CHAPTER 8

WATFR WAVES DUE TO A LOCAL DISTURBANCE

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Abstract - A model investigation of the characteristics of waves generated by a local disturbance was made in order to obtain comparison with the theories of UNOKI and NAKANO (1953) and KRANZER and KELLER (1955).
The two-dimensional model for the case of initial local elevation or depression of uniform height of the water surface showed that certain wave characteristics such as phase periods and "interference" pattern could be described reasonably well within certain limits of water depth and height and extent of the disturbed area. Beyond those limits the leading part separated from the generated wave pattern as a solitary wave or a more complicated wave system. For a certain range of conditions the leading part was preceded by a bore during the first portion of the travel.

INTRODUCTION.

The mathematical studies on the properties of surface waves generated by a local disturbance is a classical problem. It finds its application in the prediction on the wave motion resulting from an underwater seismic disturbance (tsunami) and it has recently come into prominence in the description of the wave motion resulting from an artificial near-surface, or underwater, explosion.
The mathematical description is dependent on a rather schematized assumption of the initial conditions. Most of the derived equations have never been physically checked in a systematic way as to their reliability in practical applications.

The generation of surface waves due to a local disturbance can be considered under three groups:

A. Initial elevation or depression of the surface without initial velocity.

B. Undisturbed surface with an initial distribution of a surface impulse.

C. Undisturbed surface with an initial distribution of a submerged impulse (underwater explosion).

According to these groups PRINS (1956) tabulated the authors, mentioning initial conditions, wave equations and wave characteristics. In the groups A and B the mathematical solutions are summarized from the authors LAMB (1945), UNOKI and NAKANO (1953), KRANZER and KELLER (1955) and PENNEY (1950), in group C from KIRKWOOD and SEEGER (1950) and FUCHS (1952).

Only a few model investigations on the wave performance due a looal disturbance are available. BRYANT (1950) compared the theor of PENNEY (1950) with experimental observations. Experiments on the wave generation by a surface impulse were done by JOHNSON and BERME (1949). A laboratory study of gravity waves generated by the moveme of a submerged body was done by WIEGEL (1955).

Prototype data are given in a comprehensive description in " EFFECTS OF ATOMIC WEAPONS" (1950) and on tsunami by UNOKI and NAKAN (1953).

In this paper some results of two-dimensional model investig tions for the case of initial local elevation or depression of unif height will be given. In the experiments the heights of the elevati or depression, the extent of the disturbed area and the water depth were varied. A comparison will be made with the theories of UNOKI a NAKANO (1953) and KRANZER and KELLER (1955). The leading part under oonditions not fitting the theories will be discussed.

EXPERIMENTAL EQUIPMENT AND PROCEDURE.

The model investigations were carried out in a flume one foot wide by sixty foot long. On the one end of the channel was placed a air-tight box of plexiglas with a sliding front wall, in which the water level could be elevated or depressed by means of a decreased increased pressure in the air compartment. By pulling the slide upwa in the shortest possible time it was possible to develop a free elevated or depressed area of uniform height with all the water particl effectively at rest, figure 1. The back wall of the box was conside to oause a total reflection and hence is the axis of symmetry of the system. At the opposite end of the channel a wave absorber was installed.

The extent L and the height Q of the elevations or depression and the water depth h were varied by steps:

h = 2.3, 0.5, 0.35 and 0.2 foot.

L = 1/3, 1 and 2 foot.

Q = ± 0.1, ± 0.2 and ± 0.3 foot.

The vertical movement of the water surface η was recorded as function of time simultaneously at five places along the ohannel wit a six-channel Brush Electronic Co. recorder by using parallel wire resistanoe wave gages (WIEGEL 1956), figure 2. On the same record th zero time was fixed by a resistance gage inside the box near the fro which indecated the drop, or rise, of the inside water level after pulling the slide. The charts present an x,t-plane, in which are projeoted the η,t-histories of the five gages, figure 4.

The movement inside the box and in its vioinity was filmed. T first stage of generation is shown in figure 3.

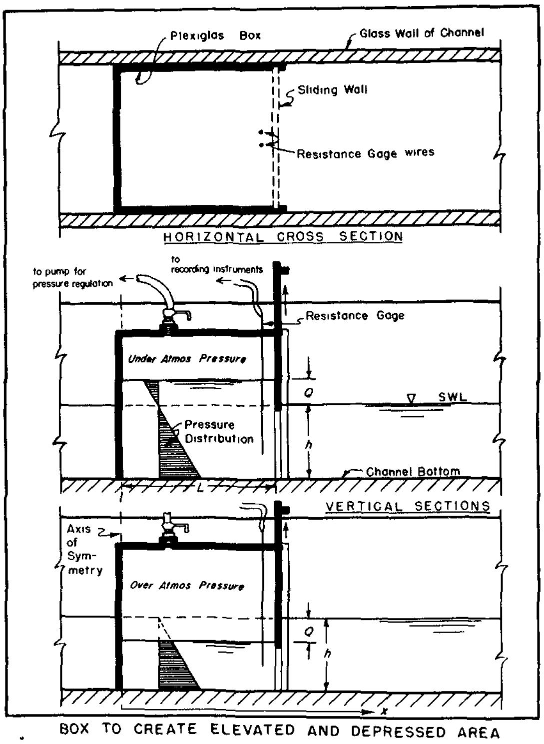

Fig. 1. Box to create elevated and depressed area.

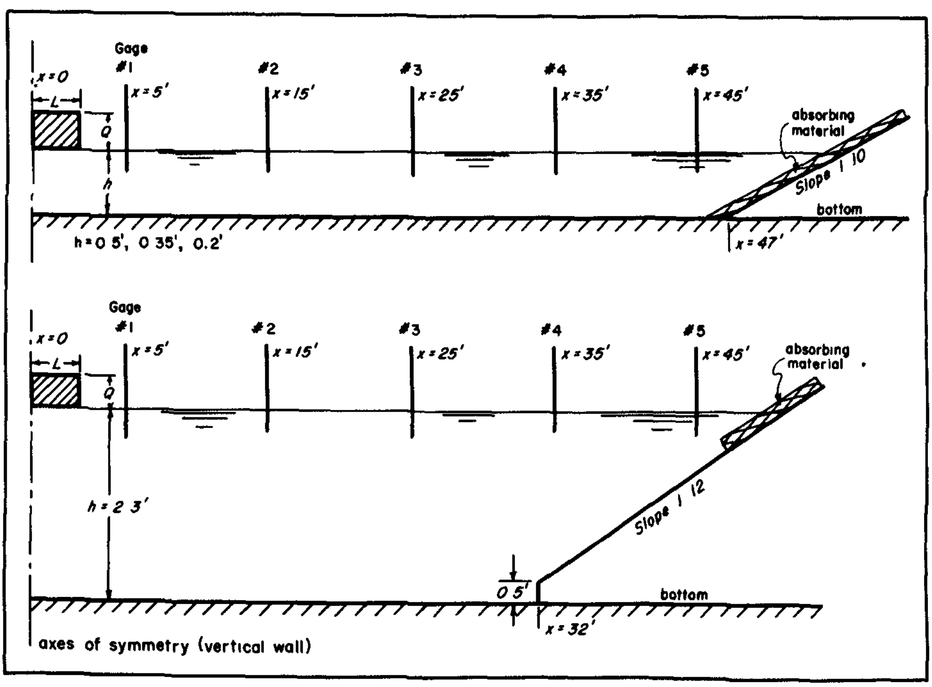

Fig. 2. Arrangement of model tests.

MATHEMATICAL DESCRIPTION AND COMPARISON WITH THE EXPERIMENTS.

The formulation of the problem in regard to the model invest gation was to describe the water surface elevation with respect to undisturbed water level as a function of place and time when a disturbance consisting of an uniform elevation or depression with a constant height Q extending over a length L in a two-dimensional system of water depth h was released.

In general $\eta = f(Q, L, h, x, t)$ (

Two cases given in the literature are applicable to the give conditions. One is the solution of the Cauchy-Poisson wave problem for the case of an initial elevation to a finite area in water of infinite depth by UNOKI and NAKANO (1953) in their approach for the description of tsunami waves. The other case is the description of surface waves produced by explosions in water of finite depth by KRANZER and KELLER (1955). A summary of their results adapted to th model investigations is given by PRINS (1956).

The simple form of the UNOKI-NAKANO equation illustrates in the best way the characteristics of the generated waves.

The equation $$\eta = \frac{4Q\,x^{\frac{1}{2}}}{\pi^{\frac{1}{2}} g^{\frac{1}{2}} t} \sin\left(\frac{gt^2}{4x} \cdot \frac{L}{x}\right) \cdot \cos\left(\frac{gt^2}{4x} - \frac{\pi}{4}\right) \quad (\text{:}$$

for $x \gg L$,

is composed of two periodic systems.

The phase period is defined by $\cos\left(\frac{gt^2}{4x} - \frac{\pi}{4}\right)$.

so that $$T = \frac{4\pi x}{gt}, \quad \lambda = \frac{8\pi x^2}{gt^2} \quad \text{and} \quad C = \frac{2x}{t} \left(= \sqrt{\frac{g\lambda}{2\pi}}\right) \quad (\text{:}$$

As the period T is dependent on x and t, we have to deal with dispersive waves. The phase period is independent of the height Q and extent L of the initial disturbance.

The term $\sin\left(\frac{gt^2}{4x} \cdot \frac{L}{x}\right)$ represents a relatively long periodic syste defining the amplitude variation of the waves with phase velocity. I will be referred to as the amplitude envelope term, with the qualities:

$$T_1 = \frac{4\pi x}{gt} \cdot \frac{x}{L}, \quad \lambda_1 = \frac{8\pi x^2}{gt^2} \cdot \frac{x}{2L} \quad \text{and} \quad C_1 = \frac{x}{t} \quad (4$$

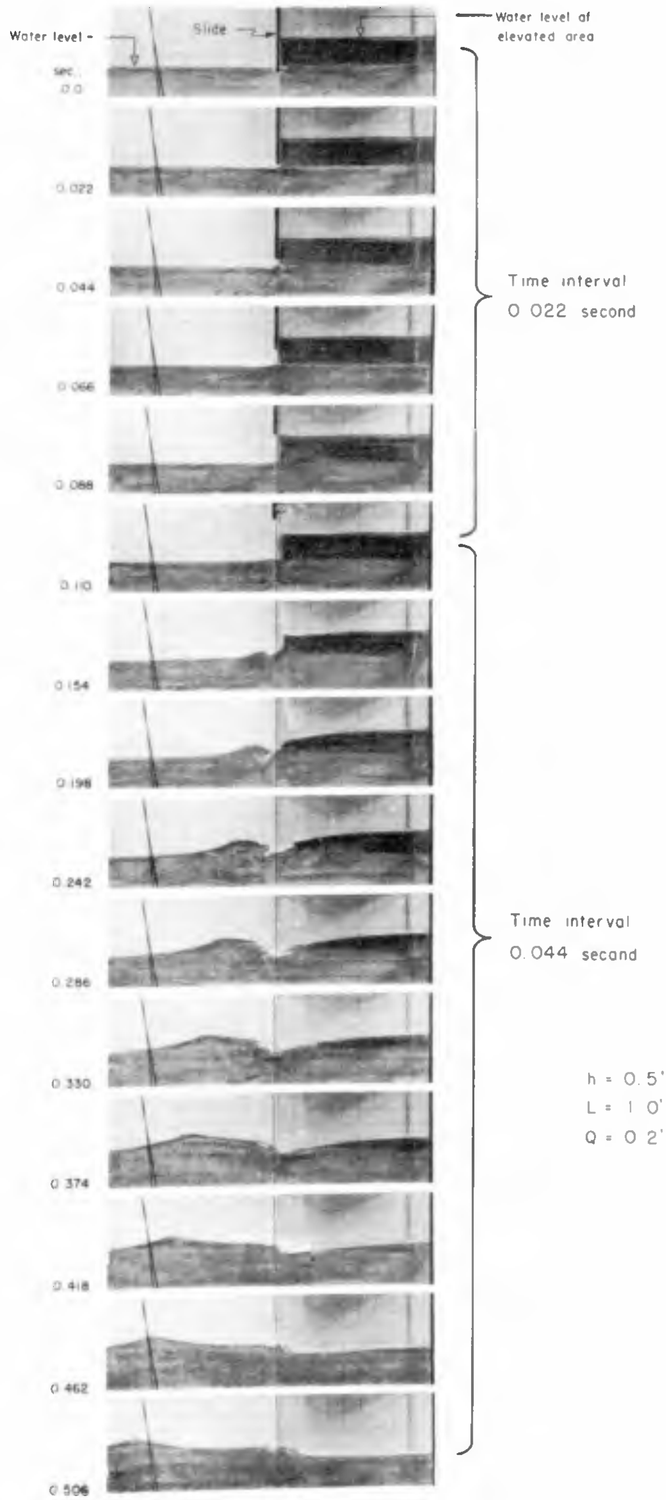

Fig. 3. Photographs showing early stages of wave generation.

The period T_1 also shows dispersive properties. It is inversely proportional to the extent L of the initial elevation.

At a certain position (x, t) the envelope velocity is one-ha the phase velocity. At infinite depth the group velocity is define likewise, so that

$$C_1 = \tfrac{1}{2}C = C_{gr} = \frac{x}{t} . \quad ($$

This equation can be written $x = C_{gr} t$ (6) which according KRANZER and KELLER is also valid for water of finite depth h, if th following expression is introduced:

$$C_{gr} = \tfrac{1}{2}\left[1 + \frac{4\pi h/\lambda}{\sinh 4\pi h/\lambda}\right] (\tanh 2\pi h/\lambda) \cdot C_0 \quad ($$

in which $C_0 = \dfrac{g}{2\pi} T.$ (

The formula $x = C_{gr} t$ shows that a wave reaching a certain place X with period T can be considered as a wave of constant lengt propagating with its group velocity from the origin to X.

The KRANZER-KELLER equation also shows relations with a dependency on the water depth h giving similar wave characteristics

The experimental data are obtained as η, t-curves at a fixed place. There is a definite distinction with respect to the leading part of the wave pattern, dependent on the ratios Q/h and L/h.

Within the limits $Q/h < 0.18$ and $L/h < 0.9$ the leading part had oscillatory characteristics and a reasonable agreement of the experimental data and the theory was found.

It was found that:

1. The generated wave pattern was of a dispersive character.
2. The variation of height and extent of the initial disturbance di not affect the phase velocities.
3. The phase periods were found to agree with the theory of KRANZER and KELLER, and for "deep water" with the theory of UNOKI and NAKANO.
4. The wave pattern shows an interference phenomenon, figure 4.
5. The position of the zero-points of the amplitude envelope curve was found to be well expressed by the theory of UNOKI and NAKANO and fairly well by the theory of KRANZER and KELLER.
6. The leading part of the wave pattern for the case of an initial elevation showed exactly the negative performance of the waves generated by an initial depression ($\eta_{elev} = - \eta_{depr}$), figure 6.
7. The amplitudes were found to be directly proportional to the height of the initial disturbance.
8. In considering the energy distribution within the wave pattern, together with the area of disturbance, it was noticed that when

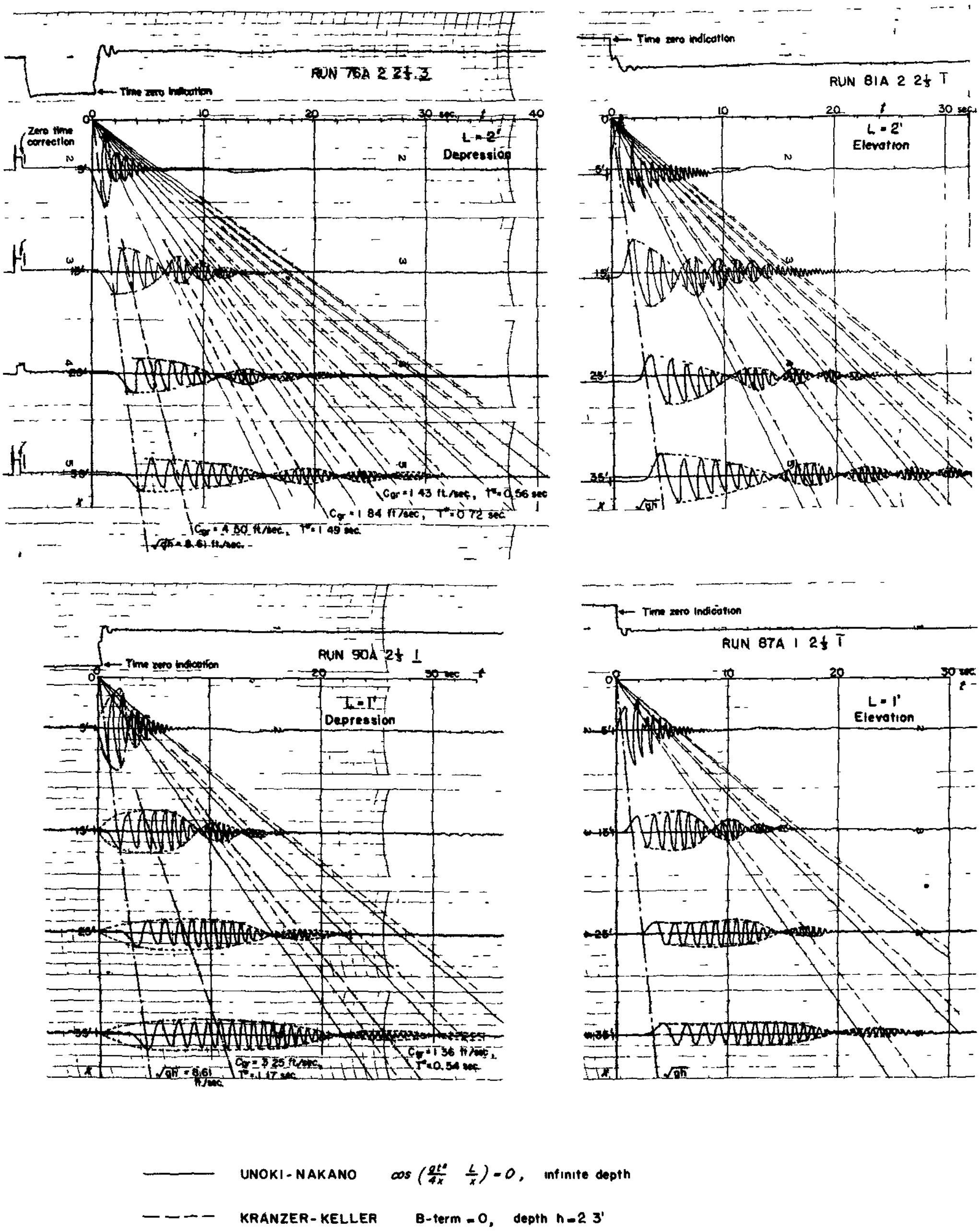

Fig. 4. Amplitude envelope zero-points, h = 2.3 ft.

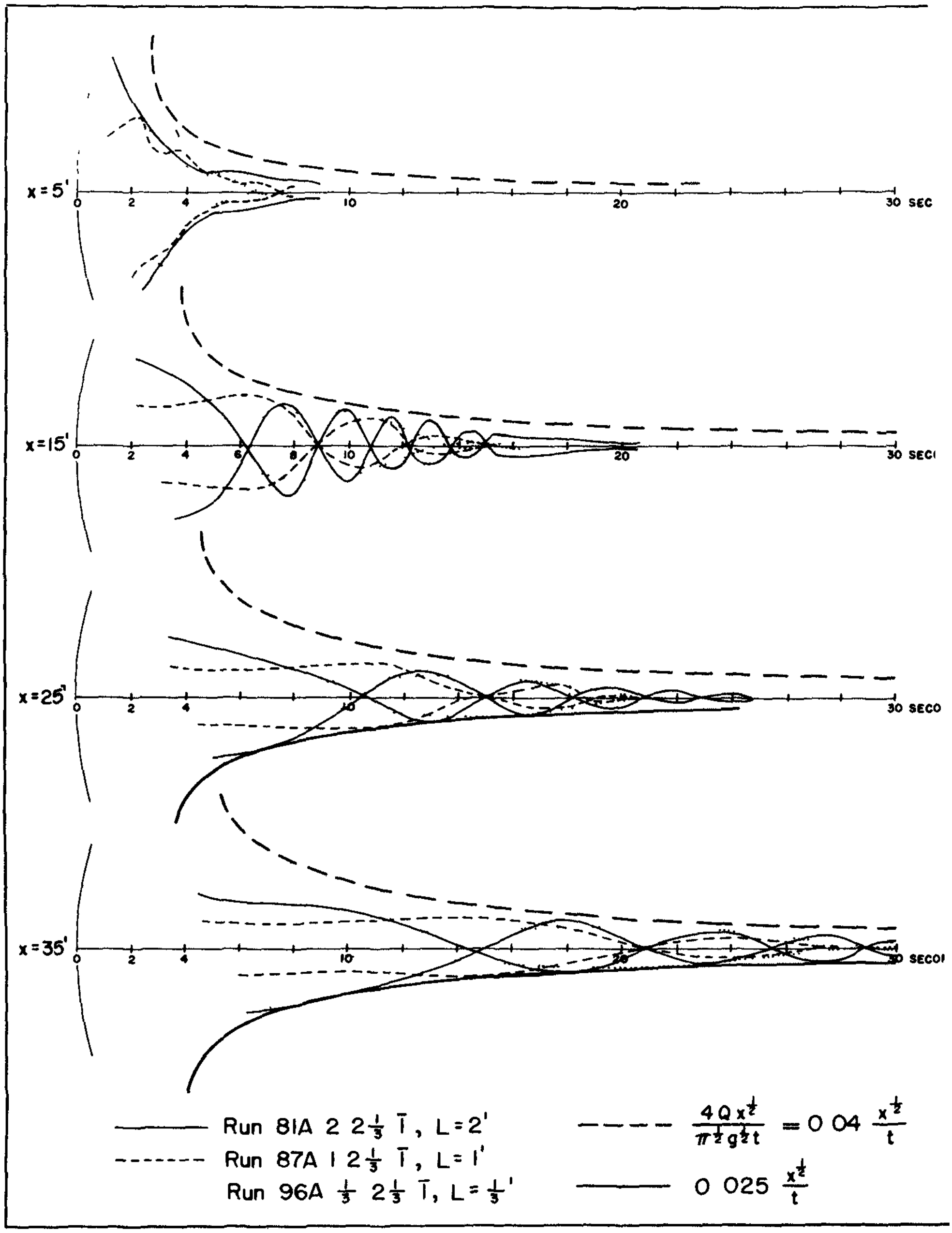

Fig. 5. Amplitude envelope curves with constant Q and variations of

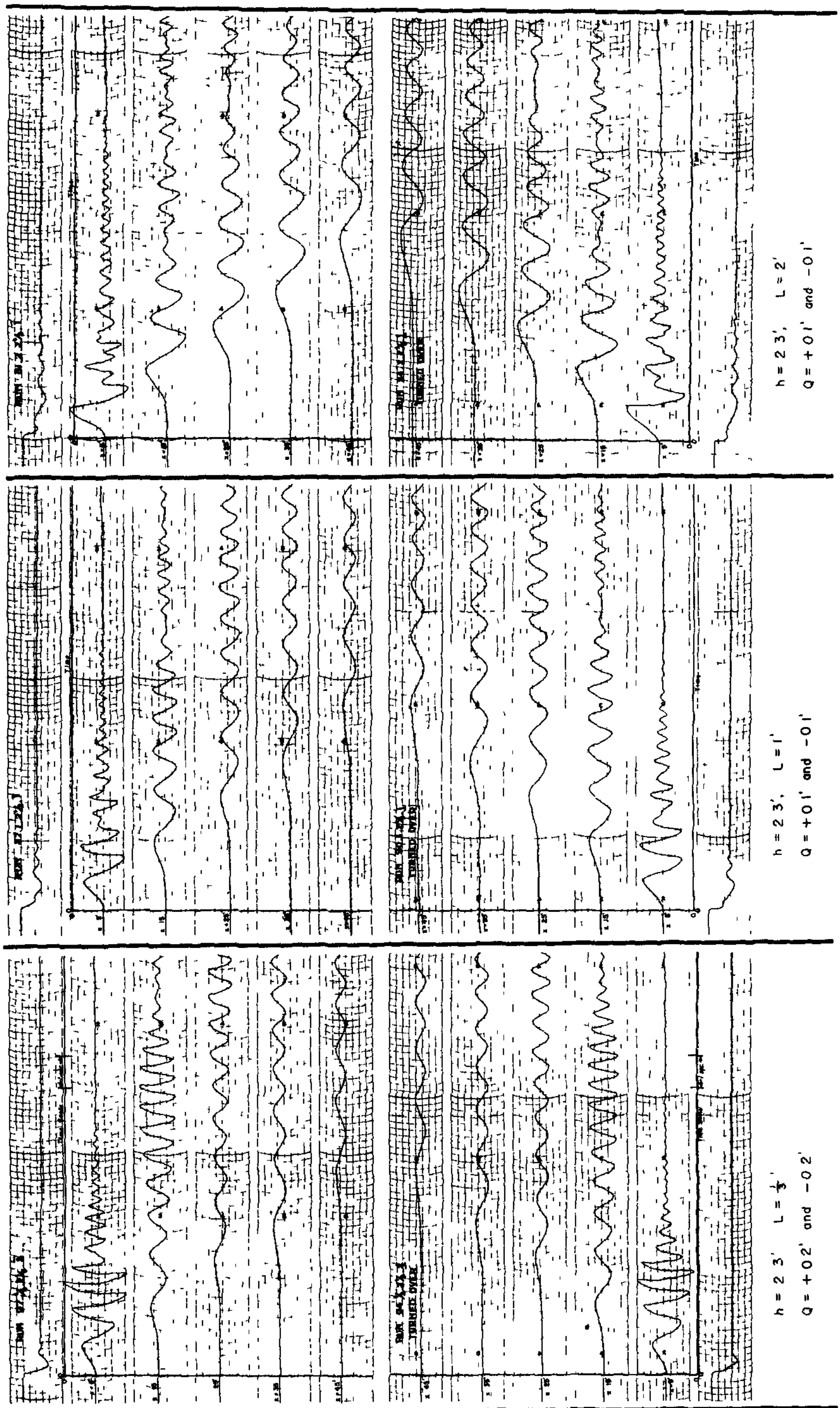

Fig. 6. Comparison of wave patterns produced by elevated and depressed areas.

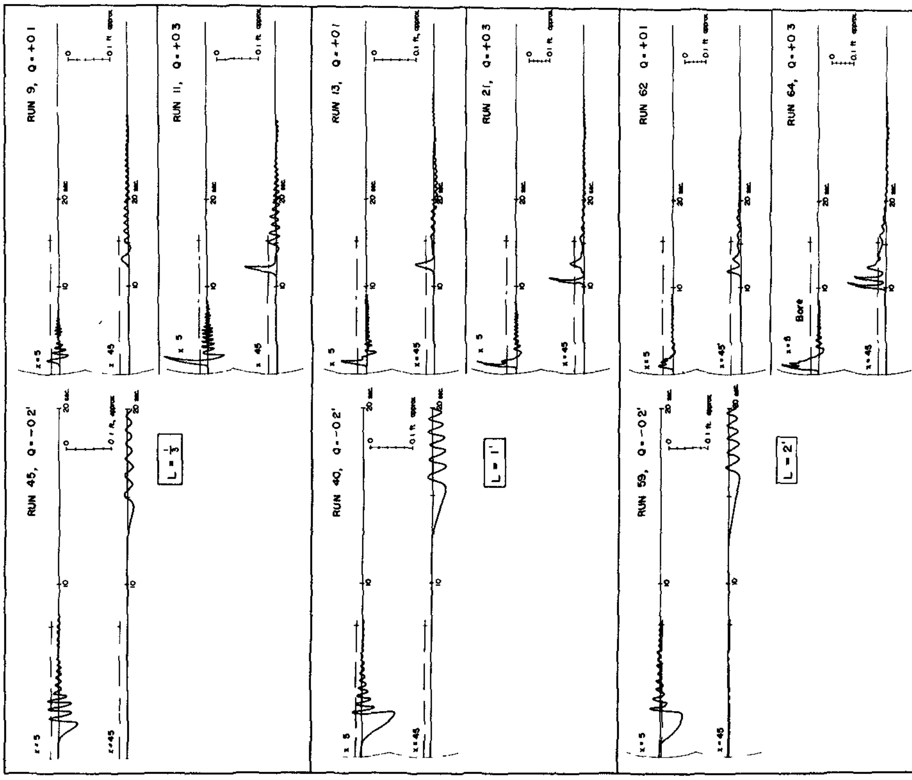

Fig. 8. Leading parts of the wave patterns at depth h = 0.25 ft

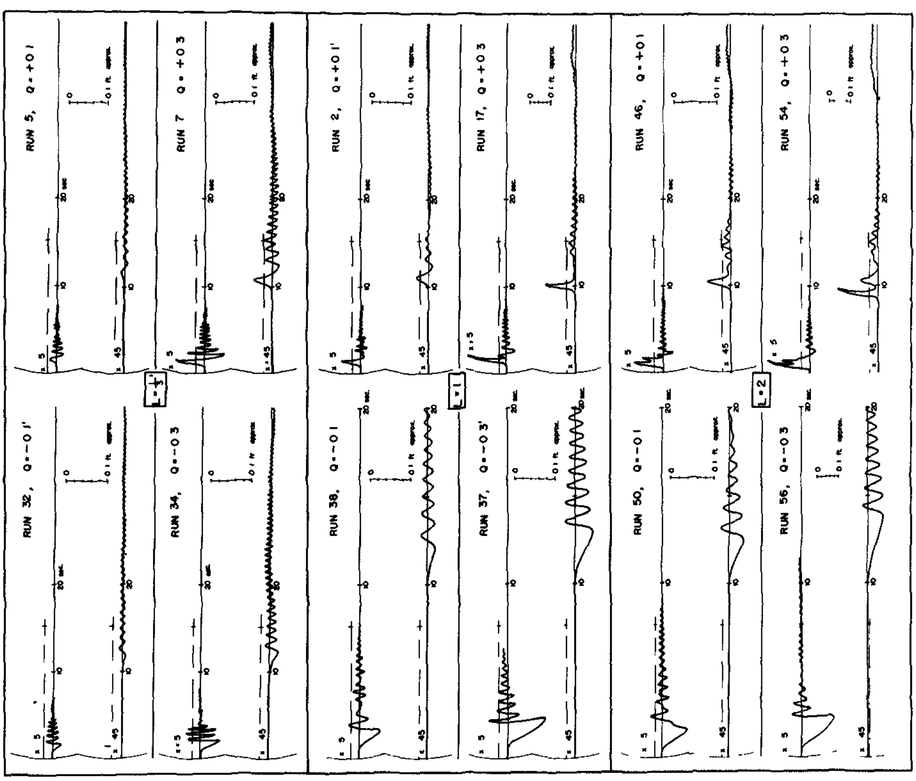

Fig. 7. Leading parts of the wave patterns at depth h = 0.50 ft

the disturbance was extended vertically, the character of the wave pattern was maintained with the amplitudes being proportional to the vertical change of disturbance. When the disturbance was extended horizontally, the wave pattern changed its appearence, but the amplitudes did not exceed those of the common envelope curve, which was independent of the variation of L, figure 5.

9. The wave amplitudes of the experiment in general were found to be smaller than the theoretical values.

CHARACTERISTICS OF THE LEADING PART WITH VARIATION OF Q/h AND L/h.

With respect to the dependency of the leading part on the ratios of Q/h and L/h four types could be distinguished:

1. Leading wave with oscillatory wave characteristics being part of the dispersive wave pattern (O), figure 6.
2. Leading wave with solitary wave characteristics with respect to its velocity of propagation followed by a trough connecting it with the dispersive wave pattern (ST), figure 7.
3. Leading wave being a single wave with solitary wave characteristics, separated from the dispersive wave pattern by a more or less flat part at the still-water-level (SS), figure 7 and 8.
4. The leading part being of complex form, which, while traveling outward, breaks up into a few waves with solitary wave characteristics, separated from the dispersive wave pattern (CS), figure 9.

For certain conditions either a bore (B) or a top bore (TB) occurred which did not seem to affect appreciably the type of leading wave.

In figure 10 the types of the leading part as a function of Q/h and L/h are given schematically, based upon the available model data for the case of an initial elevation.

In the following paragraphs some characteristics of the leading part are discussed more extensively.

In the case of waves with oscillatory characteristics the characteristics of the leading part are in close accordance with the above mentioned theories. Complete similarity is found between the performance due to an initial elevation and an initial depression, figure 6. The amplitude is direct proportional to the height of the disturbance, figure 11.

With increasing Q/h and L/h in the case of an initial elevation the leading wave tends to the solitary wave form and finally reaches the stage where its shape closely approximates the Boussinesq wave form, and it propagates with the celerity
$C = \sqrt{g(h + \eta)}$. Due to its larger velocity of propagation it separates from the wave train. The wave train still agrees reasonably well with the theory.

There is a region, which depends mainly on the ratio Q/h, where

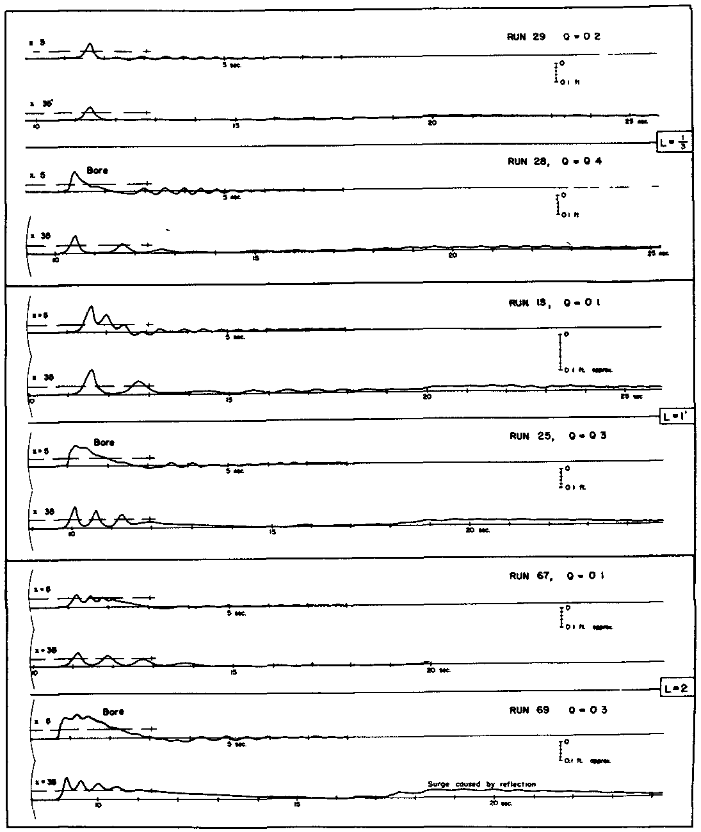

Fig. 9. Leading parts of the wave patterns at depth = 0.2 ft.

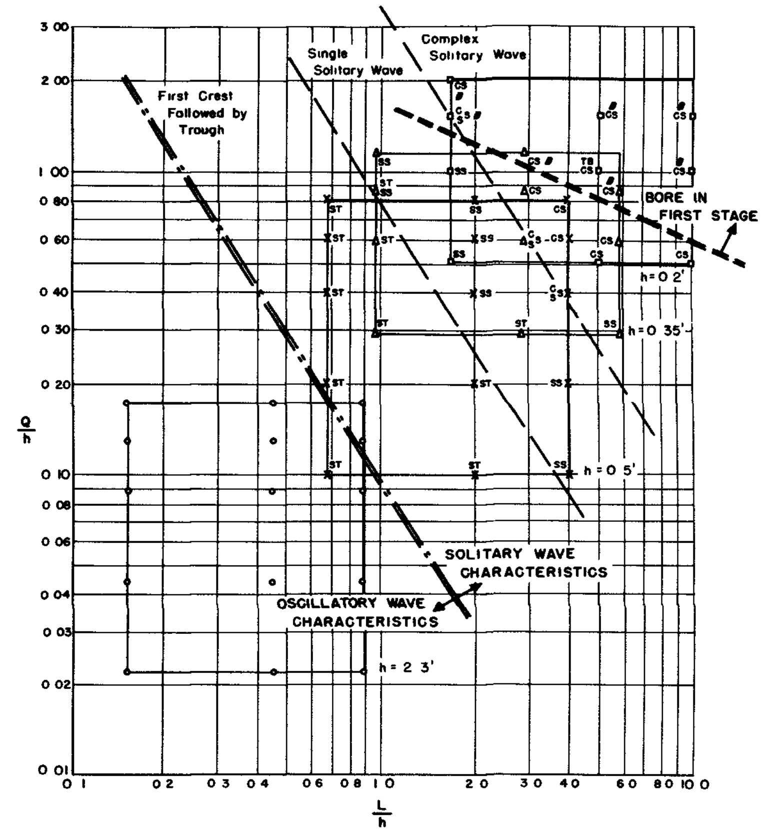

Fig. 10. Relationship between $\frac{L}{h}$, $\frac{Q}{h}$ and the characteristics of the leading wave in the case of

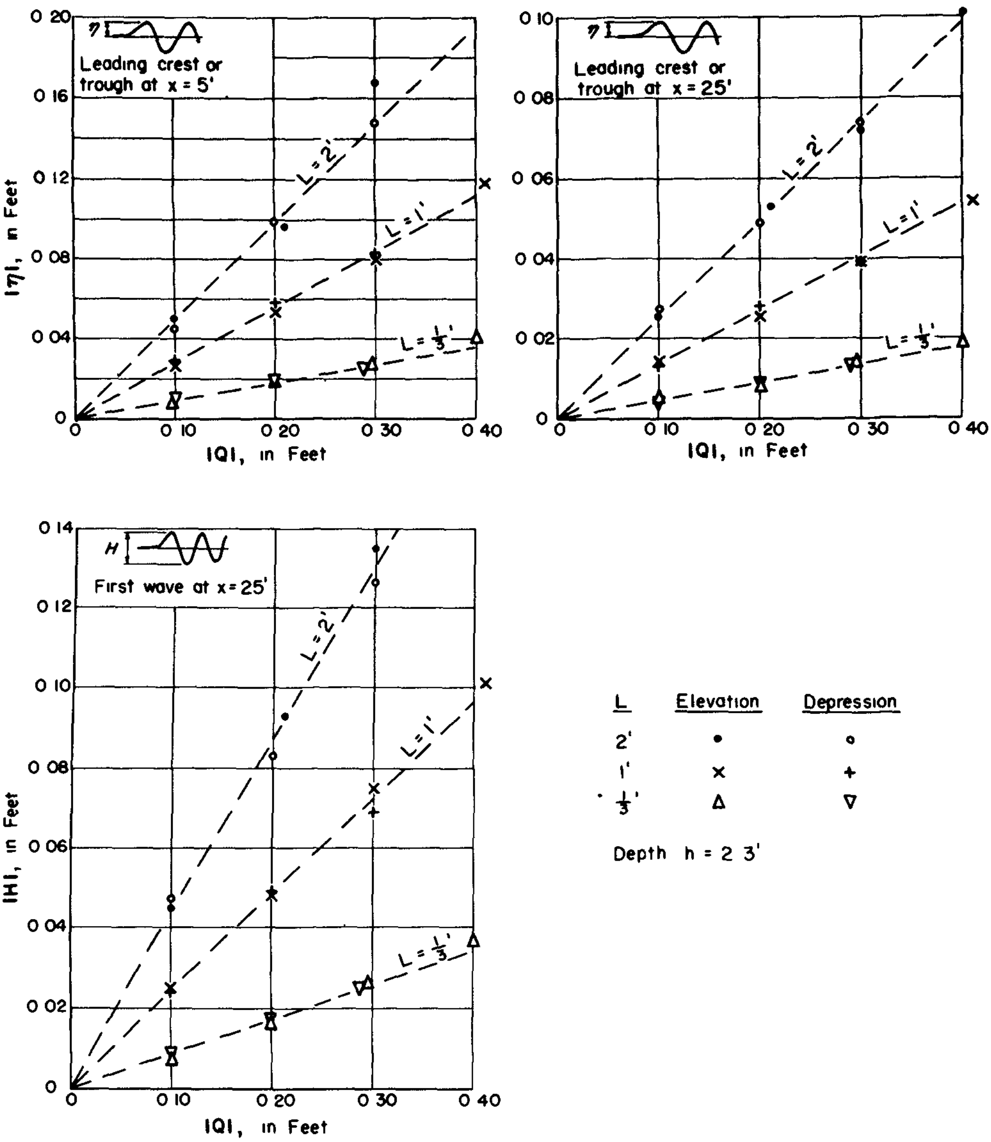

Fig. 11. ηQ - relation at constant x, t, and L.

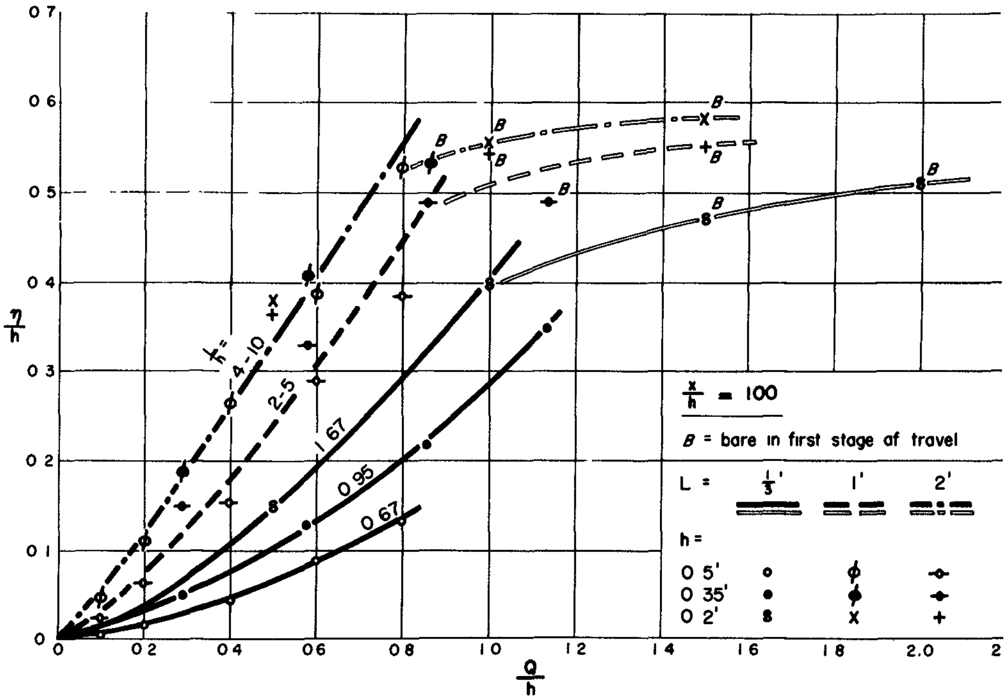

Fig. 12. η, Q - relation at x/h = 100 of leading part with solitary wa characteristics.

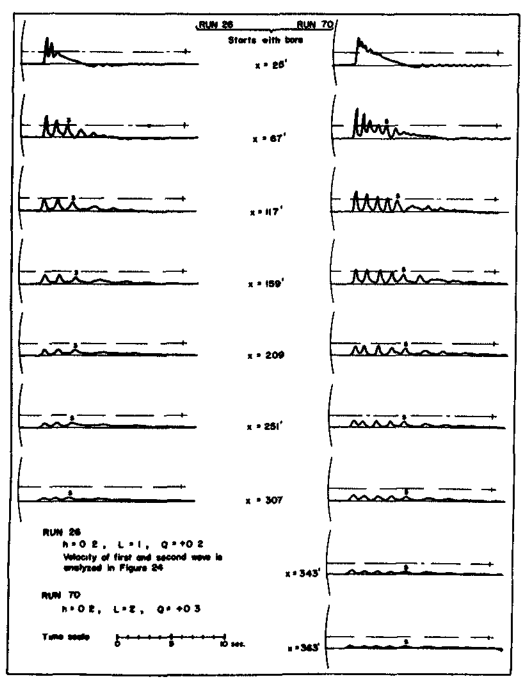

Fig. 13. "Complex Solitary Wave" group.

during the first stages a bore occurs. (Considering the initial conditions, it could be expected that every wave would have to start with an unstable front. As can be seen from the photographs on figure 3, this assumption is false as the zone of instability (the vertical front at t = o) is not propagated away from its zone of occurrence. The mechanism of generation is not analysed).

It was evident that the formation of a bore would limit the maximum possible height (amplitude η) of the leading wave, and that if the leading wave had the characteristics of a solitary wave the ratio η/h would always be smaller than the theoretical maximum value of 0.78. As Q/h can have any value, the affect of Q/h on η/h was studied. An illustration of this is given in figure 12. The height η is dependent on Q (for deep water it is directly proportional to Q) and on L through the form of the amplitude envelope curve. For L = 1' and 2' the data of the three water depths were combined in one curve. It can be seen that as soon as a bore occurred the height was cut down.

In the case of the "complex solitary wave" (CS) the first stage consisted of a serrated elevated mass of water, of which the back side gradually sloped down to about still water level, followed by a dispersive wave pattern. On traveling out from the origin the serrations (waves on top of the elevated water mass) became bigger and their troughs approached the still-water-level. They formed a group of waves with about equal heights, the number of which remained constant during their travel, so that there was no dispersion. With increasing time there was a considerable attanuation, but the troughs never went below still-water-level. It appeared that each individual wave of the group had the characteristics of a solitary wave. Figure 13 illustrates this type of leading wave pattern.

CLOSING REMARKS.

Although the model investigations were made primairily to check the theories cited, the regions beyond the validity of these theories produced quite interesting data which may be valuable in interpreting some hydraulic phenomena.

It may give an explanation concerning the difficulties met in the generation of a pure solitary wave starting from an elevated area. According to this study the solitary wave forms a portion of a dispersive wave pattern.

It may give an indication on the characteristics of the waves of shorter period originated from a breaking wave.

Further more it may explain the breaking up into a wave group of a long wave on reaching a sudden shaol.

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