#### CHAPTER 13

### A WAVE HEIGHT AND FREQUENCY METER

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#### INTRODUCTION

During World War II a group of Naval officers conducted visual measurements of ocean waves simultaneously with instrumental recordings. A comparison of the visual and instrumental values indicated ".... the natural tendency for the observer .... to record not the average wave height but a wave height based on some kind of average of the highest waves. The general experience is that an observer will give a value for the wave height which represents the average of the highest 20 to 40 per cent of the waves. (SIO, 1944). The average height of the highest one-third of the waves,  $H_{1/3}$ , was therefore suggested as the characteristic (or significant) wave height. "Characteristic wave period" was given a corresponding definition as the average period of the highest one-third waves.

An interpretation of characteristic wave height in terms of standard statistics was made by Putz (1952) and Longuett-Higgins (1952). They related theoretically rms height, mean absolute ordinate  $a_0$ , and the average height of the highest "p" per cent. It turns out that  $H_1 = 5.02a_0$ . Underlying assumptions are that the ordinates are Gaussian and the spectrum moderately narrow. A summary of observations relating to statistical properties of wave heights was recently made by Wiegel and Kukk (1957).

Measurement of the average absolute ordinate can easily be accomplished. In this paper we describe two systems which lead to a direct measurement of  $5a_0$  (= $H_{1/3}$ ). The overall situation is therefore satisfactory inasmuch as a naturally reported quantity, the characteristic wave height, can be defined in terms of fundamental statistics and objectively recorded by automatic means.

The situation is not nearly as satisfactory with regard to wave period. The original definition of characteristic wave period as the period of the highest of one-third waves could not be interpreted in terms of standard statistics, nor is it possible to design reasonably simple devices for recording this quantity. An alternate definition is

based on the average time between zero crossings. This can be interpreted in terms of standard statistics, and instruments can be devise for recording it. Moreover, a definition based on zero crossings is much in accord with the natural tendency of observers as the previou definition of characteristic period. It would seem that characteristic period is not a useful concept and should be dropped. In this paper w describe two instruments for directly measuring the average zero crossing frequency.

### OCEAN WAVE RECORD ANALYSIS

Recent research on ocean wave record analysis has dealt with two problems: (1) the description of data obtained from wave meters which record the time history of surface elevation or subsurface pressure fluctuation and (2) the formulation of a theoretical model for the analysis of the data. Two points of view have often been taken in the work on these problems, resulting in the methods of "wavewise" analysis and "ordinatewise" analysis. The first considers the record as a sequence of oscillations defined by the successive points on it which correspond to relative maxima and minima. The second treats the record as a continuous function of the time variable which may be considered either on its own merits as an isolated piece of data or as a particular sample of finite length from a continuing process. Thus, from the second viewpoint, the given time history may be analyzed by taking into account all of the information which is present in it - corresponding to the information in the ordinary Fourier spectrum (specifying amplitude and phase vs. frequency), or it may be analyzed by taking into account all the information present except that which depends upon the value of the absolute time variable corresponding to the power spectrum (specifying amplitude, but not phase.) The latter method thus seeks to describe the record in terms of the statistical distributions of the various ordinates on the timehistory curve. Such a description for a stationary Gaussian process may be considered complete if the correlation function is specified.

A wavewise analysis is often convenient to apply, whereas the ordinate-wise analysis furnishes a more complete and sophisticated description of the data. The existence of the two methods of description creates the problem of relating the measured parameters which result from the two types of analysis. It is thus of interest to see how to obtain information about the power spectrum (or, equivalently, about the correlation function) from a wavewise analysis

Wavewise analyses of time-history records have been reported by many observers. In these analyses measurements were made on the empirical distributions of wave height and wave period, referred, in each case, to troughs and crests. Certain regularities were found in these distributions, particularly those for wave heights. From

measured inter-zero-crossing period distributions the correlation function and hence, in principle, the power spectrum, has been estimated (Putz, 1957).

Studies of the Fourier spectrum of observed wave records have been made by Barber and Ursell (1948), using analog computer methods. The results of those studies are useable spectral curves, which, however, reflect the finite length of the analyzed record and the finite resolving power of the analyzing instrument.

The introduction of the notion of a stochastic process to describe ocean wave records was pioneered by Seiwell (1951) and Rudnick (1951), among others. The usefulness of the Gaussian model was recognized by Rudnick, who also computed for a number of wave records the correlation function - the element remaining to complete the picture given by the stationary stochastic process model. The study of the applicability of this stationary Gaussian model was taken up by Pierson (1952) and Putz (1953), among others.

The stationary Gaussian model was found to be in reasonably good agreement with data and to explain a number of the previouslyobserved relations brought to light by wavewise analyses. These relations were found to be derivable from the mathematical model, which had, in fact, been discussed earlier in some detail by Rice (1944-5). Wave-height distributions, predicted for a narrow-band spectrum, were found to agree with observed distributions, both as to shape and relative values of parameters. The distributions of wave periods, redefined in terms of zero-axis crossings, were found to be directly related to the power spectrum of the individual wave record, and a means of estimating the spectrum from these distributions was developed. Putz (1957a, b). In addition, various relations between parameters of these zero-crossing period distributions and the power spectrum (or, equivalently, the correlation function) were shown to hold.

#### RELATIONS INVOLVING $a_0$ AND $f_{z,c}$ .

The present report considers techniques for the measurement of two quantities, and shows how these may be used to obtain information about the power spectrum and the wave-height distribution. The first of these is the so-called mean deviation  $a_0$  of the time-history ordinate, f(t), i.e., the value of  $\overline{|f(t) - \xi_0|}$ , averaged over time, where  $\xi_0$  is the mean ordinate, such that  $\xi_0 T = \int_0^T f(t) dt$ , where T is the total duration of the record. The second is the so-called mean upcrossing frequency,  $f_0$ , i.e., the number of times the

time-history curve crosses the mean ordinate level per unit time. It convenient to define also the corresponding angular frequency,  $\omega_0$ . Thus if  $N_0$  is the total number of crossings (in both directions), then  $f_0 = \frac{N_0}{(2T)}$ , and  $\omega_0 = 2\pi f_0 = \frac{\pi N_0}{T}$ . The notation "f<sub>z:c:</sub>" is also used below for the quantity  $N_0/(2T)$ .

If  $\underline{\omega}$  is the general angular frequency and  $\Phi(\omega)$  is the spectral density function describing the power spectrum, we define the absolut spectral moment  $M_k$  of order  $\underline{k}$  by the expression  $M_k = \int_{-\infty}^{+\infty} |\omega|^k \Phi(\omega) = k = 0, 1, 2, \ldots$ . Thus the zero-order moment  $M_0$  is the total power in the spectrum. The first-order absolute moment becomes, after division by  $M_0$ , the mean angular spectral frequency,  $\mu_{\omega} = M_1/M_0$ . From  $\mu_{\omega}$  and the reduced second-order moment,  $\mu_2 = M_2/M_0$ , is derived the concept of the spectral bandwidth  $\sigma_{\omega}$ , defined by the expression  $\sigma \omega^2 = \mu_2 - \mu^2 \omega$ . Further, the relative spectral bandwidth,  $\delta_{\omega}$ , may be defined as  $\sigma_{\omega}/\mu_{\omega}$ , with the result that  $\sqrt{\mu_2} = \sqrt{\omega^2 + \mu_{\omega}^2} = \mu_{\omega} \sqrt{1 + \delta_{\omega}^2}$ .

It is well known (Rice, 1944-5) that for a Gaussian process,  $a_0 = (2/\pi)^{1/2} \sqrt{M_0}$ , and  $\omega_0 = \sqrt{M_2/M_0} = \sqrt{\mu_2}$ , so that  $M_0 = (\pi/2)a_0^2$ and  $\mu_W = \mu_0 (1 + \delta_0)^{-1/2}$ . It is seen that observation of  $a_0$ and  $\omega_0$  (or  $f_0$ ) leads to the determination of  $M_0$  and, in the case of small relative bandwidth, of an approximation (on the high side, within 5% if  $\delta_W$  does not exceed a value of 1/3, approximately) to  $\mu_W$ .

To obtain information regarding the wave-height distributions in a wave record, additional theoretical relations may be used. Thus it follows from the theory of the stationary Gaussian stochastic process that the mean height of the envelope to the time-history graph is  $(\pi/2)^{1/2}(M_0)^{1/2} = (\pi/2) a_0$ . In the case of small relative bandwidth the height of the double envelope may be taken to be the mean trough-to-crest wave height; in this case, then,  $\pi a_0$  is the mean wave height. More generally, it is known that the mean height of the ordinates at the relative maxima (or the mean depth of the ordinates at the relative minima) is given by  $\dot{\mu}_{M} = \alpha (\pi/2) a_{0}$ , where  $\alpha = f_{\Omega}/f_1$ , and  $f_1$  is the mean up-crossing frequency for the first derivative curve. The mean trough-to-crest wave height will be given by  $2\mu_{\rm M} = \alpha \pi a_0$ . A typical value of  $\alpha$  for ocean swell is 0.85. It follows also from the theory of the stationary Gaussian process that the doubled mean height of the highest one-third of the ordinates on the envelope curve is  $2(2.00 \sqrt{M_0}) = 5.02 a_0$ .

More general relationships holding for the total spectral power and the reduced second spectral moment may be obtained by considering measurement of mean ordinate deviation and mean upcrossing frequency made about an arbitrary ordinate level. Thus, if we measure the average value,  $a_h$ , of |f(t) - h|, where  $h = \xi_0 + k \sqrt{M}$  we find that  $a_h/a_0 = \exp(-k^2/2) + k$ .  $\Psi(k)$ , where  $\Psi(k) = \int_0^k \exp(-t^2/2) dt$ . Also, for the mean up-crossing frequency,  $f_h$ , at ordinate level h, we have  $f_h/f_0 = \exp(-k^2/2)$ , where, as before,  $k = (h - \xi_0)/\sqrt{M_0}$ .

It will be noted that both ratios reduce to unity when k = o. Further, the differences  $a_h - a_o$  and  $f_h - f_o$  are of higher order in k, so that the effect of a small error in locating the mean ordinate level vanishes rapidly with k. In fact, for values of k not in excess of about 0.4, the approximations  $a_h/a_o$  ( $\Rightarrow 1 + k^2/2$  (=)  $f_o/f_h$  may be used. For example, an error of 5 or 10% in the determination of  $\xi_o$  should not correspond to a value of over 0.4. for k, which yields a relative error of 8% in the value of  $a_o$ .

#### DESCRIPTION OF THE ANALYZER

The block diagram of the wave analyzer is shown in Figure 1. The wave data enters a point A as an electrical signal which varies about a zero potential in proportion to the amplitude of the wave about the mean pressure level. This signal is simultaneously applied to two channels of the analyzer. The first channel reverses the polarity of the signal each time it becomes negative to produce a full wave rectification of the incoming signal. By averaging the rectified signal with an appropriate filter a measure of the average absolute ordinate,  $a_0$ , is obtained.

The second channel produces a unit pulse for each positive zero-axis crossing of the incoming signal. By again averaging these pulses with an appropriate low-pass filter a mean zero-axis crossing frequency can be obtained.

### AN ELECTRONICALLY OPERATED ANALYZER

This analyzer (Figure 2) operates as shown in the block diagram Figure 1). All amplifiers indicated are type K2W Philbrick functional d-c amplifiers and the circuits are discussed by Philbrick (1955).

The rectifier circuit provides full-wave rectification about the zero axis with d-c coupling and a common ground between input and output. With the input connected to ground the zero adjustment potentiometer is set for zero output of the rectifier for rectification at zero input voltage level.

The pulse generator circuit consists of an overdriven amplifier and relay operated pulsing circuit. The amplifier has positive feedback with a loop gain of slighly greater than unity and therefore tends

to assume one of two stable states. When the input exceeds a critical voltage, the amplifier output is positive and the relay is energized. When the input voltage returns to a value slightly below the critical voltage the amplifier output becomes negative. With negative voltage the relay is de-energized due to the action of the shunting diode.

The difference between the level at which a positive-going voltag causes the relay to energize and a negative-going voltage causes the relay to de-energize is the circuit hysteresis. The hysteresis prevent chattering of the relays when the input voltage is zero. The amount of hysteresis is controlled by the 50M feedback resistor. For zero axis adjustment the zero adjustment potentiometer is set in the center of the hysteresis range with the amplifier input connected to ground.

Relay operated pulse generators are used in preference to electronic circuits. When relay A is de-energized a 300  $\mu$ fd electrolytic condenser is charged to 10 volts; when relay is energized the condenser is discharged through relay B energizing it for 3 seconds. A 1.35 volt pulse is therefore applied to the input of the filter for each positive zero-axis crossing.

Stability of the two filter circuits is the primary concern of their design. All electrical components must be of high quality with low temperature and aging coefficients. Sufficient stability was obtained only by using heavy negative feedback.

The time constant of the filter, being essentially equal to RC, can be made equal to 2 seconds for adjusting the circuit and 600 seconds for normal operation. With a 600 second time constant the high frequency cutoff occurs at  $1/2\pi 600$  = 1 cyc/hr. Recording millimeters can be driven directly from the output of the filter. Sensitivity of the circuits is adjusted by potentiometers connected in series with the recorders.

### A RELAY-OPERATED ANALYZER

This wave analyzer, designed to operate with a Mark IX Pressure Head (Snodgrass, 1955), consists of a thyratron-operated relay circuit which drives two panel meters filled with 12,500 centistoke silicone fluid (Figure 3). The viscous damping of the meter movement provides a response time-constant of twenty minutes. These meters are equivalent to a single stage RC filter with a high frequency cutoff at  $1/(2\pi \ 20 \ min) = 0.5 \ cyc/hr$ . (Snodgrass, Putz 1954)

The second meter receives as an input the rectified current of the a-c wave signal driving the Esterline-Angus recorder. The wave height meter is 10 times more sensitive than the Esterline-Angus recorder; its full scale reading corresponds to an average absolute



Fig. 1. Block diagram of the analyzer. Voltage signal from wave gage enters A; the average values of the rectified wave record at B and zero-axis-crossing unit pulses at C are recorded as wave height and frequency.

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ordinate value of 0.1 of full scale of the Esterline-Angus. If  $H_{1/3}$  is assumed to be equal to approximately  $5a_0$ , the characteristic wave height  $H_{1/3}$  is  $(5CR_h)/10$  where C is the full scale calibration of the Esterline-Angus recorder and  $R_h$  is the reading of the wave height meter assuming full scale to be unity.

The first meter receives a pulse of current each time the wave record passes through the zero axis in a positive direction. By proper adjustment of the circuit the meter reads one milliampere for an input signal with a frequency of 0.1 cyc/sec. The zero crossi frequency then equals  $0.1 \text{ R}_f$ , where  $\text{R}_f$  is the reading of the frequence meter.

<u>Circuit operation</u> - A combination of a d-c bias voltage and a 9 leading a-c bias voltage connected in a cathode circuit of the thyratro tube (Figure 4) prevents the tube from conducting unless the grid voltage exceeds a given value. The exact value of the grid voltage necessary to cause conduction is determined by the setting of potentiometer  $P_1$ . This voltage will be approximately 12 volts with respeto the negative side of the d-c bias supply, since the "wave-recorder potentiometer" is in its center position with no ocean-wave signal applied. Resistor  $R_1$  and Condenser  $C_1$  in the grid of the thyratron decouple the thyratron grid from the recording circuit of the wave re corder and prevent high-frequency noise from firing the tube.

Resistor  $R_2$  and Condenser  $C_2$  shift the filament voltage 90° leading with respect to the plate voltage. With this voltage connected in the bias circuit, the tube will conduct the entire positive half of the plate voltage swing as soon as grid voltage exceeds the critical value With only the d-c bias connected in the circuit, the tube would conduct only one-half of the positive plate voltage.

<u>Relay operation</u> - The relay circuit operation is as follows: Assume the thyratron grid voltage is below the critical value, the tul is not conducting, and the relays are de-energized. If a signal is supplied from the wave recorder which causes the grid voltage to increase above the critical value, the tube will conduct and relay A wil be energized, closing contacts A<sub>1</sub> and A<sub>2</sub>. Contact A<sub>1</sub> energizes Relay B, which closes contacts B<sub>2</sub> and B<sub>3</sub> and opens contact B<sub>1</sub>. Contact B<sub>1</sub> disconnects relay A, but relay A will remain energized b cause of the charge in condenser C<sub>3</sub>. The contacts of relay A remai closed for 1.4 seconds each time the circuit is actuated. Contact A<sub>2</sub> causes a unit pulse current of 7.1 ma for 1.4 seconds to flow through the "wave period" meter each time the grid signal exceeds the critic value. Contact B<sub>2</sub> prevents relay B from de-energizing when contact A<sub>1</sub> opens. Relay B, therefore, will remain energized as long as the



Fig. 3. Functional diagram of the relay operated analyzer for the Mark IX pressure gage.



Fig. 4. Schematic diagram of the relay operated analyzer. The wave height and period meters are submerged in 20,000 centistoke oil for averaging of signals.

DATE 1954	<sup>1/f</sup> z.c.	T <sub>1/3</sub>	a <sub>o</sub>	H1/3	DATE 1954	<sup>1/f</sup> z.c.	T <sub>1/3</sub>	ao	H <sub>1/3</sub>
1/26(1)	-	10.9	-	18	2/5	_	12.5	-	11
	11.1	12.0	3.2	17		12.5	12.3	3.5	17
	11.5	12.9	3.0	15		14.7	11.9	3.8	20
	-	13.3	-	16		-	11.9	-	20
1/27	-	13.7	-	17	2/6 <sup>(2)</sup>	-	11.9	-	17
	14.1	13.9	2.6	15		12.5	12.8	3.6	19
	13.2	14.6	2.9	16		14.8	11.5	2.4	13
	-	13.8	-	17		-	11.9	-	16
1/28(2)	-	13.4	-	16	2/7(2)	-	13.0	-	13
	14.1	13.9	2.7	16		14.7	13.2	3.2	17
	15.4	13.2	2.8	14		13.5	12.9	2.9	17
	-	14.6	-	15		-	12.6	-	16
1/29(2)	-	14.9	-	17	2/8	-	12.9	-	14
	15.4	15.2	2.7	15		13.3	13.5	2.7	15
	13.2	14.1	3.2	16		13.0	13.3	3.1	16
	-	15.1	-	16		-	13.6	-	16
1/30	-	16.3	-	20	2/9	-	13.2	-	16
	14.7	17.3	4.0	23		14.3	13.9	3.2	17
	12.5	16.9	4.5	19		14.5	14.9	3.6	19
	-	17.9	-	18		-	14.2	-	17
1/31	-	16.1	-	<b>24</b>	2/10	-	14.5	-	20
	12.7	16.2	4.2	22		12.5	13.9	3.0	18
	12.0	16.1	3.4	20		11.7	13.5	3.6	19
	-	14.6	-	18		-	13.2	-	18
2/1	-	-	-	15	$2/11^{(1)}$	-	14.5	-	22
	14.3	15.7	3.0	15		12.5	13.9	6.0	30
	14.9	14.7	2.6	14		11.7	13.5	6.3	30
	-	13.7	-	16		-	13.2	-	<b>2</b> 6
2/2	-	15.1	-	17	2/12(1)	-	13.9	-	<b>2</b> 8
	13.3	14.4	4.0	22		12.7	13.6	4.8	23
	14.1	13.6	3.4	17		12.5	14.0	5.1	25
	-	14.5	-	16	(-)	-	16.0	-	24
2/3	-	14.3	-	16	2/13 <sup>(1)</sup>	-	16.0	-	22
	14.1	13.3	3.2	18		12.3	15.2	11.6	58
	12.8	13.5	3.0	16		7.7	12.0	10.0	43
	-	12.7	-	15		-	14.0	-	5 <b>2</b>
2/4	-	13.2	-	13	2/14(1)	-	13.3	-	44
	13.3	12.1	3.0	16		10.7	13.9	9.2	48
	12.8	13.3	2.4	14					
	-	13.1	-	16					

### TABLE I\*. TEST RESULTS

\*Four measurements were made at 0415, 1015, 1615, and 2215 of each day.

 $f_{z.c.}$  and  $a_o$  were read from the relay-operated analyzer.  $H_1/3$  and  $T_{1/3}$  were measured from 20-minute wave records by the personnel of the Surf and Weather Station, Camp Pendleton, California.  $^1\mathrm{Local}$  storm present which generated strong secondary waves.  $^2\mathrm{Sections}$  of the record were nearly flat making zero axis crossings difficult to measure.

grid signal exceeds the critical value. Contact B<sub>3</sub> causes relay C to operate as a rectifier by reversing the polarity of the "wave height" meter. The critical grid voltage is adjusted so that the meter polarity 1s reversed as the current in the Esterline-Angus recorder passes through zero current.

Condenser  $C_4$  prevents relay B from chattering due to the halfwave rectified current flowing in the 2050 thyratron tube. Resistor  $R_4$  limits the current so that 24 volts appear across Relay B with the thyratron conducting. Resistor  $R_3$  limits the current in the thyratron to its maximum allowable value, while Relay A is being energized.

#### TESTS AND RESULTS

The relay operated analyzer was tested for 23 consecutive days at the Surf and Weather Station, Camp Pendleton, California. The readings of wave height and period obtained from the oil immersed meters were compared to the values determined by the standard manual analysis. The test indicated that  $H_{1/3}$  is approximately equal to  $5a_0$  (Figure 5).

The frequency meter indicated a wave period that was generally less than the significant wave period particularly during local storms as indicated in Table I. At the outset of the local storm high frequencies could be seen in the wave record which would considerably affect a reading on the zero crossing meter; the manual analysis of the record would fail, however, to indicate this change in sea state until the amplitude of the high frequency waves was appreciable in comparison to the underlying sea swell.

Figure 6 indicates results obtained by comparing the value of the zero-axis crossing frequency, measured by the electronic analyzer, to the value computed from the spectrum. The spectra were obtained from data consisting of 3000 values at one-second intervals read from a 50-minute wave pressure record (gage depth 14 feet), (Munk, 1957). The value at A is the mean frequency of the record, while the value at B is the theoretical zero-axis crossing frequency. The arrow labeled C indicates the range of values obtained from the analyzer during the 50-minute period of the spectrum data. Good agreement was found between the computed and measured zero-axis crossing frequency.

The continuous recording of wave height (or energy) and frequency is perhaps more important than the determination of the average values of  $H_{1/3}$  and  $f_{z.c.}$  discussed above. Large variations in  $H_{1/3}$  and  $f_{z.c.}$  can occur in relatively short periods of time as shown in Figures 7 and 8. This record indicates that 15-minute records taken a half hour apart could differ by 30 or 40 per cent.



Fig. 5. Experimental data comparing the average absolute ordinate,  $a_0$ , to the characteristic wave height  $H_{1/3}$ .



Fig.6. Experimental data comparing wave spectra, calculated mean frequency (A) and zero crossing frequency (B), to measured zero crossing frequency (C).



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The change in sea state indicated in Figure 7 was caused by a local 15-knot wind which started late October 31 and lasted through November 1. The wave record was obtained from a pressure gage 12 feet below MLLW in 20 feet of water at Scripps Pier. The zero crossing frequency increased sharply, reached a peak and then decreased before the wave energy increased appreciably.

The record shown in Figure 8 was obtained from a pressure gage 30 feet below MLLW in 35 feet of water at Camp Pendleton, California. The local winds were less than 7 knots. The waves were described as "heavy southern swell" being very low frequency and from the south. Characteristic of southern swell recordings from the analyzer, the zero crossing frequency is low and steady with large variations in wave height. Some correlation is apparent between the frequency and height recordings, with low frequency occurring at times of high energy. The values of  $H_{1/3}$  and  $T_{1/3}$ , indicated in Figure 8, were obtained by manual analysis of twenty-minute records at the time indicated.

The fluctuations of the records pose some interesting questions. Can the fluctuations be predicted for various sea states or for an aging storm? What are the expected correlations between the height and frequency records? Can the spectrum of the analyzer output be predicted from the raw wave record?

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