CHAPTER 12

DETERMINATION OF THE WAVE HEIGHT IN NATURE FROM MODEL TESTS SUPPLEMENTED BY CALCULATION

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1. INTRODUCTION

This paper deals with the problem of determining the wave characteristics in shallow water from those in deep water. In general this can be done by means of a refraction calculation. If the sea bottom topography is too irregular the height of the waves can be determined by means of a small-scale refraction model. In both cases, however, some additional effects have to be taken into account, viz. the influence of the bottom friction and the influence of the wind.

Since a small-scale model does not correctly reproduce the breaking of the waves, this should be avoided by using such small waves, that no breaking in the model occurs.
The influence of the breaking of the waves must then be studied in a separate model on a larger scale and the results of these separate tests must be taken into account as a correction factor by which the results of the refraction model have to be multiplied.

If a model is built in concrete, and the waves are long with respect to the water depth, the bottom friction in the model is not in accordance with that in the prototype. This can be compensated to a certain extent by a distortion of the model. In case of a very small scale of the model, however, this is not sufficient and additional measures have to be taken to compensate this scale effect.

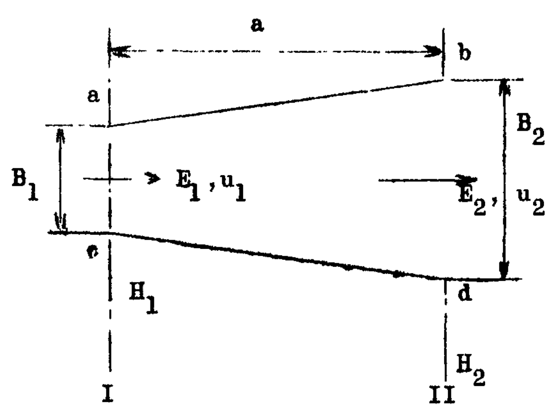
With regard to reproducing the influence of the wind on the height of the waves, it is often very difficult to generate wind in the model, in which case also this effect has to be taken into account as a correction factor.

For these reasons a small-scale refraction model cannot produce exact quantitative results and such a model will only give a correct representation of the refraction pattern.

The present paper describes a method for determining by means of a small-scale model, supplemented by calculations, the correct wave height in the prototype from the wave heights measured in the model. It is based upon the consideration of an energy balance for the prototype as well as for the model. This method may also be used if the refraction is not determined by a model but by calculations only. It can only be applied if the following conditions are fullfilled:

1) no breaking or surfing of the waves may occur
2) there must be only refraction of the waves and no reflection
3) the stretch of water, to which the method is to be applied, must be so short that the deviation of the wave height and the group velocity from the average values is not too great.

2. GENERAL PRINCIPLE



Let a b and c d be two approximately straight orthogonals and I and II two sections normal to these orthogonals, then the transport of wave energy through section II must be equal to the transport of wave energy through section I, if between the sections I and II no energy is lost nor gained.
However, neither in the model, nor in the prototype this is the case. In the prototype, as well as in the model, energy is lost due to bottom friction, while in the prototype energy is gained from the wind.

The energy balances for prototype and model can be written as follows if losses due to internal friction are left out of consideration.

Prototype

$$E_2 = E_1 - E_b + E_w \qquad \text{watt}$$

Model

$$E_2' = E_1' - E_b' \qquad \text{watt} .$$

In the above equations is:

E_w = energy supplied by the wind in the stretch I - II of the prototype

E_b = loss of energy due to bottom friction in the stretch I - II of the prototype

E_b' = loss of energy due to bottom friction in the stretch I - II of the model.

In the following it will be shown that the unknown wave height H_2 in section II of the prototype can be calculated from the known wave height H_1 in section I by means of the above energy equations, if the corresponding wave heights in the model H_1' and H_2' are measured and the losses in model and prototype, due to bottom friction are known.

3. TRANSPORT OF ENERGY IN SECTION I and II

The wave energy passing section I per second, may, with sufficient accuracy, be written as:

$$E_1 = \frac{1}{8}\rho g\, H_1^2 B_1 u_1 \qquad \text{watt}$$

and through section II:

$$E_2 = \frac{1}{8}\rho g\, H_2^2 B_2 u_2 \qquad \text{watt.}$$

Hence, without wind and without bottom friction:

$$\frac{1}{8}\rho g\, H_1^2 B_1 u_1 = \frac{1}{8}\rho g\, H_2^2 B_2 u_2 \ .$$

After substituting u_1 and u_2 by:

$$u_1 = n_1 \frac{L_1}{T} \text{ and } u_2 = n_2 \frac{L_2}{T} \qquad \text{respectively,}$$

the above equation becomes:

$$\frac{1}{8}\rho g\, H_1^2 n_1 B_1 L_1 = \frac{1}{8}\rho g\, H_2^2 n_2 B_2 L_2$$

The latter equation expresses that the wave energy present in section I on an area of $n_1 B_1 L_1$ is equal to that present in section II on an area of $n_2 B_2 L_2$.

The time required by the energy on the area $n_1 B_1 L_1$, to move from I to II amounts to $\frac{a}{u}$ sec, where u = the average energy velocity between I and II.

4. LOSS OF ENERGY DUE TO BOTTOM FRICTION

In the prototype, as well as in the model, there is a loss of energy due to bottom friction. Between section I and section II these losses amount to E_b and E_b' respectively.

According to a theory on the dissipation of wave energy by bottom friction developed by Putnam and Johnson, ref. 1, the loss of energy per unit of area, due to bottom friction, can be expressed by:

$$\Delta E_b = k\rho H^3 f\ (D,T) \qquad \text{watt/m}^2$$

where: H = wave height in m

ρ = density of the water in kg/m^3

$f(D,T)$ = a function of water depth and period

k = dimensionless coefficient of friction from the formula:

$v_* = \sqrt{k}\cdot v_D$, in which $v_* = \sqrt{\frac{\tau}{\rho}}$ m/sec

τ = shear stress in N/m^2

v_D = velocity at the bottom in m/sec.

The function $f(D,T)$ is:

$$\frac{4\pi^2}{3T^3}\left(\frac{1}{\sinh\frac{2\pi D}{L}}\right)^3 \quad \text{with the dimension of} \quad \left[t^{-3}\right].$$

The above formula for ΔE_b is valid only for an impervious sea bottom, having a coefficient of bottom friction which is independent of the magnitude of the velocity at the bottom and of the water depth in the area under consideration, while the velocity must be sinusoidal.

From wave measurements in Lake Okeechobee, ref. 2, it appeared that k has a nearly constant value: $k = 0.01$.

From the results of separate model tests on bottom friction, carried out in the Delft Hydraulics Laboratory, it appeared that the friction coefficient k for a model on a small scale is dependent on the water depth D and the period T. With the help of the in this way obtained values of k the correction for the bottom friction in the model could be determined.

The total loss of energy, due to bottom friction, in the stretch from section I to section II is then:

$$E_b = \Delta E_b \frac{B_1 + B_2}{2} a \qquad \text{watt}.$$

5. ENERGY SUPPLIED BY THE WIND

From the diagram showing the growth of wind-generated waves in shallow water, ref, 3, it appears that, after a certain period of time, for each water depth and each wind velocity, a state of equilibrium will be reached in which the waves no longer grow and the entire quantity of energy supplied by the wind is dissipated by the bottom friction.

By studying the amount of energy lost due to bottom fricti for various water depths and wave periods, an approximate constan value ΔE_w was found for the increase by wind of the wave energy p sq.m over one metre.

For deep water, ΔE_w can directly be calculated from the dia gram showing the growth of wind-generated waves in deep water, ref. 4. For each value of the wind velocity, a value for ΔE_w is th found that is independent of the wave period and of the wave heig This value does not differ much from the above mentioned value fo shallow water. This value is per metre displacement of the wave:

$$\Delta E_w \doteq \frac{\frac{1}{8}\rho g H\,(H\Delta L + 2L\Delta H)}{\Delta a L} \qquad \text{joule/m}^3$$

The total increase in energy, due to the wind, in the stret from section I to section II is then:

$$E_w = \Delta E_w \frac{B_1 + B_2}{2} ua \qquad \text{watt .}$$

6. ENERGY EQUATIONS

Based upon the foregoing considerations, the following ener equations for prototype and model can be written:

<u>prototype</u>

$$E_2 = E_1 - E_b + E_w$$

or:

$$\frac{1}{8}\rho g H_2^2 B_2 u_2 = \frac{1}{8}\rho g H_1^2 B_1 u_1 - \Delta E_b \frac{B_1 + B_2}{2} a + \Delta E_w \frac{B_1 + B_2}{2} au \qquad (1$$

<u>model</u>

$$E_2' = E_1' - E_b'$$

or:

$$\frac{1}{8}\,\rho g H_2'^{\,2} B_2 u_2 = \frac{1}{8}\,\rho g H_1'^{\,2} B_1 u_1 - \Delta E_b' \,\frac{B_1 + B_2}{2}\, a\ . \tag{2}$$

In the above equations, in the term regarding the wind energy, the product of the average width of the stretch $\frac{B_1 + B_2}{2}$ and the average energy velocity u has to be written as the average product $\frac{B_1 u_1 + B_2 u_2}{2}$; for the terms regarding the bottom friction the same may be done with fair approximation.
Thereupon both equations are divided by $B_2 u_2$ and after eliminating $\frac{B_1 u_1}{B_2 u_2}$ the following equation is obtained:

$$H_2^2 = \frac{\left(\frac{1}{4}\,\rho g H_2'^{\,2} + \Delta E_b'\, a u^{-1}\right)\left[\frac{1}{4}\,\rho g H_1^2 - \left(\Delta E_b a u^{-1} - \Delta E_w a\right)\right]}{\left(\frac{1}{4}\rho g H_1'^{\,2} - \Delta E_b'\, a u^{-1}\right)\frac{1}{4}\,\rho g} + $$

$$- \frac{\Delta E_b a u^{-1} - \Delta E_w\, a}{\frac{1}{4}\,\rho g} \tag{3}$$

In order to simplify equation (3) the following substitutions are finally introduced:

$$\frac{\Delta E_b'\, a u^{-1}}{\frac{1}{4}\rho g} = P \tag{4}$$

$$\frac{\Delta E_b\, a u^{-1}}{\frac{1}{4}\rho g} = Q \tag{5}$$

and

$$\frac{\Delta E_w\, a}{\frac{1}{4}\rho g} = R \tag{6}$$

Equation (3) then becomes:

$$H_2^2 = \frac{(H_2'^{\,2} + P)\ (H_1^2 - Q + R)}{H_1'^{\,2} - P} - Q + R \tag{7}$$

The substitution (5) contains the term

$$\Delta E_b = k\rho \left(\frac{H_1 + H_2}{2}\right)^3 f\ (D,T)\ .$$

Since the prototype wave height H_1 is given and the model wa heights H_1' and H_2' can be measured in the model, the value of H_2 in the prototype can be determined by solving the equations (5) and (7 after having inserted the values for

$$\Delta\ E_b',\quad \Delta\ E_b \quad \text{and} \quad \Delta\ E_w\ .$$

As mentioned before, the above method can only be applied if no crossing energy transport occurs and if in the stretch I - II the deviation of the wave height and the group velocity from the average values is not too great, say about ten percent.

For large stretches the total length has to be subdivided into short stretches and the calculations repeated.

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