CHAPTER 117

SUSPENDED LOAD CALCULATIONS IN A TIDAL ESTUARY

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ABSTRACT

The present work describes the derivation of equations which can represent the vertical distribution of sediment in a well mixed tidal estuary The starting point for the analysis is the two dimensional longitudinal equation of motion, including the longitudinal salinity gradient term Equations are given which represent a steady state sediment profile and it is shown that these are similar to the expression used in uni-directional flow e g Rouse's equation

It is argued that the derived equations could be applied to a real estuary, subject to certain restrictions on sediment size and estuary type

An application of the theoretical equations to the Mersey Estuary indicates reasonable agreement between observed and predicted sediment quantities for medium and coarse sand particles Agreement is shown to be worst for fine grained sediments and it is concluded that much better results can be obtained by using the non-steady one dimensional sediment distribution equation in discrete steps throughout the tidal cycle

INTRODUCTION

The calculation of the distribution of sediment in a real estuary is an exceedingly difficult task All the problems of uni-directional flow are present together with the added complications of unsteady flow, varying bed resistance and salt/freshwater density currents

The present paper endeavours to show how the distribution of certain sediment may be estimated in a particular type of well mixed estuary It is not intended to be the panacea for tidal sediment problems, but represents a comparatively simple engineering solution to certain tidal sediment distributions

METHOD

In uni-directional flow problems, the distribution of sediment is specified by assuming that a steady state is reached between sediment eroded from the river bed and sediment depositing on the river bed Clearly, if no sediment is present in the water and erosion is started then a finite time is required for a steady state to be reached The time will be dependent upon particle size, water depth, turbulence level and initial sediment distribution and will thus be expected to be small for large sized particles, whose equilibrium profile is close to the river bed and large for fine sediment, whose equilibrium is more uniformly distributed throughout the water depth A first look at the sediment distribution in a tidal environment would thus be to assume that the sediment particles reached the steady state distribution in a short time, during which the tidal flow characteristics were unaltered i e the tide is considered to be a succession of uni-directional flows The sediment distribution for the large sized particles would thus be given by the steady state profile appropriate for each elemental uni-directional flow

The effects of the salt/freshwater density profile and the inertia of the flow system must also be incorporated into the problem This can be done by assuming the tidal flow is frozen at each instant of time and then using the resultant shear stress distribution to predict the distribution of sediment

ANALYSIS

The flow of water in a tidal estuary is governed in two-dimensions by the equation[1]

$$A + F + 2D(1-\eta) - \frac{1}{\rho g H}\frac{\partial \tau}{\partial \eta} = I \qquad (1)$$

where $A = \frac{1}{g}\frac{\partial U}{\partial t}$, U = horizontal water velocity in the horizontal direction (x), t = time, g = acceleration due to gravity, $F = \frac{\partial}{\partial x}\left(\frac{U^2}{2g}\right)$, $D = \frac{H}{2\rho}\ \frac{\partial \rho}{\partial x}$, ρ = water density, $\eta = \frac{y}{H}$, y = elevation above the bed, H = water depth, τ = horizontal shear stress, I = water surface slope (positive in the ebb direction)

Equation (1) also assumes that the term $\frac{U^2}{gH}$ is small and the vertical velocity term is small

The shear stress distribution with depth is obtained from equation (1) by integration w r t η, after first specifying the distributions of A and F with depth It has been shown[1] that simple linear distributers for A and F are not vastly different than logarithmic or parabolic distributions Thus the variation of A and F are taken as

$$A = A_b + (A_s - A_b) \qquad (2)$$

$$F = F_b + (F_s - F_b) \qquad (3)$$

where b and s refer to bed and surface values respectively Equation (2) and (3) uses known or estimated values of A and F at small distances above the estuary bed and below the estuary surface

Inserting equations (2) and (3) in equation (1) and integrating w r t η gives the expression for the shear stress distribution with depth as

$$\frac{\tau}{\rho g H} = I^{'}\ (1-\eta) - B(1-\eta^2) - D(1-\eta)^2 \qquad (4)$$

$$\frac{\tau}{\rho g H} = S^{'} - 2P^{'}\eta - D^{'}\ \eta^2 \qquad (5)$$

where $I^{'} = I - A_b - F_b$, $B = \frac{1}{2}(\Delta A + \Delta F)$, $\Delta A = A_s - A_b$, $\Delta F = F_s - F_b$, $S^{'} = I^{'} - B - D$, $P^{'} = \frac{1}{2}(S^{'} - D^{'})$, $D^{'} = D - B$

The solution of equation (4) requires the knowledge of six prototype quantities at each instant of time However, equation (5) is a function of only the two quantities $S^{'}$ and $D^{'}$ These quantities can be determined by considering the velocity distribution with depth, as follows

The shear stress (τ) is first related to the velocity gradient by Prandtl's mixing length concept i e

$$\tau = \rho \ell^2 \left| \frac{du}{dy} \right| \frac{du}{dy} \qquad (6)$$

where ℓ = a mixing length, $||$ = absolute value

A suitable mixing length distribution to use in equation (6) is that found by Agnew[2] in the Haringvliet (Holland) i e

$$\ell = Ky \left[\frac{1-\eta}{1-\delta} \right]^2 \quad \text{for the region } 1>\eta>\delta \qquad (7)$$

and

$$\ell = Ky \quad \text{for the region } \delta>\eta>0 \qquad (8)$$

where K = Von Karmans Constant, $\delta = \frac{h}{H}$, h = height of position of zero nett motion of the residual salinity circulation

Equation (5), (6), (7), and (8) thus lead to the following expressions for the variation of velocity (U) with depth (η)

$$\frac{K(U-U_g)}{\sqrt{gH}} = \sqrt{1-\delta} \int_{\delta}^{\eta} \frac{([S' + D'\eta])^{\frac{1}{2}}}{\eta} d\eta \quad \text{for } 1>\eta>\delta \qquad (9)$$

$$\frac{KU}{\sqrt{gH}} = \int_{\eta_o}^{\eta} \frac{([S' - 2P'\eta - D'\eta^2])}{\eta} d\eta \quad \text{for } \delta>\eta>\eta_o \qquad (10)$$

where $\eta_o = y_{o/H}$, y_o = height at which U = 0

The solutions to equations (9) and (10) depend on the magnitude and sign of the terms S' and D' If the sign convention, ebb slopes are positive is used[1], then S' is a maximum (+ve) about mid-ebb and mid-flood (-ve)[1] while D' is negative near high water, and greater than S', and positive at low water and greater than S' The latter is to be expected as the flow reverses at the bed first at low water due to the density circulation. Values of S' and D' can be found by fitting the prototype velocity observations to equations (9) and (10) viz table 3

The sediment distribution may now be found from the general sediment diffusion equation i e in two dimensions

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(\varepsilon_x \frac{\partial C}{\partial x} - UC\right) + \frac{\partial}{\partial y}\left(\varepsilon_y \frac{\partial C}{\partial y} + \omega C\right) \quad (11)$$

where ε_x, ε_y are diffusion equations in the x, y directions ω = particle fall velocity

If attention is confined to a particular type of estuary in which the variation of U with distance (x) is small and the sediment is equally distributed along the estuary then $\partial C/\partial x = 0$ and equations (11) will reduce to

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial y}\left(\varepsilon_y \frac{\partial C}{\partial y} + \omega C\right) \quad (12)$$

An analytical solution of equation (12) is unknown for $\varepsilon_y = f(y,t)$ Finite Difference methods can be used and are being studied at present However, if only large particles are considered then a steady state profile will be quickly reached and a mathematical expression for the distribution of the sediment can be obtained from equation (12) with $\partial C/\partial t = 0$ i e

$$\ln\left(\frac{C}{C_a}\right) = -\omega\int_a^y \frac{dy}{\varepsilon_y} \quad (13)$$

where C_a = a reference concentration at level "a" above the bed The distribution curve specified by equation (13), could also be used to describe the distribution of medium to fine sand particles by replacing C_a with an observed prototype concentration value (C_t) Clearly, $C_t < C_a$ and the concentration given by the modified equation (13) will be greater than in prototype, the error being greatest for the finest sized sediments

The steady state solution is obtained from equation (13) by integration on specifying the variation of ε_y with depth This is done by relating ε_y to the momentum transfer coefficient ε_m i e

$$\varepsilon_y = \beta\,\varepsilon_m \quad (14)$$

and ϵ_m is obtained from the equation

$$\tau = \rho \epsilon_m \frac{\partial u}{\partial y} \quad (15)$$

$$\text{i e} \quad \epsilon_m = \ell^2 \left| \frac{\partial u}{\partial y} \right| \quad (16)$$

$$\text{Thus} \quad \epsilon_y = \beta \ell \sqrt{\frac{\tau}{\rho}} \quad (17)$$

$$\text{i e} \quad \epsilon_y = \beta K y \left[gH \left| S' - 2P' \eta - D' \eta^2 \right| \right]^{\frac{1}{2}} \quad \text{for } 0<\eta<\delta \quad (18)$$

$$\epsilon_y = \beta K y \frac{(1-\eta)}{(1-\delta)^{\frac{1}{2}}} \left[gH \left| S' + D' \eta \right| \right]^{\frac{1}{2}} \quad \text{for } \delta<\eta<1 \quad (19)$$

The variation of ϵ_y with depth as given by equations (18) and (19) is shown in Figure 1 for mean tide level ($S' >> D'$) and near low water ($S' = D'$ and $S' = 0$) conditions, together with field data (average tidal cycle values) taken by Bowden[3] and Sharaf[4] in the Narrows Area of the Mersey Estuary, England

Bowden's data is seen from Figure 1 to indicate smaller values in the upper half of the vertical profile and indeed it was found that the velocity equations (9) and (10) gave better answers over the low water period if the following expressions were used for ϵ_y i e

$$\epsilon_y = 2\beta \; Kh(1-\eta)^{3/2} \left[gH \left| S' + D' \eta \right| \right]^{\frac{1}{2}} \quad (20)$$

$$\epsilon_y = 2\sqrt{2}\beta \; Kh(1-\eta)^2 \left[gH \left| S' + D' \eta \right| \right]^{\frac{1}{2}} \quad (21)$$

for the region $\delta<\eta<1$

Equations (20) and (21) are also shown in Figure 1 and give a better fit to Bowden's data

The solution of equation (13) is thus dependent upon the sign and magnitude of S' and D', as were equations (9) and (10) The various solutions to equation (13) have been presented elsewhere[5] The solutions for the case $|S'|>|D'|>0$, which apply for the majority of the tidal cycle are given below i e

$$\frac{C}{C_a} = \left[\frac{(1/\eta - \alpha_o) + \left[(1/\eta - \alpha_o)^2 - (1-\alpha_o)^2\right]^{\frac{1}{2}}}{(1/\eta_a - \alpha_o) + \left[(1/\eta_a - \alpha_o)^2 - (1-\alpha_o)^2\right]^{\frac{1}{2}}}\right]^{Z} \text{ for } \eta_a<\eta<\delta \quad \ldots \quad (22)$$

where $Z = \frac{\omega}{\beta K U_*}$ and $U_*^2 = \tau_b/\rho$ where $\tau_b = \rho g H S'$

$$\frac{C}{C_\delta} = \left[\frac{(1+\sqrt{1+\eta\,(1-2\alpha_o)})\times(1-\sqrt{1+\delta(1-2\alpha_o)})}{(1-\sqrt{1+\eta\,(1-2\alpha_o)})(1+\sqrt{1+\delta(1-2\alpha_o)})}\right]^{Z(1-\delta)^{\frac{1}{2}}} \times$$

$$\left[\frac{(\sqrt{2(1-\alpha_o)} - \sqrt{1+\eta(1-2\alpha_o)})}{(\sqrt{2(1-\alpha_o)} + \sqrt{1+\eta(1-2\alpha_o)})} \times \frac{(\sqrt{2(1-\alpha_o)} + \sqrt{1+\delta(1-2\alpha_o)})}{(\sqrt{2(1-\alpha_o)} - \sqrt{1+\delta(1-2\alpha_o)})}\right]^{\frac{Z(1-\delta)^{\frac{1}{2}}}{\sqrt{2(1-\alpha_o)}}}$$

$$\text{for } \delta<\eta<1 \quad (23)$$

where $\alpha_o = \frac{P'}{S'} = \frac{1}{2}(1-D'/S')$, C_δ = concentration at height δ

Equation (23) is seen to be indeterminate for the case $\alpha_o = \frac{1}{2}$ i e $D' = 0$ If this condition is substituted in equation (18), (19) and (13), the expression for the variation of concentration with depth is

$$\frac{C}{C_a} = \left[\frac{(1+\sqrt{1-\eta})}{(1-\sqrt{1-\eta})} \times \frac{(1-\sqrt{1-\eta_a})}{(1+\sqrt{1-\eta_a})}\right]^{Z} \quad \text{for } \eta_a<\eta<\delta \quad (24)$$

and

$$\frac{C}{C_\delta} = \left[\frac{1-\eta}{\eta} \times \frac{\delta}{1-\delta}\right]^{Z(1-\delta)^{\frac{1}{2}}} \quad \text{for } \delta<\eta<1 \quad (25)$$

Equation (24) is the same as that derived by Tanaka and Sugimoto[6] for uni-directional flow i e $D = 0$, $A = F = 0$ and $S' = I$ Thus equations (22) and (23) are also similar to the expressions for uni-directional flow, except that the density circulation and inertia and kinetic terms are included It should be noted that equation (22) reduces to Rouse's equation [7] for uni-directional flow for $\alpha_o = 1\ 0$ However, equations (21-23) predict a greater quantity of sediment in suspension than found by Rouse's equation This is shown in Fig 2 It will be noticed that both Rouse's equation and that of Tanaka and Sugimoto give values almost identical to that of equation (22) below α_0 = 0 50 However, the different mixing length distribution contained in equation (19) leads to smaller sediment values in the upper half of the water flow at α_0 = 1 0 and 0 50 respectively

Application of theory to field data

The theory was applied to the Mersey Estuary in order to predict the quantity of sediment in suspension at position H (Figure 3) The latter satisfies the requirements, on the flood tide, of complete sediment cover and little velocity variation with distance Prototype sediment data was also available at position H in sufficient detail to be usable The sediment data available is shown in Figure 4

In order to apply the theory, values of C_a or C_t, α_0, S' or U_* ω, β and K must be available or capable of being calculated from the field data The determination of these quantities is now considered in detail below

(1) Determination of U_* values

This could be calculated from equation (5), with η= 0, provided sufficient velocity and tide gauge data was available A lack of data prohibits the use of this method An alternative method is to use semi-log plots of the horizontal water velocity and equate the slope of the graph to U_*/K, where K = 0 40 This is likely to over-estimate U_* on the ebb tide and under-estimate it on the flood tide A further method is to take $U_* = K\bar{U}^1$ where $\bar{U}$ is the depth mean velocity and K has the value 5 2 x 10^{-2} The method used eventually was to make semi-log plots of velocity and adjust the answer if necessary by the third method The results are shown in Table 2

Table 2 Calculated U_* values from Prototype Velocity Data - 25th Nov 1965

Mersey Estuary Position H

Time (GMT)	0801	0845	0924	1003	1044	1115
U_* (fps)	0 099	0 288	0 344	0 342	0 261	0 209

(2) Choice of β

Flume tests by Vanoni[8], Laursen[9] and River observations by Anderson[10] indicate values of β between 0 63 - 4 10 Einstein[11] uses a value of $\beta = 1\ 0$ i e the momentum (ϵ_m) and sediment (ϵ_s) transfer coefficients are equal In a tidal estuary, it is to be expected that $\beta < 1\ 0$ due to the salinity circulation It is also probable that the value of U_* to be used in the suspension exponent (Z) will also be reduced (to U_*') due to the presence of sand waves on the estuary bed The product $\beta U_*'$ will thus be expected to be less than U_*

In view of the lack of theory from which to estimate U_*', the quantity ($\beta U_*'/U_*$) was determined from two simultaneous sand samples taken at a station in the Irish Sea The samples were taken at maximum velocities from mid-depth and approximately 1ft above the bottom The value of ($\beta U_*'/U_*$) for sand sizes between 63μ - 355μ was found to be 0 62 This implies a value of $\beta = 0\ 62$ for no reduction in U_* In the present work a value of ($\beta U_*'/U_*$) of 0 658 has been used

It is interesting to note that Tofaletti's work[15] indicates a value of U_*' of 0 21 fps, which implies a β value of 1 065 for a U_* value of 0 34 Englund's work[16], however, would suggest that in deep water (68ft) the reduction in shear due to sand waves is small (<2%), this implies a value of β closer to 0 658

The value of $Z = \omega/\beta\ KU_*'$ is thus known at all points during the tidal cycle, ω was taken from standard tables[12] while a value of $K = 0\ 40$ was used since the concentration values are relatively low

(3) Determination of α_0 values

The circulation of α_0 requires a knowledge of both S' and D' These could be determined if all the terms in equation (5) are known There are, however, no adequate tide gauge readings to the North of position H α_0 values are thus computed for position C (Figure 4) and used for the calculation at position H Velocity observations from positions H, C and N were used, together with the tide gauge information from Princes Pier and Gladstone Lock[1] to determine approximate values of S' and D' The velocity expressions (9) and (10) were then used to give a better estimate of S' and D' The values found at C are shown in Table 3 in terms of the density term (D) $D = 3\ 86 \times 10^{-6}$

Table 3

S' and D' values calculated for Position C Mersey Estuary for a Spring Tide (28ft H W above Liverpool Datum)

TIME Rel to H W Princes Pier	0	$-\frac{1}{2}$	-1	-2	$-2\frac{1}{2}$	$-3\frac{1}{2}$	-4	-5	-5 3
S'	-3 39	-7 58	-13 51	-18 88	-17 86	-10 0	-5 0	-0 885	-0 22
D'	-2 4	-1 78	1 4	0 5	0 38	6 15	5 27	1 77	1 73
α_o	0 146	0 383	0 552	0 513	0 511	0 808	1 027	1 50	4 43

The α_o values for position H were found by super-imposing the time scale for the prototype observations (25th November 1965) on the values shown in Table 3 and simplifying the results - viz table 5

(4) Determination of sediment concentrations

The control concentration (C_t) was determined from the prototype data Sediment samples were taken over successive half hourly periods from a fixed distance of 18 inches above the estuary bed using a pump sampler The half hourly sampling trial was necessary in order to obtain sufficient sand for analysis

The samples were washed, dried and weighed in the laboratory and the concentration determined A grain size analysis was performed for each sample using a sedimentation tube[13], where sufficient sediment existed, and sieves where the sample size was small The results are shown in Table 4

Table 4

Mean concentration (ppm) of sand at 18" above the estuary bed

at Position H Mersey Estuary 25th November 1965

Sample	Mean Sample Time (GMT)	Mean Grain Size (μ)							Water Depth (ft)
		76	106	138	165	195	227	298	
-	0810	0	0	0	0	0	0	0	
A	0825	1 76	3 29	2 17	1 13	1 03	0 04	0 05	60
B	0856	6 20	16 3	9 4	8 36	5 4	4 08	0 71	65
D	0925	9 9	24 6	30 1	33 8	20 6	15 1	2 46	68
E	0955	3 98	16 4	21 5	17 3	36 3	18 8	1 88	72
G	1022	7 87	12 57	20 19	26 04	40 39	17 15	0 38	73
I	1050	2 55	9 5	9 5	10 25	14 3	4 44	0 26	75
J	1120	2 2	7 7	9 3	10 4	13 9	14 5	0	76
K	1148	1 26	2 06	2 4	1 75	0 89	0 34	0	77
-	1200	0	0	0	0	0	0	0	

The usefulness of the preceeding theory is checked by computing the quantity of sand that should have been caught in a suspended Delft Bottle This was kept at approximately 10ft below the water surface during the period 0830-0930 (Sample C) and 0934 - 1030 (Sample F) HRS GMT

In order to use the theoretical distribution curves, the C_t values should be instantaneous values A correction procedure was thus adopted to allow for the finite sampling time

The method consisted of first calculating the quantity of sediment that would be collected over each sampling period, assuming that the variation of C_t with time was proportional to $(U^2_*/U^2_{*c}{}^{-1})$, where U_{*c} is the value of U_* at threshold conditions The calculated quantity was then compared with the quantity that would be collected if conditions operating at the mean sampling time had prevailed over the full sampling period Clearly, the ratio of these two terms should be unity for small time intervals or for little variation of C_t with time Only those samples collected near maximum velocities showed ratios of about unity, all others indicated greater values

The correction procedure thus made the above ratio equal to unity by finding an equivalent mean time, which was a function of grain size, during the sampling interval The observed concentrations were then acredited to this equivalent mean time The concentration at the actual sample mean time was then obtained by interpolation between the equivalent mean times of all the samples

The position of the Oelft Bottle varied between 5-25% of the depth during the two sampling periods (C and F) The concentration at 25% and 5% was determined using the appropriate formula (equations 22-25) The correction procedure was then used and the concentration at the Oelft Bottle level found by interpolation The corrected concentration values are shown in Table 5

The quantity of sediment collected in the Oelft Bottle was then determined by summing up the product of the theoretical concentration values and the observed water velocity at the Oelft Bottle level The latter quantity was found by mounting an Ott Mark V Arkansas current mater on the Oelft Bottle framework

Table 5

Estimated Theoretical Concentration at Oelft Bottle Level

Time (GMT)	Mean Grain Size (μ)							% Level	
HRS	76	106	138	165	195	227	298		
0825	0 293	0 27	0 017	0	0	0	0	25	
α_o=0 70	0	0	0	0	0	0	0	5	
	0 173	0 117	0 01	0	0	0	0	16 7*	
0856	2 05	3 2	0 64	0 20	0 025	0 015	0	25	
α_o=0 60	1 24	1 45	0 20	0 036	0 004	0	0	5	
	1 66	2 36	0 43	0 1215	0 015	0 0078	0	15 4*	
0925	3 96	6 22	3 5	1 9	0 589	0 216	0	25	*Delft Bottle Level
α_o=0 50	2 89	3 93	1 69	0 712	0 175	0 051	0	5	
	3 29	4 79	2 37	1 157	0 33	0 113	0	12 5*	
0955	1 61	4 21	2 56	0 99	1 04	0 28	0	25	
α_o=0 50	1 14	2 67	1 25	0 38	0 33	0 07	0	5	
	1 184	2 81	1 37	0 436	0 396	0 0894	0	6 95*	
1022	2 79	2 617	1 716	0 97	0 684	0 133	0	25	
α_o=0 50	1 985	1 559	0 7664	0 3264	0 1757	0 027	0	5	
	2 06	1 657	0 8544	0 3859	0 2221	0 037	0	6 85*	
1050	0 70	1 344	0 447	0 1755	0 092	0 0121	0	25	
α_o=0 50	0 46	0 7024	0 1635	0 0464	0 0175	0	0	5	
	0 526	0 9044	0 2525	0 087	0 0411	0 0038	0	11 3*	

The results of the calculation for sampling periods C and F are shown in Table 6 This also includes an estimated quantity which is the measured prototype values, corrected[14] for sediment loss due to high water velocities within the Delft Bottle

Table 6

Comparison of Theoretical and Observed Sediment Quantities

Sample	Grain size (μ)	75	106	138	165	195	227	298
	Theory (gms)	4 06	5 89	2 11	0 925	0 261	0 094	0
C	Measured (gms)	1 61	3 74	2 18	2 18	1 30	0 40	0 09
	Estimated (gms)	2 48	5 20	2 84	2 70	1 56	0 465	0 095
	Theory (gms)	4 03	6 0	2 96	1 15	0 72	0 164	0
F	Measured (gms)	0 72	1 26	0 86	0 63	0 40	0 116	0 012
	Estimated (gms)	2 04	2 56	1 44	0 97	0 59	0 16	0 015

Comments

It would appear, from Table 6, that the theory predicts the sediment quantities reasonably well for grain sizes in excess of 138μ, but less well for sizes below this value It is particularly noticeable that the 75μ sand is predicted to be approximately double the observed value in both cases The lack of agreement of the theory for small sized sediment is undoubtedly due to the longer time required for the fine sediments to reach an equilibrium profile as compared with the larger sized sediments, e g the majority of the steady state profile for 195μ particles is much closer to the estuary than that of the 75μ particles

A better method of prediction, particularly for the fine sand size would be to solve equation (12) and use this solution in discrete time steps to determine the concentrations throughout the tidal cycle Preliminary work on this method indicates that the difference between the non-steady state distribution and the steady state distribution is the order of 10% for 165μ particles at a level of 10% above the estuary bed This increases to approximately 35% at the same level for 75μ particles If the method is applied to the 75μ particles and adjusted for the observed sediment concentrations, then values of 1 9 gms and 3 4 gms are obtained for samples C and F respectively These qualities are much closer to the observed values than the steady state solution

A further improvement, which is being developed at present, is to allow for variations in sedir . concentration with distance and for diffusion in the longitudinal direction

All three methods mentioned above are still dependent upon the values of β and U_* which were used in the present theory to determine the suspension exponent Z Clearly, the shape of the distribution curve assumes less significance in view of the uncertainty of β, U_* and to a lesser extent ω and K The present results do, however, suggest that in the Mersey Estuary reasonable quantitative results can be obtained by using standard values of ω and K, together with U_* values calculated from the velocity data[1] and a β value of 0 66

Conclusions

The present work leads to expressions for the vertical distribution of sediment which are similar to those of uni-directional flow The derived equations indicate the vertical distribution of sediment in a real estuary subject to restrictions on grain size and estuary type Preliminary work also indicates that the prediction of fine sediment concentrations is better represented by using the non-steady state equation in discrete steps throughout the tidal cycle

NOTATION

a	a distance above the estuary bed
A	inertia term
B	$= \frac{1}{2} (\Delta A + \Delta F)$, a composite slope term
C	concentration
D	density slope
D'	effective density slope = D - B
F	kinetic energy term
H	water depth
h	height of zero nett motion
I	water surface slope (positive when sloping downwards from land to sea)
I'	the slope term $I - A_b - F_b$
K	Von Karmans Constant (=0 40)
ℓ	a mixing length
P'	the slope term $\frac{1}{2}(S' - D')$
S'	an effective energy slope $= I' - B - D$
t	time
u	velocity in the x direction
u_*	$(\tau_b/\rho)^{\frac{1}{2}}$ = shear velocity
x	distance measured along the estuary, seawards
y	distance vertically above the bed
Z	suspension exponent $= \omega/\beta\ KU_*$
α_o	ratio P'/S'
β	Ratio of eddy diffusivities for sediment and momentum $(\varepsilon s/\varepsilon m)$
δ	$\doteq h/H$
ΔA	$= A_s - A_b$
ΔF	$= F_s - F_b$
ε	an eddy diffusivity coefficient
η	$= y/H$

ρ = density

τ = shear stress

ω = sediment fall velocity

Subscripts

a referring to level a and $t = \infty$

b bed and near bed

S near surface

t referring to time t

y level y

o level at which $U = 0$

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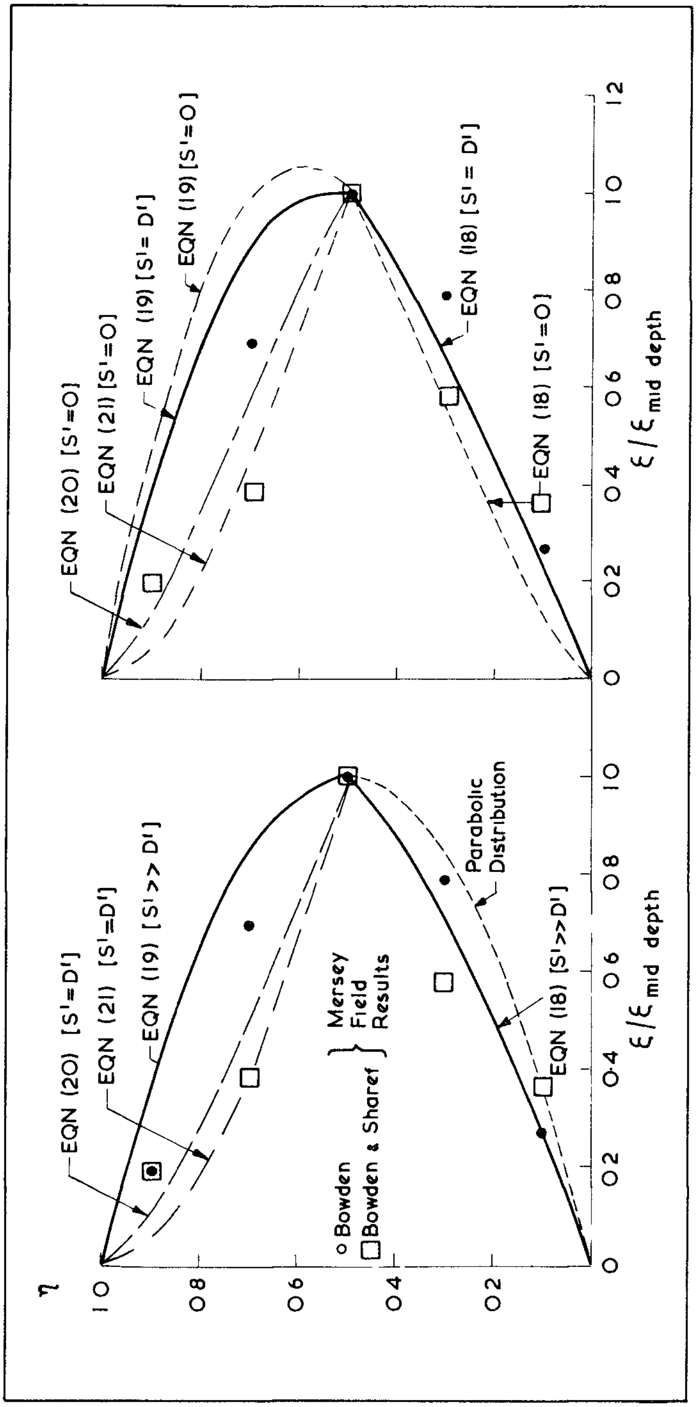

FIG 1 COMPARISON OF WITH FIELD DATA

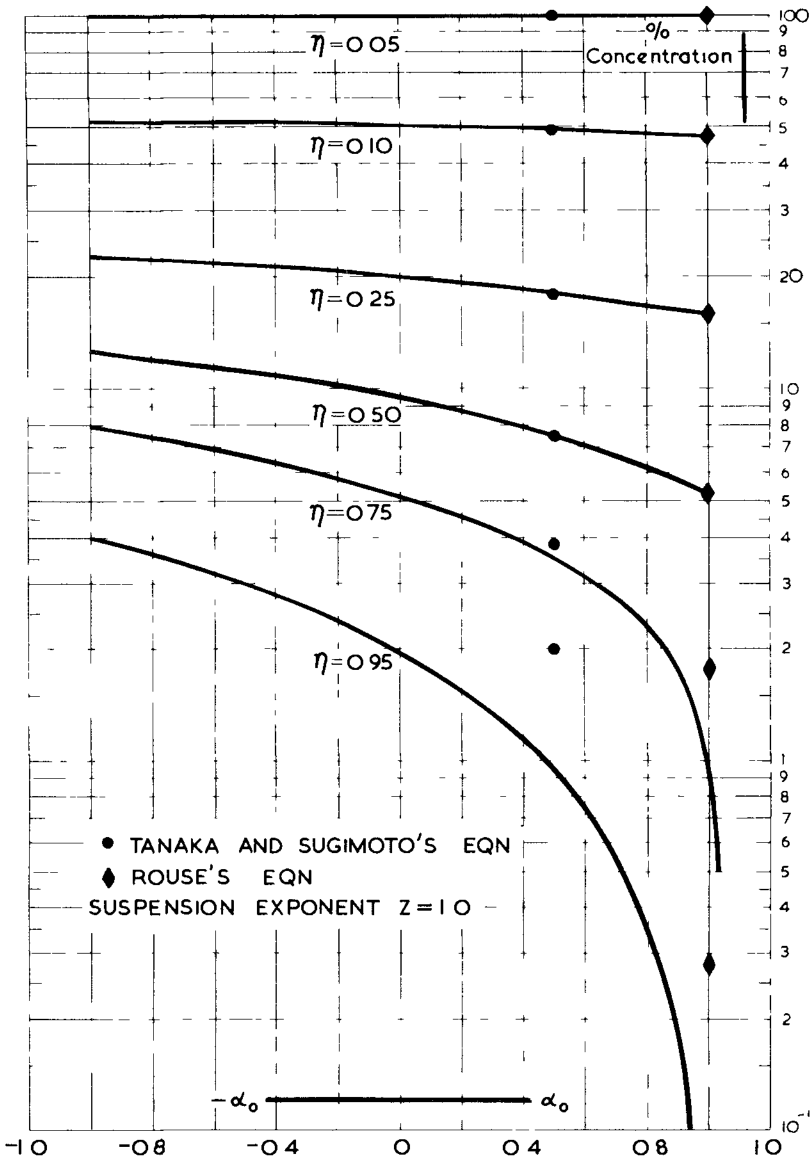

FIG 2 VARIATION OF CONCENTRATION WITH THE PARAMETER

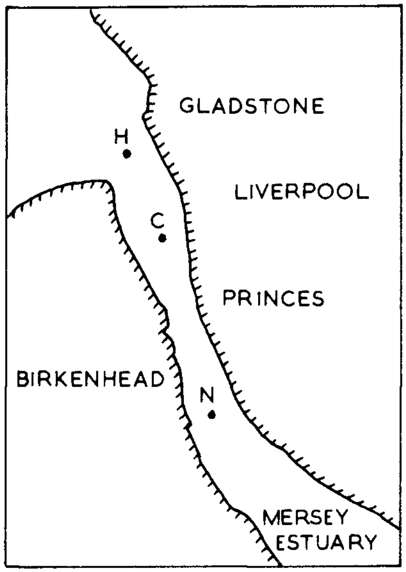

FIG 3 SELECTED OBSERVATION STATIONS - MERSEY ESTUARY

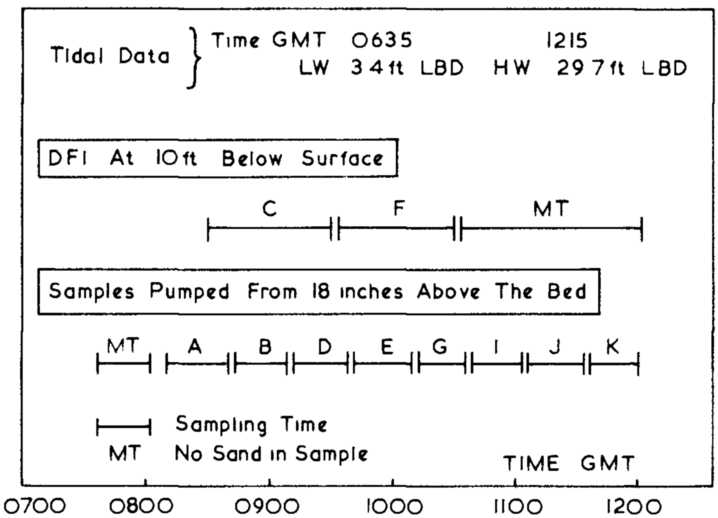

FIG 4 SEDIMENT SAMPLES AT POSITION H - MERSEY ESTUARY (25.11.65)