CHAPTER 8

RESPONSE CHARACTERISTICS OF UNDERWATER WAVE GAUGE

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ABSTRACT

The paper concerns the characteristics of the correction factor which is introduced into the relationship between the water surface elevation of progressive waves and its corresponding fluctuation of underwater wave pressure.

As a result of extensive investigations conducted both in laboratory and field, it is verified that the correction factor is well expressed by a certain function of relative water depth. By using an empirical formula proposed in this paper the power spectrum of surface elevation and its significant wave height off the Ohita Coast are estimated from the record of underwater pressure fluctuation. The estimated values, generally speaking, have a satisfactory agreement with the actual ones determined from the records of surface water elevation.

INTRODUCTION

The underwater wave gauge is the most popular device for measuring the nearshore waves. The principle of this measuring device, as well known, is to convert the record of underwater pressure fluctuation, Δp , into the corresponding surface elevation, η . According to the linear theory of gravity waves, the fluctuation of underwater wave pressure is expressed by the following equation:

$$\Delta p = -\rho \frac{\partial \phi}{\partial t} = \rho g \eta \frac{\cosh k(h+z)}{\cosh kh} \tag{1}$$

where ρ is density of fluid, ϕ velocity potential function, g acceleration due to gravity, $k = 2\pi/L$ wave number, L wave length at the water depth h , and z vertical axis taking upward from the still water level. The above equation has been recognized for many years to be inaccurate to correlate Δp with η even in the case of regular wave condition. The main reasons of the above fact will be found in the following:

1) Real gravity waves can be treated not by the small amplitude wave theory but by the finite amplitude wave

theory.

2) Actual irregular waves consist of a great number of elementary waves.

For the sake of practical conversion a sort of correction factor, n, has commonly been introduced into Eq. (1) as follows:

$$n \Delta p = \rho g \eta \frac{\cosh k(h+z)}{\cosh kh} \tag{2}$$

or

$$\eta = n \eta_p \frac{\cosh kh}{\cosh k(h+z)} \tag{3}$$

where $\eta_p = \frac{\Delta p}{\rho g}$.

In relation to the correction factor, n, the numerous values ranging between 1.06 and 1.37 have been reported on the basis of laboratory and field investigation data as shown in Table 1. The value of n = 1.3~1.35 has widely been used in Japan for the practical purpose of data processing, but the arguments on its applicability have been put forward as a result of the discrepancy between the visualized apparent wave height and its corresponding converted wave height.

The aims of the present paper are to investigate primarily the characteristic features of the correction factor, n, on the basis of the laboratory data obtained at the Coastal Engineering Laboratory, University of Tokyo, and on the basis of the field data obtained at Ohita facing Beppu Bay in Kyushu, and to present a practical method for computing the characteristics of nearshore waves.

THEORETICAL CONSIDERATION

Let us consider the system shown in Fig. 1, where $x(t)$ is the input, $y(t)$ the output, and $n(t)$ the noise, all of them are time dependent functions. While $G(f)$ is the response function of this system, which is a function of frequency f. According to the theory of spectral analysis, the following relations are introduced.

$$P_{yy}(f) = |G(f)|^2 P_{xx}(f) + P_{nn}(f) \tag{4}$$

$$P_{yx}(f) = G(f) P_{xx}(f) \tag{5}$$

In these equations $P_{xx}(f)$, $P_{yy}(f)$ and $P_{nn}(f)$ are the power spectra of input, output and noise respectively, while $P_{yx}(f)$ is the cross spectrum between input and output. In order to determine the response function $G(f)$ precisely, we have to apply Eq. (5). But if it is allowed to assume that the effect of the noise on the relation of Eq. (4) is negligible, the following approximation may be acceptable:

Table 1.

	Authors	n	Locations
	Folsom	1.07	W. E. S.
Labo.		1.1	Univ. of Calif.
	Hamada et al.	1.09	P. H. T. R. I.
	Folsom	1.06	Half Moon Bay
		1.08	
		1.18	Estero Bay
Field	Seiwell	1.31	Cuttyhunk
		1.37	
		1.37	Bermuda
		(1.35)	(Average)
	Ijima et al.	1.34	Kurihama Bay

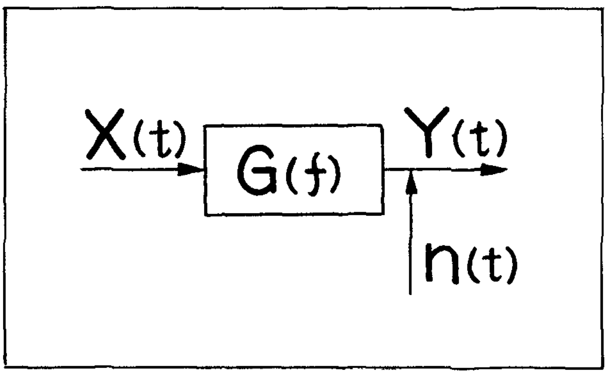

Fig. 1.

$$|G(f)| \doteqdot \sqrt{\frac{P_{yy}(f)}{P_{xx}(f)}} \tag{6}$$

Taking Eq. (3) into consideration the response function of the present case is given as follows when $z = -d$;

$$G(f) = \frac{\cosh k(h-d)}{n \cosh kh} = \frac{S(f, h, d)}{n} \tag{7}$$

Here $S(f; h, d) = \cosh k(h-d)/\cosh kh$ is a sort of hydraulic filter. Therefore the correction factor, n, can be calculated through Eqs. (5) and (7) precisely or Eqs. (6) and (7) approximately, in which $P_{xx}(f)$, $P_{yy}(f)$ and $P_{yx}(f)$ should be rewritten by $P_{\eta\eta}(f)$, $P_{\eta_p\eta_p}(f)$ and $P_{\eta_p\eta}(f)$ respectively.

On the other hand, the dimensional analysis shows that the correction factor, n, may be expressed by the following relation:

$$n = \varphi(a_i/h,\ h/L_{oi},\ d/h) \tag{8}$$

where a_i and L_{oi} are the amplitude and wave length in deep water respectively of elementary wave which has the frequency of f_i .

ANALYSIS OF DATA

LABORATORY TESTS

Extensive flume tests have been conducted at the Coastal Engineering Laboratory by using a wind flume, 36 m long, 0.6 m wide and 0.9 m high. Various types of test waves have been generated by the wind blower, or by the flap type wave generator, or by the combination of two devices. The fluctuations of surface elevation and of the underwater wave pressure are recorded simultaneously by using a parallel wire resistance type wave gauge and a pressure type transducer as shown in Fig. 2. From this example it is clearly recognized that the curve of pressure fluctuations is quite smooth even if the surface elevation curve is highly indented.

The following series of diagrams shows one example of the data processing in the present studies. Figure 3 indicates the power spectra of surface elevation and of underwater pressure fluctuation, both of which are computed by using the simultaneous records as shown in Fig. 2. In Fig. 4 are shown the response functions, $G(f)$, determined precisely and approximately by a dotted line and a solid line respectively together with the hydraulic filter function, $S(f, h, d)$. The correction factor is determined as the ratio between $S(f; h, d)$ and $G(f)$, thus the computed values of n are plotted

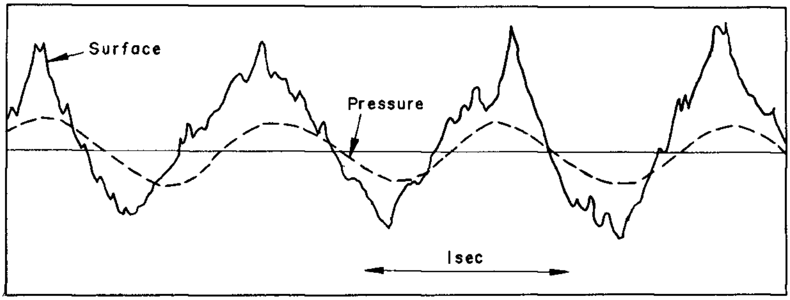

Fig. 2. Sample record of surface wave profile and its corresponding fluctuation of under water pressure.

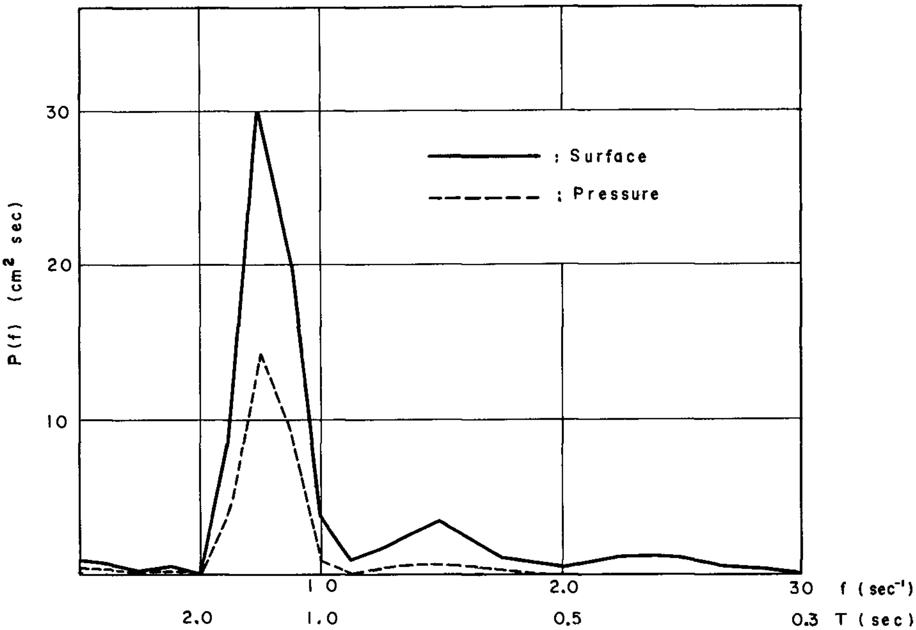

Fig. 3. Wave spectra. (Laboratory data)

Table 2.

	Laboratory	Field
Time Interval (sec)	0.03 ~ 0.15	0.66 ~ 1.32
Total Number	1000 ~ 2000	700 ~ 1500
Degree of Freedom	30 ~ 60	30 ~ 60

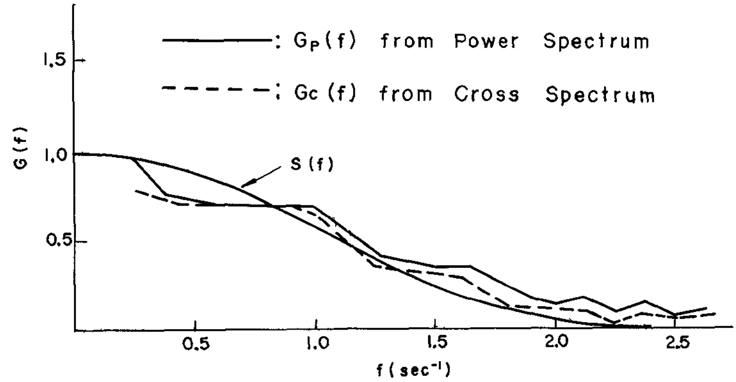

Fig. 4. Response function. (Laboratory data)

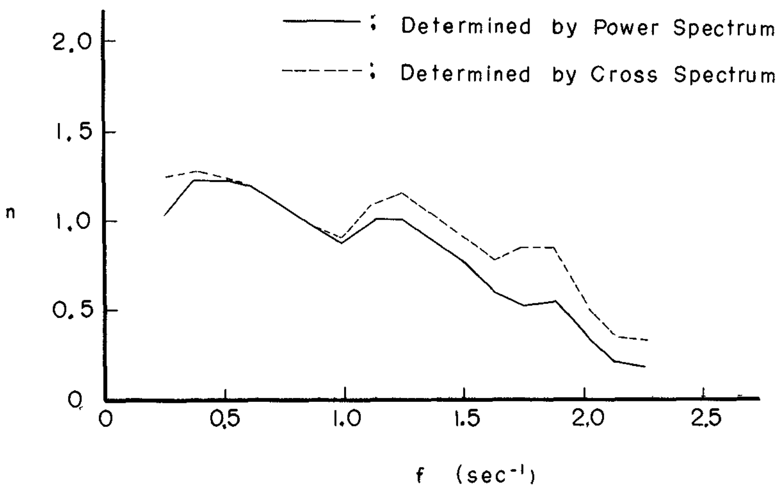

Fig. 5. Relationship between n and f. (Laboratory data)

in Fig. 5. Here the dotted line gives the curve of n determined precisely, while the solid line gives the curve determined approximately. The discrepancy between the precise one and the approximate one seems to be rather small at least in the range of small frequency f_i, that is, in the range of the small relative water depth h/L_{oi}. Therefore in the following treatment the values of n determined approximately through Eq. (6) are only used to find out the general relationship of correction factor n.

In Table 2 are given such conditions as the time interval of data reading, total number of data and degree of freedom, which were applied in the present data processing. According to the results of the preliminary investigations, it was found that

1) The elevation of wave gauge below the still water level seems to have negligible effect on n at least in the range of $1 > d/h \geq 0.375$.

2) The factor of a_i/h seems to have a little larger effect on n comparing with that of stated above.

On the basis of these results all of the computed data of n are plotted on the same graph as shown in Fig. 6. This diagram indicates that the correction factor, n, has a clear tendency of decrease with increase of f_i even though some scattering of data exists.

FIELD TESTS

Some valuable field data of simultaneous wave records were obtained by the engineers at the Construction Bureau of Ohita Maritime Industrial District. They used an underwater wave gauge and a step type wave gauge installed almost at the same site of $7.5\,m$ deep below the mean sea level in Beppu Bay under the guidance of the present writers. The waves generated by typhoon No. 23 and No. 24 in 1965 were clearly recorded.

Figure 7 shows a typical example of power spectra of surface waves and of the corresponding underwater pressure fluctuations. The conditions of computations are given in Table 2. In Fig. 8 are plotted the field data of n computed by the same procedures as in the analysis of the laboratory data. In order to compare the field data with the laboratory ones, the abscissa, f_i, in Figs. 6 and 8, is converted into the relative water depth, h/L_{oi}, and both of data are plotted on the same graph as shown in Fig. 9. The agreement between the field and the laboratory data is quite consistent in general, but some systematic difference appears between them. That is, n from the field data is a little larger comparing with the laboratory one especially in the range of small relative water depth. One of the possible reasons of the above discrepancy will be found in the fact that the incoming waves are apt to run up on the steel pile which

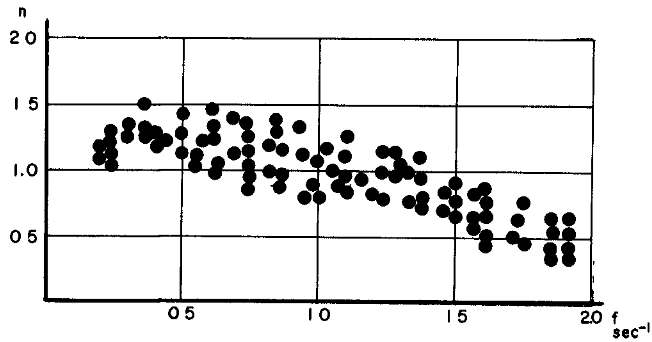

Fig. 6. Plotted data of n. (Laboratory data)

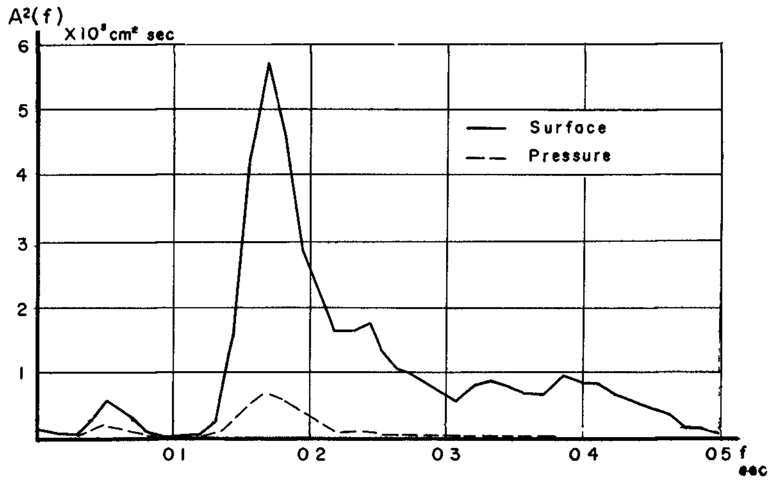

Fig. 7. Wave spectra. (Field data)

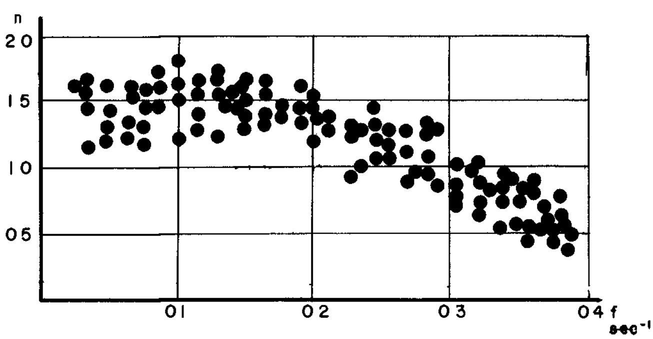

Fig. 8. Plotted data of n. (Field data)

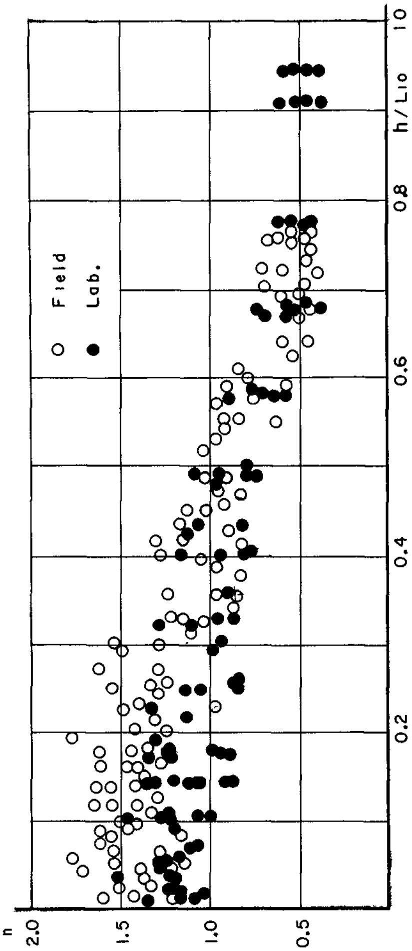

Fig. 9. Comparison between field and laboratory data.

supports closely the step gauge, so that the wave gauge records apparently higher waves instead of the actual waves.

PRACTICAL APPLICATION

EMPIRICAL FORMULA

In order to apply the foregoing result of present investigations it is desired to establish an empirical formula to express n as a function of relative water depth h/L_{oi}. Considering the tendency of plotted data in Fig. 9 we may assume the following expression of n:

$$n = A \exp\left[-\sum_{k=0}^{N} \alpha_k (h/L_{oi})^{k/2}\right] \tag{9}$$

The relative water depth h/L_{oi} is rewritten as follows:

$$h/L_{oi} = (2\pi/g)(h/T_i^2) = (2\pi/g)\, h f_i^2 \tag{10}$$

Substituting Eq. (10) into Eq. (9), we can get the next relationship:

$$\begin{aligned} n &= A \exp\left[-\sum_{k=0}^{N} \alpha_k (2\pi/g)^{k/2} h^{k/2} f_i^k\right] \\ &= A \exp\left[-\sum_{k=0}^{N} \beta_k h^{k/2} f_i^k\right] \end{aligned} \tag{11}$$

where $\beta_k = \alpha_k (2\pi/g)^{k/2}$

As a particular example we will take the site of wave observation at Ohita. The water depth of this station, where the two types of wave gauges are installed, is 7.5 m below the mean sea level, and the elevation of the pressure gauge is 1 m above the sea bottom. The bottom slope of this site is about 1/100. We will choose the following empirical expression which fits the plotted data for this particular location.

$$n = 1.55 \exp\left[-800 (f_i - 0.1)^5\right] \tag{12}$$

The above equation is equivalent to the next one expressed in a generalized form.

$$n = A \exp\left[-\sum_{k=0}^{5} \beta_k h^{k/2} f_i^k\right] \tag{13}$$

$$A = 1.55,\quad \beta_0 = -0.008,\quad \beta_1 = 0.146,\quad \beta_2 = -1.07$$

$$\beta_3 = 3.89,\quad \beta_4 = -7.11,\quad \beta_5 = 5.19$$

or
$$n = A' \exp\left[-B(f_i - b)^5\right] \tag{14}$$
$$A = 1.55,\quad B = 5\,19\,h^{5/2},\quad b = 0\,274\,h^{-1/2}$$
$$f_i : sec^{-1},\quad h : m$$

Figure 10 shows one example of comparison between the power spectrum of surface elevation and that estimated from the record of underwater pressure fluctuation by using Eq. (12). The agreement of the two curves is suprisingly good and is quite satisfactory for our present purposes. The results of sample calculation are summarized in Table 3, where the followings are given: 1) the total wave energy and the corresponding significant wave heights estimated from the record of the underwater wave gauge, and 2) the significant wave heights determined statistically from the records of the step type wave gauge. In the process of the computations for significant wave height from the wave energy spectrum, the following well-known relationship was applied.

$$H_{1/3} = 2\,832\sqrt{E} \tag{15}$$

The comparison of the above two significant wave heights indicates that the estimated wave height is about 10~20 % less than the surface wave height. The above discrepancy may be caused by the reasons that the high frequency element of wave motion is completely damped by the action of hydraulic filter and that the high frequency part of power spectrum is neglected in the calculation of total wave energy. According to the result of further study the noise seems to take a relatively important role in the response characteristic of underwater wave gauge in field comparing with that in laboratory. From this point of view more studies are required to be done in order to clarify the actual phenomena. At any rate the procedure of the present computation is still not perfect but be quite satisfactory from the practical point of view.

REVIEW OF PREVIOUS INVESTIGATIONS

After reaching the main conclusions of the present investigations, the writers could collect some more reference materials, through which they have reviewed the previous investigations.

As stated in the first section of this paper the correction factor, n , has normally been taken as a certain constant value for the data analysis, such as $n = 1.35$ (Seiwell), $n = 1.3 \sim 1.35$ (Japan) and $n = 1.25$ (Laboratoire National D'Hydraulique, France). On the other hand Draper and Glukhovskiy presented the following formulas respectively:

$$n = 1 + (0.16/\cosh\tfrac{2\pi h}{L}) \quad \text{(Draper)} \tag{16}$$

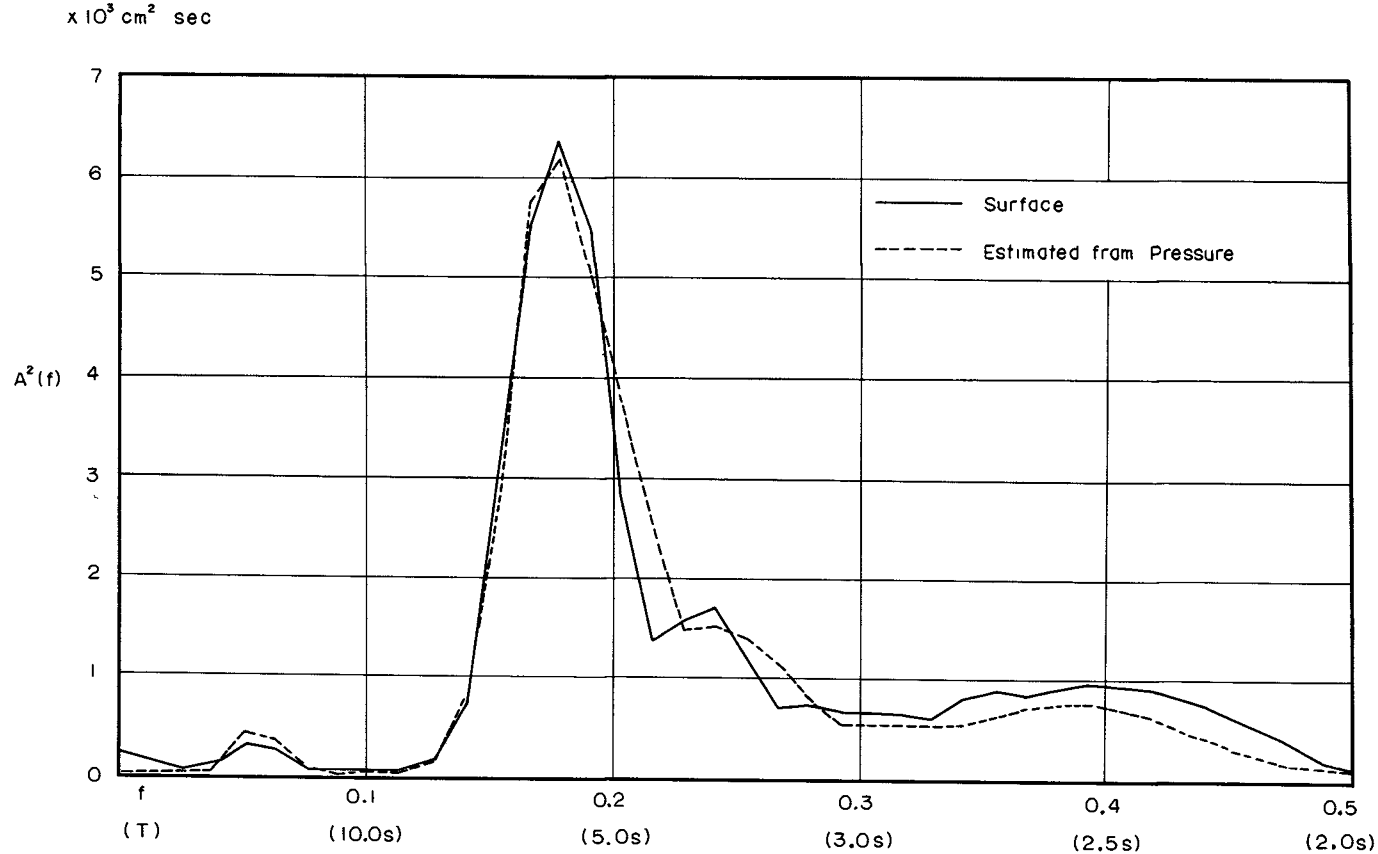

Fig. 10. Comparison between the wave spectrum of surface elevation and the estimated one.

$$n = \exp\{5.5(h/L)^{0.8} - (2\pi h/L)\} \quad \text{(Glukhovskiy)} \qquad (17)$$

The latter two formulas can be compared with the data presented in this paper. The present writers have the opinion that these formulas have a certain limitation of their applicability. The accumulation of more accurate data will be necessary to distinguish the applicability of these formulas including the writers' one.

ADDITIONAL DISCUSSION

In the above treatment the discussion is based on the applicability of Eq. (3), but the actual phenomenon is not fully expressed by the small amplitude theory. Therefore the writers try to treat the present problem by using the Skjelbreia and Hendrickson's 5th order theory of gravity waves. Figure 11 shows the result of the above computations established under the condition of $d = h$, and the result of experiments conducted by using regular waves under the various conditions of d/h. The careful comparison indicates that the factor of d/h seems to have negligible effect on the correction factor, n, as stated in the previous section, and that the agreement between the theoretical and experimental results seems to be quite consistent. However the general tendency of the theoretical curves is entirely different from that of the previous data of n given in Figs. 6,8 and 9. It means that the result of the finite amplitude theory is not fully powerful to explain the tendency of experimental and field data of irregular waves.

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Table 3 (Sept. 17, 1965).

Time	Pressure		Surface
	E (m^2)	$H_{1/3}$ (m)	$H_{1/3}$ (m)
15:20	0.1569	1.12	1.37
30	0.2237	1.34	1.52
40	0.1536	1.11	1.31
50	0.1613	1.14	1.36
16:00	0.2288	1.35	1.37
17:00	0.1797	1.20	1.30
18:00	0.1770	1.19	1.29

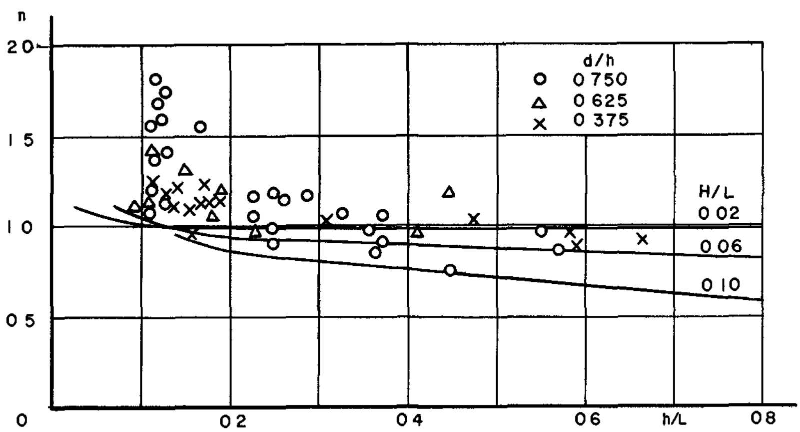

Fig. 11. Comparison between regular wave data and theoretical curves calculated under the condition of d/h = 1.0.

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