Chapter 17

WAVE ENERGY AND LITTORAL TRANSPORT

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1. GENERAL

As is known the breaking of oblique waves generates currents roughly parallel to the shore line usually designated by longshore currents. The intensity of these currents which are present almost exclusively between the breaking line and the shore depends on the characteristics of the waves (angle of approach, height and period) and on the characteristics of the shore (slope and roughness).

A certain amount of energy E is transmitted by the breaking wave along its direction of propagation. As this is a transmitted energy, it is possible to speak about its component parallel to the shoreline which would be indicated by E sen α , being the angle of approach of waves, i.e. the angle that crests make with the shoreline.

Let us consider the breaking line and two near orthogonals distant dx from one another (fig.1).

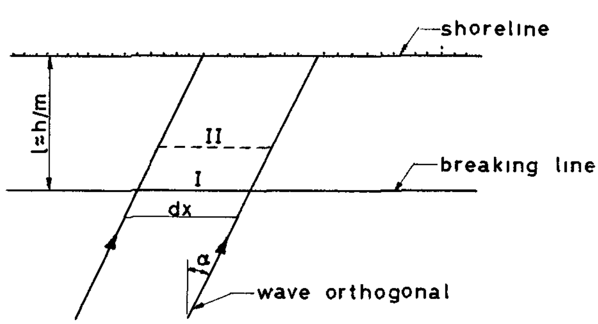

Fig.1 - Sketch of wave attack

A fraction of the energy flowing through the breaking line remains in the longshore current from which it will flow out in a continuous or concentrated form (rip currents); another part is dissipated by friction in the bottom whilst the remaining is lost by breaking (turbulence).

The distribution of the difference between the energy flow ing in at section I and flowing out at section II by these thre fractions depends firstly on the type of breaking (spilling or plun ing) and secondly on the characteristics of the wave and of th shore.

That distribution varies also along the zone where the lon shore current is present, i.e. between the breaking line and th shore.

2. DISTRIBUTION OF TRANSMITTED ENERGY

2.1 Energy dissipated in friction losses - Let us assum that the average velocity $\underline{V}$ of the longshore current betwee the breaking line and the shore(x) is known. Denoting by $\underline{k}$ th friction factor and assuming that the friction force is propor tional to the square of the velocity, the energy dissipated in beach length dx will be proportional to the third power of th velocity and can be written

$$E_d = k \rho V^3 \quad dx$$

$$\text{or} \qquad E_d = k \rho V^3 \; \frac{h}{m} \; dx \qquad (1$$

where ρ represents the unit mass of water, $\underline{m}$ being the slope c the beach and $\underline{h}$ the breaking depth.

Assimilating the breaking waves to solitary waves arriv ing each T seconds (T being the period)(xx) the transmitted er ergy parallel to the shore, per second is

$$E_t = \frac{2{,}2g\,H^3}{T} \; \text{sen}\,\alpha \; \cos\alpha \quad dx \qquad (2$$

H being the wave height.

According to some authors it is preferable to consider p riodic waves of which the deep-water characteristics are known

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(x) For the present purpose it is sufficient to consider an ave age value of the velocity of the current. In a more detaile study now under way the variation of the velocity of the long shore current in the surf zone is taken into account.

(xx) Munk: "The solitary wave theory and its application to su problems". Annals of the New York Academy of Sciences Vol.51 - 1949

Anyhow if these characteristics were known, it would always be possible to calculate the solitary wave which each T seconds transmits the same amount of energy than the periodic wave in deep water.

Therefore we shall continue to assimilate breaking waves to solitary waves.

The ratio s of the energy dissipated in friction losses in the bottom (eq.1) and the component of the transmitted energy parallel to the shore (eq.2) is

$$s = \frac{k \rho V^3 h/m \ dx}{2,2 \ gH^3 \operatorname{sen}\alpha \cos\alpha \ dx/T} \tag{3}$$

Writing the velocity of propagation of the breacking solitary wave $C = \sqrt{g(h + H)}$ with $H = 0,78 \ h$ and considering the steepness of the wave $\delta = \frac{H}{L} = \frac{H}{CT}$, equation (3)

becomes

$$s = \frac{(V/C)^3}{0.38 \ A \operatorname{sen} 2\alpha} \tag{4}$$

where A is the dimensionless parameter $A = \frac{m\delta}{k}$

2.2 Energy contained in the longshore current - According to the theory of the solitary wave, the volume of water Q leaving the breaking wave in each strip of width dx is

$$Q = 2 \ h^2 \cos\alpha \ dx \tag{5}$$

V being the mean velocity of this volume of water, its kinetic energy will be

$$E_c = \frac{1}{2} QV^2 \tag{6}$$

The ratio t of the kinetic energy of the longshore current (eq.6) to the component of the transmitted energy parallel to the shore can be written, taking equation (5) into account,

$$t = \frac{(V/C)^2}{0.64 \ \operatorname{sen}\alpha} \tag{7}$$

2.3 Energy dissipated in wave breaking(turbulence) - Th energy fraction dissipated in breaking is

$$r = 100\% - (s + t)\%$$

3. CALCULATION OF THE MEAN VELOCITY OF THE LONGSHORE CURRENT

Let us apply the momentum method for calculating the mea velocity of the longshore current.

Being Q_b the volume of water carried by the solitary brea ing wave and C_b the velocity of propagation, the component pa allel to the shore of the transmitted rate of momentum will be

$$Q_b \times C_b \quad \text{sen}\,\alpha \cos\alpha \, dx/T$$

$\underline{V}$ being the mean velocity of the longshore current, the vo ume Q_b keeps a momentum towards the shore equal to

$$Q_b V \cos\alpha \; dx/T$$

According to the momentum theorem, the variation of t rate of momentum is equal to the friction force. Therefore will be

$$\frac{Q_b C_b \,\text{sen}\,\alpha \,\cos\alpha}{T} = \frac{Q_b V \cos\alpha}{T} - k\rho V^2 \cdot \frac{h}{m} \qquad ($$

Computing $\underline{V}$ from equation (8) and substituting its value equations (4) and (7), it is possible to determine the values of and $\underline{t}$, i.e. the fractions relative to the ingoing energy respec tively of the energy dissipated in friction losses in the bottom a of the energy contained in the longshore current.

Nevertheless if equation (8) was applied as written above we should find that for some values of $s + t > 100\%$ which is possible.

This lead us to some considerations on the momentum ac ally available to generate the longshore current.

In the present paper only the case of beaches with a ve gentle slope (say 2 per cent or less) will be considered, it bei assumed that wave breaking is gradual so that the wave resumes its shape at each moment, remaining practically symmetric. This amounts to assuming that the wave height decreases linearly with depth and that the decrease of transmitted energy corresponds exactly to the energy progressively lost in wave breaking. The wave height thus follows constantly the law $H = 0.78\ h$.

In a study now in progress we have approached the case of a breaking wave giving rise to a bore.

Let us consider then two neighbouring sections I and II (fig.1). The change of momentum between them will be

$$dM = d(QC) = Q\,dC + c\,d\,Q$$

From $Q = 2\,h^2$ and $C = 1.78\,g\,h^{\frac{1}{2}}$ (solitary wave), the expressions $dQ = 4\,h\,dh$ and $dC = \frac{1}{2}\,Ch^{-1}dh$ are obtained. Hence

$$\int_{h_1}^{h_2} C\,d\,Q = \frac{4}{5}\,(Q_1C_1 - Q_2C_2)$$

$$\int_{h_1}^{h_2} Q\,d\,C = \frac{1}{5}\,(Q_1C_1 - Q_2C_2)$$

This lead us to suggest that between sections I and II the momentum available to generate the longshore current is only the volume $d\,Q$ of water available in the section under consideration times the velocity of propagation C, the momentum available between the breaking line and the shore being

$$\int_{h_b}^{o} C\,d\,Q = \frac{4}{5}\,Q_bC_b$$

The amount $\int_{hb}^{o} Q\,d\,C = \frac{1}{5}\,Q_bC_b$ would correspond to a lost momentum since the wave flowing out at section II has a velocity of propagation $C + d\,C$ instead of velocity C which corresponds to the wave flowing in at section I.

The momentum theorem will be written now

$$\frac{4}{5}\ \frac{Q_b C_b \operatorname{sen}\alpha \cos\alpha}{T} - \frac{Q_b V \cos\alpha}{T} = k \rho V^2 \frac{h}{m} \qquad (9$$

Solving equation (9) with respect to V and taking into ac count that $Q_b = 2 h^2$, $C_b = 1.78g\ h^{\frac{1}{2}}$ and $\delta = \frac{H}{C_b T}$, the followin dimensionless equation results

$$\frac{V}{C_b} = 1.30 \cos\alpha\ A \left[\sqrt{1 + \frac{1.22 \tan\alpha}{A}} - 1\right] \qquad (10$$

where $A = \frac{m\delta}{k}$

Thus for a given angle of approach , the relative veloc ity $\frac{V}{C_b}$ is a function of the parameter $A = \frac{m\delta}{k}$ alone.

4. CALCULATION OF THE ENERGY FRACTIONS DISSI PATED IN FRICTION LOSSES AND BREAKING LOSSE

By substituting the value of $\frac{V}{C_b}$ given by equation (10), is possible to obtain s and t from equations (4) and (7).

Figures 2 and 3 show the values of s and t for angles 10°, 30°, 45°, 60° and 80° in a function of the parameter $A = \frac{m}{k}$

Figure 2 shows that the fraction s of the energy dissipa ed in friction losses in the bottom reaches a maximum for a ce tain value of A which varies in accordance with .

On the other hand, for values of A not exceeding 0.3, tl fraction s is a function of the angle α , reaching a maximum nea 45° to 60°.

Figure 3 shows that energy dissipated in breaking losses always important (between 85 and 95%) for angles $\alpha = 10^o$. Fo greater angles the fraction r is important for small values of A decreasing regularly as A increases.

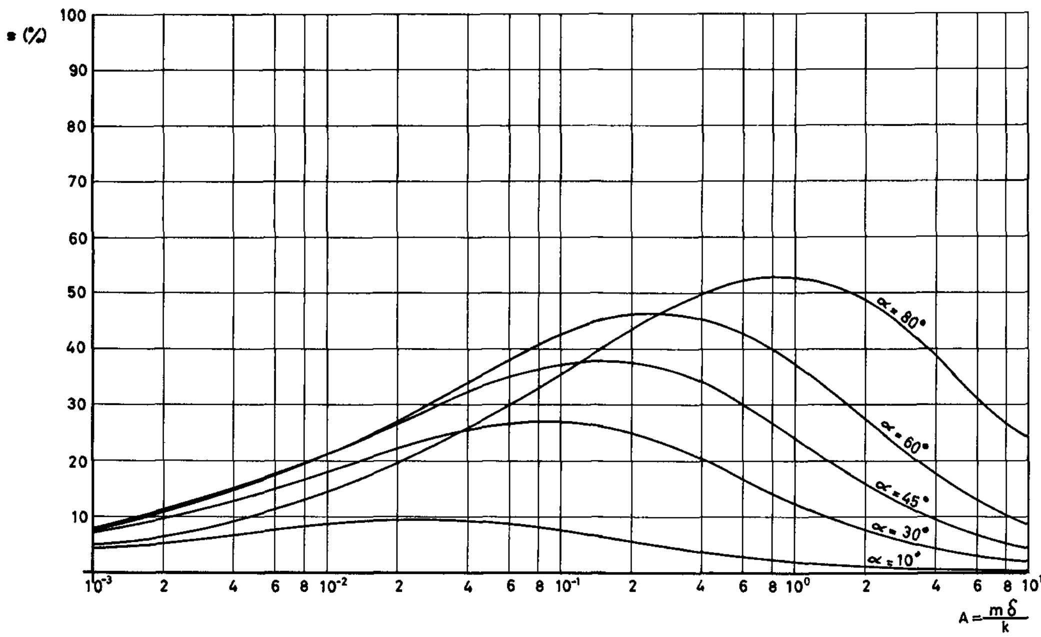

FIG 2 --PERCENTAGE OF ENERGY DISSIPATED BY FRICTION

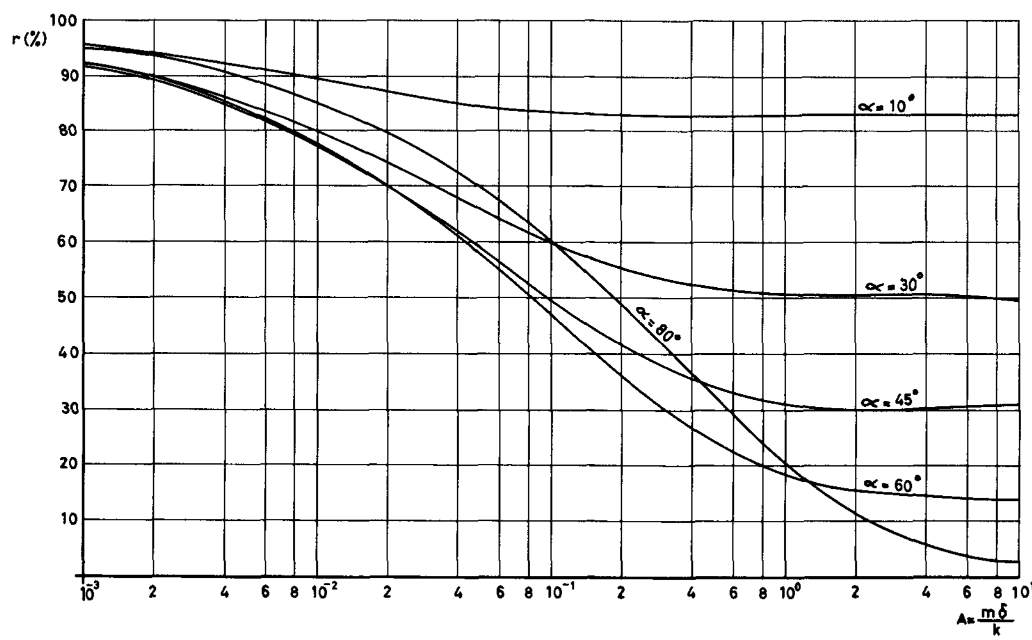

FIG 3 --PERCENTAGE OF ENERGY DISSIPATED BY BREAKING

5. SOLID DISCHARGE

For constant values of m and k the shape of the curves $s = f(\frac{m\delta}{k})$ or $s = f(\delta)$ is very similar to that of curve $Q_{solid} = f(\delta)$. The same applies to $s = f(\alpha)$, a maximum being reached between $\alpha = 45^{\circ}$ and $\alpha = 60^{\circ}$.

This seems to suggest that the solid discharge can be dimensionlessly expressed as a function of s. In a first attempt Q_{solid} could be considered as proportional to s.