CHAPTER 86

RESISTING TORQUES OR FORCES ACTING ON THE SPUDS OF THE PUMP DREDGER ON THE SURFACE WAVES

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ABSTRACT

This paper presents the analysis on the resisting torques and forces acting on the spuds mooring the pump dredger for the surface waves. Torque or force was expressed by the product of the system factor, the statical wave factor and the magnification factor.

INTRODUCTION

When violent waves attack a floating pump dredger in operation, oscillatory motion of the dredger takes place around a point moored to spuds, which is sometimes broken by this oscillatory motion. To avoid this accident, resisting torque or force acting on the spud must be calculated in relation with waves, which enables for a operator to judge the adequate moment when they have to withdraw the spuds from the sea surface and tie them along shipsides.

Recently, theoretical and observational improvements have been accomplished in connection with the behavior of moored systems, such as moored ships on long waves by Kilner(1961), submerged moored sphere by Harleman and Shapiro(1961), offshore-moored ships by Leendertse(1964), and a spread-moored landing craft by O'Brien and Muga(1964). Mooring by a single long chain or cable, or spread mooring was the case investigated by them while Harleman and Shapiro treated a mooring line bar having one degree of freedom of rotation. Ordinally, the motion of a ship have six degree of freedom, three translational and three rotational. However in the motion of the pump dredger moored by one or two spuds, surging is restricted and heaving is assumed free along the sleeves of spuds, so other four kinds of motion which are sway, pitch, yaw and rolling are to be considered. Also the features of the movement of it are noticiably influenced by the wave angle to the longitudinal ship axis. The floating body in this case oscillates around the mooring point but not around the center of floatation. This causes the additional torque of bouyancy.

This paper presents the analysis of a pump dredger moored by one spud or two spuds, the comparison between results calculated and experimental and some arguments based on the analysis.

The resisting torques and forces acting on the spuds can be calculated from the elastic or tortional deformation of them, the deflexion angle of which is equal to the inclination of the water-line of the pump dredger to the horizontal.

MOVEMENT OF PUMP DREDGER MOORED BY ONE SPUD OR TWO SPUDS ON THE SINUSOIDAL SURFACE WAVES

General features of the movement are now to be described, in connection with the theoretical investigation according to the model experiments which will be explained afterwards.

When the wave direction is coincide with the longitudinal axis of the ship, the movement is solely pitching. And when waves come with oblique angle to the ship axis, yawing takes place beside pitching. When waves attack the ship with single spud normal to the longitudinal axis, rolling is accompanied with pitching and yawing. However in the case of two spuds, sway is carried out in place of rolling.

The wave length is normally comparable to the ship length. So the configuration of waves must be taken into account.

It is assumed that waves are uniformly sinusoidal and are expressed by Airy's surface waves in shallow water. The effect of irregular waves is not considered.

ANALYSIS

EQUATIONS OF MOTION

The equations of motion of the moored ship are written as,

$$M.\ddot{s} = F_{sw}+F_{si}-F_{sh}-F_{sd}-F_{sm} \tag{1}$$

$$I_s\ddot{\Theta} = T_{\Theta w}+T_{\Theta i}-T_{\Theta h}-T_{\Theta d}-T_{\Theta m} \tag{2}$$

, where F is the force, T is the torque, M and I are the mass and the moment of inertia of the ship respectively. The first subscripts of the force and the torque refer to each degree of freedom. The subscript of the moment of inertia means the direction of axis around which the rotation of the ship takes place. Second subscripts of the force and the torque indicate the type of them, thus; F_w and T_w are respectively wave forces and their torques which exite the ship motion, F_i and T_i are respectively inertial forces and inertial torques which are usually treated as virtual mass forces and mass torques, F_h and T_h are respectively hydrostatic restoring forces and torques due to the bouyancy effect given rise by the displacement from the equilibrium condition such as the stability of the ship, F_d and T_d are damping forces and torques which are assumed here as linearly proportional to the velocity of the ship, and F_m and T_m are respectively resisting forces and torques exerted by the mooring spuds, which have components beside heave in this case, but they are not effective in surge, sway and yaw for the ships moored by the long cables and so on. These last terms are the final objects to be investigated in this paper.

RESISTING FORCES AND TORQUES BY THE MOORING SPUDS

It is assumed that the spud is a cantilever fixed at its end on sea bed. Then the moment at the point mooring the floating pump dredger is expressed as,

$$T_m + \tfrac{1}{2}F_m l_s = \frac{E_s I_s}{l_s} \cdot \theta \tag{3}$$

assuming the elastic deformation, where E_s is the modulus of elasticity, I_s is the moment of inertia of the unit density, l_s is the length of the spud measured from the mooring point of the ship to the bed point and θ is the angle of deformation to the vertical, which is equal to the declination of the waterline of the ship to the horizontal.

From eq.(3), the spud torque in pitching is given by

$$T_{pm}= \frac{E_s I_s}{l_s} \cdot \theta \tag{4}$$

and also that in rolling is given by

$$T_{rm}= \frac{E_s I_s}{l_s} \cdot \Psi \tag{5}$$

respectively by omitting F_m.

The floating pump dredger moored by two spuds oscillates as a bifilar pendulum in yawing. In this case the following equation holds,

$$T_{ym}= \frac{3E_s I_s}{2\ l_s^3} \cdot b^2 \varDelta\varphi \tag{6}$$

, where b is the distance of two spuds, and $\varDelta\varphi$ is the yaw angle.

An expression other than eq.(3) is used for the torque in sway such as,

$$T_{sm}= \frac{6E_s I_s}{l_s^2} \cdot \varDelta \tag{7}$$

and also in sway the normal force to spuds is acting such as

$$F_{sm}= \frac{12E_s I_s}{l_s^3} \cdot \varDelta \tag{8}$$

on each spud, where $\varDelta$ is the horizontal displacement of the ship normal to the longitudinal ship axis.

In yawing motion with single spud, the tortion torque T_{ym} is directly applied to the spud Thus,

$$T_{ym}= KG \frac{d\varphi}{dz} \tag{9}$$

, where G is the modulus of elasticity in shear, K is the polar moment of inertia. When the spud is fixed in the sea bed, $d\varphi/dz$ can be assumed as $\varDelta\varphi/l_s$. Then we get,

$$T_{ym}= \frac{KG}{l_s} \varDelta\varphi \tag{10}$$

HYDROSTATIC RESTORING TORQUES

The dynamic stability of an unrestrained ship is $W.\overline{GM}.\theta$, where W is the displacement of the ship, and $\overline{GM}$ is the longitudinal metacentric height. However when the dredger rotates around the point A as in fig.

1, she heaves up as α, which is assumed as $L\theta/2$, in which L is the ship length. Then the additional restoring torque is exerted and the total stability is

$$T_{ph} = W.\overline{GM}.\theta + w_o \frac{1}{4} BL^3 \theta \tag{11}$$

In rolling, this added restoring torque is exerted in opposite direction on either side of the spud and is neglected compared with the normal dynamic stability. Then,

$$T_{rh} = W.\overline{GM}_r.\Psi \tag{12}$$

, in which $\overline{GM}_r$ is the transversal metacentric radius, and Ψ is the rotational angle of the waterline of the ship to the horizontal.

In yawing, this torque is not exerted, and also in sway, the hydrostatic restoring force does not work.

WAVE FORCES AND WAVE TORQUES

Sinusoidal waves are expressed by;

$$\zeta = a \sin(mx-nt) \tag{13}$$

, in which a is an amplitude, m is the circular wave number $2\pi/\lambda$, n is a circular frequency of the wave $2\pi/T$, λ is the wave length and T is the wave period.

The ship motion is solely pitting for bow or stern waves as mentioned already. The bouyancy torque by wave is in pitching,

$$T_{pw} = \int_0^L w_o B \zeta x \, dx = w_o B \frac{aE}{m^2} \cdot \sin(nt+\varphi_p) \tag{14}$$

, where

$$E = (P_\nu^2 + Q_\nu^2)^{1/2}, \qquad \tan\varphi_p = P_\nu / Q_\nu$$

$$P_\nu = \sin(m\nu) - mL\cos(m\nu)$$

$$Q_\nu = 1 - \cos(m\nu) - mL\sin(m\nu) \tag{15}$$

in which the following relation is assumed;

$$L = p\lambda + \nu \tag{16}$$

, where p is the number of waves included in the ship length L and ν is a part of one wave included in the residual length of the ship.

When waves attack the pump dredger with oblique angle φ_o, wave length λ must be modified as $2\pi/$ m cos φ_o, or m should be replaced by

$$m \cos\varphi_o = m_c \tag{17}$$

In this case the ship yaws to and fro also around the axis φ_o. The pressure difference due to the phase shift between the exposed and the sheltered side to the wave motion is the external wave torque in yawing. This phase shift is B sin φ. Assuming as hydrostatic pressure, the pressure torque by waves is written as,

$$T_{yw} = \left| \frac{w_o}{2} \int_0^L (d - s\tan\theta + \zeta)^2 \, s \, ds \right|_{\text{exposed}}^{\text{sheltered}} \tag{18}$$

, in which d is the draught and the coupling effect of pitching is introduced by $\tan\theta$. Using $s \cos\varphi_o \pm B/2 \sin\varphi_o$ in place of x in eq. (13) and calculating eq. (18), this wave torque is expressed as,

$$T_{yw} = A_m \sin nt + B_m \cos nt + C_m \sin 2nt + D_m \cos 2nt \tag{19}$$

, where

$$A_m = 2aw_o \sin(\frac{mB}{2} \cdot \sin\varphi_o).(d.E_{L1}\cos\varphi_{L1} - \tan\theta.E_{L2}\cos\varphi_{L2}) \tag{20}$$

$$B_m = 2aw_o \sin(\frac{mB}{2} \cdot \sin\varphi_o).(d.E_{L1}\sin\varphi_{L1} - \tan\theta.E_{L2}\sin\varphi_{L2}) \tag{21}$$

$$C_m = \frac{w_o}{2} a^2 \sin(mB \sin\varphi_o) E_J \cos\varphi_J \tag{22}$$

$$D_m = \frac{w_o}{2} a^2 \sin(mB \sin\varphi_o) E_J \sin\varphi_J \tag{23}$$

$$E_{L1} = \frac{2}{m_c^2}(P_{cL}^2 + Q_{cL}^2)^{1/2}, \quad E_{L2} = \frac{1}{m_c^2}(R_{cL}^2 + S_{cL}^2)^{1/2}$$

$$E_J = \frac{1}{4m_c^2}(P_{2cL}^2 + Q_{2cL}^2)^{1/2} \tag{24}$$

$$\tan\varphi_{L1} = -Q_{cL}/P_{cL}, \quad \tan\varphi_{L2} = R_{cL}/S_{cL}, \quad \tan\varphi_J = P_{2cL}/Q_{2cL} \tag{25}$$

in which next expressions are used,

$$P_{cL}, Q_{cL} = \left| P_\nu , Q_\nu \right|_{m=m_c}, \ \nu = L \tag{26}$$

$$P_{2cL}, Q_{2cL} = \left| P_\nu , Q_\nu \right|_{m=2m_c}, \ \nu = L \tag{27}$$

$$R_{cL} = L\cos(m_c L) + \frac{(m_c L)^2 - 2}{2m_c}\sin(m_c L) \tag{28}$$

$$S_{cL} = L\sin(m_c L) - \frac{1}{m_c} + \frac{2 - (m_c L)^2}{2m_c} \cdot \cos(m_c L) \tag{29}$$

In rolling, the wave torque T_{rw} is expressed as usual,

$$T_{rw} = W.\overline{GM_r}.\ \Psi_w \tag{30}$$

, in which Ψ_w is the surface slope of the wave to the horizontal, say $d\eta/dy$.

In sway, the total hydrostatic pressure in the wave direction x normal to the longitudinal ship axis y can be expressed as,

$$F_{sh} = \left| \frac{w_o}{2} \int_o^L (d-y\tan\theta-\zeta)^2 \, dy \right|_{\text{sheltered}}^{\text{exposed}} \tag{31}$$

, in which -mB/2 is used for exposed side and mB/2 is used for sheltered side respectively in place of x in eq. (13). Then from eq.(28), we can get as,

$$F_{sh} = w_o aL \left(\frac{a}{2} \sin mB \sin 2nt - 2d \sin(mB/2) \cos nt\right) \tag{32}$$

INERTIAL FORCES AND INERTIAL TORQUES

Inertial forces in eq. (1), and inertial torques in eq. (2) are expressed by the following general formulae,

$$F_{si} = -M_o \ddot{s} + K_f \iiint_V \dot{u}_s \, dm \tag{33}$$

$$T_{\Theta i} = -I_s' \ddot{\Theta} + K_t \iiint_V r\dot{u}_{\Theta} \, dm \tag{34}$$

, in which M_o and I_s are the mass and the moment of inertia of the displaced fluid respectively, $\dot{u}_s$ is the accerelation of water in the direction s, $\dot{u}_{\Theta}$ is that in the direction Θ and dm is the mass element of water. K_f and K_t are the coefficients of the virtual mass. And V is the volume of the part of body under water.

In pitching, u_{Θ} is the vertical velocity expressed by Airy,

$$u_p = -\frac{agm}{n} \cdot \frac{\sinh m(z+h)}{\cosh mh} \cdot \cos(mx-nt) \tag{35}$$

and inertial torque in pitching T_{pi} is

$$T_{pi} = -I_p' \ddot{\theta} - K_p w_o BH \frac{ad}{m^2} (P_\nu \cos nt + Q_\nu \sin nt) \tag{36}$$

, in which H= tanh mh.

In yawing, the second term of eq. (34) is expressed as

$$-I_z' \Delta \ddot{\varphi} - K_y \rho \iiint_V \left(y \frac{du}{dt} - x \frac{dv}{dt}\right) dxdydz \tag{37}$$

, in which $\Delta\varphi = \varphi - \varphi_o$.

As we take x as the wave direction and y as normal to it in horizontal, v=0 and

$$u_y = \frac{agm}{n} \cdot \frac{\cosh m(z+h)}{\cosh mh} \cdot \sin(mx-nt) \tag{38}$$

after Airy, in which h is the water depth. Taking the value at z=0 approximately, inertial torque in yawing T_{yi} is expressed as

$$T_{yi} = -I_y' \Delta \ddot{\varphi} + K_y w_o am(A \sin nt + B \cos nt) \tag{39}$$

, where

$$A = \frac{\Psi d}{m_c^2} P_{cL} - 2\Psi \frac{S_{cL}}{m_c^2} \cdot \tan\theta - D_c \tag{40}$$

$$B = -\frac{\Psi d}{m_c^2} \cdot Q_{cL} - 2\Psi \frac{S_{cL}}{m_c^2} \cdot \tan\theta + C_c \tag{41}$$

$$\Psi = \frac{2 \sin \varphi_o}{m_s} \sin(\frac{m_o}{2} B) \tag{42}$$

$$C_c = \frac{\xi}{m_c} \left[d(1-\cos m_c L) + \tan\theta(\cos m_c L - \frac{\sin m_c L}{m_c L}) \right] \tag{43}$$

$$D_c = \frac{\xi}{m_c} \left[(d-L \tan\theta) \sin m_c L + \frac{\tan\theta}{m_c}(1-\cos m_c L) \right] \tag{44}$$

$$\xi = \frac{2\cos \varphi_o}{m_s^2} \cdot P_{sB/2} \tag{45}$$

$$m_s = m \sin \varphi_o \tag{46}$$

$$P_{sB/2} = P_\nu \big|_{m=m_s}, \quad \nu = B/2 \tag{47}$$

and P_{cL}, Q_{cL}, R_{cL} and S_{cL} are expressed by eqs. (26),(28) and (29). Again the coupling effects of pitching are shown in eqs.(40),(41),(43) and (44).

In case of rolling, inertial torque can be neglected, because the relative acceleration between water and the ship is considered so small.

In sway, the inertial force F_{si} is calculated as,

$$F_{si} = -M'\ddot{\Delta} - K_s \cdot 2w_o adL \sin(\frac{m}{2}B) \cos nt \tag{48}$$

, in which M' is the water mass.

THE DIFFERENTIAL EQUATIONS OF MOTION

Substituting all terms of forces and torques thus derived into equations (1) and (2), the differential equations of motion of the pump dredger moored by the spuds are obtained. They are,

in pitching; $$\ddot{\theta} + 2\kappa_p \dot{\theta} + \omega_p^2 \theta = P \sin(nt + \varphi_p) \tag{49}$$

in yawing; $$\Delta\ddot{\varphi} + 2\kappa_y \Delta\dot{\varphi} + \omega_y^2 \Delta\varphi = Y_1 \sin(nt + \varphi_{y1}) + Y_2 \sin(nt + \varphi_{y2}) \tag{50}$$

in rolling; $\ddot{\Psi} + 2\kappa_r \dot{\Psi} + \omega_r^2 \Psi = R \sin nt$ (51)

in sway; $\ddot{\Delta} + 2\kappa_s \dot{\Delta} + \omega_s^2 \Delta = S_1 \sin(nt + \varphi_{s1}) + S_2 \sin(2nt + \varphi_{s2})$ (52)

Table 1 shows coefficients of four modes of movement. And table 2 gives the maximum resisting forces or torques calculated by eqs. (4)-(10) using expressions of the maximum applitude of the forced oscillation which are solutions of eqs. (49)-(52). They are composed of the system factors S which depend upon the dimensions of the dredger and spuds, the statical wave factors B which are resulted from the "static" configuration of waves and are expressed as functions of L/λ, of d/λ and of h/λ and the dynamic magnification factors A which are functions of n/ω and of κ/ω.

EXPERIMENTS

EXPERIMENTAL PROCEDURE

Experiments were conducted in a concrete wave tank, 22.3m long, 5m wide and 0.4m deep in the Department of Civil Engineering, Tohoku University. The depth of water was kept to be 0.318cm before the tests. A piston-type wave generator driven by a 3-HP motor was installed at one end of this tank and a wave absorber was set at the other end. A model ship of 1/100 scale was 67.1cm long, 15.9cm breadth, 3.15cm draught and 3000g displacement. An aluminum spud specially made for this experiment had a rectangular cross section, 3mm in thickness, 10mm in breadth and 270mm in length, which was embedded in concrete at the end. The strain-gauges were KP18 made by Kyowa Dengyo Co., which gauge length was 3mm, electric resistance of which was 120± 0.3 Ω, and gauge factor was 1.96.

As a preliminary test, strain was measured for this spud by loading some known weight which gave its Young's modulus E_s as 6.3×10^5 kg/cm^2. Moment of inertia of unit mass I_s was 0.002250cm^4 and effective length l_s was 21.7cm. Moment of the spud was calculated from strain ε by.

$$M_{spud} = \frac{1}{6}\sigma bh^2 = \frac{1}{6}bh^2 E\,\varepsilon = \frac{1}{6} \times 1.0 \times 0.3^2 \times 6.3 \times 10^5 x \times 10^{-6} = 0.00945x \text{ kg-cm}$$

, in which x was $\varepsilon \times 10$.

The natural periods of the ship oscillation and the damping coefficients were calculated from the logarithmic decrements and the apparent periods of the damping free oscillation of the model ship moored to the spud. Moments of inertia were calculated by equations in table-1 using the values of natural circular frequency thus obtained. Coefficients of the virtual mass were assumed as 1.0. Table 3 shows the results of the preliminary tests.

Wave directions to the longitudinal ship axis of ship were changed in five kinds as $\varphi = 0°$, 45°, 90°, 180° and 225°. Wave periods were 0.7, 0.8, 0.9 and 1.0 sec. Wave heights were six kinds as 0.5, 1.0, 1.5, 2.0, 2.5 and 3.0cm for each wave period.

Wave profiles were recorded using two resistance type transducers which were capable of surface calibration automatically during the tests, and were set 2m apart from each other on both sides of the model ship.

After starting wave generator, ten waves have been passed before recording. Then average of the recorded wave height and of the recorded strain of spuds were compared with theoretical values.

Table 1: COEFFICIENTS OF FOUR MODES OF MOVEMENT OF THE PUMP DREDGER MOORED BY THE SPUDS

	PITCHING	YAWING	ROLLING	SWAY
Coeff. of Damping	$\kappa_p = \frac{1}{2} \cdot \frac{k_p}{I_p + I_p'}$	$\kappa_y = \frac{1}{2} \cdot \frac{k_y}{I_y + I_y'}$	$\kappa_r = \frac{1}{2} \cdot \frac{k_r}{I_r}$	$\kappa_s = \frac{1}{2} \cdot \frac{k_s}{M+M'}$
Natural Circular Frequency	$\omega_p^2 = \left(\frac{1}{I_p + I_p'}\right) \times \left(W.\overline{GM} + n\frac{E_s I_s}{l_s} + w_o \frac{BL^3}{4}\right)$ n=1: single spud n=2: twin spuds	$\omega_y^2 = \left(\frac{1}{I_y + I_y'}\right) \cdot c$ $c = \frac{KG}{l_s}$: single spud $= \frac{3}{2} \cdot \frac{E_s I_s}{l_s^3} \cdot b^2$: twin spuds	$\omega_r^2 = \frac{1}{I_r}\left(W.\overline{GM}_r + \frac{E_s I_s}{l_s}\right)$: single spud	$\omega_s^2 = \frac{1}{M+M'} \cdot \left(24\frac{E_s I_s}{l_s^3}\right)$: twin spuds
External Force	$P = \frac{w_o a B}{I_p + I_p'} \cdot (1 - K_p m d H)\frac{E}{m^2}$	$Y_1 = \frac{\left[(A_m + K\rho A)^2 + (B_m + K\rho B)^2\right]^{1/2}}{I_y + I_y'}$ $Y_2 = \frac{(C_m^2 + D_m^2)^{1/2}}{I_y + I_y'}$	$R = \frac{W}{I_r} \cdot \overline{GM}_r \cdot am$	$S_1 = \frac{2(1+K_s)}{M+M'} \cdot \times w_o a d L \sin(\frac{m}{2}B)$ $S_2 = \frac{w_o a^2 L}{2(M+M')} \cdot \sin(mB)$
Phase Lag	$\frac{\varphi_p}{n} = \frac{1}{n} \tan^{-1} \frac{P_\nu}{Q_\nu}$	$\frac{\varphi_{y1}}{n} = \frac{1}{n} \tan^{-1} \frac{B_m + K\rho B}{A_m + K\rho A}$ $\frac{\varphi_{y2}}{n} = \frac{1}{n} \tan^{-1} \frac{D_m}{C_m}$	$\frac{\varphi_r}{n} = 0$	$\frac{\varphi_{s1}}{n} = \frac{\pi}{2} = \frac{T}{4}$ $\frac{\varphi_{s2}}{n} = 0$

Table 2 MAXIMUM RESISTING FORCES AND TORQUES ACTING ON THE SPUDS IN FORCED OSCILLATION

	PITCHING	YAWING		ROLLING	SWAY
		SINGLE SPUD	TWIN SPUDS		
Max Force or Torque	$\frac{Tpm}{a} = Sp\ Yp$ $Y_P = A_P B_P$	$\frac{Tym}{a} = Sy\ Y_T$ $Y_T = A_{y1}B_{y1} + A_{y2}B_{y2}$		$\frac{Trm}{a} = S_r Y_r$ $Y_r = A_r B_r$	$\frac{Fsm}{a} = S_s Y_s$ $\frac{Tsm}{a} = S_s' Y_s$ $Y_s = A_{s1}B_{s1} + \frac{a}{4d} A_{s2}B_{s2}$
System Factor	$S_p = \dfrac{\frac{E_s I_s}{l_s}}{\frac{d}{L}\cdot\overline{GM} + \frac{d}{WL}\frac{E_s I_s}{l_s} + \frac{L}{4}}$	$S_y = \frac{WL}{B}\cdot\frac{KG}{I_s}$	$S_y = \frac{WL}{B}\,\frac{3}{2}\,\frac{E_s I_s}{l_s^3}\,b^2$	$S_r = \dfrac{\frac{E_s I_s}{l_s}}{\frac{L}{W\,\overline{GM_r}}\frac{E_s I_s}{l_s} + L}$	$S_s = \frac{W}{B},\ S_s' = \frac{l_s}{2B}$
Statical Wave Factor	$B_p = \dfrac{(P_L^2 + Q_L^2)^{1/2}}{m^2L^2}(1 - K_p mdH)$	$B_{y1} = (F^2+G^2)^{1/2},\ B_{y2} = \frac{E_J}{L^2}\sin(m_s B)$ $F = 2\frac{E_{L1}}{L^2}\sin(\frac{m_s}{2}B)\cdot\cos\varphi_{L1}$ $+2K_y\Big[\frac{P_{cL}}{m_c^2L^2}\sin(\frac{m_s}{2}B) - \frac{\sin(m_c L)}{m_s^2L^2}$ $\times\{\sin\frac{m_s B}{2} - \frac{m_s B}{2}\cos\frac{m_s B}{2}\}\Big]$ $G = 2\frac{E_{L1}}{L^2}\sin\frac{m_s B}{2}\cdot\sin\varphi_{L1}$ $+2K_y\Big[-\frac{Q_{cL}}{m_c^2L^2}\sin(\frac{m_s}{2}B) + \frac{1-\cos m_c L}{m_s^2L^2}$ $\times\{\sin\frac{m_s B}{2} - \frac{m_s B}{2}\cos\frac{m_s B}{2}\}\Big]$		$B_r = mL$	$B_{s1} = (1+K_s)\sin\frac{mB}{2}$ $B_{s2} = \sin mB$
Magnification Factor	$A_p = f(\frac{n}{p}, \frac{P}{p})$	$A_{y1} = f(\frac{n}{\omega_{ys}}, \frac{\kappa_{ys}}{\omega_{ys}})$ $A_{y2} = f(\frac{2n}{\omega_{y2}}, \frac{\kappa_{ys}}{\omega_{y2}})$	$A_{y1} = f(\frac{n}{\omega_{yt}}, \frac{\kappa_{yt}}{\omega_{yt}})$ $A_{y2} = f(\frac{2n}{\omega_{yt}}, \frac{\kappa_{yt}}{\omega_{yt}})$	$A_r = f(\frac{n}{\omega_r}, \frac{\kappa r}{\omega_r})$	$A_{s1} = f(\frac{n}{\omega_s}, \frac{\kappa s}{\omega_s})$ $A_{s2} = f(\frac{2n}{\omega_s}, \frac{\kappa s}{\omega_s})$

$$, \quad f = 1/\{(1 - \frac{n^2}{\omega^2})^2 + \frac{4\kappa^2}{\omega^2}\frac{n^2}{\omega^2}\}^{1/2}$$

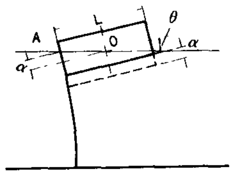

Fig.1 Pitching of the pump dredger around the point A fixed to the spud.

Table 3: RESULTS OF THE PRELIMINARY TESTS

	PITCHING	YAWING WITH A SPUD	YAWING WITH TWO SPUDS	ROLLING	SWAY
Coeff. of damping (sec^{-1})	$\kappa_p=0.934$	$\kappa_{ys}=0.0858$	$\kappa_{yt}=0.0858$	$\kappa_r=0.0467$	$\kappa_s=0.0850$
Natural circular frequency (rad./sec)	$\omega_p=10.28$	$\omega_{ys}=2.74$	ω_{yt} 0.471	$\omega_r=18.6$	$\omega_s=12.37$
Moment of inertia ($kg.cm.sec^2$)	$I_p=9.796$	$I_y=6.531$	$I_y=6.531$	$I_r=0.220$	--------

Table 4: VALUES OF $\omega^{-1}\sqrt{2\pi g/L}$ AND $2\kappa/\omega$ ADOPTED

	PITCHING	YAWING WITH A SPUD	YAWING WITH TWO SPUDS	ROLLING	SWAY
$\frac{1}{\omega}\sqrt{\frac{2\pi g}{L}}$	1.00	4.00	20.00	0.40	0.50
$2\kappa/\omega$	0.20	0.06	0.40	0.05	0.014

Table 5: RESONATED RELATIVE SHIP LENGTH IN YAWING WITH TWIN SPUDS

h/L	1/2	1/3	1/4	1/5	1/6
L/λ	0.028	0.035	0.040	0.045	0.049

EXPERIMENTAL RESULTS

Fig.2(a) shows the torque of the single spud in case of pitching by the 135 degree stern waves. Points are given by experimental results. Straight lines are drawn from the theory derived herein for cases of the wave periods of T=0.75sec and T=1.10sec. Data are scattered and do not give any distinction caused by the difference of the wave period. However calculated values seemed to be adequate.

Fig.2(b) shows the similar torque of pitching by the bow and the stern waves, in which theory seemed to give rather low values.

SOME CONSIDERATIONS

From table 2, maximum resisting forces and torques of the spuds in forced oscillation are concluded to be affected by many facters.

SYSTEM FACTORS

(1) Torques are small when the spud factor, E_sI_s/l_s or KG/l_s is small

(2) Shallow draught is unfavourable in pitching, slender ship is not recommended for yawing and sway. High $\overline{GM}$ is better for pitching but for rolling.

(3) Displacement increases forces or torques always.

STATICAL WAVE FACTORS

(1) Fig. 3 shows the relations in pitching. As the wave length increases, the statical wave factor decreases, and shallow ship of small d/L tends to increase this wave factor also. The influence of the relative depth h/L is not remarkable.

(2) Fig. 4 shows the relations in yawing. B/L is taken as constant as 0.200 and h/L is not included in functions of B_{y1} and B_{y2}. Factors of L/λ and the wave angle with the longitudinal ship axis φ_0 takes the major roll upon B_{y1} and B_{y2}. Values of B_{y1} increase as L/λ and φ_0 increase, but at $\varphi_0=30^o$, B_{y1} takes the maximum value. Values of B_{y2} also increase as L/λ in case of $\varphi_0 = 90^o$, however at small angle, they show wavy configurations.

(3) In rolling, wave factor is simply expressed as mL which is proportional to L/λ.

(4) In sway, also simple sinusoidal expressions hold to express wave factors. B_{s1} takes the maximum value of $1+K_s$ at $L/\lambda = 2.5$, but this analysis is limited in the region between 0 and 1.0 of the value of L/λ. Also B_{s2} is an increasing function of L/λ in this region.

MAGNIFICATION FACTORS

Features of the dynamic magnification factors were investigated in connection with the relative ship length L/λ.

The circular wave frequency n can be expressed as.

$$n = \left(\frac{2\pi g}{\lambda} \tanh \frac{2\pi h}{\lambda}\right)^{1/2} \tag{53}$$

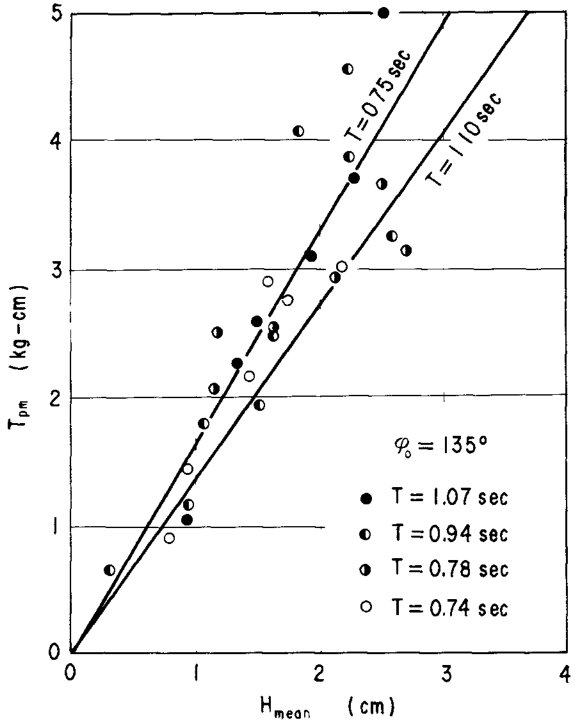

Fig 2 (a)

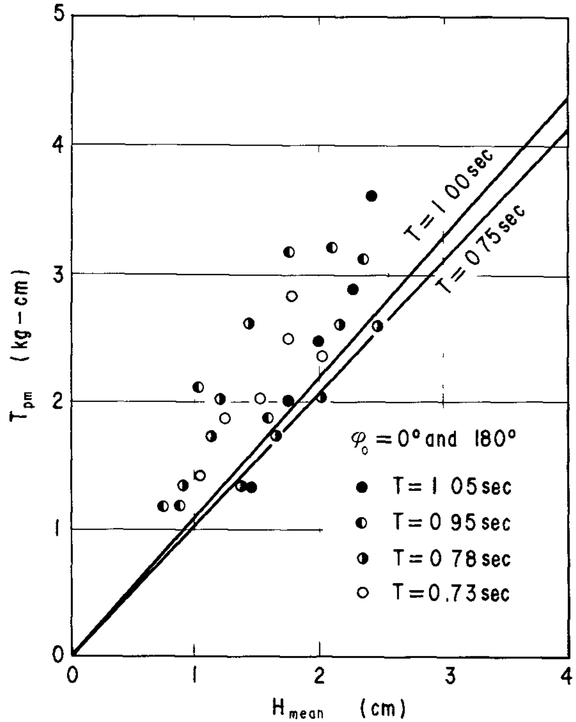

Fig 2 (b)

Fig.2 The torque of the single spud in case of pitching by the 135 degree stern waves(a) and the bow and the stern waves(b).

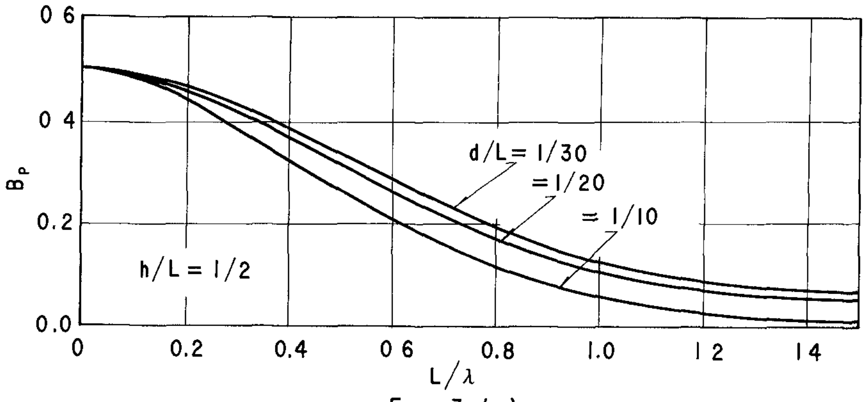

Fig 3 (a)

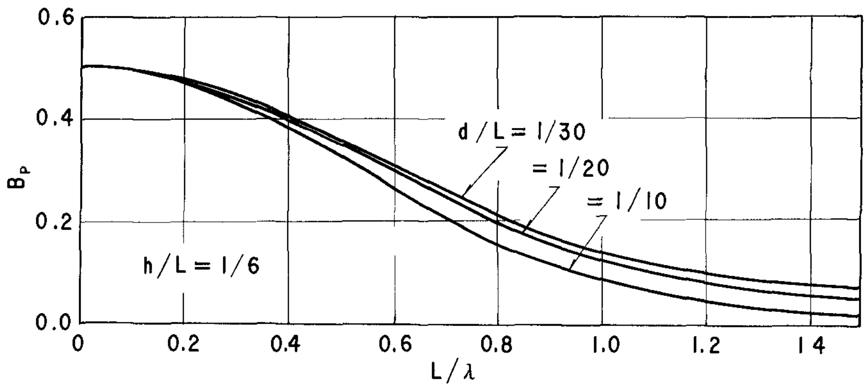

Fig 3 (b)

Fig.3 Relations between the statical wave factors in pitching B_p and the relative ship length to waves L/λ in cases of h/L are 1/2(a) and 1/6(b).

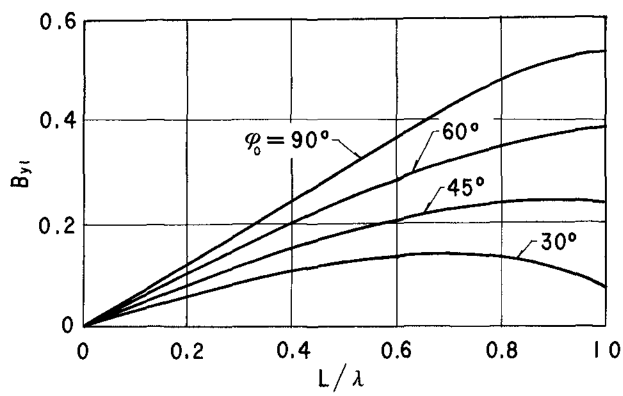

Fig.4 (a)

Fig.4 Relations between the statical wave factors B_{y1}(a) and B_{y2}(b) in yawing and L/λ.

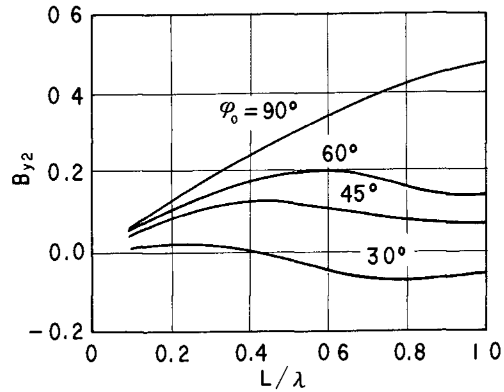

Fig 4 (b)

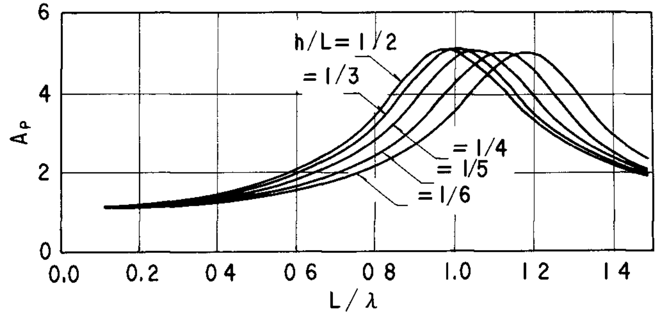

Fig.5 Relations between the magnification factors in pitching A_p and L/λ.

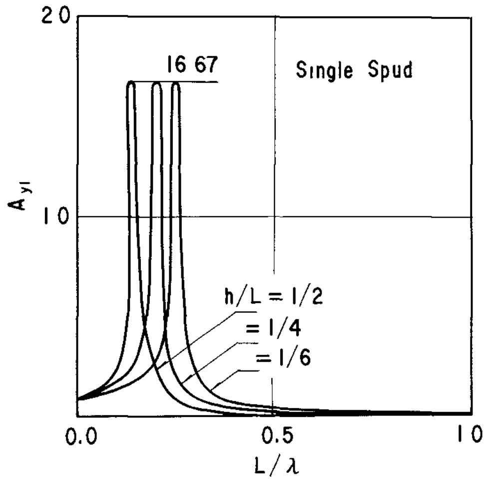

Fig.6. Relations between the magnification factors in yawing with single spud A_{y1} and L/λ.

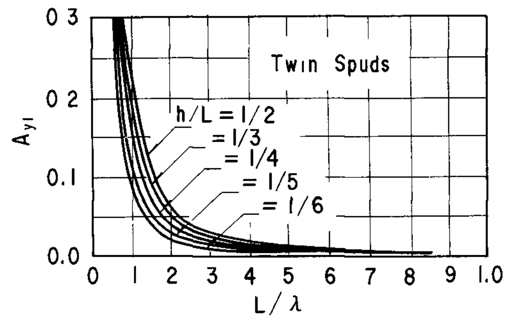

Fig.7. Relations between the magnification factors in yawing with twin spuds A_{y2} and L/λ.

for the surface waves with small amplitude. And the circular frequency of oscillation of a simple pendulum having length L is

$$n_o = 1/\,(L/g)^{1/2} \tag{54}$$

Then we have

$$\frac{n}{\omega} = \frac{n}{n_o} \cdot \frac{n_o}{\omega} = \frac{1}{\omega}\left(\frac{2\pi g}{L}\right)^{1/2} \cdot \left(\frac{L}{\lambda}\tanh mh\right)^{1/2} \tag{55}$$

as $m = 2\pi/\lambda$. When L = 67.1cm, $(2\pi g/L)^{1/2} = 9.58$ and using values of ω in table 3, $1/\omega(2\pi g/L)^{1/2}$ is calculated. In this analysis slightly changed values of them were used as shown in table 4. And then following results were obtained.

(1) Fig. 5 shows the magnification factor of pitching A_p related with L/λ. Resonance takes place when λ is nearly equal to the ship length. Maximum A_p is 0.5. As the relative depth h/L becomes smaller, the value of L/λ in resonance increases.

(2) Fig. 6 shows A_{y1} of yawing in case of single spud and fig. 7 shows that of twin spuds. Resonated relative ship length L/λ tends to be diminished as values of h/L decreases as shown in eq. (55). In case of twin spuds, L/λ of resonance is small. Then from eq. (53), following approximation holds,

$$\frac{n}{\omega} = \frac{20\sqrt{2\pi}}{\lambda}\sqrt{Lh} = 1$$

$$\frac{L}{\lambda} = 0.02/\sqrt{h/L} \tag{56}$$

Table 5 shows results of eq. (56).

(3) In rolling resonance takes place at values of L/λ larger than 1. Values of A_r for L/λ between 0 and 1 are almost same order between 1.000 and 1.156 although values of h/L are changed from 1/2 to 1/6.

(4) In sway, conditions are quite different. A_{s1} is not so different from A_r for L/λ between 0 and 1, but as L/λ approaches unity A_{s2} tends to be infinity because $2n/\omega_s$ tends to be unity and $2\kappa/\omega$ is very small.

SYNTHETIC WAVE FACTORS

Magnification factors include also wave effects. Synthetic wave effects are expressed by the products of statical wave factors and magnification factors which we define as synthetic wave factors. Thus,

$$Y_s = AB \tag{57}$$

or,

$$Y = A_1B_1 + kA_2B_2 \tag{58}$$

Values of this factor for pitching are shown from fig. 8 to 12 as ordinates Y_p. In fig. 8, the case of relative depth h/L 1/2 is shown. When the relative draught is 1/10, this factor diminishes monotonously as

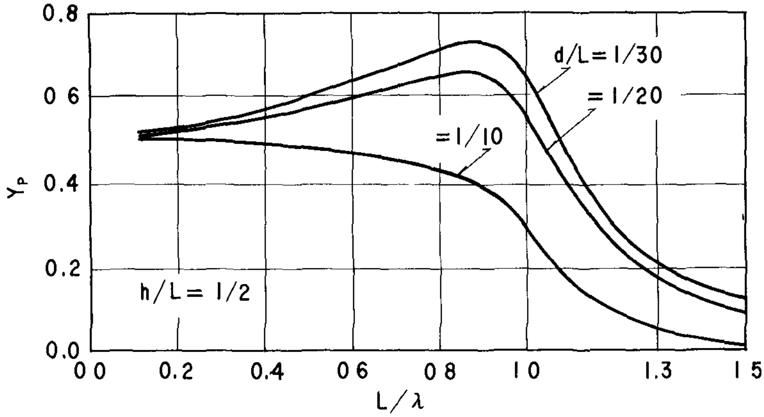

Fig.8. Relations between the synthetic wave factors in pitching Y_p and L/λ in case of h/L=1/2.

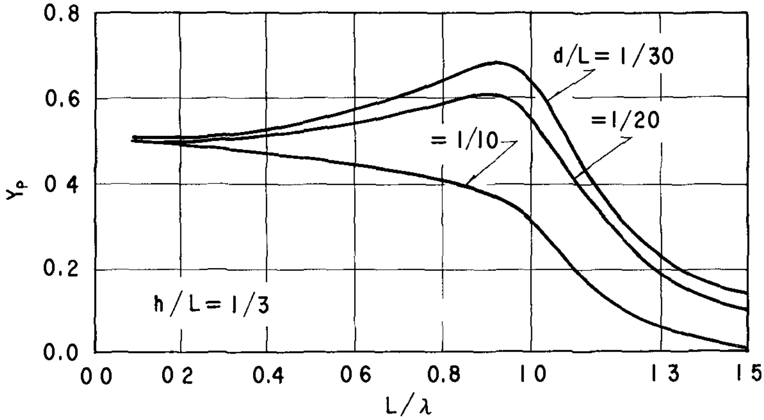

Fig.9 Relations between the synthetic wave factors in pitching Y_p and L/λ in case of h/L=1/3.

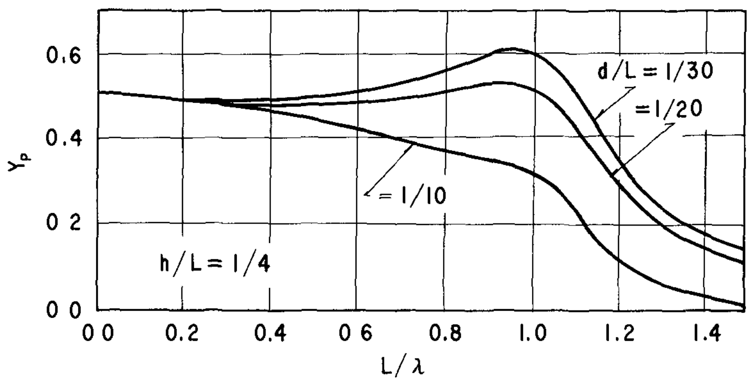

Fig.10 Relations between the synthetic wave factors in pitching Y_p and L/λ in case of h/L=1/4.

the relative ship length becomes longer. This shows more important effect of statical wave factors compared to magnification factors. However when the relative draft d/L is 1/20 or 1/30, this factor increases first, reaches maximum at about 0.9 of L/λ and then decreases rapidly as L/λ increases. This is because of the magnification factor.

When water depth becomes smaller, effects of the magnification factor becomes not so remarkable as shown in fig. 9, 10, 11 and 12. When h/L=1/6, the maximum value of Y_p is equal to or smaller than 0.5 of the value at L/λ =0.

It can be concluded that the magnification factors are remarkably modified by the statical wave factors.

Fig. 13 shows this factor Y_T for cases of yawing with single spud. When compared to fig. 6, it is clear that the magnification factor takes important roll. Taking larger the wave angle φ_0 or shallower the water depth, Y_T becomes larger. Fig. 14 shows that with twin spuds. In this case k=a/2d is taken as 0.2 for convenience. Values are remarkably small in compare to those for the case with single spud.

Fig. 15 gives the case of rolling. In this case Y_r is nearly proportional to L/λ because of the uniformity of A_r.

Fig. 16 gives the case of sway. In this case Y_s is rather small in the region of $L/\lambda < 0.9$. However as L/λ tends to be unity, resonance effects become to be remarkable.

CONCLUSIONS

Resisting forces and torques are analysed for the pump-dredger moored with spuds on sinusoidal waves with uniform wave length and wave period in shallow water. The treatment for the irregular waves in the actual sea conditions is not included here and is hoped to be studied in the future. (Leenderste 1964)

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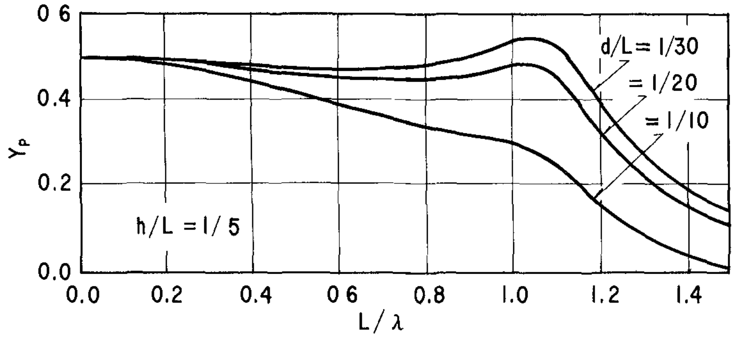

Fig.11 Relations between the synthetic wave factors in pitching Y_p and L/λ in case of h/L=1/5.

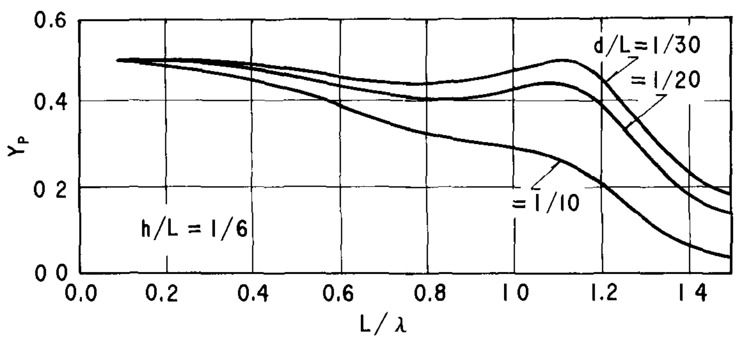

Fig.12 Relations between the synthetic wave factors in pitching Y_p and L/λ in case of h/L=1/6.

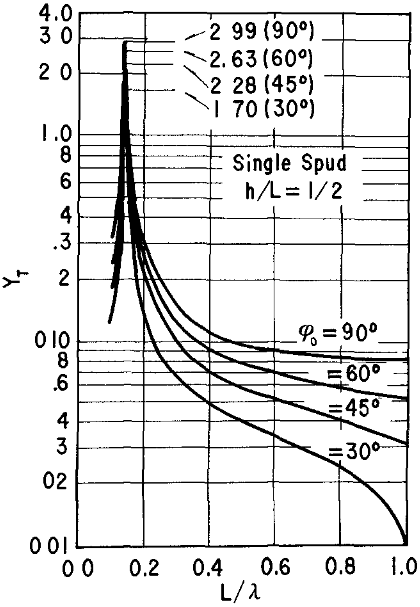

Fig.13. Relations between the synthetic wave factors in yawing with single spud Y_T and L/λ .

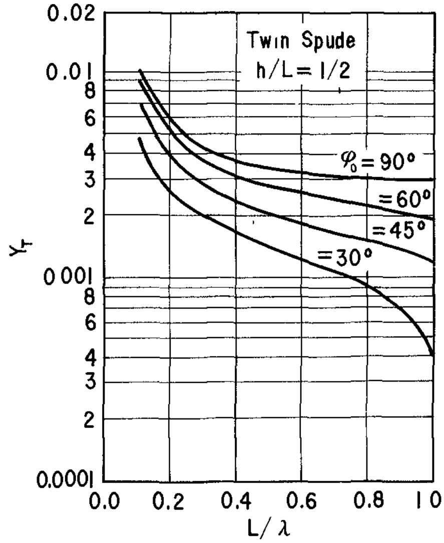

Fig.14. Relations between the synthetic wave factors in yawing with twin spuds Y_T and L/λ .

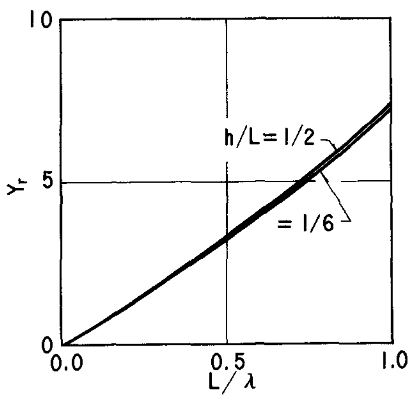

Fig.15. Relations between the synthetic wave factors in rolling Y_r and L/λ .

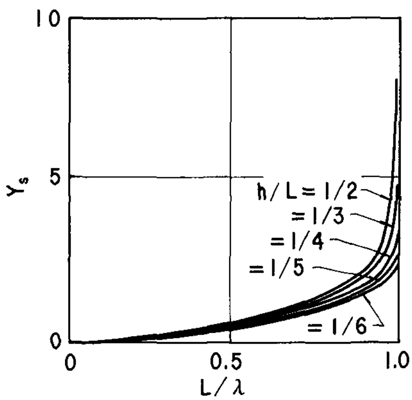

Fig.16. Relations between the synthetic wave factors in sway Y_s and L/λ .