CHAPTER 18

SHOCK PRESSURE OF BREAKING WAVE

Hisashi Mitsuyasu
Assistant Professor, Research Institute for Applied Mechanics, Kyushu University, Fukuoka, Japan

ABSTRACT

Shock pressure due to breaking wave was studied experimentally and analytically for the case of two-dimensional regular oscillatory wave.

In the first part of this paper, important characteristics of shock pressures were described, which were obtained by using the newly designed pressure gauges of very high frequency responses (10 ~ 10000 c/s). And the following two points were mainly examined 1) the detailed characteristics of pressure-time histories of shock pressures, and 2) the similarity of pressure-time histories observed simultaneously with two separated gauges.

In the second part of this paper, the dynamical models representing the generation mechanism of shock pressure were examined to explain the observed properties of shock pressures, in which the new air-cushion model extended from the original air-cushion model of Bagnold (1939) was included.

Under the assumption of small compression of the air cushion, the analytical solutions representing the pressure-time histories were obtained both for the original model of Bagnold (1939) and for the present new model. For the cases of relatively high-intensity shock pressures, the consistent results were obtained by analyzing the observed shock pressures by the new air-cushion model. The discussion of shock pressure due to finite compression of air cushion was also included.

INTRODUCTION

When water wave breaks against a vertical wall there may be chances that wave with an almost perpendicular front impinges on the wall. In such case, as the motion of certain limited mass of water is retarded abruptly, high intensity shock pressure occurs. As the shock pressure due to such breaking wave is one of the most intense force exerted on the coastal structures and the structures should be safe against it, it is very important to know the general properties of the shock pressure and to estimate its effect on the coastal structures or to make clear the conditions required for the generation of shock pressure and to avoid such conditions if practically possible. Reflecting such engineering demand, the shock pressure due to breaking wave has been the subject of
numerous investigations, for example, Bagnold (1939), Denny (1951), Ross (1955), Hayashi (1956), Mitsuyasu (1958), Nagai (1959), and Mitsuyasu (1962).

Based on those investigations, we have accumulated much knowledge about the characteristics of shock pressure due to breaking wave, such as time history of shock pressure, statistical properties of the intensity of shock pressure, vertical distribution of the peak pressure on the wall and generation mechanism of shock pressure. In addition to those investigations, Rundgren (1958) and Mitsuyasu (1962) studied the conditions required for the generation of shock pressure.

However, in spite of such numerous accumulated knowledge about the shock pressure, several difficulties are still left due to the extremely complex nature of the shock pressure of breaking wave. That is, when the shock pressure of high intensity is caused its time variation is very rapid and it shows very complicated spatial change too. From those complexities in the time and spatial change of shock pressure various difficulties are encountered in the accurate measurement of shock pressure, in its adequate description and in the evaluation of its effect on the structures. This also makes it difficult to formulate the dynamical model for the generation mechanism of shock pressure.

To avoid the difficulties arising from the complexity in the spatial distribution of shock pressure, in one of the previous study, the author directly measured the total wave force acting on the vertical wall under the various conditions, and made clear the relations between the dimensionless wave force and various non-dimensional factors such as deep water wave steepness, relative depth at the wall and beach slope. However, since the information about the spatial distribution of the shock pressure is lost in the total wave force directly observed, we need additional studies on the shock pressures locally exerted. For this purpose, the measurements of local shock pressures were conducted by using the pressure cells which have very high frequency responses (10~10000 c/s). The measurements were done simultaneously at two separate points under one of the critical conditions in which very high-intensity shock forces were measured in the previous study. The detailed experimental procedures have been published elsewhere [Mitsuyasu (1966)] and only the important results will be recapitulated here briefly. Then, the dynamical models for the generation mechanism of shock pressure are mostly examined to explain the characteristics of observed shock pressures.

CHARACTERISTICS OF OBSERVED SHOCK PRESSURE

In the upper part of Fig.1a there is shown an example of the time history of wave pressure which has been measured when a breaking wave impinged against a rigid wall. At the moment when an almost perpendicular wave front impinged against a wall the shock pressure corresponding to the first peak in \( p(t) \) is observed. But, its duration is very short and soon reaches to the ordinary wave pressure of the type of standing wave when the impinged wave begins to rise along the wall. The time history of shock pressure which we want to clarify its detailed characteristics is that near the first peak. Therefore, oscilloscope records of shock pressures, which will be discussed hereafter, have been confined only to the pressure near the first peak.
Although the measurements * were carried out under the almost same conditions by using the uniform part of a train of waves, the measured trace of wave pressure varied greatly wave by wave. Figs.1a, 1b and 1c show the typical oscilloscope records of shock pressures which are selected from the many records and are tentatively classified into five ranks according to their general patterns and peak intensities. In those records sweep velocity is 2mm/m.sec in every cases except for the record 1-3(4) in which sweep velocity is 1mm/m.sec, and timing mark of 10m.sec is overlapped on every traces by the modulation of beam intensity ( white spots in the trace ). The parenthesized value near the first peak of pressure record is the dimensionless peak pressure defined by $p_{\text{max}}/\rho g H_o$. Referring successively to the oscilloscope records in Figs.1a, 1b and 1c, the characteristics of shock pressures can be described as follows:

i) In the pressure at $P_3$ which is 7.5cm under still water level and is always in the water even at the wave trough, no shock pressure was observed. However, oscillatory pressures were observed sometimes when the distinct oscillatory pressures were happened at $P_2$ which is 0.5cm under still water level ( c.f. record 1-4(7) ). As for the pressures at $P_2$ which is near the still water level, unsocillatory pressures (c.f. record 1-3(4) ) were sometimes observed especially when the intensity of shock pressure was relatively low ( $p_{\text{max}}/\rho g H_o \leq 5$ ). However, as shown in 1-4(7), oscillatory pressure was observed in the other cases even when the intensity of the pressure was relatively low, and such oscillatory characteristics of shock pressures were seemed not necessarily depend on the intensity of shock pressure. Such characteristics were considered to be due to the generation mechanism of shock pressure. That is, the oscillatory pressure should be caused when the wave impinged on the wall with a trapped air pocket, and unsocillatory pressures should be happened in such cases that irregular wave front impinged on the wall without distinct air pocket or the trapped air had been released immediately after the impingement of the wave.

ii) When the shock pressures of medium intensity ( $p_{\text{max}}/\rho g H_o \leq 7$ ) were caused near the still water level, quite similar pressure-time histories with the same order of intensity were freqently observed at two points in the same level and apart each other by the length 2$H_o$ ( c.f. records, 2-3(3) and 2-2(3) ) **. This fact suggests that the shock pressures with peak intensity of the order $p_{\text{max}}/\rho g H_o \leq 7$ can be caused simultaneously in wide horizontal area on the wall, and we can expect not so much reduction of the total wave force due to the phase difference of peak pressure in each point.

* The measurements of wave pressures were carried out in the laboratory wave tank ( length ; 22m, width , 0.6m, water depth , 0.35m ). A vertical steel wall was placed on the beach slope ( 1/15 ) at one end of the tank. The water depth at the wall was 12.5cm, and a regular oscillatory wave ( wave period, $T = 1.90\text{sec}$, deep water wave height $H_o = 10.5\text{cm}$, deep water wave steepness $H_o/L_o = 0.019$ ) was used for the measurement of wave pressure. Wave pressure was measured by the ceramic type pressure gages ( natural frequlncy : over 10kc, pressure sensitive area $\phi = 12\text{mm}$ )

** $p_4$ is in the same level with $p_2$ and 21cm ( $\pm 2H_o$ ) apart from $p_2$. 

iii) Even when the shock pressures of medium high intensity ($p_{\text{max}}/\rho g H_0 \approx 10$) were caused, similar pressure-time histories were observed in many cases (c.f. 3-3(6)), but some differences were found in their detailed features (c.f. 3-3(5)).

iv) When the shock pressure of very high intensity ($p_{\text{max}}/\rho g H_0 \geq 20$) was observed at some point near the still water level, the time history of pressure showed quite different features for the two points even if the pressure gages were in the same level and incident wave was almost two-dimensional in offshore. This reflects that the shock pressure of very high intensity has very complicated spatial structure.

v) There must be such a chance that an water mass with fairly regular form or the breaking wave with locally vertical front impinges on the wall. In such a case the shock pressure of extremely high intensity will be caused locally, which is comparable to the pressure due to water hammer. However, if such pressure is confined only to the local area narrower than the sensitive area of the pressure gage, since the pressure gage measures the mean pressure averaged over the pressure sensitive area, their intensity must be reduced. The irregular but sharp time-history of shock pressure as shown in 2-2(9) and 2-3(6) should be caused in such circumstances.

Based on the pressure records shown in Figs.1a, 1b and 1c, we can point out its oscillatory time history following to the first peak pressure as the distinct characteristics of shock pressure due to breaking wave. In quite many cases, not necessarily in every cases but especially when the pressure intensity is high, pressure time history of the type of damped oscillation was observed, and the duration of its first peak $\gamma$ and the period of pressure oscillation $T_i$ became shorter as the increase of the intensity of first peak pressure. In our present experiment, values of $\gamma$ were in the rage of 2 ~ 17 m.sec and $T_i$ was in the range of 1 ~ 12 m.sec. The pressure oscillations were damped out approximately within 40 m.sec in many cases, and the logarithmic decrement was approximately 0.5 ~ 1. As the natural frequency of the pressure cell was over 10kc and that of front plate of the steel wall was approximately 2kc, the oscillatory pressure fluctuations measured in the present experiment can not be attributed to the oscillation of the measuring system. The similar oscillatory patterns of shock pressures were also seen in the records obtained by Bagnold(1939) and also by Ross(1955) in the different experimental conditions. Accordingly, we can say that the oscillatory pattern is one of the most distinct characteristics of the shock pressure due to breaking wave. From the fact that its frequency is quite different from that due to water-hummer pressure, it should be natural to consider that the oscillatory pressure is due to the oscillation of the air cushion enclosed between the wave front and the wall *.

* Recently, Y. Goda of Port and Harbour Technical Research Institute has obtained the experimental evidence in favor of the air-cushion model. According to his investigations, when the shock pressure of distinct damped oscillation type was observed ($p_{\text{max}}/\rho g H_0 \approx 40$), the first peak pressure was caused a little befor the contact of water to the wall. He used the small electrode for detecting the contact of water to the wall. This fact strongly support the existence of air cushion. In some case, however, when the shock pressure of unoscillatory type was observed ($p_{\text{max}}/\rho g H_0 \approx 40$), the peak pressure was caused simultaneously with the contact of water to the wall. (private communication)
DYNAMICAL MODELS FOR THE GENERATION MECHANISM OF SHOCK PRESSURE

In the light of accumulated evidence in favour of the air-cushion theory for the generation of shock pressure by breaking wave, the author has thought it desirable to examine the theory more systematically and extend it so as to fit more closely to the observations. For this purpose, an analytical solution for the original air-cushion model proposed by Bagnold(1939) is obtained first under the assumption of small compression of air cushion. Then, the exact solution for the maximum peak pressure is obtained numerically and the accuracies of various approximate solutions are checked by it. Finally, new air-cushion model with leakage of the air is examined to explain the observed time history of shock pressure which shows the distinct damped oscillation.

THE SMALL COMPRESSION OF THE AIR CUSHION

We consider the model as shown in Fig.2, which is the same to that proposed by Bagnold(1939). That is, the water mass of length $K$, which corresponds to the length of virtual mass of impinging wave, compresses the air cushion of thickness $D$ containing air initially at atmospheric pressure $p_0$ with the initial velocity $U_0$. Assuming the adiabatic compression, the pressure $p$ of compressed air cushion is given by

$$p = p_0 \left( \frac{D}{x} \right)^\gamma, \quad \gamma = 1.4$$  \hspace{1cm} (1)

On the other hand, the motion of the water mass is determined by

$$\rho \frac{d^2x}{dt^2} = p - p_0$$  \hspace{1cm} (2)

Inserting Eq.(1) into Eq.(2) and integrating it under the initial condition

$$\frac{dx}{dt} = -U_0 \quad \text{at} \quad x = D$$  \hspace{1cm} (3)

we obtain

$$\left( \frac{dx}{dt} \right)^2 = U_0^2 - \frac{2p_0}{\rho K} \left\{ \frac{D}{\gamma - 1} \left[ \left( \frac{x}{D} \right)^{\gamma - 1} - 1 \right] - (D - x) \right\}$$  \hspace{1cm} (4)

The exact solution of Eq.(4) can not be obtained, and Bagnold(1939) did the numerical integration for obtaining the solution of Eq.(4) and also pressure $p$. However, when the small compression of air cushion is assumed we can get the analytical solution of Eq.(4) in the following way.

Introducing the new variable $X$, by $X = D - x$, and expanding the right side of Eq.(4) with respect to $(X/D)$, after the slight modification of the equation, we obtain

$$\frac{dX}{dt} = \pm U_0 \left\{ 1 + \frac{2p_0}{\rho K U_0} \left[ X + \frac{D}{\gamma - 1} \left[ \left( \frac{X}{D} \right)^{\gamma - 1} - 1 \right] - (D - x) \right] \right\}$$  \hspace{1cm} (5)

Here, $-$ sign of $\pm$ corresponds to compression and $+$ sign of $\pm$ corresponds to expansion.
Taking to the second order in Eq. (5) we obtain

\[ \frac{dX}{dt} = \pm U_0 \left[ 1 - \frac{p_0 \delta}{\rho K U_0^2 D} X^2 \right]^{\frac{1}{2}} \quad (6) \]

The solution of Eq. (6) satisfying the initial condition \( X = 0 \) when \( t = 0 \) is given by

\[ X = a_1 \sin \frac{U_0}{a_1} t \quad (7a) \]

or

\[ X = a_1 \sin \sigma t \quad (7b) \]

where

\[ a_1 = \left( \frac{p_0 \delta}{\rho K U_0^2 D} \right)^{\frac{1}{2}}, \quad \sigma = \left( \frac{p_0 \delta}{\rho K D} \right)^{\frac{1}{2}} \quad (8) \]

In the same way as in Eq. (4), Eq. (1) becomes as follows by expanding it with respect to \( X/D \) and taking to the second order,

\[ p = p_0 + \frac{\gamma p_0}{D} X + \frac{p_0}{D^2} \left( \frac{\delta + 1}{2} a_1^2 \sin^2 \sigma t \right) \quad (9) \]

Substituting Eq. (7b) into Eq. (9), the pressure \( p \) is given by

\[ p - p_0 = \frac{\gamma p_0}{D} X + \frac{p_0}{D^2} \left( \frac{\delta + 1}{2} a_1^2 \sin^2 \sigma t \right) \quad (10a) \]

or

\[ p - p_0 = \frac{\gamma p_0}{D} X + \frac{p_0}{D^2} \left( \frac{\delta + 1}{2} a_1 \sin \sigma t \right) \quad (10b) \]

For the infinitesimal compression of air cushion first term represents the fairly good approximation. The first term of right-hand side of Eq. (10b) shows the characteristics of shock pressure commonly observed. That is, peak pressure is proportional to the momentum of impinging water mass and the greater the shock pressure intensity the shorter its duration and the period of pressure oscillation.

THE FINITE COMPRESSION OF THE AIR CUSHION

The assumption of small compression of air cushion is not satisfied when the momentum of water mass is large and finite compression of air cushion is expected. Even for this case, although the strict solution of Eq. (4) can not be obtained by analytical method, we can easily get the exact values of the peak pressure \( p_{\text{peak}} \) by considering the facts that the maximum peak pressure is caused at the moment of the maximum compression of air cushion and the minimum peak pressure is caused at the maximum expansion. At the moment, as the motion of the water mass is stopped, the following relation must be satisfied from Eq. (4),

\[ U_0^2 - \frac{2 p_0}{\rho K} \left\{ \frac{D}{\delta - 1} \left[ \left( \frac{x_1}{D} \right)^{\delta - 1} - 1 \right] - \frac{D - x_1}{D} \right\} = 0 \quad (11) \]

where \( x_1 \) is the thickness of air cushion at the moment \( U = 0 \). From Eq. (11) we obtain

\[ \frac{\rho K U_0^2}{p_0 D} = 5 \left( \frac{D}{x_1} \right)^{0.4} + 2 \left( \frac{D}{x_1} \right)^{-1} - 7 \quad (12) \]

By using Eq. (12) we can calculate the relation between \( \rho K U_0^2 / p_0 D \) and \( (D/x_1)^{0.4} \). And, since \( (D/x_1)^{0.4} \) correspond exactly to \( p_{\text{peak}} / p_0 \) from Eq. (1), we have derived here the exact relation between \( \rho K U_0^2 / p_0 D \) and \( p_{\text{peak}} / p_0 \), which is shown in Fig. (3). If the values of \( K, U_0 \) and \( D \) are known, \( p_{\text{peak}} \)
can be obtained by using the relation shown in Fig. 3.

On the other hand, the following relation can be derived from the approximate solution Eq. (10b):

\[
\frac{p_{\text{peak}}}{p_0} = 1 \pm 1.18 \left( \frac{\rho K U_0^2}{p_0 D} \right)^{\frac{1}{2}} + 1.2 \left( \frac{\rho K U_0^2}{p_0 D} \right)
\]

(13)

Here, + sign of ± corresponds to \( p_{\text{max}} \) and - sign of ± corresponds to \( p_{\text{min}} \). The similar relation was obtained for \( p_{\text{max}} \) by Bagnold (1939) as

\[
\frac{p_{\text{max}}}{p_0} = 1 + 2.7 \left( \frac{\rho K U_0^2}{p_0 D} \right)
\]

(14)

In Fig. 3, the approximate solution Eq. (13), its abbreviated form in which the third term of the right side of Eq. (13) is neglected, and Bagnold’s formula Eq. (14) are shown in addition to the exact solution. From Fig. 3 it can be seen that Bagnold’s formula is a good approximation within a range 2 to 10 atmosphere as already stated by Bagnold himself, that the assumption of small compression is satisfied in the range 0.8 \( \frac{p_{\text{peak}}}{p_0} \leq 4 \), and that the abbreviated form of Eq. (13) can represent the fairly good approximation only when 0.7 \( \frac{p_{\text{peak}}}{p_0} \leq 1.3 \).

THE AIR CUSHION WITH LEAKAGE

The air cushion model described in the previous sections cannot explain the observed time-history of shock pressure which shows the distinct damped oscillation. Therefore, we must examine the mechanism which causes the damped oscillation. When we carefully observe the motion of wave impinging against a wall it can be seen that a part of air trapped in the air pocket is presumably leaking upward along the wall with the spray. So we consider the model shown in Fig. 4 and assume that the velocity of leaking air \( U_i \) through the gap \( S_i \) is proportional to the pressure difference between the inside of air pocket and its outside,

\[
U_i = K_0 (p - p_0)
\]

(15)

According to the same formulation as that in the previous section the following equations can be obtained.

\[
\rho K \frac{dU_i}{dt} = p - p_0 = \frac{1}{K_0} U_i
\]

(16)

\[
p_0 D^r = p \left\{ (D - X) + \frac{S_i}{S} \int_0^t U_i(t) dt \right\}^r
\]

(17)

In Eq. (17), the second term in the bracketed term is due to the correction for the leaking air. Here, \( S \) is the crosssection of the air cushion.

Modifying Eq. (17) we can write

\[
p = p_0 \left\{ 1 - \frac{1}{D} \left[ X - \frac{S_i}{S} \int_0^t U_i(t) dt \right] \right\}^r
\]

(18)

Expanding right side of Eq. (18) and taking only to the first order we obtain

\[
p - p_0 = \frac{p_0 D}{D} \left[ X - \frac{S_i}{S} \int_0^t U_i(t) dt \right]
\]

(19)

From Eq. (16) and Eq. (19) we obtain
\[ \rho K \frac{dU}{dt} = \frac{P_0 \rho}{D} \left[ \lambda - \frac{S_1}{S} \int_{0}^{t} U_i(t) \, dt \right] \quad (20) \]

Differentiating Eq. (20) with respect to \( t \) we obtain

\[ \rho K \frac{d^2U}{dt^2} = \frac{P_0 \rho}{D} \left[ -U - \frac{S_1}{S} U_i \right] \quad (21) \]

Substituting Eq. (16) into Eq. (21) we can finally obtain

\[ \rho K \frac{d^2U}{dt^2} + \frac{P_0 \rho}{D} \frac{S_1}{S} \rho KK_0 \frac{dU}{dt} + \frac{P_0 \rho}{D} U = 0 \quad (22a) \]

or

\[ M \frac{d^2U}{dt^2} + \beta \frac{dU}{dt} + K_i U = 0 \quad (22b) \]

where

\[ M = \rho K \quad \beta = \frac{P_0 \rho}{D} \frac{S_1}{S} \rho KK_0 \quad K_i = \frac{P_0 \rho}{D} \]

The Eq (22b) is a typical equation of the free damped oscillation and three types of solutions can be obtained according to the damping conditions. After obtaining the solutions of Eq (22b) which satisfy the initial condition

\[ U = -U_0 \quad \text{and} \quad U_i = 0 \quad \text{at} \quad t = 0 \quad (23) \]

converting them to the pressure expressions by using Eq (16) we can finally obtain

\[ p - p_0 = \frac{K_i}{\sqrt{\alpha} M} \rho K U_0 e^{\frac{\beta}{2M} t} \sinh \sqrt{\alpha} t \quad (24a) \]

for \( \alpha = \left( \frac{\beta}{2M} \right)^2 - \frac{K_i}{M} > 0 \)

\[ p - p_0 = \frac{K_i}{\sqrt{-\alpha} M} \rho K U_0 e^{-\frac{\beta}{2M} t} \sin \sqrt{-\alpha} t \quad (24b) \]

for \( \alpha < 0 \)

and

\[ p - p_0 = \frac{K_i}{M} \rho K U_0 t e^{-\frac{\beta}{M} t} \quad (24c) \]

for \( \alpha = 0 \)

First solution corresponds to the case of over damping, second solution is the ordinary damped oscillation and third solution corresponds to the case of critical damping.

For the case of ordinary damped oscillation, substituting the relation of (22c) into Eq. (24b) we obtain

\[ p - p_0 = A_1 e^{-\alpha t} \sin \sigma_1 t \quad (25) \]

where the following new parameters are introduced

\[ A_1 = \rho K U_0 \sigma_1 \left[ 1 - \alpha^2 \right]^{-1} \quad \alpha = \left( \frac{\sigma_0 S_1 K_0}{2 PD} \right) \quad \sigma_1 = \left( \frac{\sigma_0 P_0}{PKD} \right)^\frac{1}{2} \left( 1 - \alpha^2 \right)^\frac{1}{2} \]

\[ \beta = \left( \frac{\sigma_0 P_0}{PKD} \right)^\frac{1}{2} \alpha \quad \sigma_1 = \left( \frac{\sigma_0 P_0}{PKD} \right)^\frac{1}{2} \left( 1 - \alpha^2 \right)^\frac{1}{2} \]

Here, parameter \( \alpha \) represents the effect of leaking air and the Eq. (25)
reduces to Eq. (10) when \( \theta = 0 \), i.e., there is no leakage of the air.

The maximum peak pressure \( p_{\text{max}} \) and the time of its occurrence \( t_i \) are given respectively by

\[
\begin{align*}
p_{\text{max}} - p_0 &= A_i e^{-\beta_i t_i} \sin \sigma_i t_i, \\
t_i &= \frac{1}{\sigma_i} \tan^{-1} \sigma_i / \beta_i.
\end{align*}
\]

The unknown factors in Eq. (25) and Eq. (26) such as \( K, D \) and \( \alpha \) can be determined by the observed pressure-time history and the motion of impinging wave, i.e., by the observed values of \( p_{\text{max}}, \ p_0 \) and \( U_0 \). Examples of the values of \( K, D \) and \( \alpha \) such obtained are shown in Table-1 in which the characteristics of corresponding shock pressures are also shown. From the results shown in Table-1 it can be seen that, since the wave height \( H \) used in the experiment is approximately 10cm, the value of \( K \) is approximately 0.2 times \( H \), that the value of \( D \), i.e., the thickness of the air cushion is the order of several milimeters when the high intensity shock pressure is caused, and that the value of \( \alpha \) is the order of 0.1. The first result agrees with that of Bagnold (1939). He determined the values of \( K \) by dividing the measured pressure time integral \( \int p \, dt \) by \( \rho U_0 \).

Although fairly consistent results were obtained for the cases of relatively high intensity shock pressures as shown in Table-1, some inconsistent results were obtained for the other cases of low intensity shock pressures. This suggest that the air-cushion model is responsible only for the generation of high intensity shock pressure.

**DISCUSSION**

As the generation mechanism of shock pressure by breaking wave we can consider several different models depending on the circumstances. That is, if the breaking wave with completely vertical front impinges against the vertical wall as shown in Figs.5(a), we can expect the occurrence of the water-hummer pressure, and the pressure is given by

\[
P = \rho U_0 C
\]

where \( C \) is the velocity of sound in the water.

However, since the actual wave front is not a plain surface but irregular surface when the wave breaks, we can not expect the occurrence of such phenomenon except for the very narrow local area on the wall.

Similarly, if the wave front is completely plain but with a small angle \( \theta \) to the wall as shown in Figs.5(b), this case is analogous to the phenomena associated with the impact of V-shaped body on water surface if we assume the uniform translation of wave front without changing the form, and we will be able to apply the theories developed for that problem, for examples, by Karman (1929) or by Wagner (1932), to our present problem.

While, in many cases, as the velocity of water at the front of breaking wave is faster near the wave crest than in the lower part, there are much possibilities that the air is enclosed between the wave front and the wall by the forward inclination of wave when the wave impinges on the wall as shown in Figs.5(c). This is the case of air-cushion model which we have
discussed in the previous section of this paper.

In actual phenomena, however, as the wave front is not so simple as shown in Figs.5(c), it may be natural to consider the water-air-solid junction as shown schematically in Figs.5(e). Therefore, various mechanisms above mentioned will possibly happen simultaneously at different local areas on the wall. This must be the main reasons why the spatial structure of the shock pressure due to breaking wave is so complicated. In some case, one of those mechanisms becomes predominant depending on the characteristic form and motion of wave front. And in our case, in which a uniform train of oscillatory wave broke against a vertical wall placed on the uniform sloping beach, it can be said that there existed much chances of enclosing the air pocket between the wall and the wave front, and thus, wave pressures are caused in many cases by the compression of air cushion.

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Fig. 1(a). Oscilloscope records of shock pressures.
Fig. 1(b) Oscillograph records of shock pressures
Fig. 1(c). Oscilloscope records of shock pressures.
The diagram illustrates the comparison between the exact solution, approximate form of the solution, and the approximate boundary formula. The equation given is:

\[ \frac{\rho K u^2}{p_0 D} = \left( \frac{1}{x/D} \right)^{0.4} + 2 \left( \frac{D}{x} \right)^{-1.7} \]

Fig. 3.

Fig. 2.

Fig. 4.
SHOCK PRESSURE

Fig. 5.