

CHAPTER 20

CONTRIBUTION TO THE STUDY OF SEDIMENT TRANSPORT ON A HORIZONTAL BED DUE TO WAVE ACTION

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Summary

With a view to explaining the phenomena of sediment transport in the open sea, outside the wave breaking area, the author carried out a laboratory investigation of wave action on a horizontal bed. He puts forward a number of new results regarding :

- 1 - The state of turbulence near the bed and the stability of the oscillatory laminar boundary layer.
- 2 - The setting in motion of materials under the influence of wave alone.
- 3 - The entrainment current caused by wave action close to the bed.
- 4 - The transport of material under wave action only.
- 5 - The indirect action of wave on the bed.

The main conclusions reached are as follows :

1/ - The results given by Huon Li regarding the onset of turbulence within the oscillatory boundary layer overestimate the range of laminar conditions. V_0 (maximum orbital velocity) and ϵ (roughness) are the principle factors governing the transition.

Test waves are either generally laminar, or are only slightly turbulent within the body of liquid, but they are, however, more often turbulent in the immediate neighbourhood of the bed.

2/ - The investigation of conditions for the onset of grain movement of the bed material shows that the action of wave can be appreciable, even at depths of several tens of metres. A wave of 6 metres amplitude, with a total length of 120 metres, would be capable of putting a 0.3 mm sand grain into motion at a depth of 60 metres.

3/ - The experimental investigation, as well as the viscous fluid theory, shows the existence, close to the bed, of an entrainment current of liquid particles which *always* works in the direction of wave propagation.

4/ - In test flumes, this entrainment current forms part of a mass transport within the liquid, the vertical distribution of which varies with the characteristics of the fluid motion. On a horizontal bed, it generally gives rise to an *effective sediment transport*, in the direction of wave propagation, as the preponderant part of the liquid velocity component, near the bed, is in this direction.

5/ - Owing to the existence of the mass transport current and the onset of suspension of material above the bed, some sediment transport can exist out to sea. These results give a explanation of why, under the action of long and regular wave, material tends to be carried in the direction of the waves and build up on the beach whereas, under storm conditions, a strong resultant turbulence produces suspension and favours erosion of the beach.

6/ - On a sloping bed, transport towards the shore is counterbalanced by the effect of gravity, currents caused by winds from seaward and density currents set up in the wave break area so that finally material eroded from land surfaces are, in part, gradually carried away towards the open sea.

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GENERAL REMARKS

The problem considered here is that of solid transport along the sea bed caused by wave action, to seaward of the breaking zone i.e. outside the zone in which the chief phenomenon occurring is that usually known as littoral transport.

a) Effect of waves on the sea bed - General comments

The extent of the effect of waves on the sea bed, from the breaking zone out to fairly considerable depths, depends on the local nature of the bed, the depth of water and the characteristics of the waves. At these depths, the bed materials are usually set in motion without there necessarily being any major displacement of solid particles, and it can be said that these displacements decrease continuously as the distance from the breaking zone increases. This decrease must generally be very rapid, for turbulence has a considerable effect on solid transport, particularly near the sea bed, and should in actual fact decrease rapidly in the bottom layers of the fluid as the distance out to sea and away from the breaking zone increases. This is why suspension generally tends to fall off quickly towards the open sea and the decanting materials become correspondingly finer and finer.

Nevertheless, it should not be forgotten that waves are not the only natural factor outside the breaking zone capable of transporting materials. The solid particles set in motion by the waves may be taken up by a random sea current, even if only very slight, that will act on solid grains already dislodged from the sea bed.

It should also be remembered that wave action on the sea bed can form or produce favourable conditions for density currents, which carry the fine particles out to sea. The finer the particles and the steeper the sea bed, the farther and faster they travel.

Finally, and contrary to general belief, this bed zone can be the seat of quite considerable exchanges of solid materials. A knowledge of these movements is of great value when seeking to better understand the movements of solids in a beach profile and when tackling the problem of bar formation at the seaward end of a channel, or trying to find the best place to dump dredged or other materials at sea.

b) Experimental Techniques

Most of the Authors who have carried out experimental research on turbulent states or sediment transport due to wave propagation in shallow

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water and an horizontal sea bed (HUON LI⁽¹⁾*, MANOHAR⁽²⁾, BAGNOLD⁽³⁾ etc. have used an oscillating bed in conjunction with a still mass of water.

Although the movements near the bed produced by this method are about as extensive as in real life, the effect of a number of phenomena caused by progressive waves (such as entrainment currents, acceleration of fluid particles, pressure fluctuations in the immediate vicinity of the sea bed, development of turbulence, etc ..) on a stationary sediment layer are not completely brought into evidence.

We therefore thought it preferable to use a conventional wave flume, in which the bed materials were subjected to the effective action of progressive waves, although the results obtained would not be so directly applicable to real life conditions because of the comparatively restricted range of velocities considered.

SYMBOLS

The following symbols are used in the text :

- 2a Total wave amplitude at free surface (vertical distance from trough to crest)
- 2a_d Total wave amplitude corresponding to the onset of the movement of a grain of material.
- d Diameter of single grain of material in the bed layer ;
d is such that the diameter of 50 % of the grains is < d
- d₁ Total travel of a fluid particle in the immediate vicinity of the bed.
- $$d_1 = \frac{2a}{sh \frac{2\pi}{h/L}} \quad (\text{Theoretical value, 1st order, perfect fluid})$$
- δ Thickness of oscillatory laminar boundary layer
- ε Characteristic dimension of a grain of powdery bed material (in actual fact, ε ≅ d for material with a sufficiently uniform granulometry)
- g̃ Apparent gravity field $\bar{g} = g \frac{\rho_s - \rho_f}{\rho_f}$
- h Height of water above bed
- L Wave length (horizontal distance between two successive crests)
- ν Kinematic viscosity of water
- ρ_s Specific mass of grain
- ρ_f Specific mass of fluid
- T Wave period (ω = 2π/T)

*.See References at end of text.

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u	Maximum intensity of entrainment current near the bed at laminar conditions (mean value)
$u(z)$	Flow velocity in the oscillatory laminar boundary layer at height z above the bed level
$\tilde{u}_z$	Strength of mass transport current at height z above the bed level.
V	Velocity of fluid particle at a distance δ from the bed
V_0	Maximum value of V , being equal to the theoretical maximum orbital velocity in the immediate vicinity of the sea bed.
W	Rate of fall of a grain of diameter d in calm water

I - TURBULENT STATE IN THE VICINITY OF THE BED- STABILITY OF THE OSCILLATORY LAMINAR BOUNDARY LAYER

a) Thickness of laminar boundary layer

The pattern of the hydrodynamic forces acting on a grain of bed material depends chiefly on the flow conditions in the immediate vicinity of the bed, i.e. the actual nature of the flow around the grain.

In a viscous fluid, the overall flow pattern - even if oscillatory - causes a boundary layer - itself oscillatory - with a very steep velocity gradient to develop on the bed.

The appearance of turbulence at this oscillatory boundary layer can in general be considered as an important factor for the sediment transport in the vicinity of the bed, having a particularly marked effect on the onset of grain movement.

It is easy to imagine that the nature of the flow around a grain depends on the ratio δ/ϵ , i.e. on the depth of immersion of the grain into the oscillatory laminar boundary layer.

At a distance δ from the bed, the velocity of a fluid particle is a direct function of the upper flow and is given by an expression of the following form, for waves in a finite depth, and as a first order approximation :

$$V = \frac{\omega d_1}{2} \cos \omega t$$

Velocity V decreases very rapidly in the laminar boundary layer and becomes zero on the bed. The velocity distribution in an oscillatory laminar boundary layer has the following form (cf. LAMB, VALEMBOIS ⁽⁴⁾) :

LAMINAR

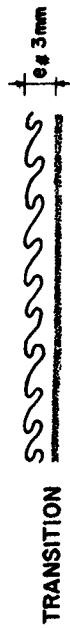


Fig. 1. Transition criterion

$\bullet$ $h = 100 \text{ cm}$
 $+$ $h = 80 \text{ cm}$
 $\circ$ $h = 60 \text{ cm}$
 $\times$ $h = 40 \text{ cm}$

$T = 1,9$

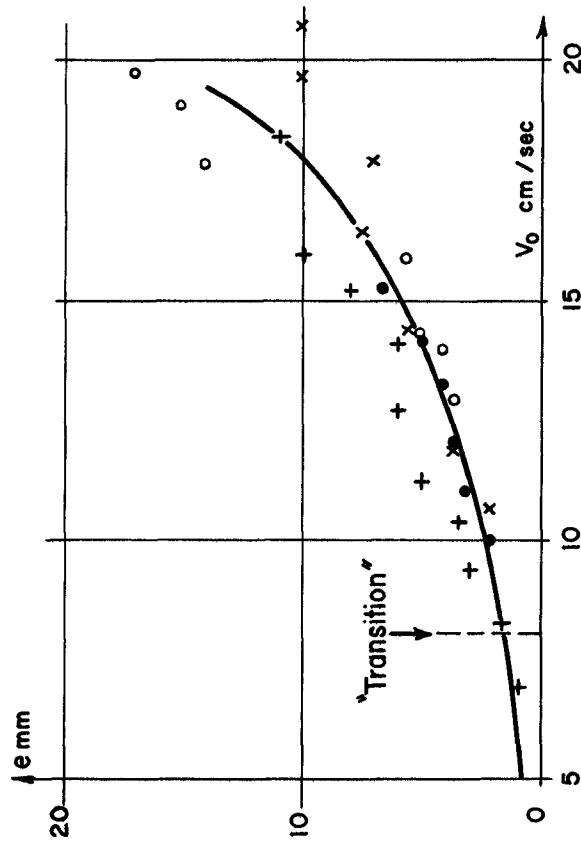


Fig. 2. Thickness of coloured "cloud" e , in mm, for varying

$$V_0 = \frac{\pi D_1}{T} \quad \text{.sand (10)} \quad \epsilon = 0.023 \text{ cm}$$

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$$u = V - \frac{\omega d_1}{2} e^{-(1/\lambda)z} \cdot \cos \left(\omega t - \frac{z}{\lambda} \right)$$

where the fictitious wave length λ has the form $(v/\omega)^{1/2}$ and characterises the thickness of the oscillatory laminar boundary layer ($\delta = K\lambda$) to within one coefficient. For a given fluid, δ therefore depends essentially on the oscillation period of the movement.

b) Stability of Boundary Layer - Work done by HUON LI

A very interesting experimental study of the oscillatory laminar boundary layer has recently been carried out in the United States by HUON LI⁽¹⁾ and completed by MANOHAR⁽²⁾.

Considerable difficulties are encountered in the study of this phenomenon in progressive waves if the aim is to reproduce, even if only schematically, the boundary conditions due to the action of waves on a natural bed by means of a model.

HUON LI carried out his experimental tests in the same way as BAGNOLD, by making the bed oscillate in an horizontal plane, with the fluid mass at rest.

In this method, the periodic accelerations acting on the fluid particles* and the pressure variations occurring near the bed are not taken into account, although both very probably tend to some extent to "burst" the laminar boundary layer, i.e. to accelerate the onset of turbulence in it. HUON LI's results can therefore be expected to exaggerate the relative importance of the laminar regime compared to what it really is when the oscillatory boundary layer is formed by waves effectively progressing along a stationary bed.

The results obtained from our experimental study confirm the above comment, besides bearing out the theory that transition occurs more rapidly as the characteristic grain size, the travel of a fluid particle in the immediate vicinity of the bed and the frequency of the oscillatory movement increase. (in other words, the velocity of a fluid particle in the immediate vicinity of the bed).

c) Definition of a "transition" criterion (Fig. 1)

When the oscillatory boundary layer is *laminar*, a fine particle of colouring matter placed on the bed is seen to generate a thin coloured cloud (the thickness of this cloud varies slightly at a given point, to the rhythm of the waves) with an apparently perfectly smooth top surface. (One end of the cloud gradually travels along in the direction of wave propagation).

The mean thickness e of the coloured cloud increases slightly as the velocity of the fluid particle near the bed increases. (Fig.2)

* cf. R.MICHE in particular

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At a certain oscillation rate, slight discontinuities appear on the top surface of the cloud. These form when the pressure at the bed is near its minimum value and inflect in the opposite direction to the waves, due to the momentary local flow. As the velocity increases further, they develop into small individual " flames " that become gradually more extensive.

In our view, the appearance of these " flames " is characteristic of " transition ".

All other things being equal, the rougher the bed, the lower the oscillation rate at which these " flames " appear.

There is also a detectable laminar film in immediate contact with the bed. This relatively thin film becomes still thinner as local turbulence increases.

If the boundary layer is turbulent, a grain of coloured material of approximately equal density to that of the fluid will, if released at the free surface, sink slowly, describing a very individual coloured line through the fluid down to a distance from the bed that decreases with the " turbulence ".

The above seems to show that, at regular wave conditions, ⁽⁶⁾ the turbulence produced by the bed does not propagate readily through the fluid mass.

d) Results obtained by HUON LI

According to the work carried out at BERKELEY, the bed would be smooth - in the hydraulic sense of the word - if : $\frac{\delta}{\epsilon} > 30$ and " rough " if : $\frac{\delta}{\epsilon} < 18.5$.

HUON LI expresses the thickness of the oscillatory laminar boundary layer in the following terms :

$$\delta = 6.5 \left(\frac{\nu}{\omega} \right)^{1/2} \text{ soit } \delta = 2.6 \cdot \sqrt{\nu T}$$

Although this value is probably high we have nevertheless retained this expression for purposes of comparison.

According to HUON LI, if the bed is " smooth ", the oscillatory boundary layer becomes turbulent at local Reynolds numbers where : $\nu d_1 (\omega/\nu)^{1/2} = 800$, apparently even at relatively low values of : $\frac{\nu}{\omega}$

If the bed is " rough ", the transition Reynolds number $\omega d_1 \epsilon/\nu$ is a constant for each roughness factor, or, for each characteristic bed material grain size.

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e) Results obtained at SOGREAH

The values $d_1 = f(T)$ at which, in accordance with the criterion defined above, a turbulent state appears in the oscillatory laminar boundary layer, are plotted in Fig. 3. The values obtained at BERKELEY for a "smooth" bed and various materials are shown dotted and the continuous lines give the values obtained from our tests at progressive wave conditions.

These show that, with two materials with similar d (or ϵ), and if the oscillatory movement has a given period, turbulence appears at a lower total oscillation amplitude if the relative movement in the vicinity of the bed is caused by wave action. (For instance, if we consider two similar values of ϵ , as for materials (3) and (10), or (4) and (13), we find that, according to HUON LI, at constant T , transition begins to appear at a value of d_1 that is about 5 times greater than the one observed by us.)

This discrepancy in experimental results appears to be fairly general and occurs at all absolute bed "roughness" values (the roughness being due to grains of a material of characteristic dimensions ϵ , or, in other words, d).

We cannot state categorically that this difference also occurs with a "smooth" bed, because it is very difficult to characterise the state of such a surface. Even though the beds we considered were relatively very smooth by HUON LI's standards ($\delta/\epsilon > 30$), consisting of a smooth metal plate ("smooth" in the physical sense) and meticulously smoothed out concrete, their behaviour from the transition point of view appeared to be that of a rough bed, inasmuch as transition occurred at a Reynolds number that still slightly depended on surface conditions.*

Our results confirm that, for a bed with some roughness, the transition Reynolds number $\omega d_1 \epsilon / \nu$ is comparatively very low, being expressed by the values given below, all of which are obtained from our various test results :

$$1) \text{ sand } n^{\circ} 4, \epsilon = 0.023 ; \frac{\omega d_1}{\nu} \epsilon \neq 36 ; \frac{\delta}{\epsilon} < 19$$

$$2) \text{ sand } n^{\circ} 2, \epsilon = 0.046 ; \frac{\omega d_1}{\nu} \epsilon \neq 65 ; \frac{\delta}{\epsilon} < 8.5$$

$$3) \text{ sand } n^{\circ} 1, \epsilon = 0.063 ; \frac{\omega d_1}{\nu} \epsilon \neq 60 ; \frac{\delta}{\epsilon} < 6.2$$

* If we, like HUON LI, had considered a smoother bed ("smooth" in the physical sense) such as a sheet of polished glass for instance, we might probably have reached hydraulically smooth conditions. The results obtained with a metal plate and smooth cement seem to indicate that the Reynolds number $\omega d_1 \nu^{-1/2}$ characterising transition for a hydraulically smooth bed would not have been less than 800, from which the Reynolds number $\omega d_1 \nu^{-1/2}$ for transition on an hydraulically smooth bed would appear to be at least 800..

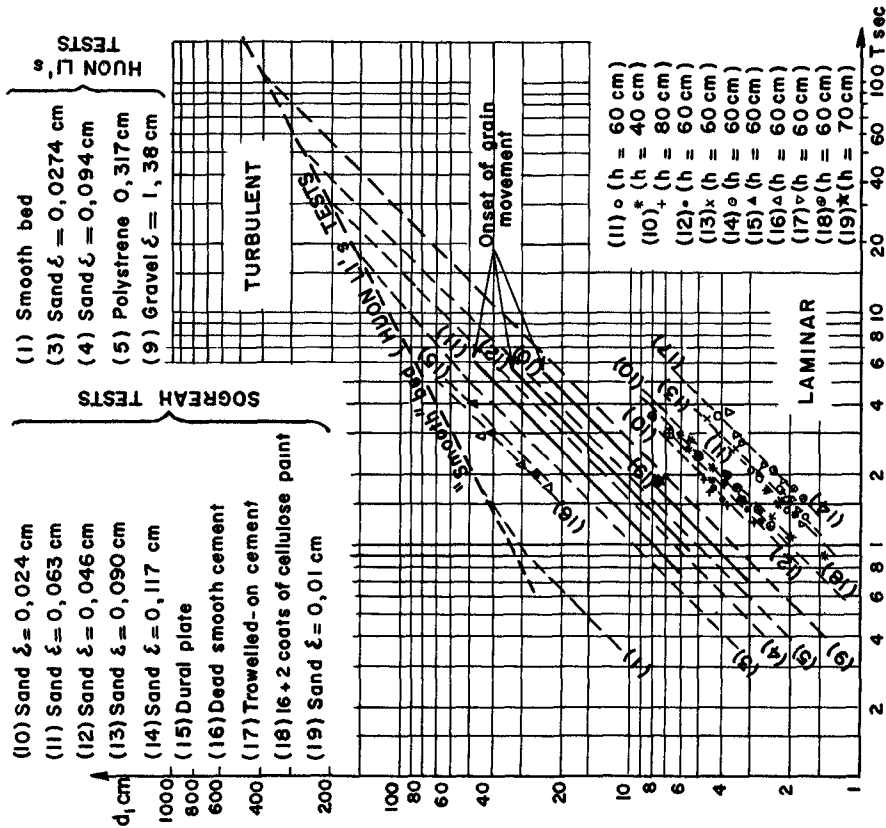


Fig. 3. Onset of turbulence in the oscillatory laminar boundary layer in progressive waves.

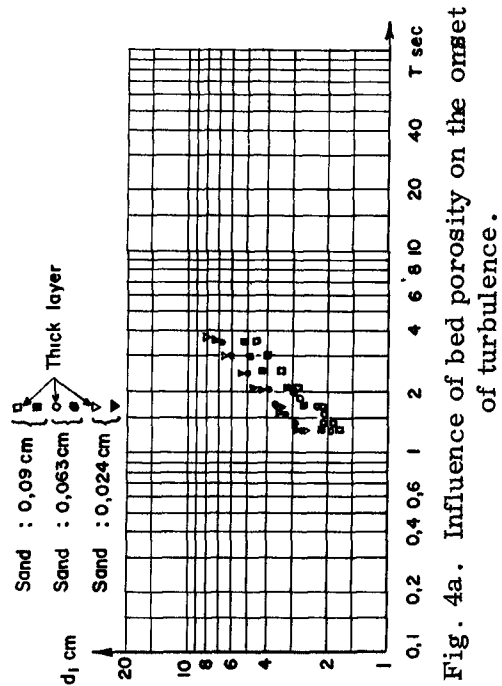


Fig. 4a. Influence of bed porosity on the onset of turbulence.

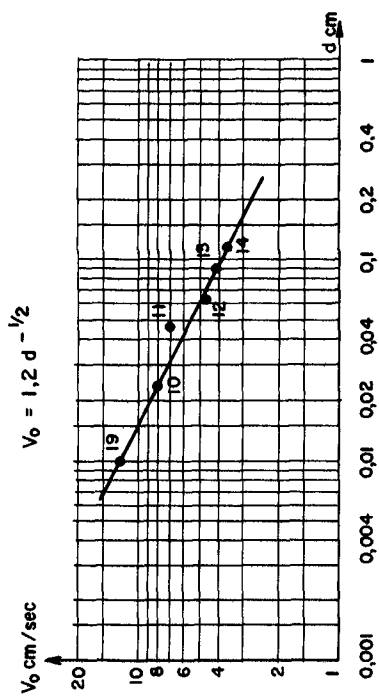


Fig. 4b. Relationship between V and d (sand)

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Although it is quite possible that a slight difference between the transition criteria adopted at BERKELEY and GRENOBLE could to some slight extent explain the discrepancies observed, it is more likely that they are due to the different experimental methods used. If, as an experiment, we plot the values of d_1 at which the onset of grain movement in the bed material occurs on the transition characteristic curve $d_1 = \psi(T)$ for the three sands tested ((10); (11) and (12)), we find that the value of d_1 at which the grains begin to move is itself lower than that used by HUON LI for characterising transition. And yet, with these materials, movement was already beginning to occur, although the boundary layer showed every sign of being turbulent.

Summing up therefore, we would say that the values obtained at BERKELEY appear to exaggerate the importance of the laminar conditions.

EFFECT OF BED POROSITY ON THE ONSET OF TURBULENCE

The tests carried out with sands (10), (11) and (13) were repeated, but this time with a "thick" layer (thickness of bed material layer 5 cm) and a "thin" layer (only a few grains thick, with the bottom layer grains stuck to an impervious smooth surface). The results of these tests are shown on the graph in Fig. 4a, from which it is seen that bed porosity facilitates the onset of turbulence to some extent. The influence of bed porosity however becomes very small for sands with a grain size below 0.024 cm.

EFFECT OF GRAIN SIZE

Neglecting the thickness of the bed layer, the grain diameter d , and V_0 , are the main characteristic transition parameters. According to our test results, a relationship of the form $V_0 = 1.2 d^{-1/2}$ (Fig. 4b) can be defined between these two parameters.

II - ONSET OF GRAIN MOVEMENT

Considering the evolution of movement in the immediate vicinity of the bed from the appearance of the laminar boundary layer to the instant at which well defined solid transport appears, the following phenomena can usually be observed:

- 1) Development of the oscillatory laminar boundary layer
- 2) Appearance of turbulence in the boundary layer
- 3) Initial setting in motion of first grains of material* (the first to be transported are often the coarsest of the bed materials)

*

It may nevertheless very well occur that the onset of grain movement precedes the appearance of turbulence if the material is very fine. According to our own experimental results, this possibility seems to arise at periods of about 2 seconds when $\varepsilon \leq 0.01$ cm. This comment is directly inspired by the results shown in Fig.3 and Fig.5, where it is seen that, at a given wave period, for transition, d_1 decreases as d increases, whereas in the case of nascent grain movement d_1 increases with d .

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- 4) General grain movement
- 5) Appearance of riffles, which are subsequently regularised.
- 6) Sediment transport with characteristic conveyance and saltation in the direction of wave propagation.
- 7) Slow progression of turbulence towards the mass of the fluid, from the bed towards the surface.
- 8) First signs of transport in suspension in the lower part of the fluid mass and in the opposite direction to that of wave propagation.
- 9) Lengthening and gradual disappearance of riffles.

a) Earlier Results

As far as we know, R.A. BAGNOLD (3) was the first to tackle this problem by studying the setting in motion of grains of material in a layer of uniform thickness resting on a cylindrical surface oscillating in a mass of still water.

Tests in progressive waves were carried out in 1954 at the Lille Institute of Fluid Mechanics by MM. MARTINOT-LAGARDE and FAUQUET (6) (7).

In the United States, Arthur T. IPPEN and Peter S. EAGLESON (7)(8), and later Madhar MANOHAR (2), have examined the problem during very recent years. Since then, MM. LARRAS (9) and VALEMBOIS have studied the effects of waves and clapotis on sandy beds, and as a result of these tests, M. LARRAS has been able to obtain a relationship between V_0 , T and W to characterise the onset of movement.

b) Criterion for the onset of movement

Experience shows that it is essential to state the criterion for the onset of movement very clearly when referring to nascent movement of a material in a moving bed ; this is all the more necessary when comparing experimental results obtained by a number of different methods.

BAGNOLD, and apparently MANOHAR too, observed the oscillation characteristics of the plate carrying the test material when the first grain started to move.

FAUQUET observed that the onset of entrainment of a few grains a smooth bed (glass) was characterised by the displacement of a least half the grains.

During our tests, the onset of movement was also characterised by displacement of the first grains. This was borne out later, when, by

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studying sediment transport proper along the bed, we were able to deduce at what hydraulic characteristics the solid discharge is near or at zero, with the aid of the curve defining the solid discharge Q in terms of hydraulic characteristics.

These definitions are still imprecise in some cases ; the onset of movement occurs in two distinct stages in the case of some of the very fine sands, as follows :

- Initial movement without riffling (the only case considered here), followed by spontaneous riffle formation before any appreciable sediment transport occurs.
- Initial movement with riffle formation at a fairly appreciably lower velocity at the bed. (Here, the onset of grain movement is definitely facilitated by the increased turbulence caused by the riffles).

c) Materials investigated

BAGNOLD used steel, quartz and coal grains.

FAUQUET used sand of various grain sizes as well as spherical limestone grains.

MANOHAR used various grades of sand, polystyrene and glass spheres.

We ourselves used two grades of granulated pumice and pollopas (granulated plastic) and sand in various grain sizes, their specific gravities being 1.38, 1.46 and 2.65 respectively (specific gravity of effectively transported grains).

d) Experimental results obtained at SOGREAH

The graph in Fig. 5, which is plotted on a logarithmic scale, shows the values of d_1 for various values of T at which the onset of grain movement occurs. (d_1 is the theoretical value for the total travel of a particle in the vicinity of the bed, not the observed value.)

Within the range considered, d_1 (T) can be correctly expressed by a relationship of the following form :

$$T^{-1} d_1 = \psi = \frac{V_0}{\pi}$$

where ψ is a function depending solely on the material and the fluid (kinematic viscosity)

For a given material, the relative depth h/L , i.e. the depth of water, clearly plays a certain part.

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This " dispersion " can either be due to the fact that the theoretical value d_1 only provides an approximate value for the total effective travel of the fluid particle near the bed (recent tests have shown discrepancies depending on h/L) or that, in general d_1 and T cannot by themselves fully characterise the onset of grain movement in a given material (It can be observed, for instance, that V_0 increases slightly with T for a given material and fluid). This can be explained by the fact that the thickness of the boundary layer also increases with T .

We shall see later that the maximum orbital velocity near the $V_0 = \pi d_1 T^{-1}$ (first order of approximation) is only an approximate value of the true velocity.

e) Influence of the physical characteristics of the grains

It would be misleading to try to derive a relationship between the velocity at which the grains begin to move and the physical characteristics of the grains from a set of insufficiently complete results. There are many reasons for saying this, chiefly that it is very difficult to characterise grain of material, whatever its shape, by a simple relationship, and further because of the influence of local turbulence on grain behaviour, which is usually difficult to characterise ; these reasons hold good, however many test results may be available.

We shall therefore merely give a few comments on our results, resisting the temptation to consider certain applications and compare the results with those already published (particularly in (2,3,7 & 9)), then deducing a relationship between the physical characteristics of the grains and the depth of water and the characteristics of the waves setting them in motion.

M A T E R I A L	d cm	$V_0 + \bar{u}$ cm/sec	W cm/s	$\frac{V_0 + \bar{u}}{W}$ *
Sand N° 1	0.063	28.50	9.4	3.03
Sand N° 2	0.046	20.50	6.0	3.42
Sand N° 3	0.024	16.50	3.3	5.00
Sand N° 4	0.010	12.50	0.8	15.60
Pumice N° 1	0.160	11.00	8.9	1.24
Pumice N° 2	0.120	8.50	7.3	1.16
Poñopas N° 1	0.110	10.50	5.6	1.86
Poñopas N° 2	0.039	7.00	2.1	3.30

Using the results shown above, we have plotted $V_0 + \bar{u}/W$ against d on logarithmic scales (Fig. 6).

* $\bar{u}$ being the mean velocity of an entrainment current acting on the fluid particles in the immediate vicinity of the bed.

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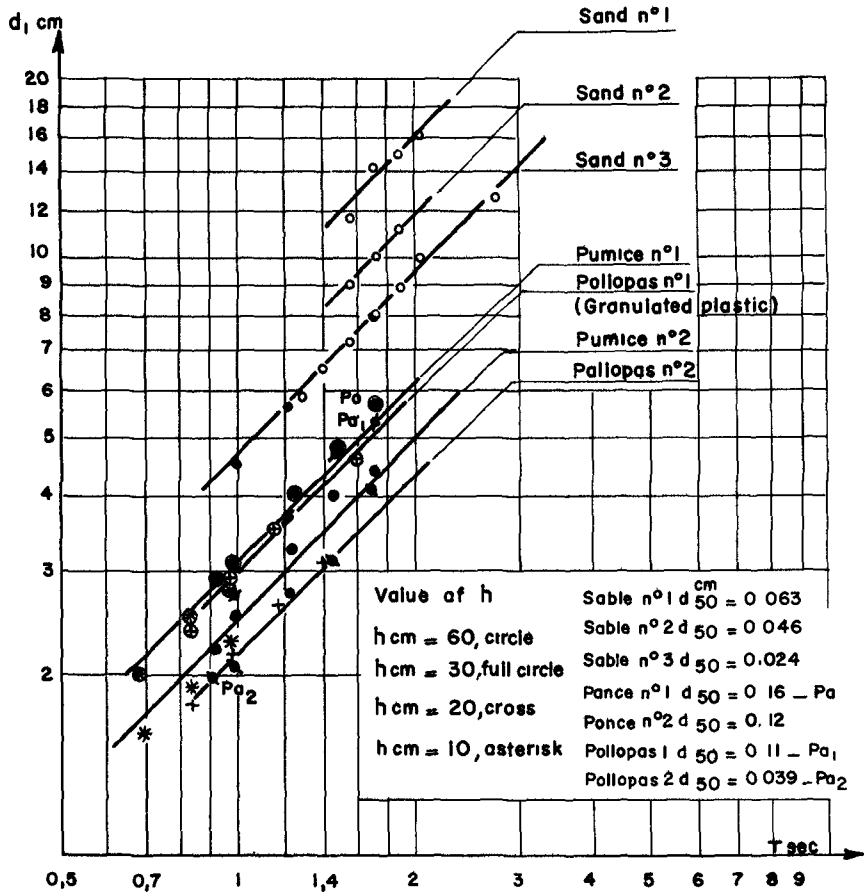


Fig. 5. Onset of grain movement.

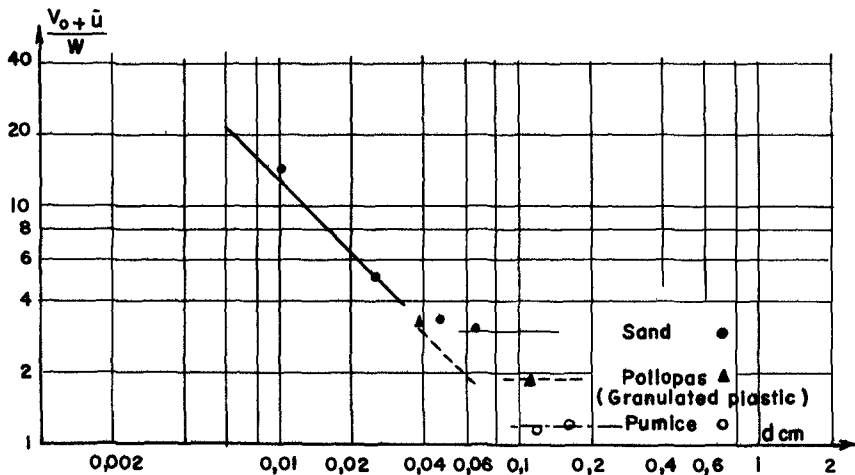


Fig. 6. Graph of $\frac{V_0 + \bar{u}}{W} = f(\tilde{g}, d)$

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A relationship of the form $\frac{V_0 + \bar{u}}{W} = \bar{F}(d^n)$, can be found

for a material of a given specific mass, with m varying as d . This result can be compared to M. VALEMBOLIS' comments on the results obtained by M. LARRAS in connection with the influence of d/d_0 .

At low d/d_0 i.e. when the grain of material is practically completely enclosed by the oscillatory laminar boundary layer at the onset of movement, $\frac{V_0 + \bar{u}}{W}$ appears to assume the form $\frac{V_0 + \bar{u}}{W} = \frac{k}{d}$ for a material with a given specific mass and for a given fluid (water). However, when d/d_0 reaches values at which the grain of material is no longer protected from the upper flow by the laminar boundary layer (either because the latter is relatively too thin or partly turbulent) $(\frac{V_0 + \bar{u}}{W})$ tends to become independent of d .

For a given fluid, the relationship between $\frac{V_0 + \bar{u}}{W}$ and d therefore generally seems to depend on the relative value d/d_0 , or, finally on T . With a *natural* sandy bed, where the flow in its vicinity should usually be turbulent, $\frac{V_0 + \bar{u}}{W}$ would be independent of d , depending rather to some extent on $\hat{g}$, as indicated by Fig. 6.

f) Turbulent state near the bed at the onset of grain movement

The setting in motion of the grains obviously approximatively satisfies a relationship of the form $d_1 T_i = \psi$, i.e. a similar relationship to that characterising transition in the boundary layer.

By analogy with HUON LI's "transition" Reynolds number, we can define a "setting off" Reynolds number of the form $\omega d_1 \epsilon / \nu$, which is, on the average, a constant for each degree of roughness.

$$\frac{\omega d_1}{\nu} \epsilon = 160 \text{ (pumice n° 2)} ; \frac{\omega d_1}{\nu} \epsilon = 55 \text{ (pollopas n° 2)}$$

$$\epsilon = 0.12 \qquad \qquad \qquad \epsilon = 0.039$$

$$\frac{\omega d_1}{\nu} \epsilon = 300 \text{ (pumice n° 1)} ; \frac{\omega d_1}{\nu} \epsilon = 165 \text{ (sand n° 2)}$$

$$\epsilon = 0.16 \qquad \qquad \qquad \epsilon = 0.046$$

$$\frac{\omega d_1}{\nu} \epsilon = 70 \text{ (sand n° 3)} ; \frac{\omega d_1}{\nu} \epsilon = 305 \text{ (sand n° 1)}$$

$$\epsilon = 0.024 \qquad \qquad \qquad \epsilon = 0.063$$

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The " transition " and " setting off " Reynolds numbers are similar for the materials considered, although the former is lower than the latter. This tends to confirm the important effect of the onset of turbulence in the vicinity of the bed on the setting in motion of grains with $d > 0.01$.

The oscillatory boundary layer is usually already turbulent when the grains are set in motion (although this is not necessarily the case if $\epsilon < 0.01$ cm)

In real life, the thickness of the laminar boundary layer can generally be expected to be greater than that observed during our tests, since it varies as $T^{1/2}$.

For a given fluid and material, the real life and model δ/ϵ will not be the same ; the real bed will be " smoother ". On the other hand, since V_0 is usually greater, the boundary layer will be turbulent in the majority of cases.

III - ENTRAINMENT VELOCITY IN THE IMMEDIATE VICINITY OF THE BED

MAXIMUM FLUID PARTICLE VELOCITY IN THE IMMEDIATE VICINITY

OF THE BED AT THE ONSET OF GRAIN MOVEMENT

We have seen that the expression $d T^{-1} = V_0$ approximately characterises the setting in motion of grains of a given material. In fact, the maximum resultant velocity of a fluid particle in the immediate vicinity of the bed differs slightly from the theoretical value V_0 characterising the velocity of a fluid particle on the bed in a perfect fluid (first order of approximation).

In a recent calculation, LONGUET-HIGGINS (10) has shown that, in a viscous fluid in a laminar state and in the immediate vicinity of the bed (or more exactly, in the boundary layer) progressive waves were accompanied by an entrainment current carrying in the direction of propagation.

The existence of such an entrainment current has been confirmed by flume tests, during which it was observed that, taken by and large, the fluid particles in the immediate vicinity of the bed moved in the direction of wave propagation.

The mean maximum value of this entrainment current has been given the symbol $\bar{u}$.

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In this way, the velocity of the fluid particles acting on the grains of material in the immediate vicinity of the bed can be considered as the resultant of the two following velocities :

- The entrainment current accompanying the progressive waves
- The pulsating component, a resultant of the general oscillatory motion

Immediately near the bed, the maximum values of these two velocity components add algebraically, both being parallel to the bed ; furthermore, they are horizontal in this case.

a) Value of entrainment current in the immediate vicinity of the bed

According to LONGUEE-HIGGINS's calculations, which have been confirmed by tests as we shall see, the maximum value of the entrainment current in progressive waves is given by an expression of the following form : (page 568)

$$\bar{u}_{\max} = \frac{1.376 T}{L} \left(\frac{2\pi a}{T} \right)^2 \frac{1}{\sinh^2 \frac{2\pi h}{L}}$$

or, in other words :

$$\bar{u}_{\max} = \frac{1.376 T}{L} V_{o \max}^2$$

- * As a first order approximation, the movement appears to be symmetrical ; at a given point, the velocity of a fluid particle in the direction of propagation (passing a crest) is equal to the velocity in the opposite direction (passing a trough).

As a second order, which is closer to real conditions, $V(t)$ for a given particle is no longer symmetrical, the maximum velocity of the particle passing a crest being greater than its velocity passing a trough. We also know that a mass transport current can then occur.

In real waves and in a finite depth, tests confirm that the wave orbits are no longer symmetrical at a certain distance from the bed. If the relative depth h/L is small and the waves very steep fronted, a double asymmetry can be clearly distinguished on the orbit (vertical and horizontal asymmetry). On one orbit, the curvature is more pronounced at the upper part than at the bottom (Fig. 8a) and the orbit is not closed except at a certain distance above the bed, where the vertical dissymmetry disappears.

This double dissymmetry shows up on the characteristic curve for the velocity of a fluid particle along an orbit ; the shape of the curve differs quite appreciably from the sinusoidal (Fig. 8b).

This difference in the velocities of the particles travelling in the direction of propagation and in the opposite direction tends to favour sediment transport towards the shore.

This difference becomes more marked as h/L decreases, i.e. as L increases at constant depth. In the limit, the waves are often comparable to solitary waves.

With shallow beds, therefore, considering the overall effect of the entrainment current and the dissymmetry of the orbital velocities in the direction of propagation and against it, long shallow-fronted waves can apparently cause quite considerable shoreward sediment transport.

This would tend to explain why " fair weather waves " (long, shallow waves) that cannot cause powerful rip currents or any appreciable seaward transport of suspended solids, are particularly likely to cause accumulations on beaches.

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where $V_{o_{\max}}$ is the maximum theoretical value of the horizontal component of the orbital velocity (first order, perfect fluid).

The entrainment current measurements carried out in the immediate vicinity of the bed during our tests confirm this expression with fair approximation. They are plotted on Fig. 7 for indicative purposes, which shows that the difference between the observed velocity and the theoretical value given by the LONGUET-HIGGINS formula is generally small at laminar boundary layer flow conditions (range of validity of the theory). This discrepancy increases as turbulence develops.

b) Maximum velocity of fluid particles in the immediate vicinity of the bed at the onset of grain movement

The resultant of the velocity of the fluid particles near the bed finally takes the form shown by Fig. 8c.

Let U_d characterise the *velocity at the onset of movement* of a given grain. The grain begins to move when the maximum value of the absolute velocity of a fluid particle in the immediate vicinity of the bed becomes at least U_d , with respect to a stationary grain of material.

As a second order approximation (perfect fluid) and in the immediate vicinity of the bed, this can be considered valid at the upper boundary of the boundary layer, the maximum velocity of a fluid particle being of the following form :

$$V_o \pm [a \text{ second order term of the form } V_o^2 f(h, L)]$$

The velocity $\bar{u}$ of the LONGUET-HIGGINS entrainment current (2nd order in the boundary layer) adds to this "periodic" term for a viscous fluid.

At the onset of movement we can neglect the second order term between the square brackets and say that the grain begins to move when :

$$U_d = \bar{u} + V_o$$

We have calculated the respective values of V_o , $\bar{u}$ and $(V_o + \bar{u})$ for the various materials considered. Fig. 9 shows the values of $\bar{u} + V_o$ for various values of L for seven different materials.

$V_o + \bar{u}$ is clearly practically constant, at the *onset of movement* of grains of a given material, irrespective of the factors giving rise to these velocities (we have already seen however that V_o increases

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slightly with $\bar{U}$ and explained it by the increase in the thickness of the boundary layer).

The means of the resultant initial velocities are as follows, for the cases considered :

- sand n° 1 ($d_{50} = 0.063$; $\rho_s = 2.65$) ; $V_0 + \bar{U} = 28.5$ cm/sec
- sand n° 2 ($d_{50} = 0.046$; $\rho_s = 2.65$) ; $V_0 + \bar{U} = 20.5$
- sand n° 3 ($d_{50} = 0.024$; $\rho_s = 2.65$) ; $V_0 + \bar{U} = 16.5$
- pumice n° 1 ($d_{50} = 0.160$; $\rho_s = 1.38$) ; $V_0 + \bar{U} = 11.0$
- pumice n° 2 ($d_{50} = 0.120$; $\rho_s = 1.38$) ; $V_0 + \bar{U} = 8.5$
- pollopas n° 1 ($d_{50} = 0.110$; $\rho_s = 1.46$) ; $V_0 + \bar{U} = 10.5$
- pollopas n° 2 ($d_{50} = 0.039$; $\rho_s = 1.46$) ; $V_0 + \bar{U} = 7.0$

A quick comparison between these initial movement velocities for a given material in progressive waves and permanent flow conditions shows some variation. The initial movement velocity appears to be considerably lower at progressive wave conditions ; however it is not always easy to determine the velocity near the bed at permanent flow conditions.

Furthermore, in view of the form of the resultant velocity in the immediate vicinity of the bed, the trajectory of a grain of material in the direction of propagation of the waves will be greater than in the opposite direction. Once in motion, the grain progresses by successive forward and backward bounds, the former being greater than the latter (Fig. 8d), with the net result that, on an horizontal bed, the grains progress in the direction of wave propagation.

c) Simultaneous action of a permanent current and the waves on a bed of materials

It often occurs in practice that the action of the waves on the bed is superimposed upon that due to ordinary currents. The waves can either run in the same direction as the current or obliquely across it, or even in the opposite direction. This is often the case for instance near a flat tidal coast or just off an estuary.

Out at sea, the ordinary currents are seldom strong and usually by themselves insufficient to dislodge the bed materials. This does not apply however if the materials have already been set in motion or suspension by the waves ; in such cases, even the smallest ordinary current can cause quite appreciable solid transport.

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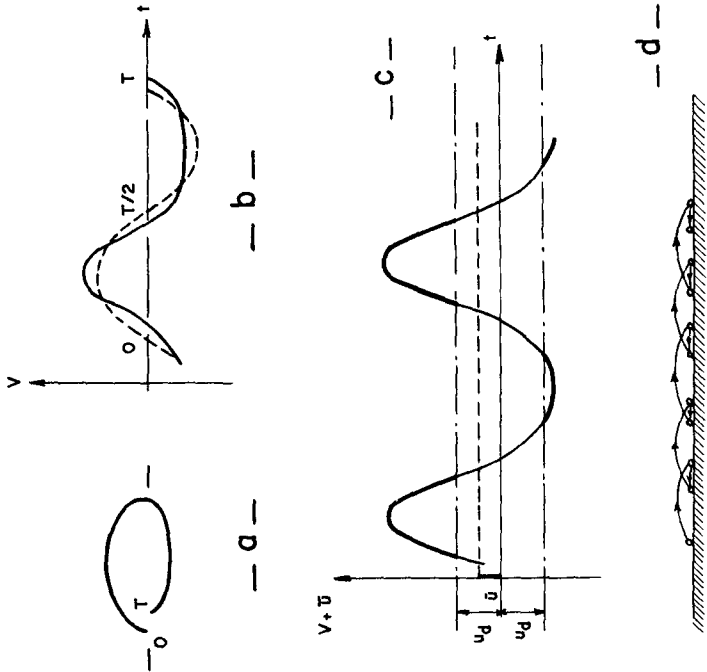


Fig. 8

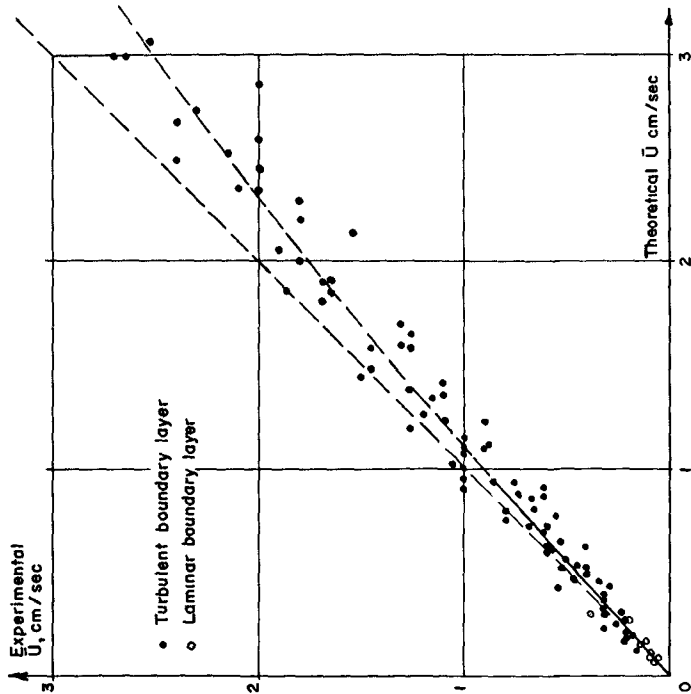


Fig. 7. Maximum velocity of entrainment current in the immediate vicinity of the bed. Comparison between experimental and theoretical values (Longuet-Higgins).

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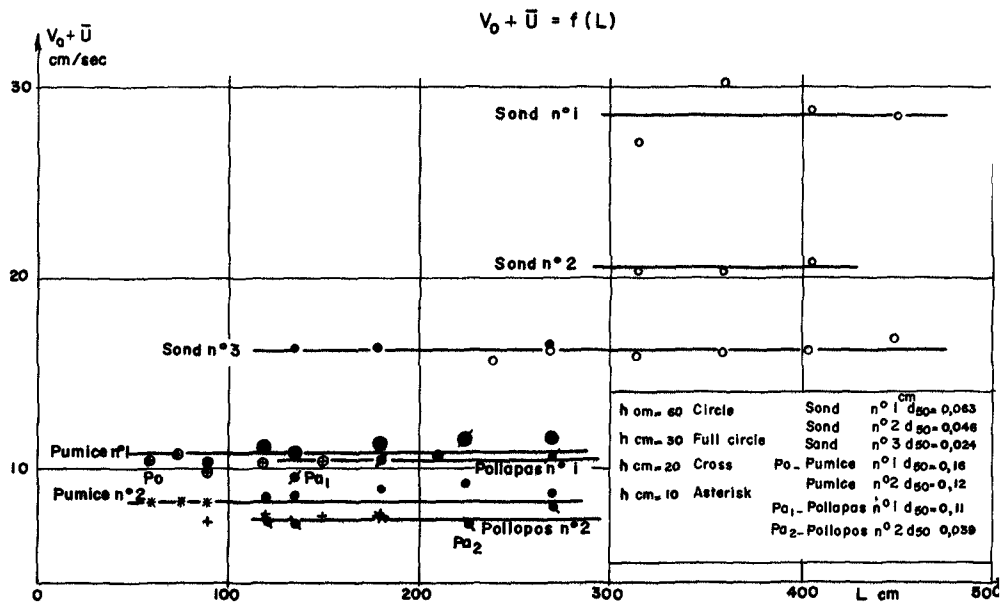


Fig. 9. Velocity at which grains begin to move.

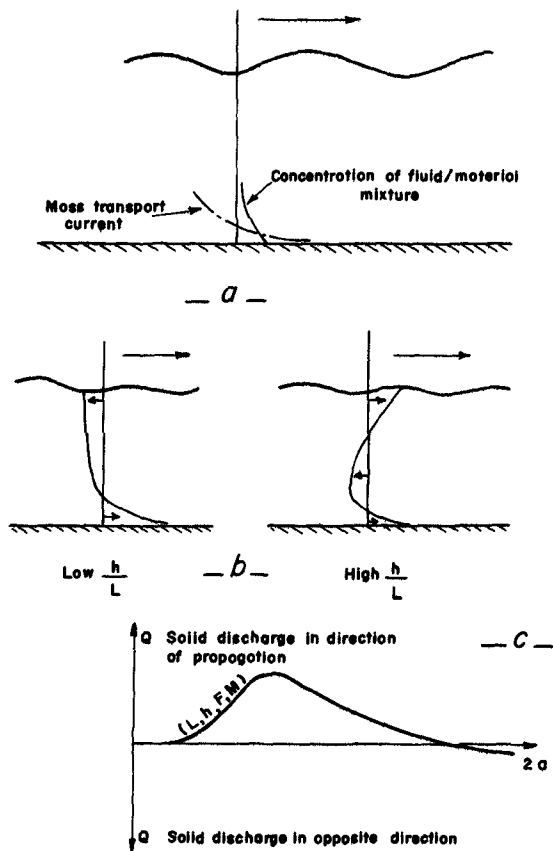


Fig. 10

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The effect of the waves when superimposed on the current is either to considerably reduce the critical entrainment tractive force for the material, or, as we have already seen, to cause the grains to begin to move.

It is often most desirable to study the combined action of waves and ordinary currents ; tests have been recently carried out at SOGREAH on this subject.

IV - SEDIMENT TRANSPORT AS AFFECTED BY WAVES ALONE

Due to the fact that, particularly in the immediate vicinity of the bed, the resultant of the velocities in the direction of propagation is greater than that acting in the opposite direction, the onset of grain movement is accompanied by *real solid transport*.

a) Mass transport current in a pure single-period wave

It can be seen during flume tests that, for two-dimensional waves, the oscillatory movement of the particles in the fluid mass is accompanied by a general movement.

The vertical distribution of the velocities of this transporting current has the following characteristics :

- Near the surface, the current carries in the direction of wave propagation ; a carrier current in the opposite direction has been observed at low values of h/L during tests.
- Near the bed, the current always conveys in the direction of propagation and is particularly pronounced in a relatively thin zone in the immediate vicinity of the bed (entrainment current).
- In between these two zones the current flows in the opposite direction ; in a closed flume, where the mean discharge is zero, the discharges are equal in both directions.

Two typical distributions for this current, both of which were observed during closed flume tests, are shown diagrammatically in Fig. 10b.

* This mass transport current appears in the equations for waves in a perfect fluid, from the 2nd order onwards, but the vertical velocity distribution is arbitrary.

Currents of this kind have been produced in test flumes, especially by CALIGNY (1878), MITCHIN (1939), BAGNOLD (1947), KING (1948), and the LABORATOIRE DAUPHINOIS D'HYDRAULIQUE (1949)

Such currents may well occur at sea, providing the waves are sufficiently regular ; they may be caused by friction at the interfaces which, as is well known, has a definite value in a real fluid.

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b) Inversion of the overall solid transport on a horizontal bed

As we have seen, flume tests show that, in general, for a practically pure wave on a horizontal bed, the mass transport travels in the direction of wave propagation.

However, at certain conditions, the overall solid discharge, i.e. the solid discharge integrated across a vertical and not only that in the immediate vicinity of the bed - begins to increase with wave amplitude passes through a maximum, decreases and then, given certain particular circumstances, may invert.

This phenomenon occurs in the test flume at high turbulence, with the layer of suspended material near the bed reaching the intermediate layers of the fluid in which the transporting current is directed out to sea.

Similar effects still occur if the waves are not pure, i.e. in the case of partial clapotis; in any case, the vertical distribution of the transporting current appears to be highly sensitive to the "purity of the waves".

The overall solid discharge may therefore flow either with or against the direction of propagation, depending on its relative values in both directions, i.e. the distribution u_z (resultant of the velocity) and $c_m(z)$ (mean concentration of material).

Functions $\bar{u}$ and c_m usually take the form shown in Fig. 10a in the considered zone.

Lastly, the curve for the overall solid discharge Q plotted against z can take the diagrammatic form shown in Fig. 10c in certain cases.

It is easy to imagine the value of such results, which, together with density currents, provide an apparently satisfactory explanation for the existence of seaward movements of solids in a given profile.

c) Attempt at establishing an empirical relationship from the test results

In the course of our consideration of wave action on a horizontal bed in this article, we have brought a number of parameters exerting a direct influence on the solid discharge into evidence, such as the orbital velocity of the particles, entrainment currents, turbulence, et

Once sufficiently far out to sea beyond the breaking zone most of these parameters are usually insufficient in themselves to cause sediment transport and can at most produce favourable conditions for it; in the absence of an ordinary marine current one might think that only the entraining current could transport sediment effectively. The position is generally different in shallow zones, where the orbital velocity is appreciably

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dissymmetrical, or in *very shallow* zones where the oscillation wave can take the form of the translation wave that produces a considerable difference in velocity when of the wave crest or trough is passed, particularly near the bed.

This being the case, one might expect to be unable to find a simple law relating sediment transport to the hydraulic characteristics on one hand, and to the characteristics of the materials on the other, this particularly in the zones considered as shallow for the wave length considered.

Establishment of the relationship $Q_{sol} = f(V_0)$

We have seen that the expression $d_1 T^{-1}$, i.e. V_0 (maximum orbital velocity, 1st order) is a fundamental parameter characterising the onset of movement of a given material. V_0 plays an analogous part with respect to sediment transport solely caused by wave action.

In order to illustrate the principal part played by this factor, we have plotted, as an example, the $Q_{solid} = f(V_0)$ relationship on the graph in Fig. 11 for a medium-fineness sand, very fine sand, and pumice. We have purposely chosen two materials forming bed ripples and suspension to a varying degree (sands) and a third material not causing either, at least within the fairly wide range of hydraulic conditions considered (pumice).

These results show that :

- 1) For a given material and fluid, the solid discharge not only depends on V_0 , but also on the depth of water h and the period T of the wave (or the wave length L). At a given V_0 , Q_{sol} increases with h and with decreasing T (this apparently contradictory result shows up the effect of V_0 very well).
- 2) For each material (L and h being known), Q_{sol} is expressed by a relationship of the form $Q_0 = K V_0^m$, with m not independent of V_0 ; at all values of L or h , m changes at a definite value of V_0 , i.e. at a certain value of a Reynolds number $2 V_0 \rho / \nu$ (or $\omega d_1 \rho \nu$) that can characterise the turbulence around the grain.

Since the effects are the same for both materials, one producing ripples and suspension and the other neither, one might be tempted to think that turbulence near the bed plays an important part during the first stages of sediment transport.

In actual fact, sediment transport caused by waves apparently divides into three distinct phases, as follows :

- 1) - Slight turbulence near the bed, with transport caused solely by bed load transport.
- 2) - Medium turbulence, with transport chiefly due to bed load transport

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and suspension ; the latter does not however extend farther than the fluid layers in which the mass current component acts in the opposite direction to the wave propagation.

- 3) - Strong turbulence, transport caused by bed load transport and suspension, the latter extending for quite a height above the bed. We have seen that, the *overall* sediment transport is not necessarily very large in this case.

With the exception of gales, during which suspension can reach a sufficient height above the bed to show up in the zone in which the mass current runs in the opposite direction to the wave propagation, the intermediate condition is the one most frequently encountered in real life.

The essential characteristic of transport during the first and second stages is that the solid discharge proper varies very rapidly with V_0 . With fine sand, for example, $Q_0 \neq K V_0^6$ in the range observed during our tests and corresponding to the second stage referred to above (Q_0 varies very little with h and T , at constant V_0 , in this particular case).

Solid discharge in a laminar or slightly turbulent medium

When the flow near the bed is laminar or slightly turbulent, the entrainment current seems to be the main driving element in the transport pattern, acting almost as a continuous current, with the superimposed oscillatory movement merely facilitating the transport. The following reasoning can be applied in this case :

The velocity gradient through the boundary layer will, if assumed constant, create the following tangential force in the boundary layer :

$$\tau = \mu \frac{\partial u}{\partial y} = \mu \frac{u\delta}{\delta} = \mu \frac{\text{cte } T}{L} V_0^2 \left(\frac{\nu T}{2\pi}\right)^{1/2} *$$

By analogy with the expressions given for solid transport in a permanent current, we can write :

$$q_s = A \tau (\tau - \tau_d)$$

or, for a given fluid and material :

$$q_s = \text{cte } f(h, L) \cdot a^2 (a^2 - a_d^2) \quad \text{where}$$

$2a_d$ characterises the wave amplitude for the onset of motion.

Plotting :

$$q_s = f [a^2 (a^2 - a_d^2)]$$

* $u_{01} = \frac{\text{cte } T}{L} V_0^2$ (LONGUET-HIGGINS) and $\delta_1 = A \sqrt{\nu/\omega}$

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$Q_{Solid} = f(V_0, h, T)$
for 3 separate materials

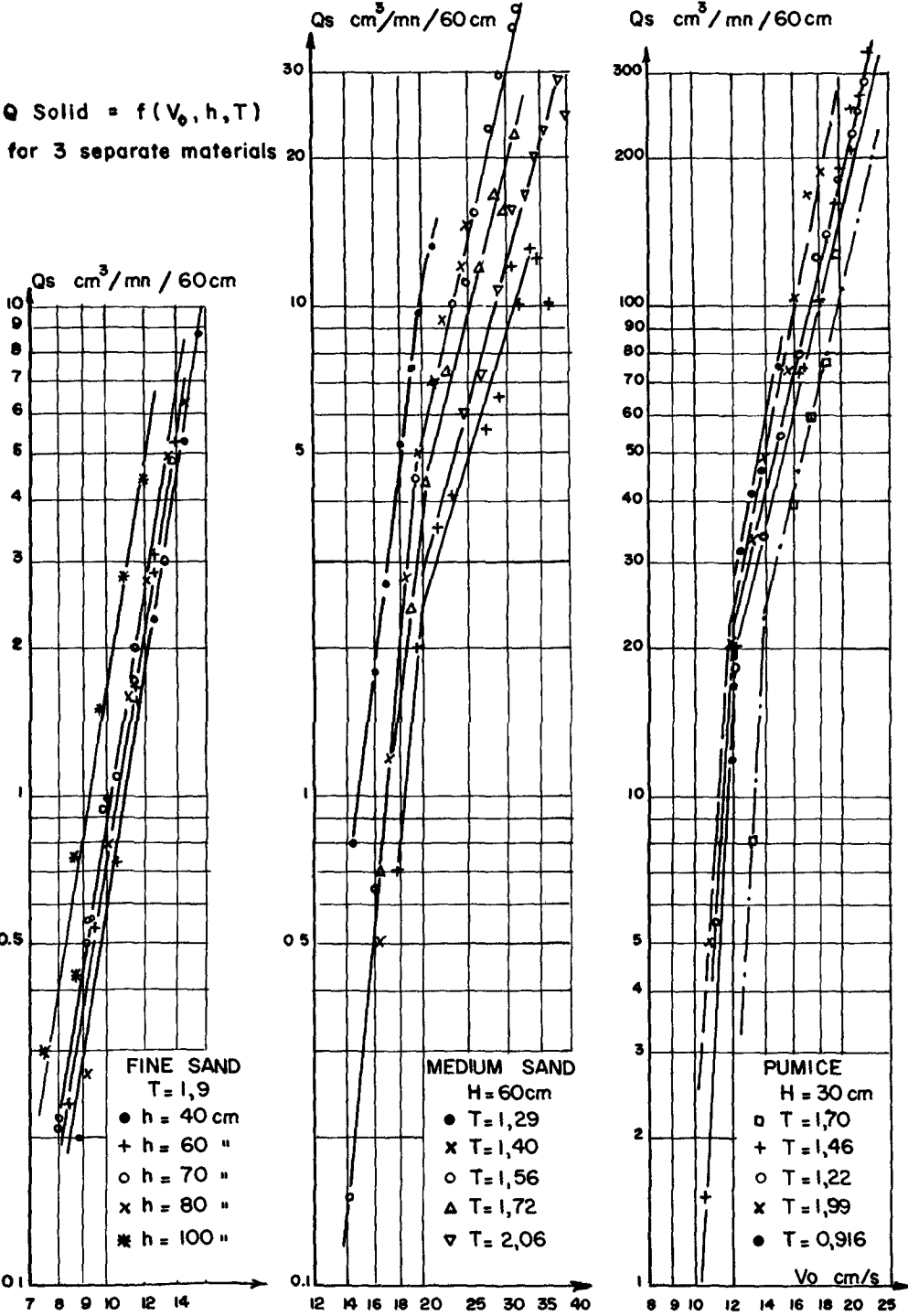


Fig. 11. Wave-generated solid transport on an horizontal bed.

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we find that the form of q_s is as follows :

$$q_s = K a^2 (a^2 - a_d^2)$$

For a given fluid and material, K is a constant for each value of h and L (or T), so that we can express q_s as follows :

$$q_s = a^2 (a^2 - a_d^2) \times \varphi (M, h, L) \quad (++)$$

where M is the characteristic of the material.

This expression is very well confirmed for the majority of the results we have obtained in a relatively low turbulence range.

This reasoning is strictly speaking only theoretically valid for laminar conditions, but still appears to remain applicable in cases where the flow near the bed is only slightly turbulent, an assumption that seems to be borne out by the results.

At this stage of the solid transport, q_s apparently has the following form :

$$q_s = F (\text{Material}) a^2 (2a^2 - 2a_d^2) L h^{-4}$$

where q_s is a solid discharge per unit width (in this case unit width is that of the test flume, i.e. 60 cm) its dimensional form being $L^2 T^{-1}$. The materials function therefore has the dimensions of a velocity (e.g. falling velocity).

V - INDIRECT EFFECT OF WAVES ON THE BED

The foregoing investigation clearly shows that, near the bed, the waves transport materials in the direction of their propagation, i.e. towards the coast in real life. In this zone, the shallower the water and the finer and lighter the material, the greater generally the sediment transport in the direction of propagation, for a given wave type. One might be led to believe therefore that the materials travel indefinitely towards the coast, but in fact, the reverse very often happens, the material eroded from the coast being carried far out to sea and dispersed.

We have seen during this investigation that the overall solid discharge on a vertical line could be zero or even invert ; we have attributed this inversion of sediment transport to the leading part sometimes played by the seaward component of the transporting current.

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However, we think that there are other reasons besides (not including the obvious effect of gravity, particularly near the shore where the bed slope is sometimes quite steep).

To the previously mentioned action of the waves on the bed, i.e. their capacity for developing density currents carrying the finer particles seawards (see beginning of this article) we can add the irregularity of the natural waves, particularly during gales. It was very noticeable during our tests how much the form of the transporting current for instance could be affected by a partial reflection or an irregularity in the period.

The sediment movements out to sea can also be favoured by the sometimes violent action of the surface wind, which can, particularly during a gale, set up quite an appreciable shoreward current at the surface that moves materials near the bed out to sea.

The sometimes violent currents appearing in the surf area also play a part that cannot be neglected on sediment transport out to sea, if only by creating suitable conditions for the development of density currents. There is every reason to believe that, by virtue of the principle of continuity, these " littoral currents " integrate intimately into the system of currents outside the breaking zone and which we have particularly stressed here.

Seaward sediment transport by density current

Experience shows that the waves produce density currents, similarly to flow by gravity.

When the waves are high (e.g. gale conditions), a considerable quantity of material can be dislodged from the bed and put into suspension in the breaking zone. Obviously, the finer the materials concerned and the more recent the origin of the materials on the beach subjected to the waves, the more pronounced this effect will be (Density currents chiefly consist of solid particles of approximately 50 μ or less, which are themselves able to entrain larger particles by virtue of their nature.

Because it is relatively denser than the surrounding medium, water carrying sediment sinks to the bed where it then flows towards the greater depths.

Tests (11) carried out in a wave flume show that the flow of the turbid mass towards the bed is accompanied by a counter-current of clear water in the upper fluid layers, so that the water at the surface remains clear.

In the case of waves, this phenomenon is complicated by the presence of currents linked to the very existence of the natural waves and effective right into the breaking zone.

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It is probable, however, that the wave currents occurring in real conditions only have a secondary effect upon the turbid currents. The density currents running seawards particularly tend to develop near the end of a gale, when the currents connected with the waves are in the process of disappearing. These currents may at most delay the flow of the density current, because, in the vicinity of the bed, they generally head towards the shore, thus directly opposing the development of the density current.

When a gale is dying out, therefore, the density current probably progressively gains an ascendancy over the mass transport current in the layers near the bed.

It is also probable that, during a gale, some of the finest particles may be able to reach the open sea, travelling in the intermediate fluid layer in which the seaward component of the mass transport current is active.

Finally, it would not be unexpected to find a fairly well-defined dividing line between cloudy and clear water a comparatively short way out from the shore. This is because the counter-current of the density current due to wind action and the mass transport current caused by waves all tend to bring the clear water from the sea in towards the shore along the surface.

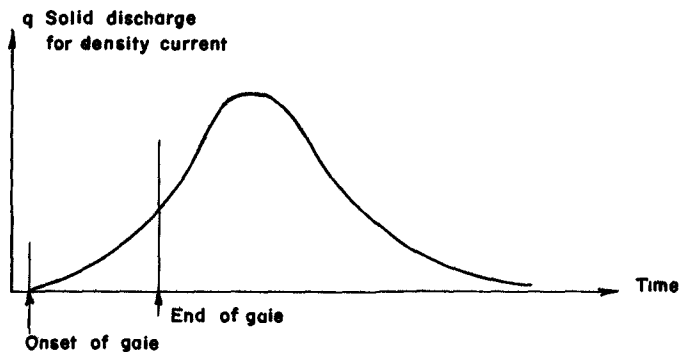


Fig. 12. Seaward movement of materials due to density currents.

Since turbulence in the breaking zone gradually decreases after a gale, the solid flood of the density current itself decreases, with a probable lag between the two events. Since the density current probably originates right at the beginning of the gale, it seems reasonable to draw the shape shown diagrammatically in Fig. 12 for the curve for the solid discharge carried out to sea by density current with respect to time.

This is why the bed materials become progressively finer further out to sea. Density currents, the progressive dying out of turbulence ... etc, contribute to this tendency for the finest materials to decant the farthest out from the shore. A correlating fact for this is that one only comes across comparatively few very fine particles on beaches, particularly after spells of bad weather.

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