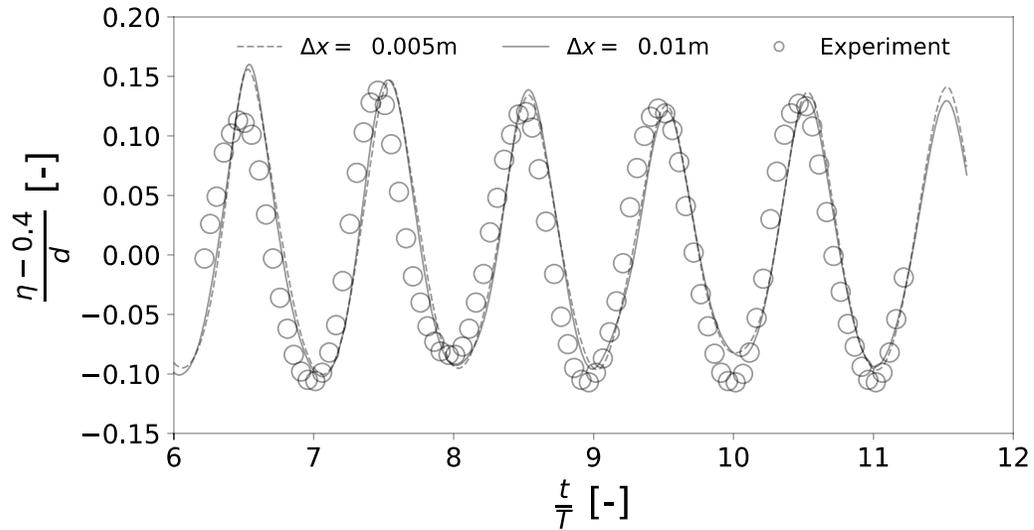


2D Breakwater Motion in Waves

- Experiments from Ren et al., 2015, Applied Ocean Research
- 2D barge:
 - $\rho = 500 \text{ kg/m}^3$
 - $h = 0.2 \text{ m}$
 - $l = 0.3 \text{ m}$
- Incoming waves:
 - $\lambda = 1.936 \text{ m}$
 - $H = 0.04 \text{ m}$
 - $T = 1.2 \text{ s}$



2D Breakwater Motion in Waves



2D Moored Breakwater Motion in Waves

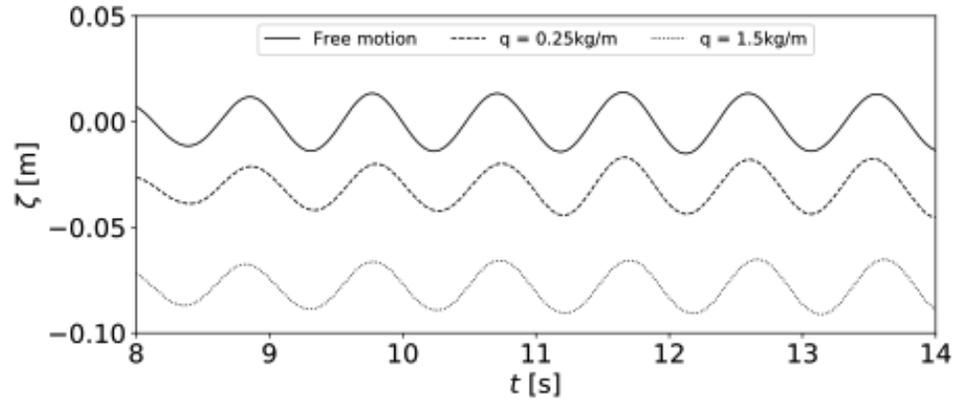
- Comparison of free-floating barge with moored-floating barge
 - Mooring system of two mooring lines
 - Two different line configurations with different specific weight



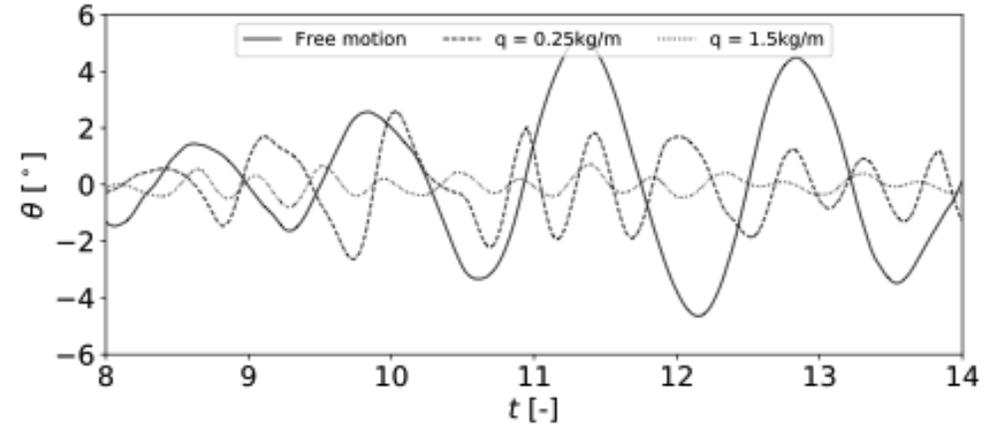
Colours shows u_x in the fluid and tension forces in the lines

2D Moored Breakwater Motion in Waves

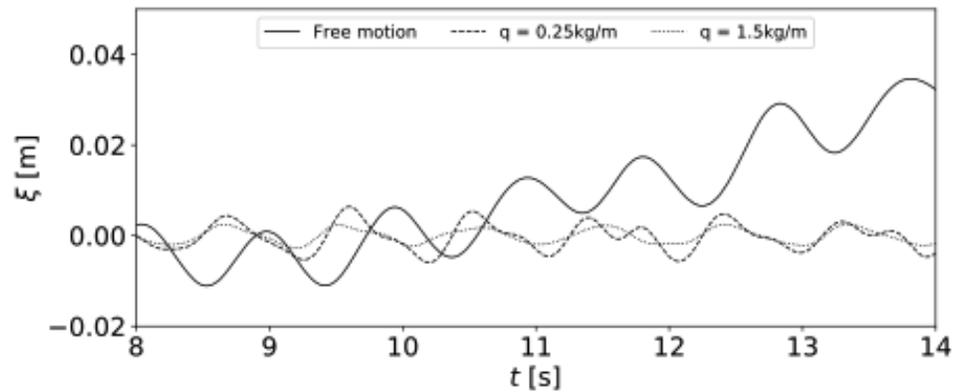
- Comparison of the three motions of freedom



(a) HEAVE MOTION.



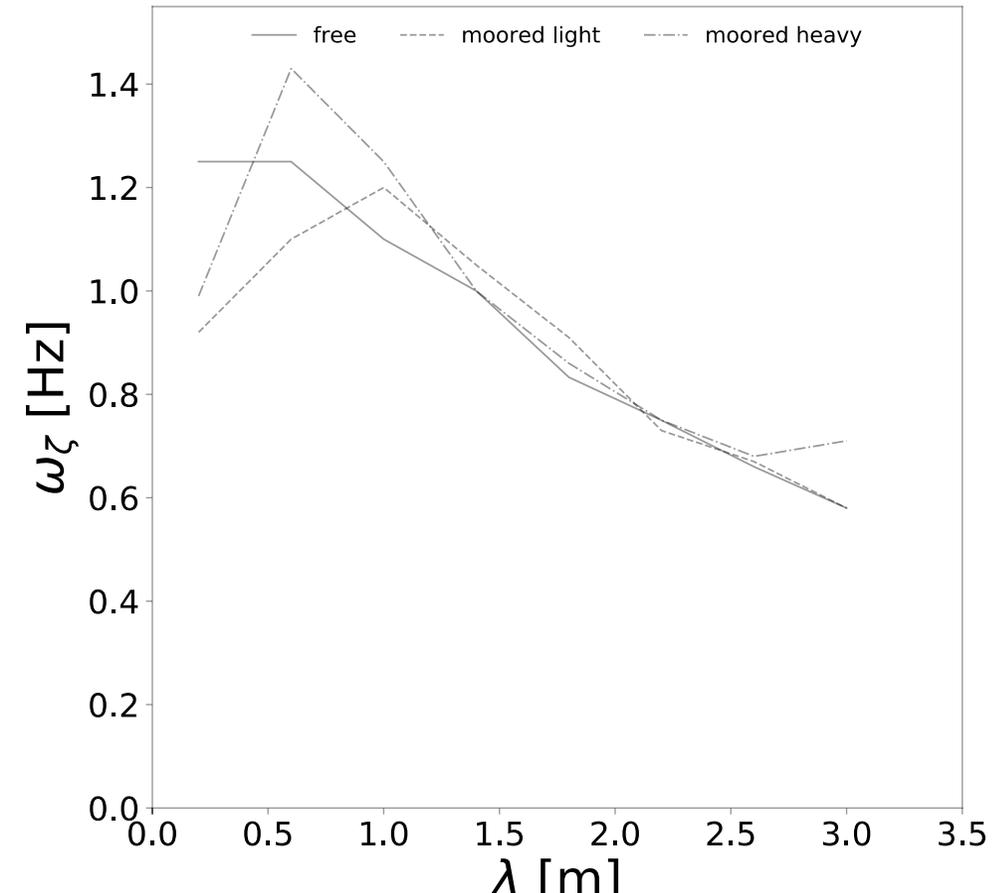
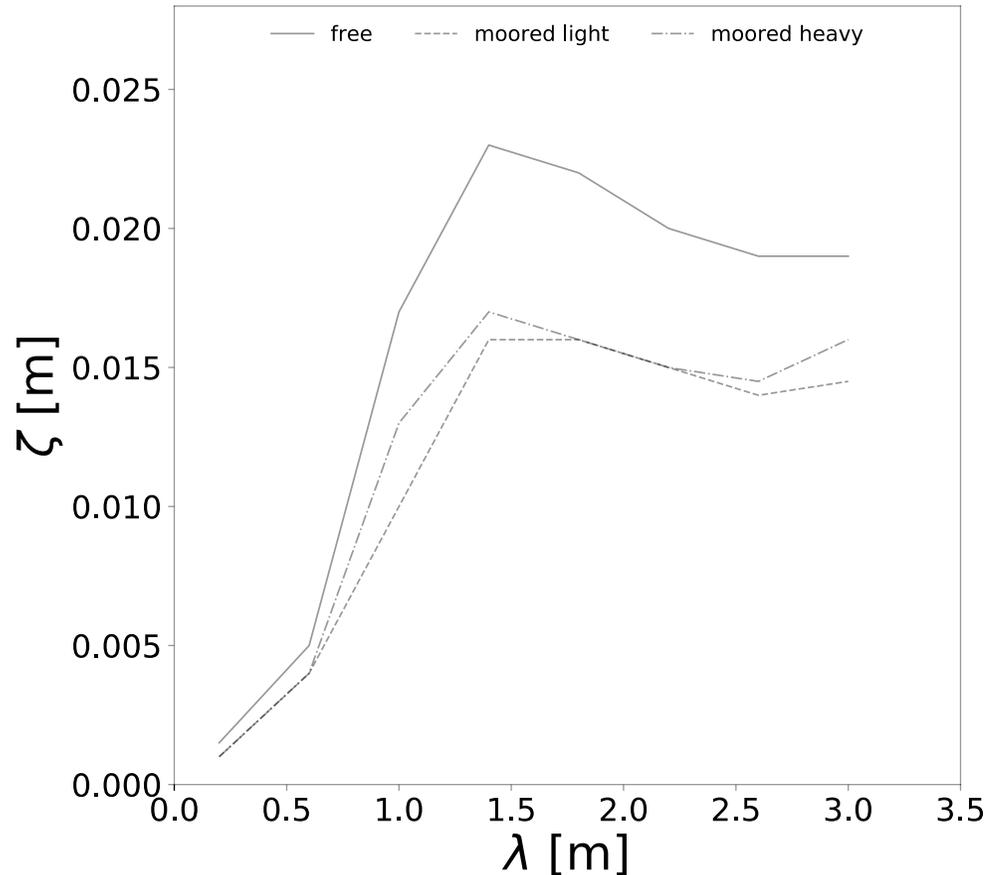
(c) PITCH MOTION.



(b) SURGE MOTION.

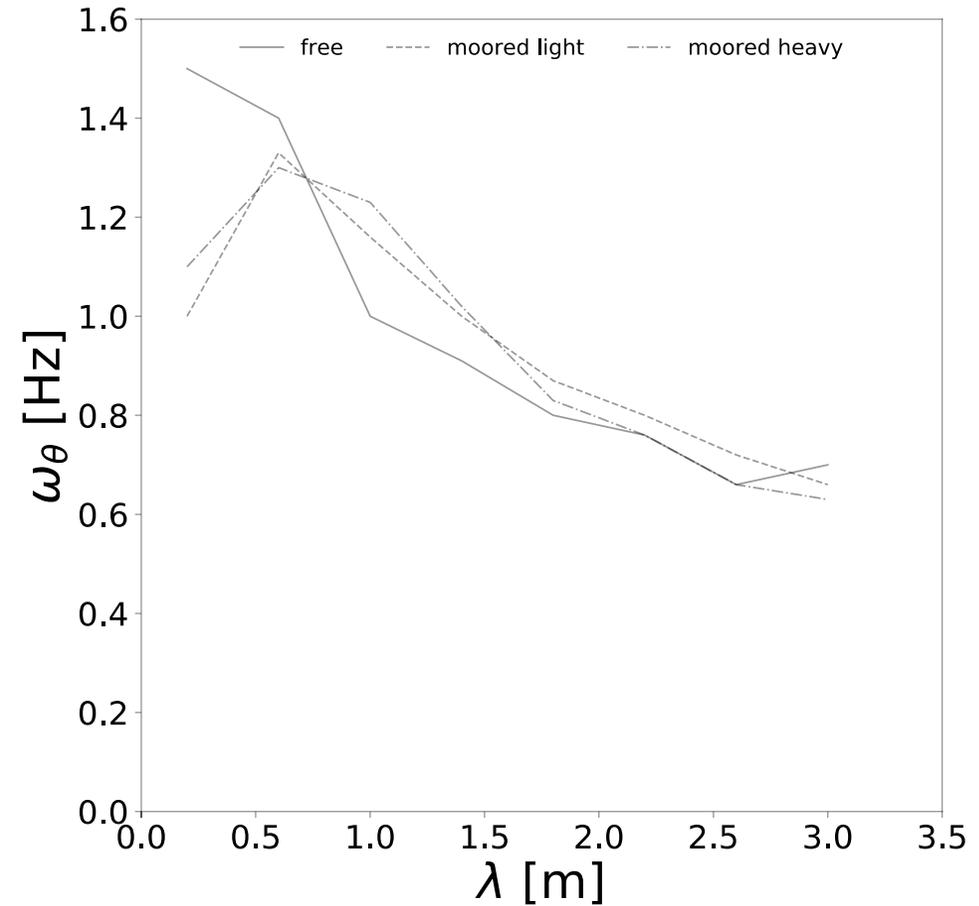
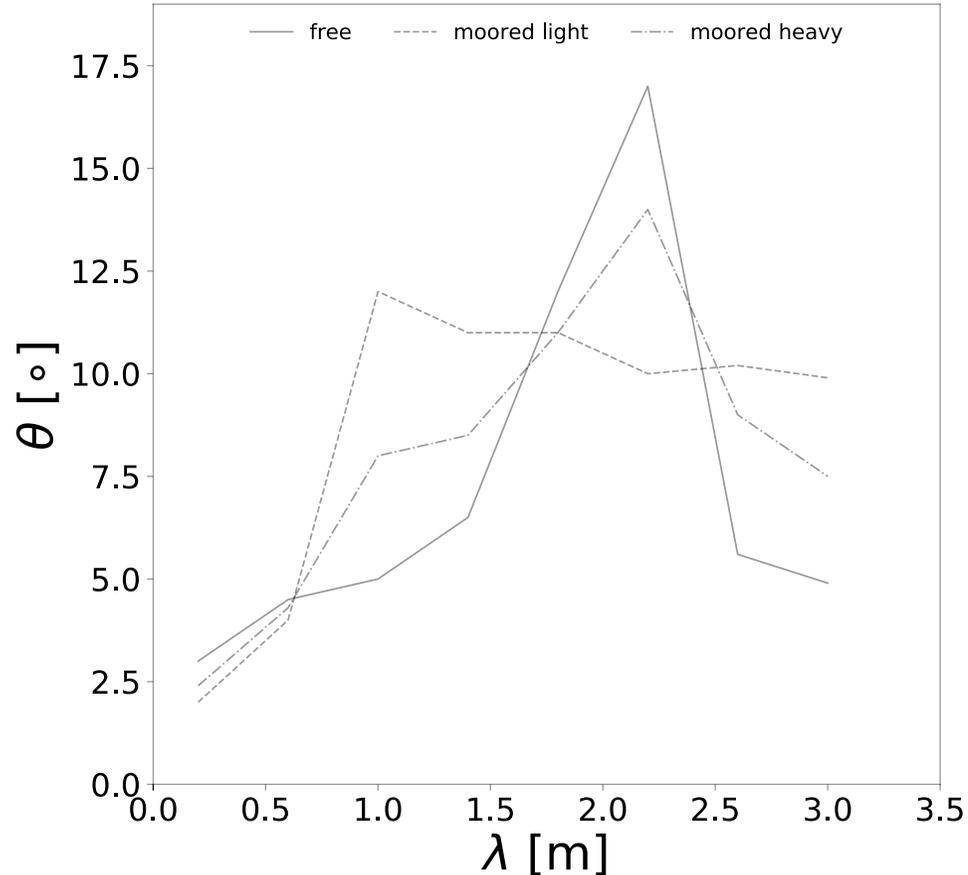
2D Moored Breakwater Motion in Waves

- Influence of different mooring systems on heave motion in waves of different wave lengths
 - Decreasing amplitude irrespective to weight
 - Changing frequency near eigenfrequency



2D Moored Breakwater Motion in Waves

- Influence of different mooring systems on pitch motion in waves of different wave lengths
 - Changing amplitudes over wave length
 - Changing frequencies at small wave lengths



Conclusion

- 6DOF Algorithm
 - Based on ghost cell immersed boundary method
 - Weakly coupled algorithm shows good convergence
- Mooring model
 - Coupled to 6DOF algorithm through forces
 - Quasi-static algorithm for efficient calculations
 - Suitable for slack and tensed configurations
- Mooring systems influence all motions of the breakwater in both amplitude and frequency
 - Surge: Weight decreases the motion
 - Heave: Amplitude of motion mainly depends on the angle at the mooring point
 - Pitch: Weight can significantly influence the behaviour at different wave lengths