CHAPTER 8

PERIOD BY THE WAVE-GROUP METHOD

Warren C. Thompson, Professor of Oceanography
Naval Postgraduate School, Monterey, California

ABSTRACT

The wave-group period, $T_g$, is obtained by a simple manual procedure from the periodic wave groups and sequences that appear in strip-chart records. A dozen or more values are commonly derived from a 20-minute record. When plotted as a time-series, $T_g$ values form patterns that represent individual wave trains, each generated by a synoptic weather event. Swell trains and wind-wave trains can be distinguished by their characteristic period distributions.

For swell trains, the mean wave-group frequency, $f_g$, obtained from the best-fit line to the time-series plot of $f_g$ values (reciprocal $T_g$ values), is equivalent to the frequency of maximum energy density, $f_{m}$, obtained from spectral analysis. The distance and time of origin for a swell train can be determined from the time rate of change of $f_g$, and with the use of weather maps the generating area can be identified and the deep-water arrival direction of the swell train at the wave gage determined.

The zero-line crossing period, $T_z$, represents an integration of the waves from all wave trains present and has no synoptic significance.

INTRODUCTION

Period is ordinarily obtained from a wave record by computer analysis of the wave information in digitized form to produce an energy density spectrum, or by manual analysis of an analog trace written on a strip chart to produce one or more of a half dozen kinds of period measures. Computer algorithms have also been developed for picking off crests, troughs, and zero-line crossings from analog data for use in obtaining several of the latter period measures.

Spectral analysis has an important advantage over other methods in that it yields the energy-density distribution over the full range of frequencies considered, although from this writer’s observations it is only the period associated with the dominant frequency peak present that is used in most practical applications. In addition, there is adequate demonstration of the fact that prominent maxima in the spectrum represent wave trains from individual generating areas. The latter frequently can be followed in the wave records for several days.

Among the non-spectral procedures for processing analog records, the wave-group method has received little attention in the literature (Harris, 1970; Thompson, 1970). Nevertheless, as will be demonstrated, the
wave-group period is equivalent to the spectral peak period for swell, and in addition it is similarly identified with individual wave trains and is therefore a synoptically significant period parameter. The zero-line crossing method, which may be regarded as the standard non-spectral method of analysis by reason of its common use and frequent reference in the literature, is also examined in relation to the wave-group and spectral methods.

The results presented in this paper are principally drawn from the analysis of swell generated in moving cyclonic storms and propagated over distances of several hundred miles or more to wave gages on the California coast located in the general direction of storm advance. The data were recorded using pressure-type sensors mounted in 25 to 40 feet of water.

Period and frequency are referred to interchangeably throughout the paper in accord with conventional usage and ease of visualization.

WAVE-GROUP METHOD

Those who work with analog strip-chart records are familiar with the sequences of more or less periodic waves that appear at intervals in the record. Examples are illustrated in Figure 1. All wave sequences that can be identified in a record are here termed wave groups, whether they appear as discrete groups, as in Figure 1(a), or combined with other waves to form an irregular profile.

The wave-group period, $T_g$, can be obtained by identifying the first and last waves in a periodic sequence, and dividing the time interval between by the number of waves in the sequence. The measurement is made between those equivalent parts of the wave form that the analyst judges to give the best measure in each case, e.g., between selected crests, troughs, centers of wave mass, or zero-line crossings. Measurement between centers of mass are most useful when the wave shape is irregular and crest or trough measures are unsatisfactory (Figure 1(b)). Zero-line crossings should in general be avoided because most wave sequences are not symmetrical with respect to the zero line of the record due to superimposition on other waves that are ordinarily present. Occasionally, more than one wave sequence can be identified from the same time interval (Figure 1(d)). A quick check of the period of each wave in the sequence is recommended to avoid sequences containing phase changes.

The identification of periodic sequences requires judgment and seems at times to be quite subjective; nevertheless, even $T_g$ values derived from waves of highly irregular form ordinarily prove to be associated with a recognized wave train. Rapid improvement in the identification and evaluation of wave sequences comes with a little practice, and the measurement of $T_g$ for a selected sequence is a very quick operation. Best results in both quantity and quality of the data obtained requires the judgment of an analyst having a basic understanding of the fundamentals of wave interference, such as are given by Manley (1945).

A 20-minute record will frequently contain a dozen or more sequences of 3 or more waves. The $T_g$ values obtained should be identified whenever
possible with individual wave trains present, as described in the next section. Averaging of the values from a given wave record without segregating them will yield an integrated period measure of uncertain significance, and a primary advantage of the wave-group method will have been discarded. The wave-group method is meant here to include both the mechanics of obtaining the wave-group period from a wave sequence and the association of the $T_g$ values with wave trains present so as to yield synoptically significant period information.

SYNOPTIC WAVE TRAINS

Values of $T_g$ plotted as a time series usually form well-defined patterns, each representing an individual wave train generated by a synoptic weather event, such as a cyclonic storm, occurring locally or at a distance from the recording station. These synoptic wave trains can ordinarily be recognized, from their distinctive period distributions, either as swell generated at a distance from the wave gage or as wind waves and associated young swell of local origin. Wave trains may appear singly in the $T_g$ data, but two or more frequently are present at once. Most of the time individual trains can be distinguished from one another, but at times their period distributions merge and cannot be separated, and at other times during periods of very low waves the $T_g$ values fall in a random pattern across the spectrum and no wave trains can be recognized. The identification of wave trains and their recognition as sea or swell from the period data are usually correctly confirmed by the associated wave-height data and weather maps.

Figure 2 presents values of $T_g$ derived from 5-minute records made hourly for 7 days at Monterey, California, and reveals three successive wave trains arriving during otherwise quiet sea conditions. The continuous decrease of period with time from the initial appearance of each train clearly marks them as swell. The $T_g$ distribution in a typical swell train also displays a relatively small scatter about the mean, which is consistent with the narrow spectral bandwidth character of swell. About the time of appearance of the first $T$ values in a newly arriving swell train, the wave height normally rises above the energy level of the existing waves; however, on uncommon occasions during intervals of very low wave height (half a foot or less) the period distribution has unmistakably revealed the presence of a swell train when its height did not rise above the energy level of the background. When one swell train replaces another in time, the $T_g$ values associated with each train during the interval of overlap (when the energy levels are of the same order of magnitude) may appear either as independent distributions, as in the case of Trains 1 and 2 of Figure 2, or they may form a continuous transition zone, as occurs from Train 2 to Train 3. Whether the period distribution is continuous or discontinuous appears to be determined by the magnitude of the period separation between the trains. The swell properties described here are typical of swell generated in approaching cyclonic storms.

Wind waves and associated local swell are revealed in the $T_g$ data by the initial appearance of short periods, which increase with time as the sea grows until the wave height peak is reached. The $T_g$ values display wide scatter, consistent with the broadband spectral nature of a sea, but tend to be concentrated near the upper envelope of the distribution. Following
passage of the wave peak, the $T_g$ values for the young swell trailing in
after the wind has fallen off diminish with time, and the period distribution
takes on more the appearance of a distant swell train. No clearcut example
of a wind-wave train is available for illustration.

$T_g$ values obtained from the analysis of a series of wave records can be
put to practical use best by fitting a curve to the time-series data associ-
ated with each wave train identified. In the case of a swell train, a curve
fitted to the $T_g$ values will represent the mean wave-group period for the
train, $T_g$. If reciprocal $f_g$ values are plotted instead, as shown in Figure
3 for the swell trains of Figure 2, the best-fit curve will be, similarly, the
mean wave-group frequency for the train, $f_g$; this curve should properly be
linear for distant swell trains, as discussed in a later section. The $f_g$
curve, as will be shown in the next section, agrees closely with the frequency
of maximum energy density, $f_m$, for the swell train as determined by
spectral analysis. In the case of wind-wave trains, no comparative analyses
of the same wave records have been made by the wave-group and spectral
methods; however, inductive evidence indicates that a curve drawn near the
upper envelope of the $T_g$ distribution and through the densest concentration
of values will give a reasonable approximation of $T_g$ for the wave train.

In this manner, a time series of $T_m$ curves (or $f_m$ curves) can be pro-
duced from which the user can pick off the value or values prevailing for
any selected time. When more than one wave train is recognized to be
present at a given time, the dominant train can be identified by the fact
that its period will be associated with the wave sequences of largest ampli-
tude. The dominant train can often be identified also from examination of
the wave-height distribution in relation to the period data. For most
practical applications, when more than one wave train is present the ocean
surface can be characterized by using the observed wave height along with
the period of the dominant train.

**EQUIVALENCE OF $T_g$ AND $f_m$ FOR SWELL**

In the case of swell trains generated in wind areas located several
hundred miles or more from the recording station, the mean wave-group
frequency, $f_g$, and the spectral peak frequency, $f_m$, derived from the same
wave records by the respective methods of analysis have been found to
agree closely in all comparisons made.

By way of demonstration, a swell train recorded at Monterey during
otherwise quiet wave conditions was selected, and 20-minute strip-chart
records made every 6 hours for its 2-day life were analysed by the two
methods. The analog traces, written at a chart speed of 2 inches per
minute, were converted for spectrum computation to digital data using a
digitizer, and analysis was performed using the Blackman-Tukey (1958)
method. The energy densities were computed to an upper limit of 0.24
Hertz using a frequency increment of 4.16 milliHertz, with 90 per cent
confidence limits estimated at 0.60 and 2.10. The results are presented
in the form of spectral density topography in Figure 4.

The prominent ridge in the topography extending across the lower part
of the figure represents the swell train of interest. The frequency along
the ridge line, or spectral peak, is \( f_m \) and is seen to increase with time in the characteristic manner of swell. The swell train reached its maximum height of two feet at 0200Z on 29 November. Several hours later, a new swell train with considerably more energy and of lower frequency began to arrive and caused the frequency peak to shift (just off the graph) in the same manner as shown in Figure 3 between Trains 2 and 3.

Values of \( f_g \) obtained from periodic sequences in the wave records are shown as dots in Figure 4, and are seen to be concentrated about the ridge line of the swell train in the lower part of the graph and to be scattered about a dying wave train in the upper left corner. This association of the \( f_g \) values with the spectral masses of the two trains is also illustrated in Figure 6, which shows the 14 \( f_g \) values obtained from Wave Record No. 2.

The \( f_g \) values shown in Figure 4 for the swell train of interest were averaged to obtain \( f_{m} \), and these values are plotted in Figure 5. Also plotted are the values of maximum energy density frequency, \( f_m \), obtained from the spectral computation for each wave record. The two sets of values are also tabulated in Table 1. \( f_m \) exceeded \( f_g \) in six of the seven records by an average amount, expressed as period, of 0.3 seconds. Agreement within one second is typical of all comparisons we have made on swell trains.

\( f_{m} \) for a given wave record can be obtained, as just illustrated, by averaging the \( f_g \) or \( T_g \) values associated with the swell train of interest. It can also be obtained in the form of the mean or the median of the distribution of frequency or period of the individual waves composing the appropriate wave sequences. These procedures each give results that agree closely. The simplest and most practical means of obtaining \( f_g \) for an entire swell train, however, is to fit a straight line visually to the time plot of the \( f_g \) values for the train, as illustrated in Figure 3.

It is pertinent here to note that Harris (1970) compared six kinds of period measures and found the highest correlation to occur between the most prominent period in a 7-minute wave record and the spectral peak period. He stated that the period correlation might be improved if the data were stratified in some way so as to make the data samples more homogeneous. The present study indicates that stratification of a basic sort can usefully be performed on Pacific Coast wave data by differentiating between swell and wind waves where possible before manipulating the data, and that correlation can be further improved by use of a larger sample of periods such as is derived from the wave sequences appearing in wave records.

ZERO-LINE CROSSING METHOD

The zero-line crossing method (also called zero-upcrossing and zero-crossing method) involves determining the mean water level or zero line, usually visually, then counting the number of intervals between upcrossings (or downcrossings) of the zero line and dividing this number into the duration of the wave record to obtain \( T_{z} \). The zero-line crossing method and the wave-group method are similar in that both basically involve the measurement of wave intervals relative to a datum surface, but differ in
that the datum used in the first method is of constant elevation and represents the still water level for the entire wave record (after the tides are filtered out) and the datum in the second method is the ordinarily irregular water surface on which a wave sequence is superimposed. Because of the difference in datums, \( T_z \) can be obtained by use of a computer algorithm that picks off zero crossings (Wilson, Wu, and Baird, 1972), but the task of discriminating individual wave sequences and measuring \( T_z \) remains beyond the accomplishment of computer programming.

The relationship of the zero-line crossing period, in the form of its reciprocal, \( f_z \), to \( f_g \) and \( f_m \) is illustrated for the swell train of interest in Figures 5 through 7. In Figure 5, and in Table 1, \( f_z \) is seen to differ considerably from the other two values and to display large variability from record to record. In the first four wave records \( f_z \) migrates from near the spectral peak of the dying wave train in Record No. 1 toward the spectral peak of the growing swell train. Subsequently, \( f_z \) roughly approximates \( f_g \) and \( f_m \) for the dominant swell train. The relationship of \( f_z \) to the spectra of the first four records is best shown in Figure 7.

This example serves to illustrate the fact that \( T_z \) is not equivalent to \( T_g \) or \( T_m \) in the case of a swell train (wind waves have not been examined sufficiently), and that it has no synoptic significance, i.e., it gives no information about the number of wave trains present, their dominant periods, or their identification as sea or swell, and unlike \( T_g \) and \( T_m \) it yields no information about the wind field that produced the swell (discussed in next section). Since a single period value is derived from a given record, \( T_z \) is clearly an integration of the waves associated with all wave trains present. It is evident that the zero-line crossing period should be used with these characteristics and limitations in mind.

In other evaluations, Harris (1970) found a poor correlation between the zero-line crossing period (\( T_{ZUC} \)) and the period corresponding to the frequency of maximum energy density per unit frequency (\( T_{FM} \)) from wave records recorded at Atlantic City, and presented a short series of comparative data showing \( T_z \) to be smaller than \( T_m \) in every record analysed.

Wilson, Wu, and Baird (1972) compared \( T_z \) and \( T_m \) from recordings made at Western Head, Nova Scotia and Ocean Station Papa and stated they found no strong correlations.

**SOME APPLICATIONS**

**Swell Source and Direction Determination**

Using spectrally analysed wave data, Munk, et al., (1963), showed that swell trains arriving from generating areas located at distances greater than a few hundred miles from the wave gage are revealed on a frequency-time graph by ridges in the spectral energy density topography, and noted that the ridge axes are linear or very nearly so. A straight ridge line implies a point source for the swell train in space and time. In view of the fact that a storm system produces swell for the duration of its passage over the ocean, the point source computed from wave records appears to represent that location and moment in the storm history at which the maximum energy density in the sea directed toward the wave gage was generated.
The distance from the wave gage to the effective swell source, \( D_0 \), can be computed from the slope of the ridge line, \( \frac{df_m}{dt} \), using the relationship

\[
D_0 = \frac{g}{4\pi \frac{df_m}{dt}}
\]

or

\[
D_0 = \frac{1.515}{\frac{df_m}{dt}}
\]

where \( D_0 \) is in nautical miles, \( f_m \) is in Hertz, and \( t \) is in hours. The effective origin time, \( t_0 \), is given by the intercept of the ridge line with zero frequency.

In view of the equivalence of wave-group frequency and spectral peak frequency in swell, it is evident that the effective origin of a swell train can also be computed using frequencies obtained manually through application of the wave-group method. In regard to the swell train presented in Figure 4, for example, a straight line fitted to the \( f \) data has a slope of 0.4220 milliHertz per hour, giving a source distance of 3590 nautical miles from the wave gage at Monterey and an origin time of 0918Z/23 November, or about 5-1/2 days before the swells were first detected. A straight-line fit to the spectral peak frequencies derived from the same wave records (Figure 5) gives \( \frac{df_m}{dt} = 0.4175 \) milliHertz per hour, \( D_0 = 3622 \) nautical miles, and \( t_0 = 0654Z/23 \) November. The swell source in space and time computed from the two data sets is seen in this example to agree within approximately 1/2° latitude and 2-1/2 hours.

With regard to application of this procedure, it should be pointed out that the peak frequency theoretically can shift as a wave train passes through shoal water to the wave gage due principally to differential amplification of the spectral components by refraction. A significant shift could produce an erroneous computed swell source; however, it can be demonstrated that because of the narrow-banded character of swell spectra no shift effectively occurs, even when the rate of change of the refraction factor with frequency for a given shoal-water site is large. It may be concluded that nearly everywhere in shoal water the peak frequency of swell is conserved for purposes of swell origin computation.

Records from a single wave gage provide no directional information that can be used to locate the wind area in which the swell was generated. The storm responsible can be identified, however, using the computed origin distance and time derived from the wave records combined with weather charts. An arc of great-circle radius \( D_0 \) drawn on a weather chart of or near time \( t_0 \) will be found to intersect the storm system. The effective swell source point within the storm can be taken as the point of intersection of the arc of radius \( D_0 \) and the axis of the maximum winds directed toward the recording station, as illustrated in the example in Figure 8.

When the location of the source point is established, a great-circle trajectory drawn from the point to the wave gage may be considered for practical engineering purposes to give the deep-water swell direction \( \psi_0 \), for the wave-gage site (and for any other location on the adjacent coast).
Because the wave-group method yields information equivalent to that from spectral analysis for determining swell origin and deep-water swell direction, this method has particular attraction as a practical tool in economic situations and geographic regions where computer facilities are unavailable or technical expertise is limited. In this regard, it may be noted that a wave gage is not a necessity, since the mean wave-group period of the dominant swell train present can be obtained from the larger wave sequences by an observer with a stopwatch.

Additional discussion of the applications described here is given by Thompson (1970) and Austin (1972).

Surf Forecasting

Height and period measured during the early stages of arrival of a swell train can be used to make a forecast for the remainder of the train. Measurement of the dominant period, $T_g$ or $T_m$, from wave records or visually over an interval of 6 to 12 hours when converted to frequency will give the rate of change of frequency with time, from which can be obtained an accurate forecast of frequency until the train has passed. The swell height can be predicted to increase continuously until the peak is observed to pass, then to decrease continuously until the train is gone or masked by a new train. Swell trains generated in distant wind-areas are generally observed to last 2 to 5 days. This procedure has proven useful to the Marine Corps, Camp Pendleton, California for forecasting surf.

AREAS FOR FURTHER INVESTIGATION

The distinctive period signature of oceanic swell trains described herein is typical of swell recorded on the west coast of North America. These waves originate in generally approaching cyclonic storms which cross the Pacific in the latitudes of the prevailing westerlies in both hemispheres. Similar swell properties can be expected on any coast where an approaching storm situation occurs, i.e., on most coasts of the oceans. This investigator has not examined wave records from coasts which receive swell trains generated in retreating storms, as occurs on the east coasts of the continents in middle and higher latitudes. There is reason to believe that the period signature of such swell trains may be different, and this question merits investigation.

It is evident from the limited attention given in this paper to wind-wave trains that the relationship between $T_g$ values and wind-wave spectra obtained from analysis of the same wave records needs to be probed further.

The height of the larger waves composing a wave sequence is observed to be related to the height of the wave train with which they are associated in the case of swell. When more than one swell train is present, the dominant train is readily identified by its larger wave groups. This relationship should be examined quantitatively. Possible relationship between the height of a wave sequence and the energy density in the frequency band represented by the wave sequence also appears to merit investigation, and may prove of particular significance in regard to broadband spectra.
There is an important area of engineering application in which the wave-group method may provide information that cannot be provided by computation of the spectrum. Wilson, Wu, and Baird (1972) pointed out that "some marine structures react to individual or groups of waves rather than spectra and there is a real requirement for parameters other than r.m.s. water surface deviation, the moments of the spectrum and some sort of characteristic period". It would appear that such parameters should be developed from knowledge of the wave sequences appearing in wave records. The properties of wave groups and their relationship to the wave train have not been examined to the writer's knowledge beyond what is presented in this paper.

ACKNOWLEDGMENTS.

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REFERENCES


Table 1: SWELL TRAIN PERIOD (FREQUENCY) BY DIFFERENT METHODS

<table>
<thead>
<tr>
<th>Wave Record No.</th>
<th>Arrival Time/Date (GMT/1971)</th>
<th>Period in seconds (Frequency in $10^{-5}$ Hertz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Spectral $T_m$ ($f_m$)</td>
</tr>
<tr>
<td>1</td>
<td>1204/27 Nov</td>
<td>21.83 (4580)</td>
</tr>
<tr>
<td>2</td>
<td>1808/27 Nov</td>
<td>20.70 (4830)</td>
</tr>
<tr>
<td>3</td>
<td>2355/27 Nov</td>
<td>19.69 (5080)</td>
</tr>
<tr>
<td>4</td>
<td>0600/28 Nov</td>
<td>19.05 (5250)</td>
</tr>
<tr>
<td>5</td>
<td>1207/28 Nov</td>
<td>17.79 (5620)</td>
</tr>
<tr>
<td>6</td>
<td>1810/28 Nov</td>
<td>17.54 (5700)</td>
</tr>
<tr>
<td>7</td>
<td>2356/28 Nov</td>
<td>16.21 (6170)</td>
</tr>
</tbody>
</table>

Note: The zero-line method integrates all wave trains present.
Figure 1: PERIODIC WAVE SEQUENCES
Figure 2. WAVE DATA FOR THREE SWELL TRAINS

Wave Period, T (sec)  Significant Height, H (ft)
FIGURE 3: FREQUENCY-TIME DISTRIBUTION FOR THREE SWELL TRAINS
Figure 4: VALUES OF \( f_g \) SUPERIMPOSED ON SPECTRAL DENSITY TOPOGRAPHY
Figure 5: COMPARISON OF FREQUENCIES OBTAINED FROM MANUAL AND SPECTRAL ANALYSES
Figure 6: FREQUENCY SPECTRUM OF WAVE RECORD NO. 2

VALUES OF \( f_g \)

ENERGY DENSITY (Ft²/Sec)
Figure 7: FREQUENCY $f_z$ IN RELATION TO SPECTRA FOR WAVE RECORDS 1 THROUGH 4