# CHAPTER 4

## NUMERICAL CALCULATION OF WIND WAVES IN SHALLOW WATER

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### SYNOPSIS

For the purpose of estimating the waves raised by typhoons approaching continental shelf and inland seas, one of the authors (I960) devised graphical method to the forecasting the waves in the fetches travelling over shallow water area in I960. The method has been widely adopted to evaluate the waves of the bays and inland seas in Japan and the western coast of Taiwan, since it was proved that calculated results considerably agreed with measured records.

On the account of the spread of electronic computers, numerical analysis will be more expedient than graphical operations nowadays. Wilson s numerical integration method (I96I)(I962) has been extended to facilitate the calculation of the waves of shallow water area. The procedures of calculation are described and example of hindcasting of waves in typhoon by the machine run are also submitted in this paper.

### PROPOSED RELATIONSHIPS GOVERING SHALLOW WATER GENERATION

Based on the measured data of Bretschneider (I958), the sibnificant wave height H and period T are expressed by the following equations in shallow water.

$$\frac{gH}{U^2} = \alpha \tanh\left[k_3\left(\frac{gD}{U^2}\right)^{3/4}\right] \tanh\left[\frac{k_1\,(gx/U^2)^{1/2}}{\tanh k_3\,(gD/U^2)^{3/4}}\right] \quad \ldots(I)$$

U: wind velocity　D: water depth

g: gravity　x: fetch length

$\alpha = 0.26$　$k = 0.0I$　$k = 0.578$

$$\frac{gT}{2\pi U} = \beta \tanh\left[k_4\left(\frac{gD}{U^2}\right)^{3/8}\right]\tanh\left[\frac{k_2\,(gx/U^2)^{1/3}}{\tanh k_4\,(gD/U^2)^{3/8}}\right] \quad \ldots \quad (2)$$

$$\beta = 1.40 \qquad k_2 = 0.0436 \qquad k_4 = 0.520$$

Both of the equations are approaching to Wilson's while $D \longrightarrow \infty$

$$\frac{gH}{U^2} = \alpha \tanh\left[k_1\left(\frac{gx}{U^2}\right)^{1/2}\right] \quad \ldots\ldots\ldots \quad (3)$$

$$\frac{gT}{2\pi U} = \beta \tanh\left[k_2\left(\frac{gx}{U^2}\right)^{1/3}\right] \quad \ldots\ldots\ldots \quad (4)$$

Equations (1) and (2) are illustrated by Fig. 1 and 2.

The ratio between group velocity and wind velocity in shallow water is:

$$\frac{G}{U} = \frac{1}{2}\left(1 + \frac{4\pi D/L}{\sinh 4\pi D/L}\right)\frac{gT}{2\pi U}\tanh\frac{2\pi D}{L} \quad \ldots\ldots \quad (5)$$

G: group velocity L: wave length

Let

$$S = \frac{gD/U^2}{gT/2\pi U} = \frac{2\pi D}{L_0} = \frac{2\pi D}{L}\tanh\frac{2\pi D}{L} = y \tanh y \quad \ldots \quad (6)$$

L: wave length in deep water

$$y = \frac{2\pi D}{L}$$

Next, designate

$$M = \frac{G/U}{(gD/U^2)^{1/2}} \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$$

From eq. (5)

$$M = \frac{S - S^2 - y^2}{2yS^{1/2}} \quad \ldots\ldots\ldots\ldots \quad (8)$$

If S approaches to 0, $y^2 = S$, $M = 1 - S/2$, whereas $S = \pi$, is the case of deep water wave, the ratio between group velocity and wind velocity is expressed by:

$$\frac{G}{U} = \frac{1}{2}\,\frac{gT}{2\pi U} = \frac{\beta}{2}\tanh\left[k_2\left(\frac{gx}{U^2}\right)^{1/3}\right] \quad \ldots\ldots\ldots \quad (9)$$

In the region of $0 < S < \pi$, following equation can be approximately established

$$1 - M = a_1 S + a_2 S^2 + \ldots + a_6 S^6 \quad \ldots\ldots\ldots \quad (10)$$

$a_1 = 0.4536$ $a_2 = 0.0931$

$a_3 = -0.2745$ $a_4 = 0.17033$

$a_5 = -0.04760$ $a_6 = 0.005067$

From this equation, in case $S \geq \pi$, $M \leq 0.288$, and S can also be expressed as follows if $M > 0.288$

$$S = b_1 (I-M) + b_2 (I-M)^2 + \ldots\ldots + b_7 (I-M)^7 \quad \ldots\ldots(II)$$

$b_1 = 2.464857$ $\qquad b_2 = -7.35305$

$b_3 = 52.7458\dot{3}$ $\qquad b_4 = -I62.\dot{2}$

$b_5 = 275.8\dot{3}$ $\qquad b_6 = -247.\dot{2}$

$b_7 = IOI.I9046$

The group velocity and period of shallow water wave can be calculated when S and M are worked out.

CALCULATION PROCEDURES

The calculation of waves in shallow water is also to be carried out by stepwise method from the lattice of wind field, only evaluating S instead of calculating period or celerity directly. At the initial point of the fetch, Wilson's method will be adopted all the same. If the wave height Ha, group velocity Ga at the point "a" on the space time wind field are known, the problem is to calculate the wave features of the point "b" in the lattice as shown in Fig. 3.

At first, compute the velocity Ua of the wind blowing over point "a" from the lattice or by a formula U = U(x, t), which can be drived from the pressure distribution of typoon or hurricane as well as its moving direction and velocity, also the water depth of this point Da is to be detemined from the lattice or some approxmate function D = D(x, t).

Secondary, calculate $Ma = \frac{Ga}{Ua} \frac{I}{(gD_a/U_a^2)^{1/2}}$, if $M < 0.288$, the waves at this point are still to be deep water wave, and Wilson's method should be used, whereas $M < 0.288$ following procedures are to be adopted.

CALCULATION IN CASE OF WAVE DEVELOPMENT

By differentiating Eq. (I)

$$\frac{dH}{dx} = \frac{k^2}{\alpha} \times$$

$$\times \frac{[\alpha \tanh k_3(gD/U^2)^{3/4} + gH/U^2]\,[\alpha \tanh k_3(gD/U^2)^{3/4} - gH/U^2]}{[\tanh k_3(gD/U^2)^{3/4}][\ln(\alpha \tanh k_3(gD/U)^{3/4} + gH^2/U^2) - \ln(\alpha \tanh k_3(gD/U^2)^{3/4} - gH/U^2)]} \quad \ldots\ldots\ldots(I2)$$

at point "a", if the waves are developing, it must be that $\alpha \tanh k_3(gD/U^2)^{3/4} > gH/U^2$, and the wave height $H_b$ can be calculated by:

$$H_b = H_a + \left(\frac{dH}{dx}\right)_a \Delta x$$

the choosing of $\Delta x$ is the same as Wilson's method.

Prior to the evaluation of group velocity, calculate Sa from Ma by eq.(11).

Since $S = \frac{gD}{U^2} / \left(\frac{gT}{2\pi U}\right)^2$ from eq.(2)

$$S = \frac{gD}{U^2}\left[\beta \tanh k_4 \left(\frac{gD}{U^2}\right)^{3/8}\right] \tanh\left[\frac{k_2 (gx/U^2)^{1/3}}{\tanh k_4 (gD/U^2)^{3/8}}\right] \quad \ldots (13)$$

therefore

$$\frac{dS}{dx} = \frac{8k_2^3}{3\beta}\frac{g}{U^2}\frac{S}{(1/S\times gD/U^2)^{1/2}}\left[\tanh\frac{1}{k_4 (gD/U^2)^{3/8}}\right] \times$$

$$\frac{[\beta \tanh\{k_4(gD/U^2)^{3/8}\} + (1/S\times gD/U^2)^{1/2}][\beta \tanh\{k_4(gD/U^2)^{3/8}\} - (1/S\times gD/U^2)^{1/2}]}{\{\ln[\beta \tanh\{k_4(gD/U^2)^{3/8}\} - (1/S\times gD/U^2)^{1/2}] - \ln[\beta \tanh\{k_4(gD/U^2)^{3/8}\} - (1/S\times gD/U^2)^{1/2}]\}^2} \quad \ldots. (14)$$

For developing waves, naturally $\beta \tanh\{k_4 (gD/U^2)^{3/8}\} > (1/S\times gD/U^2)^{1/2}$, and Sb will be determined by following equation,

$$Sb = Sa + \left(\frac{dS}{dx}\right)_a \Delta x \quad \ldots\ldots (15)$$

Mb can be calculated by eq.(10) and

$$Gb = Mb\ Ua \left(\frac{gDa}{Ua^2}\right)^{1/2} = Mb (gDa)^{1/2} \quad \ldots.. (16)$$

also

$$Tb = \left(\frac{4\pi^2 Da}{gSb}\right)^{1/2} \quad \ldots.. (17)$$

## CALCULATION IN CASE OF WAVE DECAY

If $\alpha \tanh\{k_3 (gD/U^2)^{3/4}\} < gH/U^2$ and/or $\beta \tanh\{k_4 (gD/U^2)^{3/8}\} < (1/S\times gD/U^2)^{1/2}$ are recognized at point "a", it means that the wave series whose height is Ha reaching this point with group velocity Ga can not grown any more under the circumstance Ua and Da. In other words, the waves have already larger than the wind Ua can generate in shallow water area of depth Da. As shown in Fig. 4, Ha is located above the curve H(Ua, Da). In such a case, following consideration are being made.

1) If the wind of velocity Ua blew over deep water area, the wave height would increase $\Delta H_1$, while the fetch was being prolonged $\Delta x$, $\Delta H_1$, can be calculated by

$$\Delta H_1 \approx \left(\frac{dH}{dx}\right)_a \Delta x$$

$$= \frac{k^2}{\alpha}\frac{(\alpha + gHa/Ua^2)(\alpha - gHa/Ua^2)}{\ln(\alpha + gHa/Ua^2) - \ln(\alpha - gHa/Ua^2)}\Delta x \quad \ldots. (18)$$

2) Actually the waves should decrease their height for being suffered by bottom friction at shallow water of depth $D_a$. It is necessary to evaluate the wind velocity $U_a'$ which makes the fully arisen wave height just equals $H_a$ in depth $D_a$ as shown by the curve OM in Fig. 4, from eq. (1), let

$$\frac{gH_a}{Ua'^2} = \alpha \tanh\left\{k_3\left(\frac{gDa}{Ua'^2}\right)^{3/4}\right\} \quad \ldots\ldots.. (19)$$

$U'_a$ can be found out by Newton's method then, $\Delta H_2$ can be calculated as follows

$$\Delta H_2 = \frac{k_1^2}{\alpha} \frac{(\alpha+gHa/U_a'^2)(\alpha-gHa/U_a'^2)}{\ln(\alpha+gHa/U_a'^2) - \ln(\alpha-gHa/U_a'^2)} \Delta x \qquad \ldots(20)$$

While the wind of velocity $U_a'$ is acting on a wave series with a height $Ha$ over $\Delta x$ length in deep water, the height should be increased $\Delta H_2$, however, in shallow water of depth $Da$, the wave height remain constant, the energy obtained from the wind and lost due to bottom friction are in equilibrium, namely, the lost height of wave series of height $Ha$ travelling over $\Delta x$ is $\Delta H_2$. So that $Hb$ is:

$$H_b = H_a + \Delta H_1 - \Delta H_2 \qquad \ldots\ldots(21)$$

The same consideration can be applied for evaluating group velocity, calculate $\Delta G_1$ from following equation first.

$$\Delta G_1 = \frac{8}{3} \frac{k_2^3}{\beta} \frac{g}{Ua} \frac{(\beta/2+Ga/Ua)(\beta/2-Ga/Ua)}{[\ln(\beta\ 2+Ga/Ua)-\ln(\beta/2-Ga/Ua)]^2} \Delta x \qquad \ldots(22)$$

The relationship of $G/U$ and $gD/U^2$ when $x\to\infty$ can be approximately expressed as bellow:

$$gD/U^2 > 0.06 \qquad \frac{G}{U} = \frac{\beta}{2} \tanh\left\{k_5 \left(\frac{gD}{U^2}\right)^{1/3}\right\} \qquad \ldots\ldots\ldots\ldots(23)$$

$$k_5 = 0.566$$

$$gD/U^2 \leqq 0.06 \qquad \frac{G}{U} = \frac{\beta}{3} \left(\frac{gD}{U^2}\right)^{2/5} \qquad \ldots\ldots\ldots\ldots(24)$$

The wind velocity $U_a''$ which makes the fully arisen group velocity at depth $Da$ just equals $Ga$ can be worked out by solving eq. (23) or (24) and $\Delta G$ is

$$\Delta G_2 = \frac{8}{3} \frac{k_2^3}{\beta} \frac{g}{U} \frac{(\beta/2+Ga/U_a'')(\beta/2-Ga/U_a'')}{[\ln(\beta/2+Ga/U_a'')-\ln(\beta/2+Ga/U_a'')]^2} \qquad \ldots(25)$$

$G_b$ will be calculated by

$$G_b = G_a + \Delta G_1 - \Delta G_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(26)$$

The period can be calculated as follows.

$$M_b = \frac{G_b}{U_a} \frac{1}{(gDa/Ua)^{1/2}} \ldots(27), \qquad T_b = \left(\frac{4\pi^2 Da}{gS_b}\right)^{1/2} \qquad \ldots(28)$$

$$S = b(1-M) + b(1-M)^2 + \ldots + b(1-M)^7 \qquad \ldots(29)$$

The position of point 'b' on the lattice are determined by following equations.

$$\text{if} \quad Ga > \lambda/\tau \quad \Delta x = \lambda \quad \Delta t = \lambda/Ga \qquad \ldots(30)$$

$$\text{if} \quad Ga < \lambda/\tau \quad \Delta t = \tau \quad \Delta x = Ga\tau \qquad \ldots(31)$$

$$x_b = x_a + \Delta x \qquad t_b = t_a + \Delta t \qquad \ldots(32)$$

Same Procedures will be applied to calculate waves of Point "c" from point "b". The flow chart is to be used for Programming as Fig.5.

## DISCUSSION ON THE EFFECT OF REFRACTION

The refraction effect of waves in shallow water must not be neglected. In following examples, calculations on refraction have been made by amending the contour line to be parallel. The difference is only a few percent both the wave height, group velocity and wave propagation line in comparison with the result of calculation without considering refraction. It is not unnatural because the refraction of waves is caused by decreasing of celerity as the waves advancing to shallow water, however, in this calculation, the decrement in wave celerity owing to shoaling is almost balanced by the increase from wind effect. The wave celerity remains nearly constant, therefore the refraction effect seems not to be appeared. This is very noticeable phenomenon in wind waves of shallow water, further investigations are needed.

## CALCULATION EXAMPLE

The waves along the northern coast of Seto Inland Sea raised by typhoons ″Suō″ (Aug. I946) ′Ruth′ (Oct.I95I) ′Doya″(Sept. 1954) as well as the waves attacked western coast of Taiwan caused by Typhoon ″Parmela′ (Sept. I96I) have been hindcasted by this method. The result of the calculation of typhoon ″Suō″ is submitted here.

## FOUNDAMENTAL CONDITIONS

The route of typhoon and topographical features of western part of Seto Inland Sea are described in Fig. 6, the fetch length of various direction of every calculated point are also shown in the same figure.

Along this coast, the tidal range is rather large, extraordinary hightide will be recognized as the typhoon center approaching, in this calculation, the deviation of water level by the extrahightide has been considered and added into water depth.

Waves diffracted from outer sea have not been considered in this calculation. All waves to be calculated are generated in shallow water area.

## CALCULATION CONDITIONS

In general, the pressure distribution in typhoon cycle is as below.

$$P = P_c + a \exp\left(-\frac{r}{r_0}\right) \qquad \text{.........}(33)$$

$P_c$ : pressure at typhoon center (m b)
$r$ : distance from typhoon center (km)
$r_0$ : radius of the largest gradient wind velocity circle (km)
$P$ : pressure at the circle with radius $r$
$a$ : constant

a and r are different in each typhoon.

The gradient wind velocity Vg is

$$V = \sqrt{\frac{a}{\ }\,\frac{r}{r}}\exp\left(-\frac{r_0}{2r}\right) - \frac{fr}{2} \qquad .....(34)$$

: air density

$f = 2\omega\sin\phi$ : Coriolis coefficient

Actual wind in typhoon is the resultant of symetrical wind $U'$ and wind of field $U''$, taking typhoon center as the origin, wind velocity of the point (x, y) can be calculated from the following equations.

$$Ux = U'x + U''x, \qquad Uy = U'y + U''y \qquad .....(35)$$

$$U'x = -0.6(Vg\ r)(x\sin\alpha + y\cos\alpha) \qquad .....(36)$$

$$U'y = 0.6(Vg\ r)(x\cos\alpha + y\sin\alpha) \qquad .....(37)$$

$$U''x = (0.6\ Vgmax)\ Vg\cdot Vx \qquad .....(38)$$

$$U''y = (0.6\ Vgmax)\ Vg\ Vy \qquad .....(39)$$

$\alpha$ is the angle of symetrical wind direction and the tangent of isobar. It will be different in lattitude, in this calculation $\alpha = 30°$ is adopted. Vx, Vy are the components of progressing velocity of typhoon.

The origin of fixed coordinate is set at I3I E and 33°4 N, EW and NS direction are taken as X-Y- axes respectively. If the linear fetch is at an angle of  to the X axis the component of wind velocity can be calculated by following equation

$$U = Ux\cos\theta + Uy\sin\theta \qquad .......(40)$$

The positions of the center of typhoon ″Suō″, when she was in the vicinity of Seto Inland Sea at every hour, are listed below

| Date | hour | X(km) | Y(km) |
|---|---|---|---|
| Aug.27 | I4 | -IOI | -207 |
| | I5 | -88.5 | -I65.5 |
| | I6 | -89.5 | -I25.5 |
| | I7 | -89.5 | -9I |
| | I8 | -9I | -65 |
| | I9 | -85 | -24.5 |
| | 20 | -77.75 | I5.5 |
| | 2I | -72.25 | 55.5 |
| | 22 | -32.50 | I05 |
| | 23 | -25.0 | I47 |
| Aug.28 | 0 | -I8.50 | 20I.5 |

During calculation, the unit distance $\lambda$ on lattice of wind diagram is to be 2km windward and Ikm in the region of depth less than 10m leeward, but the time unit $\tau$ is remaining 30 minutes.

## CALCULATION RESULTS

Wave which attacked the estuary of Yoshida river and other point from various direction have been calculated by the electronic computor. Fig. 7,8 illustruted the waves on the SE fetch of Yoshida estuary.

In addition, for the purpose of investigating the distribution of waves over the west part of Seto Inland Sea, a number of parallel linear fetch with a distance of 10km have been set as shown in Fig. 6. Waves on such fetch lines have been calculated and the contour of wave heights and periods are to be delineated for every hour as shown in Fig. 9 and 10.

## REFERENCES

1) Sakamoto, Ijima, Sato, Aono (1960): "Graphical Approach to the Forecasting of wind waves in Shallow Water", proceedings of 7th Japanese Coastal Engineering Conference, (in Japanese) Japan Civil Engineering Society.
2) Ijima, Sato, Aono "Waves raised by Typhoon Ise" Ibid. (In Japanese) (1961)
3) Wilson, B. W. (1961) "Deep Water Wave Generation by Moving Wind Systems", Proc. A. S. C. E. WW2
Wilson, B. W. (1962) "Deep Water Wave Generation by Moving Wind Systems" Proc. A. S. C. E. WW3
4) Bretschneider, C. L. (1958) : Revisions in Wave forecasting, Deep and Shallow water, Proc. 6th Conference on Coastal Engineering.

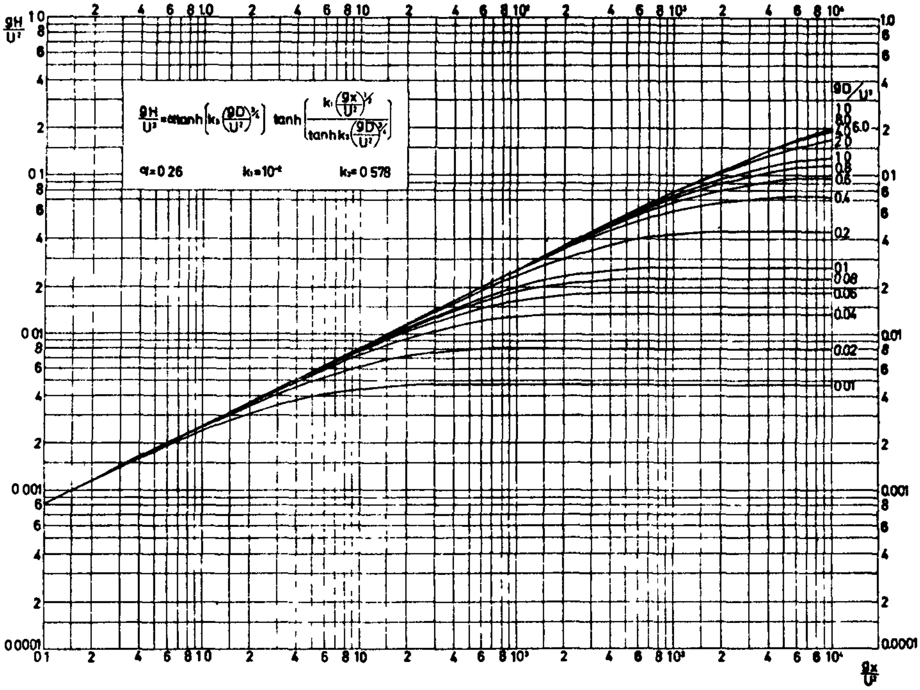

Fig. 1. Relation of wave height and wind for shallow water.

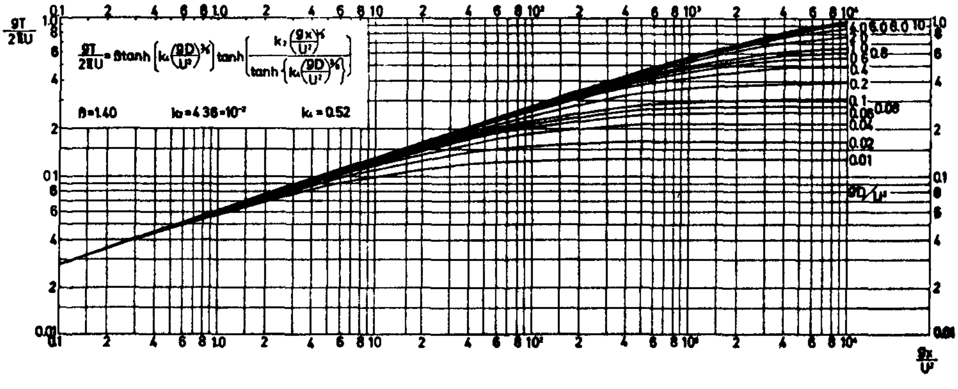

Fig. 2. Relation of wave period and wind for shallow water.

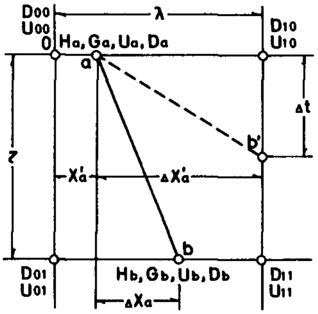

Fig. 3. Process of calculation.

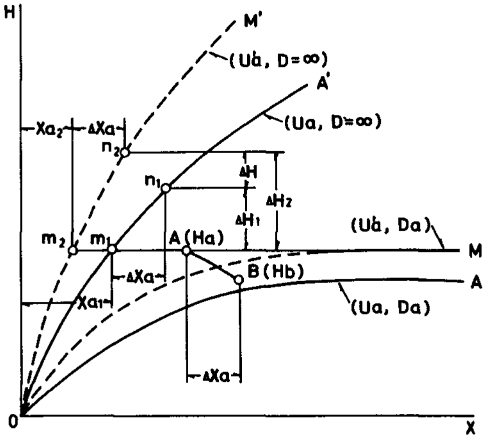

Fig 4 Calculation of wave height decrease.

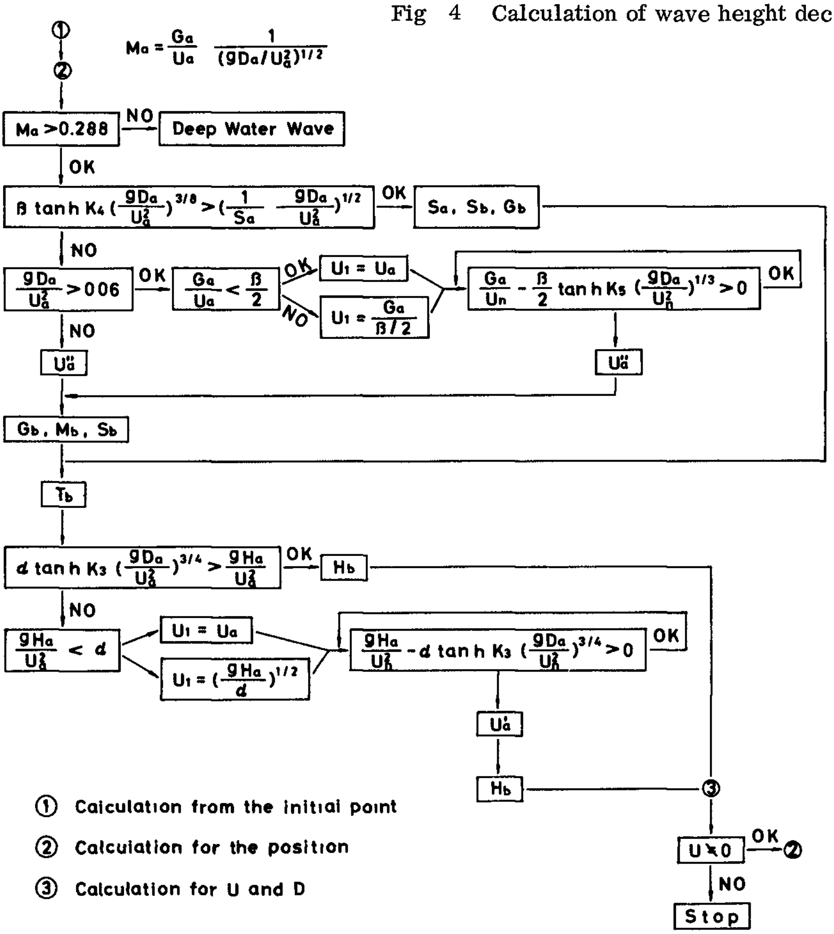

Fig. 5. Flow chart

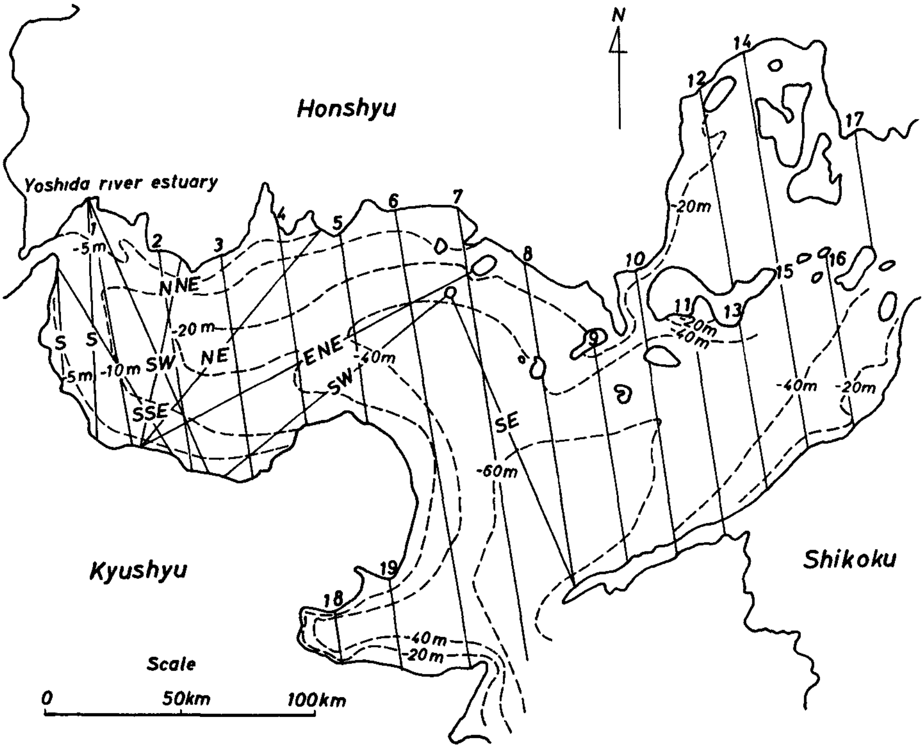

Fig. 6. West part of inland sea of Seto, and locations of linear fetch.

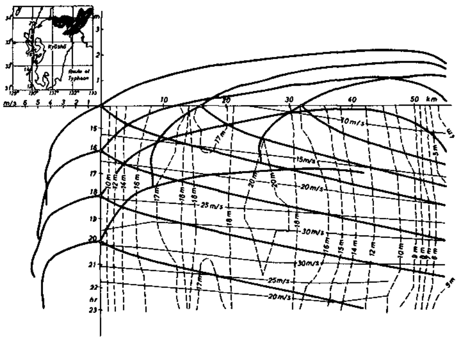

Fig. 7. A result of numerical calculation Yoshida River Estuary, typhoon Suō Nada SE.

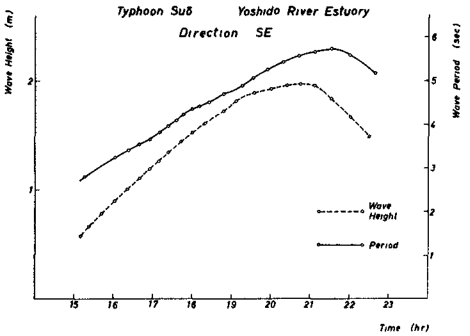

Fig. 8. Time change of wave height and period.

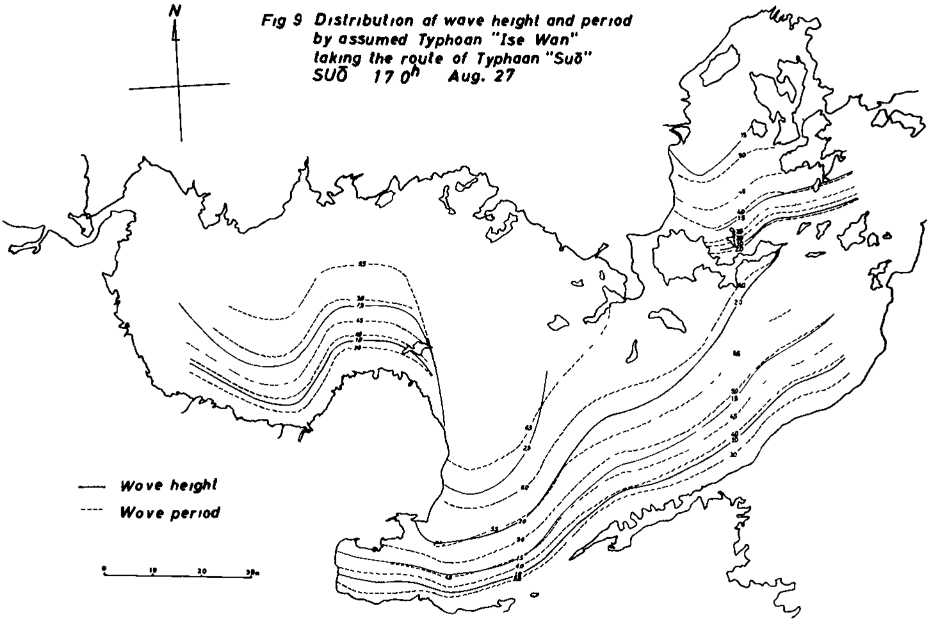

Fig 9 Distribution of wave height and period
by assumed Typhoon "Ise Wan"
taking the route of Typhoon "Suō"
SUŌ 17 0^h^ Aug. 27

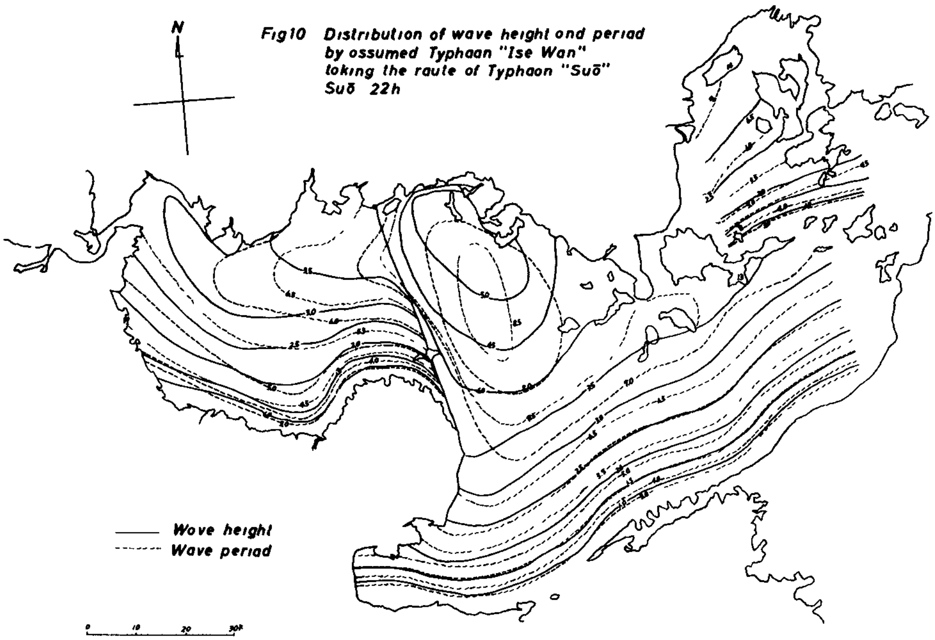

Fig10 Distribution of wave height and period
by assumed Typhoon "Ise Wan"
taking the route of Typhoon "Suō"
Suō 22h

Fig. 9, 10. Distribution of wave height and period by assumed typhoon "Ise Wan" taking the route of typhoon "Suō".