EQUILIBRIUM CONDITIONS IN BEACH WAVE INTERACTION

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ABSTRACT

Laboratory studies were conducted in an attempt to find out a relationship between beach and wave characteristics when equilibrium conditions are reached in beach wave interaction for the simple case of regular waves acting normal to the beach. Experimental results indicate the existence of stable points on beach profiles where the coordinates of the profile do not change with time when waves of constant characteristics act on the beach. Empirical relationship between the wave and beach properties are proposed. A new criterion for classification of beach profiles is indicated.

INTRODUCTION

In beach-wave interaction and the resultant process of mutual modification there is a tendency to attain equilibrium conditions under which beach and wave no longer cause any further change on each other. In nature however, the ever-changing meteorological factors which cause rapid changes in waves, tides and currents seldom permit this interaction to reach equilibrium conditions. Nevertheless, certain dominant tendencies in the wave pattern may persist over a season or part of a season thereby enabling us to delineate a few significant characteristics of waves to work out their influence on beach modification problems. This in turn offers us an opportunity for a meaningful determination of beach response to wave action, provided necessary numerical relationships are available for this purpose. Accurate formulae for determination of the nature and extent of changes on beach profiles for given wave conditions, though of great importance in a large number of shore problems, are not available at present. It appears that the evolution of theoretical relationships, which can be used on practical problems with a reasonable degree of accuracy, may have to wait till better tools are available to deal with problems of sediment transport in an unstable and oscillating wave velocity field. This is discussed in detail by Silvester (1,2). Empirical relationship can however be derived for certain conditions of profile.
changes. In this paper an attempt is made to derive empirical relationships for the determination of the type and quantum of beach profile changes in the simple case of regular waves acting on a beach of uniform initial slope aligned normal to wave action.

STABLE POINTS ON BEACH PROFILES

Available experimental and field data on beach profiles indicate that usually there is a stable point on beach profiles near the breaker zone which does not undergo any considerable change during the process of profile modification due to incident waves of essentially constant characteristics. In the case of storm profiles a second stable point may exist in the offshore zone depending on the steepness of the incident waves (See Fig. 1). The stable point (or the second stable point in case two stable points exist) acts as a fulcrum about which the waves try to keep the material distribution in balance - erosion onshore and deposition offshore - within the active zone of the profile. For normal profiles the stable point does not exist in the strict sense, but there is one point on the modified profile which seem to serve the same function as above (See Fig. 2). In this case there is erosion offshore and deposition onshore.

Neutral points on beach profiles where, material of a given size will be in oscillating equilibrium, with no net movement, have been discussed in a few previous works. Cornaglia (1898) was perhaps the first to make this observation(3). Inman's(4) observations on sediment sorting and studies of Miller and Zeigler (5) showed that there are zones of oscillating equilibrium for each sediment size for the given wave conditions. Eagleson et al (6) in their theoretical analysis of the problem of sediment transport on sloping beaches under waves equated the forces acting on a bed particle and arrived at a criterion for oscillatory equilibrium. Wells (7) defined a neutral point on the beach profile such that the skewness of the probability distribution of horizontal water velocity of second order gravity waves will be zero at the point.

The neutral positions of particle motion described by these authors indicate that particles of the same size will move in opposite directions when placed on either side of the neutral point. (A sediment particle on the seaward side will move seaward while one on the shoreward side will move shoreward of the neutral point for the same sediment size). The stable points mentioned in this paper are those which act as a fulcrum about which the profile swings while material is moved from one side of the point to the other. The net sediment motion is unidirectional, either towards shore or away from it, for appreciable distances on either side of the stable point. A directional variation is expected
only due to changes in velocity field imposed by reflection or due to changes in material size. The influence of the former is comparatively small and can be easily identified by the formation of nearly stationary humps and hollows on the bed whose spacing will be closely related to the wave length. This is easily understood from the profiles shown in Figures 3 to 8 and from the profiles given in the references cited.

Fig. 3 shows the stable point on a set of natural beach profiles taken at a shore which is in equilibrium. Fig. 14.22 of Ref (8) shows 12 sets of profiles on an actual beach. Each set contains 5 profiles taken during the course of two to three months at a single station. It is seen that in most cases the profiles remain stationary at a depth of 5 ft. (1.52 m).

Figures 4 to 7 show the stable points on experimental beach profiles of the storm type and Figure 8 shows the modified version of the stable point on a normal beach. In the experimental beach profile shown in Fig. 14 of Ref (7) the stable point is visible at 17.5 cm depth. At this point there is no change for the original profile irrespective of the time of propagation of waves, although adjacent areas are getting altered with time of wave action.

EXPERIMENTAL SET-UP AND PROCEDURE

The experiments were conducted in a wave flume 31 m long 90 cm wide and 90 cm deep. The plunger type wave maker powered by a variable speed motor is fixed at one end for wave generation. At the other end of the flume, model beaches are laid to uniform initial slope using sand of required size. All tests in the present series were done under a constant water depth of 35 cm at the toe of the beach.

Waves of constant characteristics were generated and allowed to work on the beach till the profile showed no significant change in time. In order to locate the stable points and to find out the pattern of modifying process the beach profile was measured at 5 to 10 hour intervals. Normally it took 40 to 50 hours for the profile to reach equilibrium conditions. Wave characteristics were measured by resistance type wave gauges and a recorder unit.

EXPERIMENTAL RESULTS AND DISCUSSION

Even though experiments were done with sand of different sizes and coal powder and with different initial slopes, the results of tests on 0.3 mm sand alone is reported in this
paper. Other profiles also show the same pattern. But
detailed analysis with respect to stable point is done
only for 0.3 mm sand for different wave conditions and
initial slopes and hence this is discussed in detail.

Typical storm type profiles for slopes 1/8, 1/10,
1/12 and 1/15 are shown in Figures 4, 5, 6 and 7 indicat-
ing the progressive development of profiles towards
equilibrium. Similarly Fig.8 shows the development of a
normal type profile on a 1/10 slope.

TYPES OF PROFILE CHANGES

It is found that mainly there are three types of
profile changes:

(i) Storm type profiles with erosion on the shore
and deposition in deeper waters. Erosion takes place shore-
ward of the second stable point when it exists or shore-
ward of the first stable point when the second one is
absent. Longshore bar is present.

(ii) Normal profiles with deposition taking place on
the shore from the material dug out by waves from offshore
of the breaker by waves. Longshore bar is absent.

(iii) Storm type profile with longshore bar, but
deposition takes place on the shore. Shoreward of the
second stable point erosion takes place and seaward of it
deposition is seen.

The last mentioned of these is an interesting phenomenon
in that it is customary to think of storm profiles as
indicative of shore erosion. Examination of this type
of profiles show that onshore transport in storm profiles
takes place when the distance between the two stable points
is more than one local wave length. It appears that in
such a case more than one set of mass transport circulation
cells will be formed with possibility of a reversal in the
direction of transport. It may be recalled that Longuet-
Higgins (9) showed that mass transport circulation cells
may be formed in pairs at spacings equal to half wave
length for standing waves. In the present case there is
reflection from the beach and hence there will be a partial
standing wave. This incidentally explains why flat beaches
usually show accretion on the shore while steep beaches
erode for the same wave and bed material characteristics.
It is interesting to note that the second stable point being
the offshore limit of erosion of the original beach profile,
its location is controlled by the local wave characteristics,
which is primarily dependent on the depth below still water
level, and bed material characteristics. The first stable
point is a product of the breaker at the longshore bar and hence is controlled by the slope of the beach also. Therefore the distance between these points is a function of the slope also. As the slope becomes flatter, the distance increases, increasing the possibility of creating more than one mass transport circulation cell. This helps in reversing the direction of transport at some point between the offshore bar and the second stable point, thereby permitting deposition on the shore.

**SIGNIFICANCE OF STABLE POINTS**

A stable point can exist only on a beach which is in equilibrium - i.e. a beach with a shore line which may oscillate between certain shoreward and seaward positions due to seasonal changes in waves while maintaining its mean position without significant change when considered over a period of one or two decades. This means that there should not be a net loss or gain of material to the beach in its active zone either due to onshore-offshore transport or longshore transport mechanism. The presence of a stable point on a beach profile can be taken as an indication that the beach is in equilibrium.

The presence of stable points on natural beaches Fig.3 and Ref. (8) indicate that small changes in wave characteristics as may occur within a season for short periods do not annihilate the stable point or the corresponding alongshore stable zone.

**EQUILIBRIUM PROFILE AND STABLE POINTS**

If the local wave steepness \( H/L \), where \( H \) is the local wave height and \( L \) the local wave length calculated from the linear theory with Rayleigh's assumptions, is plotted against the ratio of \( x/h \) for a straight uniformly sloping beach, a straight line graph will be obtained for \( h/L_0 \) ratios less than nearly 0.15. Here \( x \) is the horizontal distance from a suitable origin to be chosen on the still water line (SWL) to any desired point on the bed profile and \( h \) is the depth of that point below SWL. (See Fig.1 for definition). The slope of this line is controlled by the location of the origin on the still waterline as can be seen from Fig.9. Fig.9 gives plots of \( H/L \) Vs \( x/h \) for the case of uniform beach slope of 1/10 when a wave of deep water wave height, \( H_0 = 11.28 \) cm and wave period, \( T = 1 \) sec acts on the beach for different locations of origins. The straight vertical line is for origin at the intersection of the SWL with beach slope. This is the point of zero chainage also. The plot becomes flatter and flatter as the origin is shifted further and further offshore.
Available experimental results indicate that a plot of H/L vs x/h prepared for the final equilibrium profile, even though not falling strictly into a straight line, can be approximated to one for portions of profile offshore of the longshore bar. The dashed line in Fig. 9 is the line of best fit drawn in this fashion for the final profile resulting from the action of the above wave on the 1 on 10 initial beach slope. In this case the projection of the 1st stable point on SWL at ch. 104 cm is taken as the origin. This line will then correspond to a straight line which represents the mean profile in the offshore part.

Fig. 10 gives plots of H/L vs x/h for the final equilibrium profiles resulting from waves of different characteristics acting on the 1 on 10 initial beach slope. In all cases the origin is chosen as the first stable point. The lines marked 1 to 4 represent storm type profiles and 5 and 6 represent normal profiles. It will be noted that the lines for the same type of profile, either storm or normal are nearly parallel to each other. Also the distances between these lines along the ordinate are nearly equal to the differences in deepwater wave steepness in the case of profiles of the same type. This perhaps, indicates the possibility of obtaining the mean profile resulting from any incident wave if H/L vs x/h relationship for one wave condition is known for the same initial beach slope.

LOCATION OF STABLE POINTS

From the available experimental results concerning an initially uniform slope of 1/10 with material having a median dia of 0.3 mm it is found that the depth at the first stable point is a function of H0 and T for the given slope. The depth at the stable point is found to increase linearly with the nondimensional parameter (H0 g T)/μ where g is the acceleration due to gravity and μ is the kinematic viscosity of water. This is shown in Fig. 11.

The second stable point, when it exists, corresponds to the point of intersection of the plots H/L vs x/h for the initial and final profiles calculated with the origin at the first stable point for both cases. Fig. 12 shows H/L vs x/h relationships for initial and final conditions, for the four profiles shown in figures 4 to 7. These four profiles have four different initial slopes. The wave characteristics are given in the drawing. It can be easily seen that the intersection of the initial and final plots of H/L vs x/h represents the second stable point.
CLASSIFICATION OF BEACH PROFILES

A new criterion for Johnson's classification of beach profiles as 'storm and normal' is briefly examined. If the formation of a normal profile is examined it will be seen that transport of material is onshore everywhere along the profile. This would suggest that the fluid particle movement at bottom, at least predominantly if not always is towards shore. A type of wave which satisfies this condition is the solitary wave which in its ideal case has only forward movement everywhere across its depth. In shoaling water it is possible that wave condition may approximate to solitary wave. If the point of threshold movement of bed material is onshore of the point at which the wave becomes nearly solitary, only onshore movement of sediments can take place.

Taking Bagnold's criterion $T = \frac{2\pi}{\frac{\sqrt{h}}{g}}$ to find the depth $h_s$ at which a progressive wave may be approximated to a solitary wave for a given period $T$ and finding the depth $h_T$ at which threshold movement starts by equating the maximum instantaneous horizontal velocity from linear theory to the threshold velocity for material movement as given in Ref. (11), the criterion for classification of profiles can be written as:

$$\frac{h_s}{h_T} \geq 1 \quad \text{normal profile}$$

$$\frac{h_s}{h_T} < 1 \quad \text{storm profile}$$

Here, the quantity $mh$ is given as a function of the ratio of amplitude $m$ to depth below SWL, $h$ by Bagnold (10) for easy extraction of the same for calculation purposes. Available experimental and field data generally confirm the validity of this test. A detailed description of the same is not attempted in this paper.

CONCLUSIONS

Available experimental and field data indicate the existence of stable points on beach profiles of equilibrium beaches. On a normal beach two stable points may be present, one near the breaker zone and one at the offshore limit of erosion of original profile seaward of the longshore bar.

The variation of the local wave steepness $H/L$ with the ratio of coordinates $x/h$ with the projection of stable point on SWL as the origin is nearly linear for the final equilibrium...
profile. For the same initial slope and material characteristics, different wave characteristics give nearly parallel plots of $H/L$ vs $x/h$, the distance between these lines along the $H/L$ axis being equal to the deepwater wave steepness. This indicates the possibility of getting the mean final equilibrium profile if the depth at the first stable point for the profile is known along with one plot of $H/L$ vs $x/h$ for the same initial slope and bed material and any other wave characteristics.

Experiments on a 1/10 initial beach slope using 0.3 mm dia sand show that the depth at the stable point is proportional to the dimensionless parameter $(H_0 g T)/\lambda$.

The second stable point can be fixed if the first one is determined and the plot of $H/L$ vs $x/h$ plot can be drawn as already described.

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REFERENCES

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2. Silvester, R., "Beach Profiles and Littoral drift assessment", La Houille Blanche/No.6-1969, pp 615-622


NOTE: 1, 2 indicate successive stages in profile modification.

FIG. 1. PROFILE MODIFICATION FOR STORM BEACH PROFILE

FIG. 2. PROFILE MODIFICATION FOR NORMAL BEACH PROFILE
FIG. 3. STABLE POINT ON UNPROTECTED BEACH IN EQUILIBRIUM – BEACH PROFILE AT 300 m. SOUTH OF THOTAPALLY, BAR-KERALA (INDIA)
EQUILIBRIUM CONDITIONS

Deep water wave height \( H_0 = 6.8 \text{ cm} \)
Wave period \( T = 1 \text{ sec} \)
Deep water wave steepness \( H_0 / L_0 = 0.0434 \)

FIG. 5.. BEACH PROFILE CHANGES
EQUILIBRIUM CONDITIONS

\[ \frac{U}{J} \approx x_0 < \frac{1}{11} \]

ELEVATION IN cm

DISTANCE IN cm

FIG. 7. BEACH PROFILE CHANGES

Stable point 1

Initial uniform slope (1 on 15)

Stable point 2
FIG. 8. BEACH PROFILE CHANGES

Initial uniform slope (1 on 10)

Stable point

DEEP WATER WAVE HEIGHT

DEEP WATER WAVE STEEPNESS

WAVE PERIOD

DISTANCE IN cm.

ELEVATION IN cm.
EQUILIBRIUM CONDITIONS

\[ H_0 = 11.28 \text{ cm}; \ T = 1 \text{ sec.} \]
\[ H_0/Lo = 0.0722; \ Lo = 156.1 \]

\[ H/L \]

\[ x = 0 \text{ at ch. 60} \]
\[ x = 0 \text{ at ch. 40} \]
\[ x = 0 \text{ at ch. 20} \]
\[ x = 0 \text{ at ch. 0} \]

\[ x = 0 \text{ at ch. 104} \] (Stable point)

**NOTE:**
1. Firm lines are for initial uniform slope 1 on 10
2. Dashed line is for the final equilibrium profile
3. Chainage starts from intersection of SWL with initial slope
4. Chainage of stable point = 104 cm
5. There is only one stable point for this profile

**FIG. 9.** \( H/L \) \( \text{vs} \) \( x/h \) for different locations of origin
FIG. 10. $H/L$ Vs $X/h$

$H/L$ Ratios correspond to local wave parameters; $x/h$ as defined in fig. 1.
EQUILIBRIUM CONDITIONS

$H = \text{Deep water wave height}$

$T = \text{Wave period}$

$g = \text{Acceleration due to gravity}$

$\nu = \text{Kinematic viscosity of water}$

$h_{ST} = \text{Depth at first stable point}$

$h_{toe} = \text{Depth at toe of beach}$

$10^5 \left( \frac{H_o \cdot T \cdot g}{\nu} \right)$

FIG.11. $(H_o \cdot g \cdot T) / \nu$ Vs $h_{ST} / h_{toe}$ FOR 1 ON 10 INITIAL SLOPE
**FIG. 12.** $H/L$ vs $x/h$ for initial and final profiles

**NOTE:**

Firm line is for initial uniform slope and dashed line for final modified profile.

<table>
<thead>
<tr>
<th>PROFILE</th>
<th>SLOPE</th>
<th>$H_0$ cm</th>
<th>$T$ sec</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>1 ON 8</td>
<td>6.8</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>1 ON 10</td>
<td>6.8</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>1 ON 12</td>
<td>11.08</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
<td>1 ON 15</td>
<td>6.49</td>
<td>1.0</td>
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