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The State of the Art and Science of Coastal Engineering

Study on the Proudman resonance of waves induced by a moving atmospheric pressure disturbance

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Background

- **Meteotsunami**

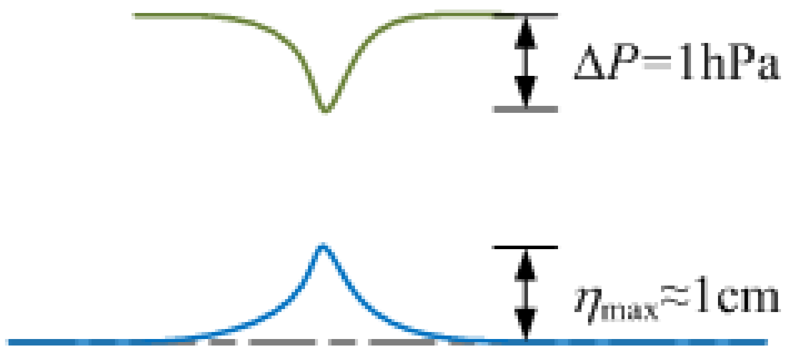
Beaches, bars and terraces in Majorca and Menorca were swallowed by 1.5m Meteotsunami on July 16, 2018

http://www.dailymail.co.uk/news/article-5959831/Mini-TSUNAMI-strikes-Spanish-resorts-Majorca-Menorca-flooding-beach-bars-terraces.html

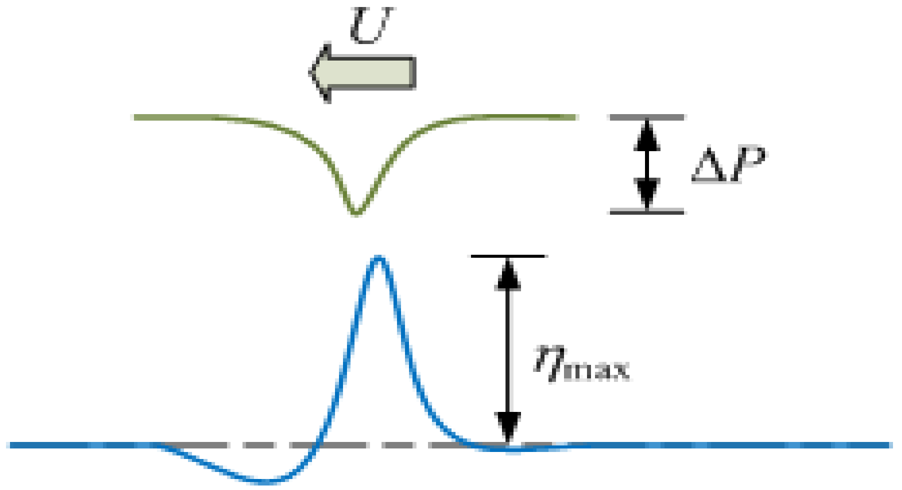
Background

- **Atmospheric pressure disturbances over water surface**

Proudman resonance ($Fr=U/c=1$)



Static equilibrium



Fast moving disturbance

Idealized problem and numerical model

Governing equation

Nonlinear shallow water wave equations

$$\frac{\partial \eta}{\partial t}+\frac{\partial (hu)}{\partial x}+\frac{\partial (hv)}{\partial y}=0$$

$$\frac{\partial hu}{\partial t}+\frac{\partial (huu)}{\partial x}+\frac{\partial (huv)}{\partial y}+gh\frac{\partial \eta}{\partial x}+\frac{h}{\rho}\frac{\partial P_a}{\partial x}+\frac{gu\sqrt{u^2+v^2}}{C^2}-h\left(\frac{\partial}{\partial x}\left[v_e\frac{\partial u}{\partial x}\right]+\frac{\partial}{\partial y}\left[v_e\frac{\partial u}{\partial y}\right]\right)=0$$

$$\frac{\partial hv}{\partial t}+\frac{\partial (hvu)}{\partial x}+\frac{\partial (hvv)}{\partial y}+gh\frac{\partial \eta}{\partial y}+\frac{h}{\rho}\frac{\partial P_a}{\partial y}+\frac{gv\sqrt{u^2+v^2}}{C^2}-h\left(\frac{\partial}{\partial x}\left[v_e\frac{\partial v}{\partial x}\right]+\frac{\partial}{\partial y}\left[v_e\frac{\partial v}{\partial y}\right]\right)=0$$

Chezy friction coefficient $C=R^{1/6}/n$

- ADI (Alternating direction implicit difference algorithm)

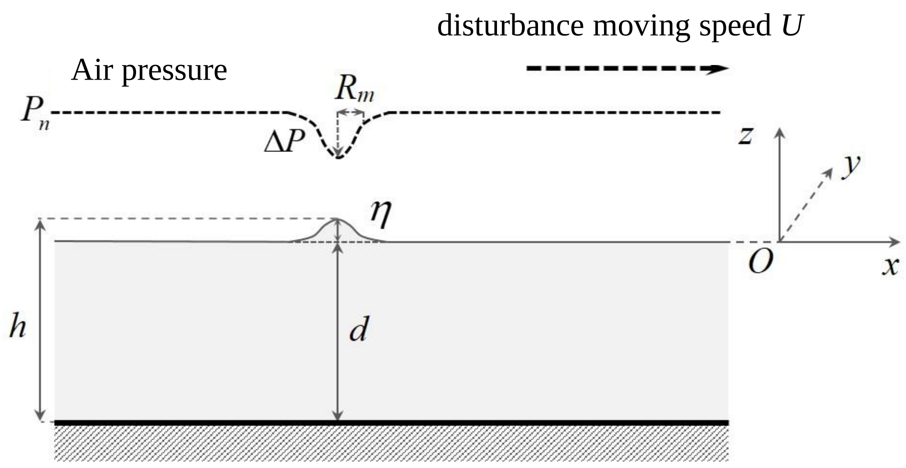


Illustration of the idealized physical problem

$$P_a=P_n-\Delta P\left[1-\exp\left(-\frac{R_m}{\sqrt{x^2+(y-Ut)^2}}\right)\right]$$

P_n : neutral atmospheric pressure
ΔP : central pressure drop
R_m : radius / spatial scale

- Large computing domain
- Sponge layer
- Mesh size smaller than $1/20R_m$

Accumulation of energy during wave evolution

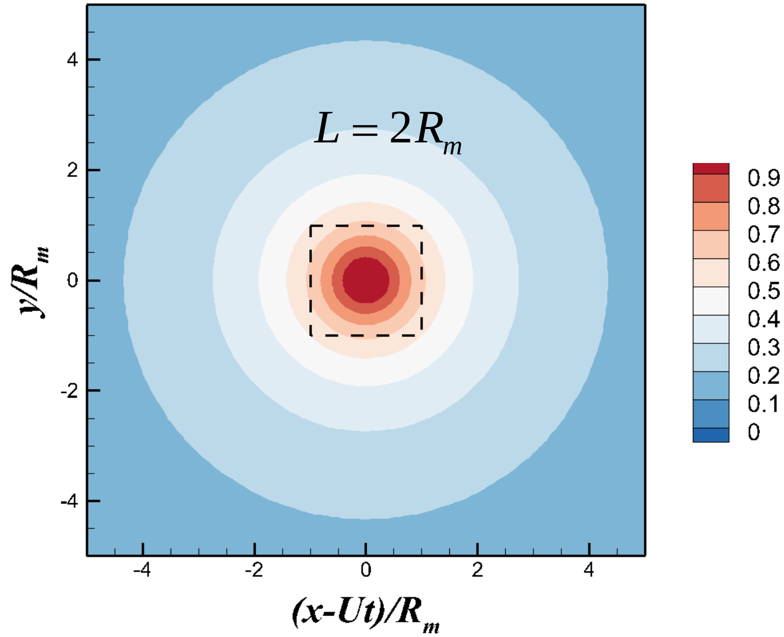
Potential Energy and kinetic energy

$$E_T = E_K + E_P = \int_S e_K \mathrm{d}A + \int_S e_P \mathrm{d}A$$

$$e_K = \int_{-d}^{\eta} \left(\frac{1}{2} \rho |\mathbf{u}|^2 \right) \mathrm{d}z$$

$$\approx \frac{1}{2} \rho \left(u^2 + v^2 \right) \left(\eta + d \right)$$

$$e_P = \int_{-d}^{\eta} \left(\rho g z \right) \mathrm{d}z + \frac{1}{2} \rho g d^2 = \frac{1}{2} \rho g \eta^2$$



Work done by pressure

$$Q_p = \int_S q_p \mathrm{d}A$$

$$q_p = \left(P_a - P_n \right) \frac{\partial \eta}{\partial t}$$

Energy spreading away

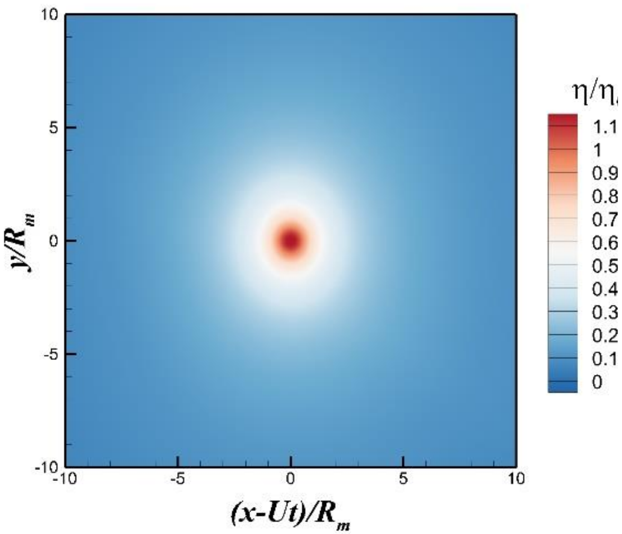
$$Q_f = \int_S \nabla \cdot \mathbf{\Phi} \mathrm{d}A = \int_\Gamma \mathbf{\Phi} \cdot \mathbf{n} \mathrm{d}s$$

$$\mathbf{\Phi} = \int_{-h}^{\eta} \mathbf{u} \left(\frac{1}{2} \rho |\mathbf{u}|^2 + p + \rho g z \right) \mathrm{d}z$$

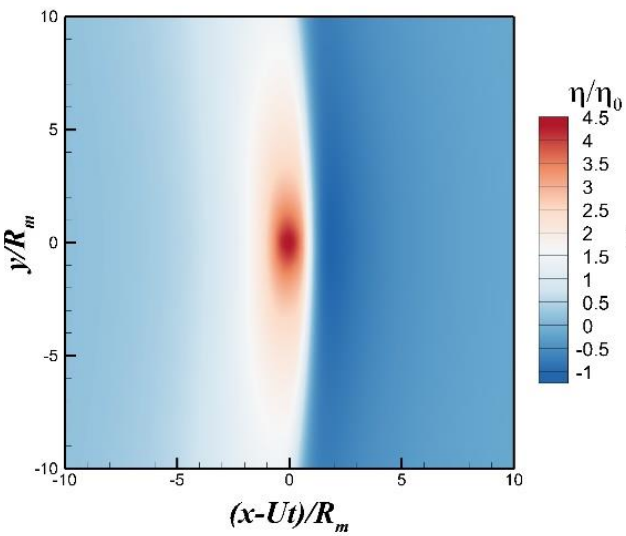
$$\approx \left(\overline{u}\mathbf{i} + \overline{v}\mathbf{j} \right) \left[\frac{1}{2} \rho \left(\overline{u}^2 + \overline{v}^2 \right) + \left(P_a - P_n \right) + \rho g \eta \right] \left(\eta + d \right)$$

Wave pattern caused by moving pressure disturbances

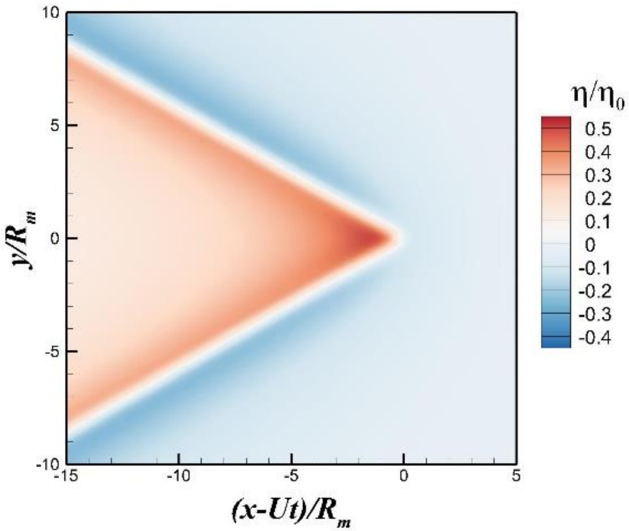
***Fr*=0.5**



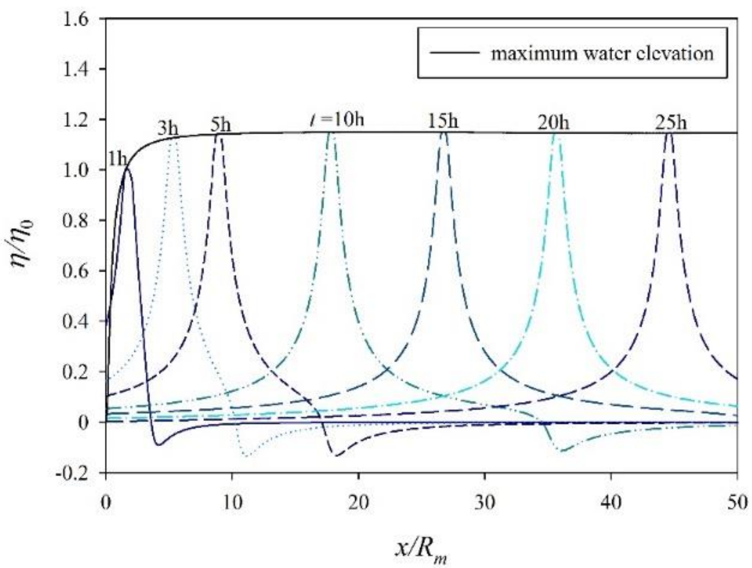
***Fr*=1.0**



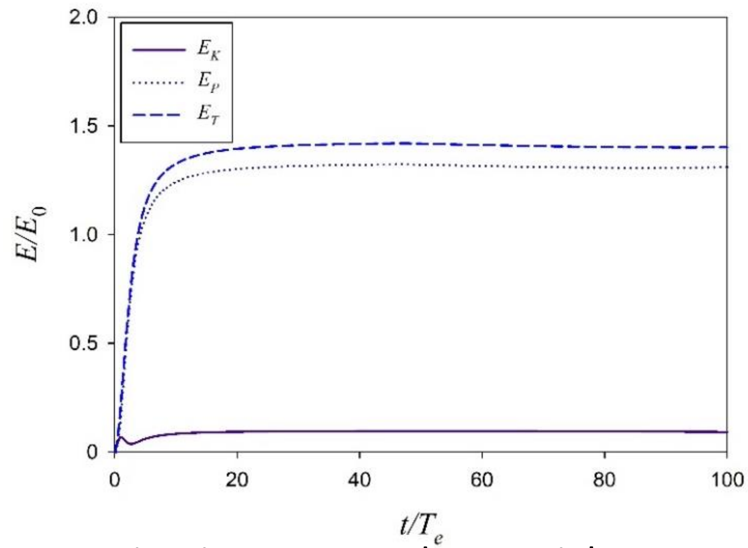
***Fr*=2.0**



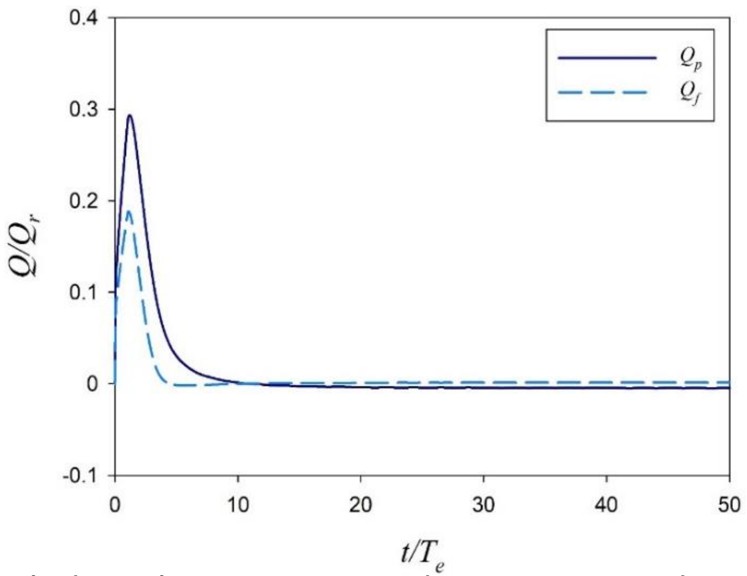
Wave evolution and energy accumulation Fr=0.5



Water elevation along the trajectory of pressure center at different time

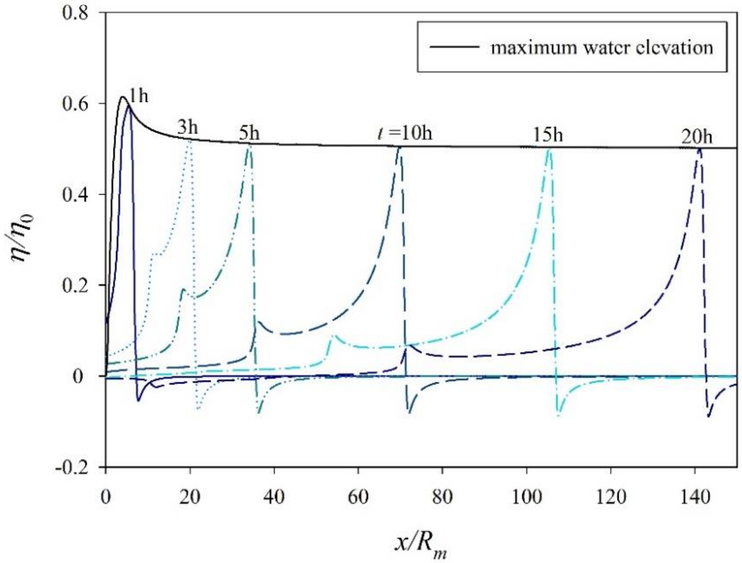


Kinetic energy and potential energy

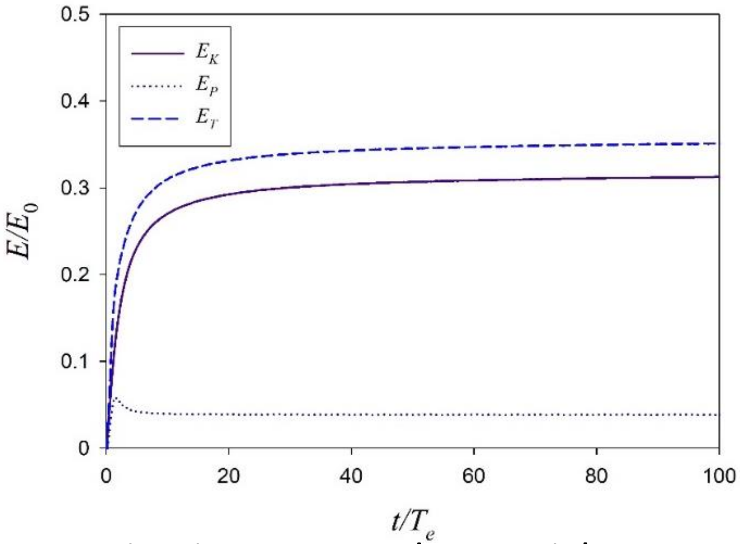


Work done by pressure and energy spreading away

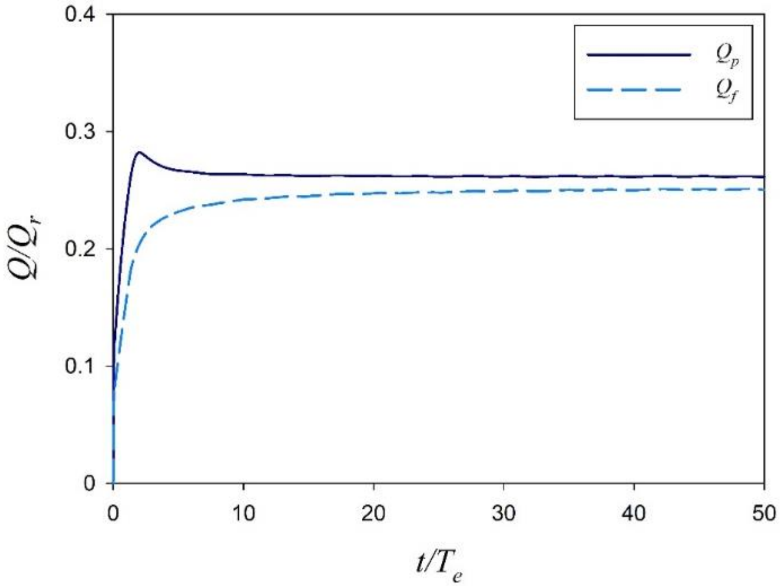
Wave evolution and energy accumulation Fr=2



Water elevation along the trajectory of pressure center at different time

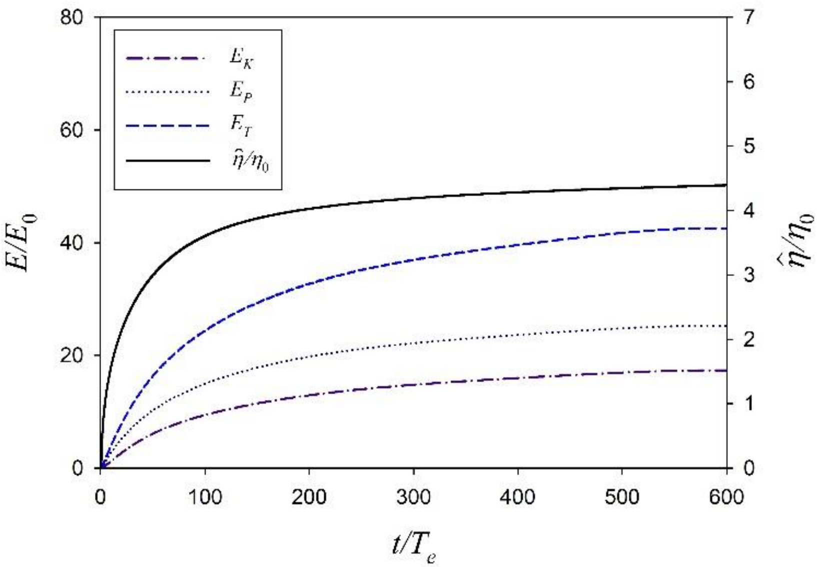


Kinetic energy and potential energy

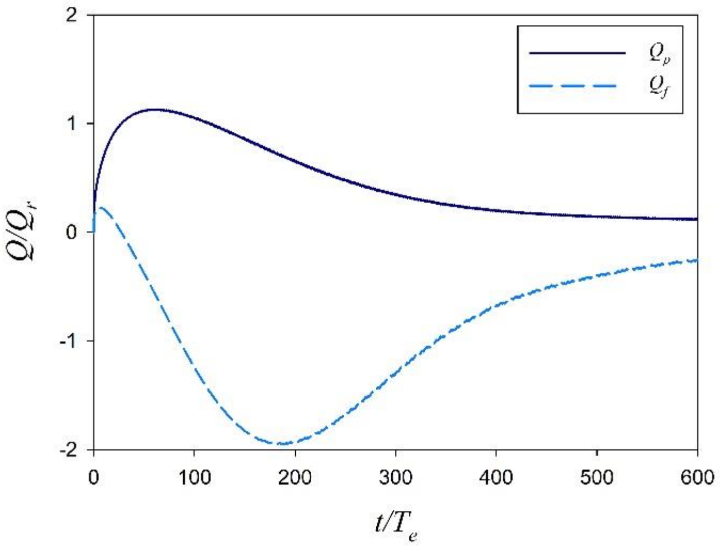


Work done by pressure and energy spreading away

Wave evolution and energy accumulation Fr=1

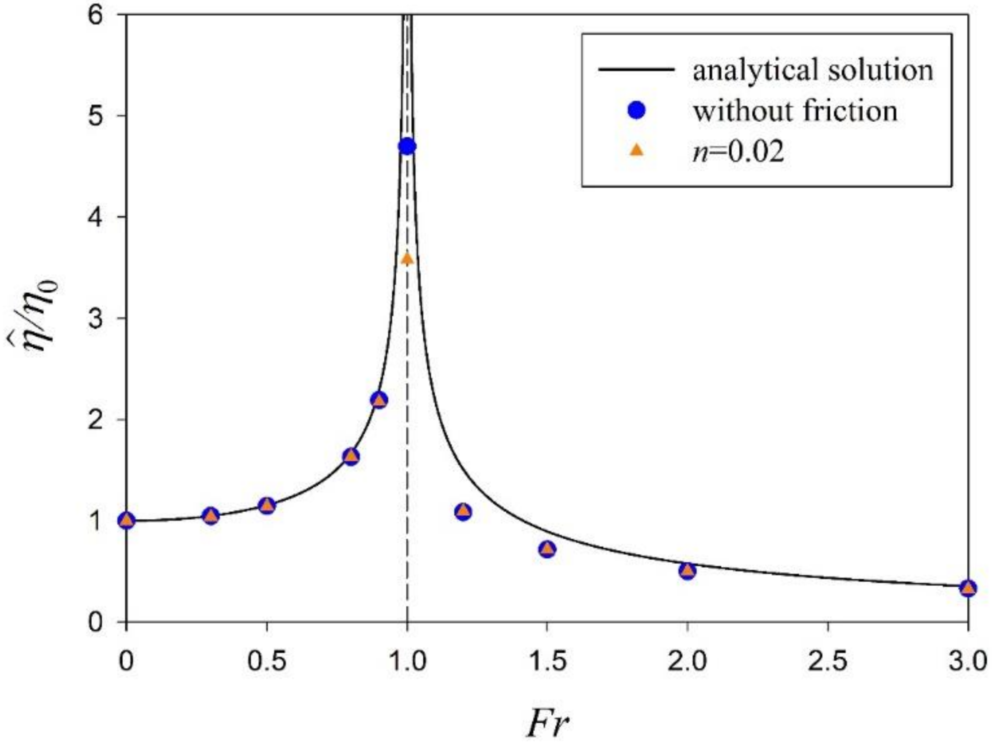


Variation of kinetic energy and potential energy



Variation of work done by pressure and energy spreading away

Maximum water elevation vs Fr



Leading part of analytical solution

$$\eta/\eta_0 = \frac{1}{\sqrt{\left|1-(U/c)^2\right|}}$$

- **The relative maximum water elevation increase with Fr when Fr<1 and then decrease when Fr>1.**
- **The maximum water elevation at Fr=1 depends on many factors.**

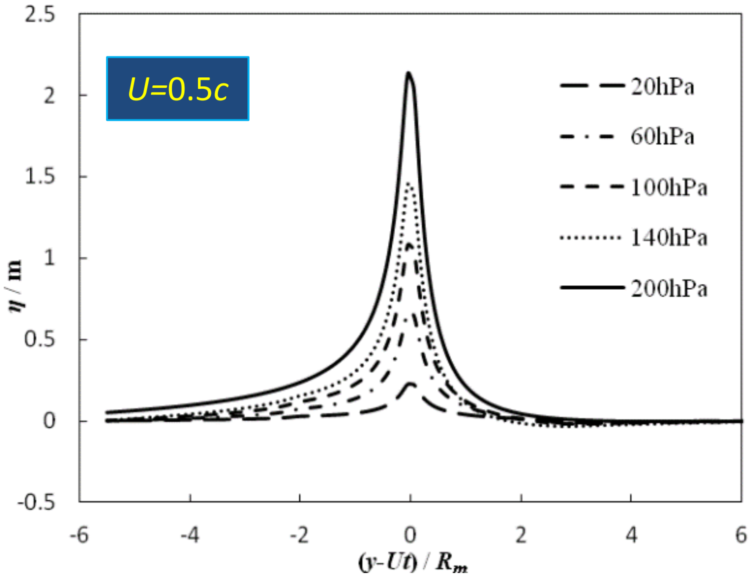
Contributions of each term in shallow water equations

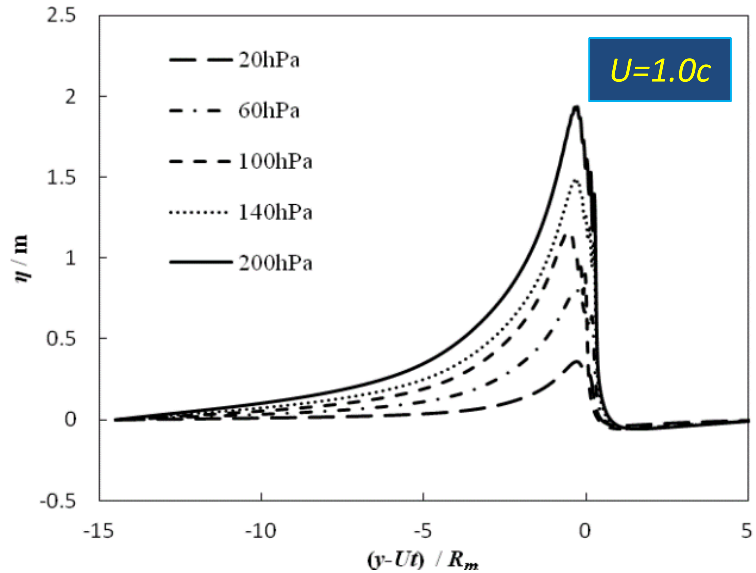
No.	Numerical case in contrast with the benchmark case	Difference
1	Convective term ignored	$+0.07\eta_0$
2	Bottom friction n=0.01	$+0.48\eta_0$
3	Bottom friction ignored	$+0.76\eta_0$
4	Water viscosity ignored	$+10^{-3}\eta_0$

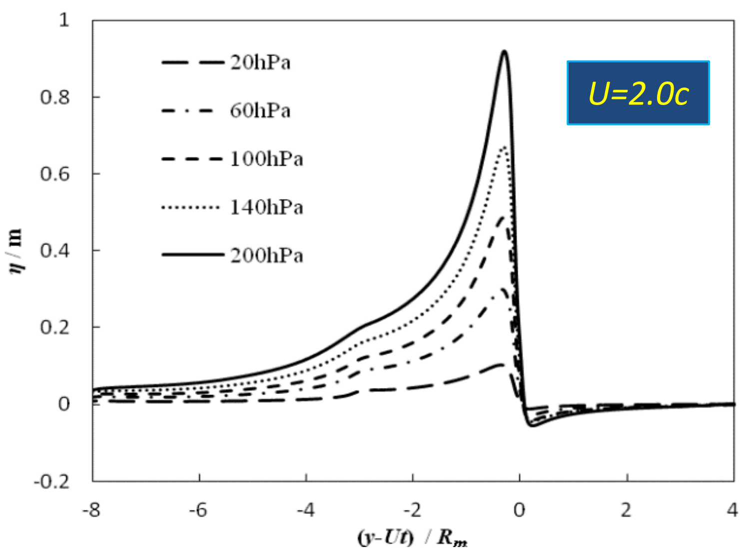
Benchmark case: R_m=10km, ΔP=20hPa, d=10m, U=1.0c, n=0.02, v_e=10^{-6}m^2/s, η=2.92 η_0

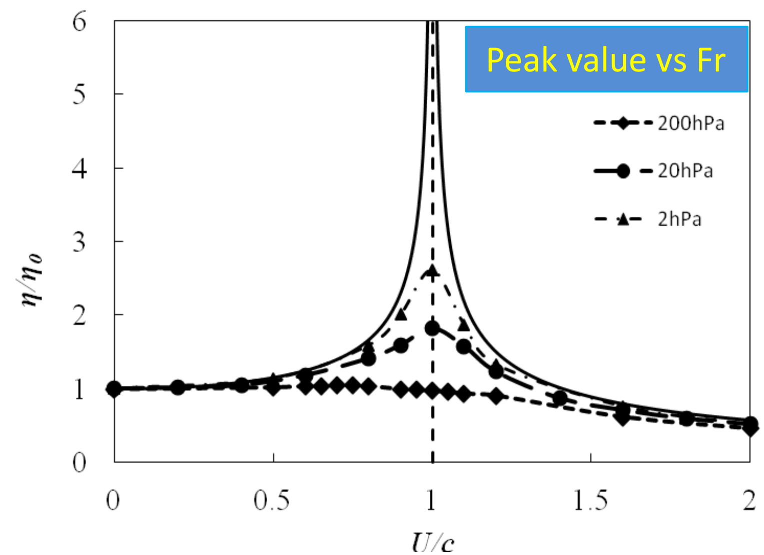
Influences of disturbance parameters

- **Central pressure drop ΔP**



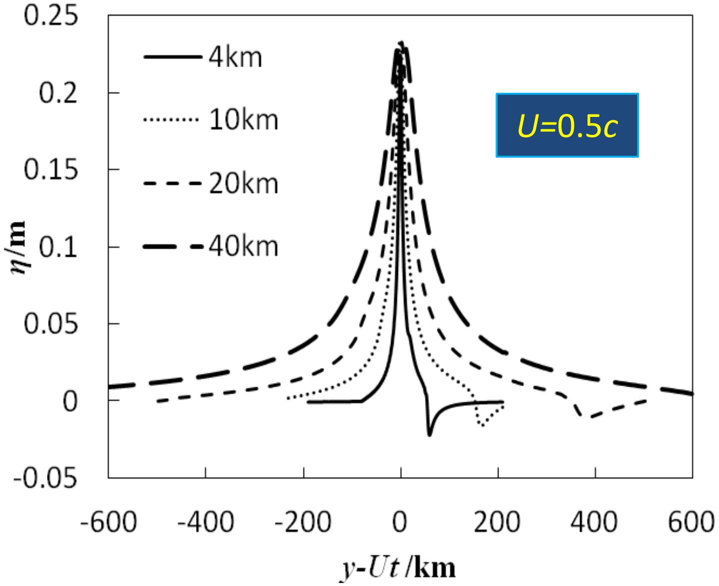


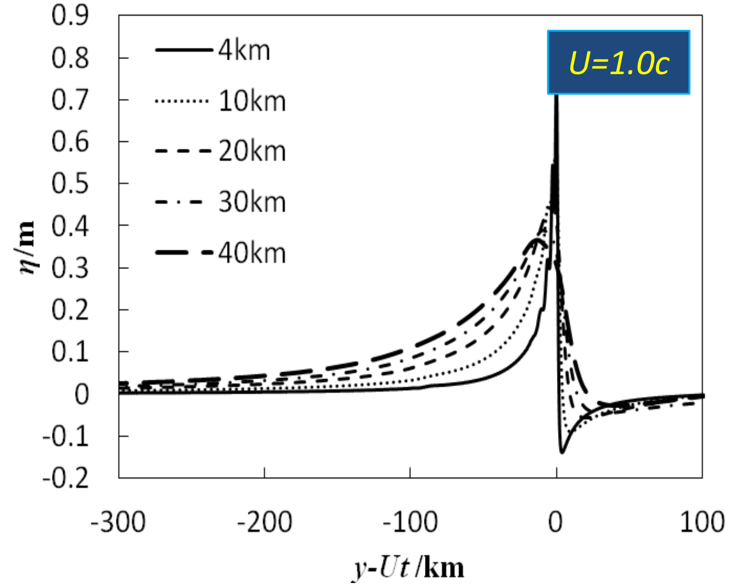


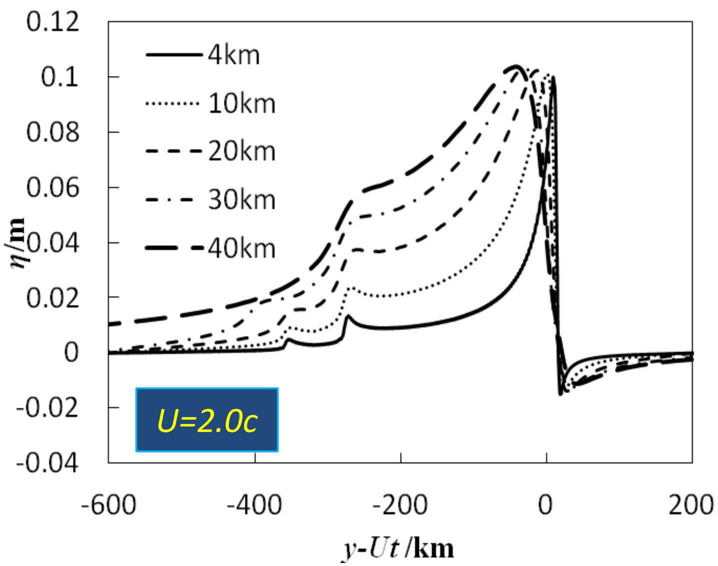


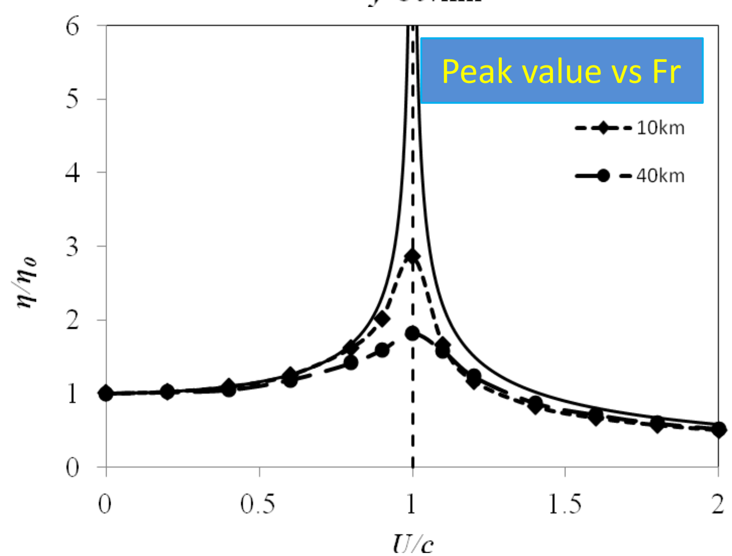
Influences of disturbance parameters

- **Radius of the pressure disturbance R_m**









Conclusions

- Based on the nonlinear shallow water wave model, the forced wave induced by an atmospheric pressure disturbance have been studied.
- The characteristics of energy accumulation within the central region moving along with pressure disturbance are shown.
- Bottom friction has larger impact on the maximum water elevation, and its impact is only significant when Fr is close to 1.
- The maximum water elevation when Fr=1 is approximately in proportion to the central pressure drop, and slightly affected by the spatial scale of pressure disturbance. A pressure disturbance with smaller spatial scale and smaller central pressure drop gives a larger η/η_0.

Thanks for your attention!