CHAPTER 134

TIDAL MOTION IN BAYS

by

O H Sheddin, Associate Professor
R M Forney, Research Assistant

Department of Coastal and Oceanographic Engineering
University of Florida, Gainesville, Florida

ABSTRACT

A method is proposed to investigate periodic tidal motion in single or multiple basins connected to the ocean by an inlet. Non-sinusoidal tidal motion in the ocean and square friction law in the inlet are both considered. The method is applied to Boca Raton inlet, Florida. The calculated tidal elevation and velocity in the inlet are found to be in reasonable agreement with measured values. The bottom shear friction coefficient is defined

\[ T_{to} = \rho r \, u |u| \]

where \( \rho \) is the water density, \( T_{to} \) is the shear stress, and \( u \) is the average velocity in the inlet. The results of the study yield \( r = 0.0039 \), and predict net transport of sediment into the bay.

INTRODUCTION

The dynamics of tidal motion in a bay connected to the ocean by an inlet was investigated by Brown (1928) who considered only sinusoidal tidal motion in the ocean and linear bottom friction in the inlet. Later, Keulegan (1951) treated the same problem but included the square friction law in the inlet and predicted a non-sinusoidal oscillation in the bay elevation. Van de Kreeke (1968) developed a scheme which predicted tidal oscillations in bays in the presence of freshwater inflow by rainfall or streams. The above investigations were all limited to a sinusoidal tidal oscillation in the ocean and all neglected flow acceleration in the inlet.

An extensive treatise on tides and tidal propagation is given by Dronkers (1964). It is known that tidal motion is not simple but has many harmonic constituents. The principal components are shown in Table I.
Table 1 Principal Tidal Components

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Component</th>
<th>Periods (hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M2</td>
<td>Principal lunar semidiurnal</td>
<td>12 421</td>
</tr>
<tr>
<td>S2</td>
<td>Principal solar semidiurnal</td>
<td>12 000</td>
</tr>
<tr>
<td>N2</td>
<td>Lunar elliptic semidiurnal</td>
<td>12 668</td>
</tr>
<tr>
<td>K1</td>
<td>Lunisolar declinational diurnal</td>
<td>23 936</td>
</tr>
<tr>
<td>O1</td>
<td>Lunar declinational diurnal</td>
<td>25 819</td>
</tr>
</tbody>
</table>

The relative importance of the different constituents depends on geographical location and depth of water. The principal lunar semidiurnal component (M-2) has usually the highest amplitude.

This study proposes a general method for analyzing tidal oscillation in bays connected to the ocean by inlets. It includes acceleration of flow in the inlet, square friction law, and a non-sinusoidal tidal oscillation in the ocean. The method utilizes the general equations of motion and approximates the periodic tidal oscillation by a series in circular functions. Use is made of complex variables to simplify computations. The method can be applied to multiple basins connected to each other and to the ocean. The present study is restricted to relatively deep bays in which negligible spatial variation in water elevation exist. The inlet area is assumed constant and equal to the mean area during one tidal cycle. The method can be applied, however, to shallow bays. The method has a resemblance to Dronker's (1964) harmonic method of tidal propagation although he did not specifically apply the method to inlets. A method proposed by Sidjabat (1970) for describing the nonlinear friction is employed in this study.

THEORETICAL APPROACH

The theoretical development to follow describes the dynamics of flow in an inlet with a constant cross sectional area connected on one side to a basin with uniform water level and on the other side to an ocean which has harmonic but non-sinusoidal tidal oscillation. The flow in the inlet is assumed one-dimensional. Resonance and bottom friction in the bay are neglected. Quantitatively bays satisfying these conditions must be at least 20 feet deep when the longest dimension equals 5 miles.

A Single Bay Coupled to Ocean

A definition sketch is shown in Figure 1. The equation of motion in the x-direction (direction of inlet flow) is used to describe the inlet flow.

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial^{2} v}{\partial z},
\]

where \( u \) is the inlet velocity, \( p \) is the pressure, \( \rho \) is the water density, and \( \frac{\partial^{2} v}{\partial z} \) is the viscous shear stress. Denoting
TIDAL MOTION

\[ u(x,y,z,t) = u_m(x,z,t) + u'(x,y,z,t) \]
\[ w(x,y,z,t) = w'(x,y,z,t) \]

where \( u_m \) denotes the mean flow over a length of time, \( T \), which is small compared to tidal period by long compared to time scale turbulence. The overbar is taken to denote averaging over period \( T \).

\[ \bar{u} = \frac{1}{T} \int_0^T u dt = u_m \]

Equation (1) becomes

\[ \frac{\partial u_m}{\partial t} + \frac{\partial}{\partial x} (u_m^2 + \bar{u}^2) = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial z} (\tau_v + \tau_t) \] (2)

where \( \tau_v \) is the turbulent Reynolds stress defined by

\[ \tau_t = -\rho \bar{u}' \bar{w}' = -\rho \int_0^T u' w' dt \]

Next, a vertical average over the depth \( h \) is specified. We define

\[ U(x,t) = \frac{1}{h} \int_0^h u_m(x,z,t) dz \] (3)

Integrating Equation (2) over depth and assuming hydrostatic pressure distribution

\[ \frac{dU}{dt} + \alpha \frac{\partial}{\partial x} (U^2) = -g \frac{\partial}{\partial x} \left( \frac{1}{\rho h} (\tau_v + \tau_t) \right) \] (4)

where

\[ \alpha U^2 = \frac{1}{h} \int_0^h (U_m^2 + \bar{u}'^2) dz \]

and the subscript \( o \) denotes bottom values \( (z = 0) \). From open channel considerations \( \alpha \) is the momentum factor and is of the order of 1.05. The value of 1.0 will be assumed sufficiently accurate. In inlets, the flow is mostly turbulent except when it approaches zero during a tidal reversal. The bottom shear stress will be assumed to be primarily due to turbulence rather than viscosity \( (i.e. \, \tau_v << \tau_t) \) and to have the empirical square-law form.
\[ \tau_{to} = \rho r U|U| \]  
(5)

where \( r \) is the friction coefficient Equation (4) now becomes

\[ \frac{3U}{\partial t} + \frac{3}{\partial x} U^2 = -g \frac{3h}{\partial x} - \frac{r}{h} U|U| \]  
(6)

Equation (6) is applied to the inlet shown in Figure 1. Integration of Equation (6) along the x-direction yields an equation which relates the ocean level to that in the basin. Assuming the inlet is finite in length and the velocity accelerates towards the inlet but achieves a constant velocity, \( U_{ls} \), in the inlet, Equation (6) is integrated to yield

\[ h_s = h_{b1} + \frac{A_{b1} L_1}{g} \left( 1 + \frac{r}{h_1 L_1} \right) \frac{U_s U_{ls}}{g}, \]  
(7)

where \( h_s, h_{b1}, L_1 \) and \( h_1 \) are defined in Figure 1. The equation of mass conservation consistent with the above assumptions becomes

\[ U_1 A_1 = A_{b1} \frac{dh_{b1}}{dt}, \]  
(8)

where \( A_1 \) and \( A_{b1} \) are as defined in Figure 1. Equation (8) further assumes that the changes in \( A_{b1} \) and \( A_1 \) due to depth changes are small over a tidal cycle. Eliminating \( U_1 \) in Equation (7) yields

\[ h_s = h_{b1} + \left( \frac{A_{b1}}{A_1} \right) \frac{L_1}{g} \frac{dh_{b1}}{dt} + \left( 1 + \frac{r}{h_1 L_1} \right) \frac{1}{g} \left( \frac{A_{b1}}{A_1} \right)^2 \frac{dh_{b1}}{dt} \left| \frac{dh_{b1}}{dt} \right| \]  
(9)

The Linearized Solution for Simple Motion

The tidal motion in the ocean is described by complex variables to simplify computations

\[ h_s(t) = \hat{h}_s e^{i \sigma t}, \]  
(10)

where \( \hat{h}_s \) is a complex number and \( \sigma \) is the tidal frequency. Let

\[ h_{b1} = -1 \hat{h}_{b1} e^{i \sigma t}, \]
where $h_b$ is a real number representing the bay tidal amplitude. Linearization of the last term in Equation (9) implies

$$\frac{dh_b}{dt} = \frac{dh_b}{dt} = \frac{8}{3\pi} a^2 h_b e^{\sigma t} + \ldots,$$

which yields the solution

$$h_s = h_b^{\prime \prime} \left[ e^{\sigma t} - e^{-\sigma t} \right], \quad (11)$$

where

$$\sigma = \frac{8}{3\pi} \left( 1 + \frac{r_1 L_1}{h_b} \right) \frac{1}{g} \left( \frac{A_b}{A_1} \right)^2 a^2,$$

and

$$n = \left[ 1 - \frac{A_b}{A_1} \frac{L_1}{g} a^2 \right].$$

The term $(A_b/L_1) a^2$ represents the flow acceleration and has been traditionally neglected. It need not be always negligible compared to $1/0$, it was found to be equal to 0.25 for Macquarie Harbor which was studied by Van de Kreeke (1968). The amplitude and phase shift of the ocean fluctuation are given respectively,

$$|\hat{h}_s| = h_b^{\prime \prime} \sqrt{(\hat{h}_b^{\prime \prime})^2 + n^2}, \quad (12)$$

and

$$\tan \theta = -\frac{n}{\hat{h}_b^{\prime \prime}}. \quad (13)$$

O'Brien (1969) proposed an empirical relationship between the tidal prism, $P$, and inlet area $A_1$

$$A_1 = CP, \quad (14)$$

where $C$ is a constant. The present analysis yields an analytical representation for $C$ which can be used to test the empirical constant of O'Brien

$$C = \frac{\left( \frac{8}{3\pi} \left( 1 + \frac{r_1 L_1}{h_b} \right) \frac{1}{g} \left( \frac{A_b}{A_1} \right)^2 a^2 \right) \tan \theta}{2 \left[ \frac{A_b}{A_1} \frac{L_1}{g} a^2 - 1 \right]} \quad (15)$$
2 Solution for Non-sinusoidal Motion

The tidal motion in the ocean is not simple as indicated previously so that sinusoidal simplifications introduce errors. Since the bay motion is affected by nonlinear friction, motion is not simple even when the ocean motion is. The latter case was investigated in detail by Keulegan (1951) when the tidal motion in the ocean is periodic but not simple, the bay motion is even more complex since each ocean harmonic constituent generates its own harmonic as well as multiple harmonics due to nonlinear friction. Thus a linearized procedure becomes invalid, in fact it is unrealistic and suggests negative friction at the higher harmonics.

In the following a method is proposed to describe the harmonic motion in both bay and ocean. The complex representation is employed

\[ h_\text{s} = \sum_n \frac{1}{2} (r_n - is_n) e^{i\omega t} \]

and

\[ h_{\text{b}} = \sum_n \frac{1}{2} (c_n - id_n) e^{i\omega t} \]

where the summation ranges over both positive and negative values of the index \( n \) which specifies the principal tidal frequencies, linear combinations thereof, and higher harmonics. The terms corresponding to \( n = 0 \) reflect mean levels in ocean and bay. Since \( h_\text{s} \) and \( h_{\text{b}} \) are real functions, the following relationships must be satisfied (Lee, 1967):

\[ r_n = r_{-n}, \quad s_n = -s_{-n} \]
\[ c_n = c_{-n}, \quad d_n = -d_{-n} \]

(18)

The average velocity is specified by

\[ U(t) = \sum_n \frac{1}{2} (a_n - ib_n) e^{i\omega t} \]

where

\[ a_n = a_{-n}, \quad b_n = -b_{-n} \]

The representation of \( T_o \) in series form was considered by Sidjabat (1970) in conjunction with tidal propagation in wide shallow bays. His description of the nonlinear term is conveniently adopted. Equation (15) may be expressed

\[ T_{to} = \rho r \left[ \sum_{m} \sum_{n} \frac{1}{4} (a_m + ib_m) (a_n + ib_n) e^{i(\omega + \beta)m} \right] \frac{1}{2} \sum_n (a_n - ib_n) e^{i\omega t} \]
which is equivalent to

\[ r_{to} = \rho r \left[ \sum_{j} \frac{1}{2} (a_{j}^{2} + b_{j}^{2}) + \sum_{j \neq m} \frac{1}{4} (a_{j} - ib_{j})(a_{m} - ib_{m}) e^{i(j+m)t} \right] \]

\[ = \sum_{n} (a_{n} - ib_{n}) e^{in\omega t} \quad (20) \]

Denote

\[ \lambda^{2} = \sum_{j=-\infty}^{\infty} \frac{1}{4} (a_{j}^{2} + b_{j}^{2}) = \frac{1}{2} \sum_{j=0}^{\infty} (a_{j}^{2} + b_{j}^{2}) \quad (21) \]

Then

\[ T_{to} = \rho r \lambda (1+c)^{1/2} \sum_{n} (a_{n} - ib_{n}) e^{in\omega t} \quad (22) \]

where

\[ \epsilon = \frac{1}{\lambda^{2}} \sum_{j} \sum_{m \neq n} \frac{1}{4} (a_{j} - ib_{j})(a_{m} - ib_{m}) e^{i(j+m)t} \]

It was found by Sidjabat (1970) that in areas where M2 is the dominant tide \( \lambda \) is determined primarily by the M-2 component and \( \epsilon \) does not exceed the value 0.25. Equation (22) is approximated by

\[ T_{to} = \rho r \lambda \sum_{n} \frac{1}{2} (a_{n} - ib_{n}) e^{in\omega t} \quad (23) \]

The error in the friction term is less than 12.5% and corresponds to smaller errors in computing tidal elevation. In studies where such an error is significant, it is possible to calculate \( \epsilon \) and to include its effect in Equation (22). The above method will be applied to Boca Raton inlet, the terms comprising \( \epsilon \) will be neglected subject to comparison with measured tidal elevations.

The nonlinear term in Equation (9) becomes according to the above analysis (see Equation (17))

\[ \frac{|dh_{|}}{dt} \frac{|dh_{d|}}{dt} = \lambda_{1} \sum_{n} \frac{1}{2} (\nu_{c} c_{n} + \nu_{d} d_{n}) e^{in\omega t} \quad (24) \]

where
Substituting for $h_s$, $h_{b_1}$, and $|\frac{dh_{b_1}}{dt}|$ (\frac{dh_{b_1}}{dt}) and equating the real and imaginary coefficients, the following algebraic equations for the coefficients of different harmonic constituents are deduced

$$r_n = [1 - n^2 \sigma^2 (\frac{A_{b_1}}{A_1}) \frac{L_1}{g}] c_n + \frac{\lambda_1}{g} (1 + \frac{r}{h_1} L_1) (\frac{A_{b_1}}{A_1})^2 n \sigma d_n$$

$$s_n = [1 - n^2 \sigma^2 (\frac{A_{b_1}}{A_1}) \frac{L_1}{g}] d_n - \frac{\lambda_1}{g} (1 + \frac{r}{h_1} L_1) (\frac{A_{b_1}}{A_1})^2 n \sigma c_n$$

Equations (26) relate the bay motion to the ocean tidal oscillation. The non-linear friction is specified by the term $\lambda_1$, which also couples the different harmonic constituents of the ocean elevation.

The procedure for solution depends on the available information. If the tidal elevations in both ocean and bay are measured, it is possible to evaluate the friction coefficient $r$. When only the motion in the bay is known, it is possible to predict the tidal motion in the ocean for any given $r$. However, when the tidal motion in the ocean is given, the motion in the bay can only be computed by an iterative procedure for any given $r$.

### B. Two Bays Coupled to Ocean

A definition sketch for two bays coupled together and to the ocean is shown in Figure 2. The second bay is not connected to the ocean. The tidal elevations in the ocean and first bay are specified by Equations (16) and (17), respectively. The tidal motion in the second bay is specified by

$$h_{b_2} = \sum_n \frac{1}{2} \left( \rho_n - i q_n \right) e^{i \omega t}$$

Subject to all assumptions stated previously, the equation which relates the motions in the two bays becomes

$$h_{b_1} = h_{b_2} + \left( \frac{A_{b_2}}{A_2} \right) \frac{L_2}{g} \frac{d^2 h_{b_2}}{dt^2} + (1 + \frac{r}{h_2} L_1) \frac{1}{g} \left( \frac{A_{b_2}}{A_2} \right) \frac{dh_{b_2}}{dt} \left| \frac{dh_{b_2}}{dt} \right|$$

where the equation of continuity in the second inlet

$$U_2 A_2 = \frac{dh_{b_2}}{dt}$$

was used. The resulting algebraic coefficient equations relating the two bays become
TIDAL MOTION

\[ c_n = \left[1 - n^2 \sigma^2 \frac{A_{b_2}}{A_2} \frac{L_2}{g}\right] p_n + \frac{\lambda_2}{g} \left(1 + \frac{r}{h_2} L_2\right) \frac{A_{b_2}}{A_2}^2 \sigma_n, \]

\[ d_n = \left[1 - n^2 \sigma^2 \frac{A_{b_2}}{A_2} \frac{L_2}{g}\right] q_n - \frac{\lambda_2}{g} \left(1 + \frac{r}{h_2} L_2\right) \frac{A_{b_2}}{A_2}^2 \sigma_p, \]

where

\[ \lambda_2 = \frac{1}{\sqrt{2}} \left[ \sum_{k=0}^{\infty} \sigma^2 k^2 \left(p_n^2 + q_n^2\right) \right]^{\frac{1}{2}} \] (31)

The motion of the first bay may now be related to the motions of both ocean and second bay. Equation (7) remains valid for the first inlet, but the continuity equation now takes the form

\[ U_1 A_1 = A_{b_1} \frac{dh_{b_1}}{dt} + U_2 A_2 \]

Using Equation (29)

\[ U_1 = \frac{A_{b_1}}{A_1} \frac{dh_{b_1}}{dt} + \frac{A_{b_2}}{A_1} \frac{dh_{b_2}}{dt}, \]

and substituting for \( U_1 \) in Equation (7) the equation which relates the first bay to the ocean is obtained

\[ h_s = h_b + \frac{L_1}{g} \frac{A_{b_1}}{A_1} \frac{d^2 h_{b_1}}{dt^2} + \frac{L_1}{g} \frac{A_{b_2}}{A_2} \frac{d^2 h_{b_2}}{dt^2} \]

\[ + \left(1 + \frac{r}{h_1} L_1\right) \frac{1}{g} \left(\frac{A_{b_1}}{A_1} \frac{dh_{b_1}}{dt} + \frac{A_{b_2}}{A_1} \frac{dh_{b_2}}{dt}\right) \left(\frac{A_{b_1}}{A_1} \frac{dh_{b_1}}{dt} + \frac{A_{b_2}}{A_1} \frac{dh_{b_2}}{dt}\right) \] (33)

Using the representation for \( h_s, h_{b_1}, \) and \( h_{b_2}, \) given by Equation (16), (17) and (27), respectively, the following algebraic coefficient equations are deduced

\[ r_n = c_n \left[1 - n^2 \sigma^2 \frac{L_1}{g} \left(\frac{A_{b_1}}{A_1}\right)\right] p_n \left[n^2 \sigma^2 \frac{L_1}{g} \left(\frac{A_{b_2}}{A_1}\right)\right] \]

\[ + \frac{\lambda_2}{g} \left(1 + \frac{r}{h_1} L_1\right) \left[\left(\frac{A_{b_1}}{A_1}\right) \sigma_n + \left(\frac{A_{b_2}}{A_1}\right) \sigma_p\right] \]

\[ s_n = d_n \left[1 - n^2 \sigma^2 \frac{L_1}{g} \left(\frac{A_{b_1}}{A_1}\right)\right] q_n \left[n^2 \sigma^2 \frac{L_1}{g} \left(\frac{A_{b_2}}{A_1}\right)\right] \]

\[ - \frac{\lambda_2}{g} \left(1 + \frac{r}{h_1} L_1\right) \left[\left(\frac{A_{b_1}}{A_1}\right) \sigma_n + \left(\frac{A_{b_2}}{A_1}\right) \sigma_p\right] \]

where
The procedure for solution again depends on the available information. The friction factor can be determined when the motion in the ocean and both bays are known. The elevations in the first bay and ocean can be obtained deterministically for any given \( r \) when the motion in the second bay is known. The motion in the bays can be determined by an interactive scheme for any given \( r \) when the motion in the ocean is known.

**Application - Boca Raton Inlet**

The theoretical method outlined above was applied to Boca Raton Inlet, Florida. The inlet connects the Atlantic Ocean to Boca Raton Lake which is also connected to Lake Wyman. A plan view of these lakes is shown in Figure 3. The intercoastal waterway which connects Lake Wyman and Boca Raton Lake extends to South Lake Worth Inlet north and Hillsboro Inlet south. Comparisons of tide records obtained at Boca Raton Lake and at a station on the Intercoastal Waterway south of Boca Raton indicated no possible flow to or from the Intercoastal Waterway south of Boca Raton Lake. Boca Raton Inlet primarily influences Boca Raton Lake, Lake Wyman, and areas occupied by boating marinas north of Lake Wyman.

Boca Raton Inlet was chosen for this study because tide records were available at the Inlet North Jetty (denoted by Station 1 in Figure 3), at Boca Raton Lake (denoted by Station 2), and at Lake Wyman (denoted by Station 3). Actually tide records at three stations around Boca Raton Lake were also available. These records indicated no special variation of tide elevation in Boca Raton Lake and verifies the representation of conservation of mass given by Equation (8). The tide elevations recorded at Stations 1, 2 and 3 are shown in Figure 4. They indicate the magnitude and phase shift of tidal motion in the two bays relative to the ocean. Other available data included velocity measurements over a tidal cycle at Station 1 to be shown later in comparison with computed velocities.

Since tidal records were available in both bays and at the inlet, the multiple bay analysis was used to determine the friction factor \( r \) for the system. The tidal record in Lake Wyman was used to predict the tidal elevation in Boca Raton Lake using Equation (30). The computed elevation was then compared with the measured one. With the tidal elevation in Boca Raton Lake known, the ocean elevation in the ocean (Station 1) was computed using Equations (34) and the results compared with the measured values. The variances between the computed and measured elevations were computed at Stations 1 and 2 for different friction coefficients. The quantities which describe the bay system (see Figure 2) \( A_1, L_1, A_b, h_1, A_s, L_2, h_2 \) were all known. The area \( A_b \) was not known. The computed motion which gave the best fit to measured tidal elevations at Stations 1 and 2 corresponded to \( r = 0.0039 \) and \( A_b = 23 \times 10^6 \) ft$^2$. The latter area is much larger than Lake Wyman's area and suggests that the tidal motion extends to areas occupied by marinas north of Lake Wyman. Table 2 summarizes the physical properties of the above two-bay system.

\[
\lambda_3 = \frac{1}{\sqrt{2}} \left[ \sum_{k=0}^{n} \left( \frac{A^b_k}{A^b_1} \right) \cos k \tau m + \left( \frac{A^b_1}{A^b_1} \right) \cos k \tau n \right]^2 + \left( \frac{A^b_1}{A^b_1} \right) \cos k \tau n \right]^2 \right]^{1/2} \tag{35}
\]
TABLE 2 Properties of the Two-Bay System

<table>
<thead>
<tr>
<th></th>
<th>Boca Raton Lake</th>
<th>Lake Uyman</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet area (ft²)</td>
<td>1 40 x 10³</td>
<td>1 68 x 10³</td>
</tr>
<tr>
<td>Bay Area (ft²)</td>
<td>3 74 x 10⁶</td>
<td>23 00 x 10⁶</td>
</tr>
<tr>
<td>Inlet depth (ft)</td>
<td>7 0</td>
<td>8 0</td>
</tr>
<tr>
<td>Inlet length (ft)</td>
<td>2 67 x 10³</td>
<td>7 33 x 10³</td>
</tr>
<tr>
<td>M-2 measured amplitude (ft)</td>
<td>1 57</td>
<td>1 40</td>
</tr>
</tbody>
</table>

The variances between computed and measured tidal elevations at Boca Raton Lake (station 2) and at the Ocean (station 1) were 1 9 x 10⁻² and 2 0 x 10⁻², respectively for r = 0 0039. The difference in computed and measured phase shifts of the M-2 component were 2 degrees for Boca Raton Lake and 4 degrees for the ocean.

SIGNIFICANCE OF NON-SINUSOIDAL TIDAL COMPUTATION

A comparison between the measured tide record at station 1 and the M-2 constituent is shown in Figure 5. The agreement is reasonable only over part of the tide cycle. A comparison between the measured tide elevation at station 1 and the computed elevation using six harmonics is shown in Figure 6. Better agreement is recognized over most of the tidal cycle. The importance of considering the different harmonics in tidal computations appears more dramatically in velocity computations, however. In Figure 7, the measured velocity at station 1 is compared to the computed velocity using the M-2 constituent only. Near the maximum three velocity peaks appear in the measured record but are absent in the sinusoidal computation. These peaks appear in computations which include different harmonics as shown in Figure 8. Since the contribution of each harmonic to velocity depends on frequency and amplitude of that harmonic the higher harmonic contributes significantly to velocity even when the amplitude is small.

The velocity cycle has an important influence on sediment transport in the inlet. The sand trapping capability of an inlet may play an important role in shoreline stability in the vicinity of the inlet. The trapping capability of Boca Raton Inlet is investigated in what follows.

The theoretical basis for describing movable beds is empirical. Many relationships exist that relate volume of sediment transport to bottom shear stress. While there is no one relationship which is far superior to others, the empirical result arrived at by Einstein (1942) is used

\[
q_s' = 40 \rho \frac{U_\ast}{[g(s_\ast - 1)]^{3/2}},
\]

(36)

where \(q_s'\) = the weight rate (in water) of sediment transport per unit width, \(\rho = \) water density, \(U_\ast = \) shear velocity (=\(U_{to}/\rho\)), \(g = \) gravitational acceleration,
$s_s = \text{specific gravity of sand, and } F \text{ is given by}$

$$F = \left[ \frac{2}{3} + \frac{36 \nu^2}{gd^3(s_s - 1)^2} \right]^{1/2} - \left[ \frac{1}{gd^3(s_s - 1)} \right]^{1/2}, \quad (37)$$

where $\nu = \text{kinematic viscosity of water}$ Equation (36) was used successfully by Shemdin (1970) in modeling of sediment flow in the coastal zone. Equation (5) relates the average velocity to the shear velocity

$$U_* = r^{1/2} U$$

For a wide channel with width $B$ the weight rate of sediment transport, $q_s$, becomes

$$q_s = q_s'B = 400FB \frac{r^3 \nu^6}{[g(s_s - 1) d]^{3/2}} \quad (38)$$

The net sediment transport into Boca Raton Inlet was computed from results similar to those shown in Figure 8. Tide records were found to fluctuate in amplitude and typical records were used for two different days. The net transport into the inlet for the two days was calculated to be 15 and 47 (yd$^3$/day) which correspond to 5,400 and 17,000 yd$^3$/yr, respectively. A recent dredging operation in Boca Raton Inlet have been removing 30,000 - 40,000 cubic yards of sand per year from the inlet. The computed transport in the inlet is of the same order of magnitude. Further research on inlets may fruitfully include field measurements of sand transport.

CONCLUSIONS

A non-linear coupled procedure is proposed to analyze tidal motion in inlets and bays. The importance of the different tidal constituents is shown to be more important in velocity computation compared to surface elevation. The exchange of sediment between bays and the ocean is dependent on the velocity variation over the tidal cycle and can only be computed accurately by considering different harmonic constituents in a tidal record. The procedure is applied to Boca Raton Inlet and the result indicate that more sand transport occurs during the flood period compared to the ebb period. The inlet consequently behaves like a sand trap.

REFERENCES


7 Shemdin, O H (1970), River-Coast Interaction Laboratory Simulation, J Waterways and Harbors Div., Vol 96, WW4

8 Sidjabat, M M (1970), The Numerical Modeling of Tides in a Shallow Semi-enclosed Basin by a Modified Elliptic Method, Ph D Dissertation Submitted to the University of Miami, Coral Gables, Florida

9 Van de Kreeke, J (1968), Water Level Fluctuations and Flow in Tidal Inlets, J Waterways and Harbors Div., Vol 93, No WW4, pp 97 - 106
Figure 1  Definition sketch for a single bay coupled to ocean

Figure 2  Definition sketch for two bays coupled to ocean
Figure 3  Plan view of Boca Raton Inlet and lake, and Lake Wyman

Figure 4  Tide elevations at Boca Raton Inlet, Boca Raton Lake, and Lake Wyman
Figure 5  Comparison between measured and computed (M-2) component of tidal motion at Boca Raton Inlet

Figure 6  Comparison between measured and computed tidal motion using six harmonics at Boca Raton Inlet
Figure 7  Comparison between measured and computed velocity using (M-2) component only at Boca Raton Inlet
Figure 8  Comparison between measured and computed velocity using six harmonics at Boca Raton Inlet