**FREQUENCY DISPERSION IN DEPTH-INTEGRATED MODELS THROUGH MACHINE LEARNING SURROGATES**

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INTRODUCTION

Boussinesq- type wave models have the accuracy to resolve wave propagation in coastal zones, having the ability to capture nearshore dynamics that include both nonlinear and dispersive effects for relatively short waves. The accuracy of Boussinesq type models over their counterparts which utilize the non- linear shallow water (NLSW) equations provides a clear advantage in studying nearshore processes. However, the computational expense of finding the Boussinesq solution over the NLSW solution hinders fast and/ or real time simulation using Boussinesq type models.

MACHINE LEARNING WAVE MODEL

To capture the dispersive effects in the Boussinesq solution while maintaining the computational efficiency of the NLSW solution, we propose a numerical scheme that solves the non- linear shallow water equations using traditional methods but uses a data- driven machine learning (ML) surrogate to “add” frequency dispersion. The ML component corrects for the difference in the NLSW and Boussinesq solution, effectively providing a close approximation to the Boussinesq solution while incurring the computational expense of a nonlinear shallow water solution. This machine learning correction is a two-dimensional convolutional neural network (CNN) trained on the differences in the depth-integrated mass fluxes in both x and y directions as well as the difference in free surface elevation between the NSLW and Boussinesq solution for a variety of input conditions.

METHODS

The inputs to the CNN at every analysis point are derived from the values necessary in numerically solve the extended Boussinesq equations at each time step in a numerical predictor- corrector time marching scheme. The CNN contains the structure to approximate the Boussinesq solution from the NLSW solution. Inputs to the CNN include x,y location within the computational domain, bottom elevation, free surface elevation, depth integrated mass fluxes in x and y, and bottom friction terms.

RESULTS AND FUTURE WORK

Initial results of this CNN on a 1D model provided an RMSE between the numerically calculated free surface elevation and CNN-estimated free surface elevation of <1% when tested on 2000 timesteps from a testing dataset independent from the CNN training and validation data. This simple case uses with solid wall boundary conditions and a simple Gaussian initial condition over a constant water depth. Subsequent tests of the simply trained model on varying bathymetry and initial conditions, including multiple Gaussians and sine waves, provided similar results. At this conference, we will present the results of our two-dimensional model and demonstrate the efficacy of this method on a variety of common boundary conditions and initial conditions.

The ultimate objective of this work is to allow coastal wave and flow simulations the ability to use a NLSW solver as the physical and numerical foundation, but correct that foundation with computationally efficient ML modules. These modules will approximate second-order corrections such as frequency dispersion, wave breaking, internal source wave generation, and possibly absorbing boundary conditions.

Chart, line chart

Description automatically generated

Figure 1 – CNN predicted difference between the NLSW solution and the Boussinesq solution for 1000 timesteps past training data on a 1D grid.

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