CHAPTER 5

A PROBABLE LEVEL OF WAVE CREST FOR THE DESIGN OF COASTAL STRUCTURES

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ABSTRACT

The height of sea water level in front of a coastal structure changes by the astronomical tide, the meteorological effect and the short period wave. From three histograms of the astronomical tide, meteorological tide and wave height, the probable level of wave crest in the coast will be estimated by the method presented in this paper.

INTRODUCTION

For the design of a coastal structure, the height of its crown must be determined rationally and economically, taking into consideration the water level of the sea and its occurence probability. The water level in the sea is mainly referred to the astronomical tide, the meteorological effect and the short period wave. If component height according to these elements are given as;

x_1 ; the tidal level,
x_2 ; the level rise caused by meteorological origin,
and x_3 ; the half height of wave,

the level of the wave crest X at a certain tidal condition is shown by following equation under several assumption:

$$X = x_1 + x_2 + x_3 \qquad (1)$$

In the case of the occurence number of the class mark of x_1, x_2 and x_3 measured from the observation in a period, three histograms of x_1, x_2 and x_3 can be obtained. The probability P(X) of the design level of wave crest will be calculated from those histograms by the method presented in this paper.

ASSUMPTION

On the astronomical tidal change in a period, the height of the tidal level above the lowest level is shown by x_1. And observed values of x_1 is not sufficient on the rational probability theorem because of small numbers of samples. (And numbers of x_2 and x_3 are also generally small.) The x_1 must be observed in the coastal site in consideration, because the value of it is affected by the topographical feature etc.

The meteorological water level is affected by the season and climate, the topographical condition etc. The value of level raise by the meteorological phenomena is given by x_2 from the observation at the site.

The half height of wave of short period is shown by x_3. The x_3 is referred to the wave generated by wind, swell, seich, earthquake etc. And x_3 is also must be observed in the coast, because the wave height is affected by the scale and kind of waves.

For the calculation of a probability P(X) of X obtained from equation (1), several assumptions as following may be permitted; those are

(a) The correlation of x_1, x_2 and x_3 is assumed to be small. Therefore variables x_1, x_2 and x_3 are assumed to be independent each other or non-correlated.

(b) The level of wave crest is given approximately by the sum of x_1, x_2 and x_3, or $X = x_1 + x_2 + x_3$.

(c) The value of difference of the class mark of x_1, x_2 and x_3 is shown as D that is common and constant for three x.

The occurence frequencies of x_1, x_2 and x_3 are shown as N_1, N_2 and N_3 respectively in a period of an observation, such as shown in following table:

x	x_1	x_2	x_3
(0 - 0.99..)D	${}_1N_1$	${}_1N_2$	${}_1N_3$
(1 - 1.99..)D	${}_2N_1$	${}_2N_2$	${}_2N_3$
(2 - 2.99..)D	${}_3N_1$	${}_3N_2$	${}_3N_3$
....	...	...	...
	${}_iN_1$	${}_jN_2$	${}_kN_3$
Total	ΣN_1	ΣN_2	ΣN_3

CALCULATION

(a) Coefficient table

In $X_3 = (0 - 2.99..)D$, $\quad X_3 = {}_1x_1 + {}_1x_2 + {}_1x_3$,

total probable frequencies of X_3 is $\quad {}_1N_1 + {}_1N_2 + {}_1N_3$

and its coefficient $\quad$ 1 1 1 .

In $X_4 = (3 - 3.99..)D$,

$$X_4 = {}_1x_1 + {}_1x_2 + {}_2x_3 = {}_1x_1 + {}_2x_2 + {}_1x_3 = {}_2x_1 + {}_1x_2 + {}_1x_3,$$

total probable frequencies of X_4 is

$$\begin{aligned} &{}_1N_1 + {}_1N_2 + {}_2N_3 \\ +&{}_1N_1 + {}_2N_2 + {}_1N_3 \\ +&{}_2N_1 + {}_1N_2 + {}_1N_3 \end{aligned}$$

$$= 2{}_1N_1 + {}_2N_1 + 2{}_1N_2 + {}_2N_2 + 2{}_1N_3 + {}_2N_3$$

and its coefficient

2 1 2 1 2 1 .

In $X_5 = (4 - 4.99..)D$, total frequencies of its are

$$\begin{aligned} &= 3{}_1N_1 + 2{}_2N_1 + {}_3N_1 \\ &+ 3{}_1N_2 + 2{}_2N_2 + {}_3N_2 \end{aligned}$$

$$+ 3_1N_3 + 2_2N_3 + 3^N_3 ,$$

and its coefficients are

3	2	1
3	2	1
3	2	1 .

....

....

Therefore, the coefficients table shown as Table-1 can be generally obtained.

For an example the coefficient table in the case of i=4, j=6 and k=15 can be shown as Table-2.

From such coefficient table, if N are multiplied by the coefficient respectively, these products are summed up for each X and the summation is divided by the total frequencies of all X, each probability of X can be calculated.

(b) Example

If the number of occurence of x_1, x_2 and x_3 are obtained from the observation in 303 days at a coastal site, the calculation of probability of X is shown as Table-2 and Table-3. The histograms of x_1, x_2 and x_3 are shown in Figure-1, Figure-2 and Figure-3. From the calculation of such as Table-2 and Table-3, the probability curve of X can be drawn in Figure-4.

It seems generally that the probability curve of X from that method is situated between the actual observed curve of X' and the rational curve of X''.

CONCLUSION

In that example, a probability of one by 303 x 100 = 0.33 %, and the X of the probability less than 0.33% seems to be negligibly small in this observation period. Therefore, the maximum level of wave crest in the site, P = 0.248 % and

$$X_{max} = (27 - 28)D \fallingdotseq 27.5 \times 0.20 = 5.50 \text{ m} ,$$

and the minimum level of it, P = 0.777 %

$$X_{min} = (5 - 6)D \fallingdotseq 5.5 \times 0.20 = 1.10 \text{ m}$$

are obtained from Table-3. Then, it is thought that a structure lower than 1.1m must be always in water and a structure higher than 5.5m may be not economical in that coast. And the most frequent height of the wave crest, that is most probable 7.041 % in this example is $X_{most} = (11 - 12)D \fallingdotseq 11.5 \times 0.20 = 2.3$ m. And at the level about 2.3 m, the water surface affects most frequently to the structure in this coast.

As the probability of X around the peak point changes by a gentle curve, the probability of a comparatively large X about 3.753 % at the level of $X_{comp} = (21 - 22)D \fallingdotseq 21.5 \times 0.20 = 4.3$ m is not so small. For a criteria of a coastal structure, the high water level is appropriate to 4.3 m in the coast.

These values of X and their probabilities can be used for the effective design of a coastal construction.

Table 1.

x / N / X	x_1					x_2					x_3				
	0	D	2D	.....	(i-1)D	0	D	2D	.....	(j-1)D	0	D	2D	.....	(k-1)D
	D	2D	3D	......	iD	D	2D	3D	......	jD	D	2D	3D	......	kD
	1^{N_1}	2^{N_1}	3^{N_1}	....	i^{N_1}	1^{N_2}	2^{N_2}	3^{N_2}	....	j^{N_2}	1^{N_3}	2^{N_3}	3^{N_3}	....	k^{N_3}
0-3D	1					1					1				
3D-4D	2	1				2	1				2	1			
4D-5D	3	2	1			3	2	1			3	2	1		
5D-		....					....					....			
....			....					....					....		
....				....					....					....	
(i+j+k-1)D -(i+j+k)D					1					1					1
SUM.	jk	....				ki	....				ij	....			

Table 2.

x / N / X	x_1				x_2						x_3														
	0–D	D–2D	2D–3D	3D–4D	0–D	D–2D	2D–3D	3D–4D	4D–5D	5D–6D	0–D	D–2D	2D–3D	3D–4D	4D–5D	5D–6D	6D–7D	7D–8D	8D–9D	9D–10D	10D–11D	11D–12D	12D–13D	13D–14D	14D–15D
	N_{11}	N_{12}	N_{13}	N_{14}	N_{21}	N_{22}	N_{23}	N_{24}	N_{25}	N_{26}	N_{31}	N_{32}	N_{33}	N_{34}	N_{35}	N_{36}	N_{37}	N_{38}	N_{39}	$N_{3,10}$	$N_{3,11}$	$N_{3,12}$	$N_{3,13}$	$N_{3,14}$	$N_{3,15}$
0– 3D	1				1						1														
3D– 4D	2	1			2	1					2	1													
4D– 5D	3	2	1		3	2	1				3	2	1												
5D– 6D	4	3	2	1	4	3	2	1			4	3	2	1											
6D– 7D	5	4	3	2	4	4	3	2	1		4	4	3	2	1										
7D– 8D	6	5	4	3	4	4	4	3	2	1	4	4	4	3	2	1									
8D– 9D	6	6	5	4	4	4	4	4	3	2	3	4	4	4	3	2	1								
9D–10D	6	6	6	5	4	4	4	4	4	3	2	3	4	4	4	3	2	1							
10D–11D	6	6	6	6	4	4	4	4	4	4	1	2	3	4	4	4	3	2	1						
11D–12D	6	6	6	6	4	4	4	4	4	4		1	2	3	4	4	4	3	2	1					
12D–13D	6	6	6	6	4	4	4	4	4	4			1	2	3	4	4	4	3	2	1				
13D–14D	6	6	6	6	4	4	4	4	4	4				1	2	3	4	4	4	3	2	1			
14D–15D	6	6	6	6	4	4	4	4	4	4					1	2	3	4	4	4	3	2	1		
15D–16D	6	6	6	6	4	4	4	4	4	4						1	2	3	4	4	4	3	2	1	
16D–17D	6	6	6	6	4	4	4	4	4	4							1	2	3	4	4	4	3	2	1
17D–18D	5	6	6	6	3	4	4	4	4	4								1	2	3	4	4	4	3	2
18D–19D	4	5	6	6	2	3	4	4	4	4									1	2	3	4	4	4	3
19D–20D	3	4	5	6	1	2	3	4	4	4										1	2	3	4	4	4
20D–21D	2	3	4	5		1	2	3	4	4											1	2	3	4	4
21D–22D	1	2	3	4			1	2	3	4												1	2	3	4
22D–23D		1	2	3				1	2	3													1	2	3
23D–24D			1	2					1	2														1	2
24D–25D				1						1															1

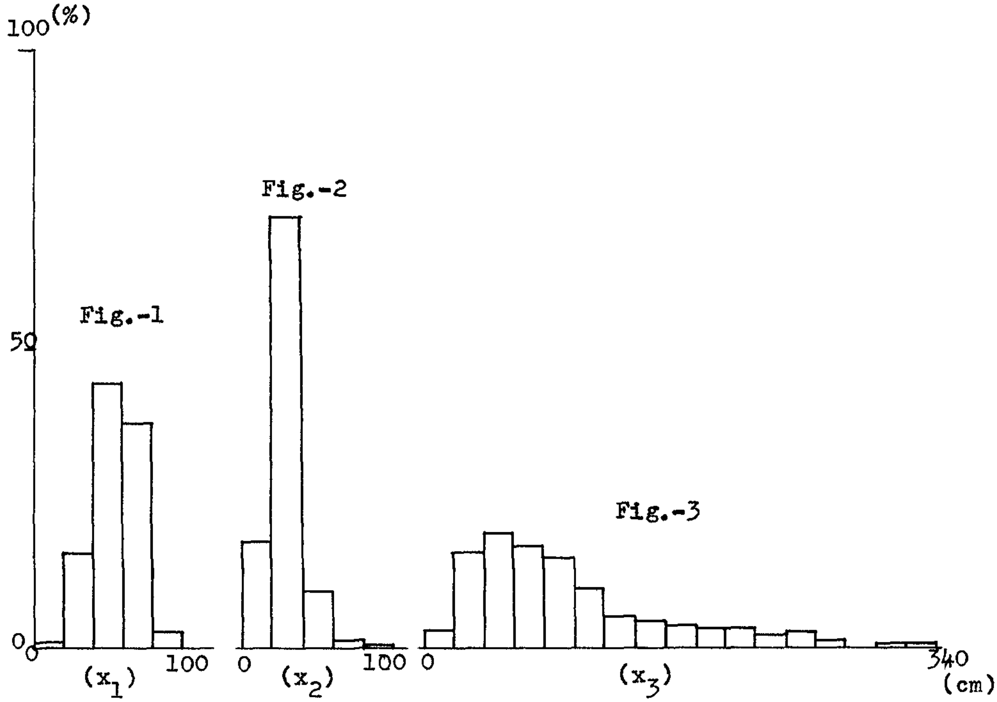

Fig.-1 Fig.-2 Fig.-3

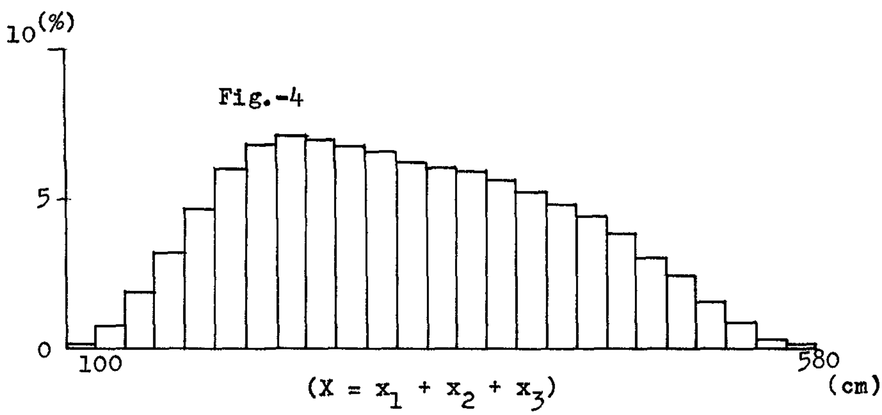

Fig.-4

Table 3.

X \ x, N	x_1					x_2								
x	D	2D	3D	4D	5D	2D	3D	4D	5D	6D	2D	3D	4D	5D
N	1	48	133	113	8	54	217	28	3	1	7	47	56	49
2D- 5D	1					54					7			
5D- 6D	2	48				108	217				14	47		
6D- 7D	3	96	133			162	434	28			21	94	56	
7D- 8D	4	144	266	113		216	651	56	3		28	141	112	49
8D- 9D	5	192	399	226	8	270	868	84	6	1	35	188	168	98
9D-10D	5	240	532	339	16	270	1085	112	9	2	28	235	224	147
10D-11D	5	240	665	452	24	270	1085	140	12	3	21	188	280	196
11D-12D	5	240	665	565	32	270	1085	140	15	4	14	141	224	245
12D-13D	5	240	665	565	40	270	1085	140	15	5	7	94	168	196
13D-14D	5	240	665	565	40	270	1085	140	15	5		47	112	147
14D-15D	5	240	665	565	40	270	1085	140	15	5			56	98
15D-16D	5	240	665	565	40	270	1085	140	15	5				49
16D-17D	5	240	665	565	40	270	1085	140	15	5				
17D-18D	5	240	665	565	40	270	1085	140	15	5				
18D-19D	4	240	665	565	40	216	1085	140	15	5				
19D-20D	4	192	665	565	40	216	868	140	15	5				
20D-21D	4	192	532	565	40	216	868	112	15	5				
21D-22D	3	192	532	452	40	162	868	112	12	5				
22D-23D	2	144	532	452	32	108	651	112	12	4				
23D-24D	2	96	399	452	32	108	434	84	12	4				
24D-25D	1	96	266	339	32	54	434	56	9	4				
25D-26D		48	266	226	24		217	56	6	3				
26D-27D			133	226	16			28	6	2				
27D-28D				113	16				3	2				
28D-29D					8					1				

x_3													X	
6D	7D	8D	9D	10D	11D	12D	13D	14D	15D	16D	17D	18D		
43	30	15	12	10	9	9	5	6	3	0	1	1	TOTAL	P(X)%
													62	0.111
													436	0.777
													1027	1.831
													1783	3.179
43													2591	4.620
86	30												3360	5.994
129	60	15											3785	6.749
172	90	30	12										3949	7.041
215	120	45	24	10									3909	6.970
172	150	60	36	20	9								3783	6.745
129	120	75	48	30	18	9							3613	6.442
86	90	60	60	40	27	18	5						3465	6.178
43	60	45	48	50	36	27	10	6					3355	5.982
	30	30	36	40	45	36	15	12	3				3277	5.843
		15	24	30	36	45	20	18	6	0			3169	5.560
			12	20	27	36	25	24	9	0	1		2864	5.107
				10	18	27	20	30	12	0	2	1	2669	4.759
					9	18	15	24	15	0	3	2	2464	4.343
						9	10	18	12	0	4	3	2105	3.753
							5	12	9	0	5	4	1658	2.956
								6	6	0	4	5	1312	2.339
									3	0	3	4	856	1.526
										0	2	3	416	0.745
											1	2	137	0.248
												1	10	0.018