CHAPTER 133

COMPUTER STUDIES OF ESTUARY WATER QUALITY

by

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ABSTRACT

The convection and dispersion of pollutants in a deltaic estuary system are calculated using several interfaced computer programmes. The basic programme is a Dronkers type one dimensional hydrodynamic model, which is interfaced with a model of the salt water intrusion in the seaward reach.

The importance of Pitt Lake, a large fresh water but highly tidal lake, is discussed, inasmuch that it integrates water quality changes over long periods of time.

The salt wedge analysis reveals the nature of the interfacial stress which is not a function of velocity or velocity squared. It is shown that the interfacial stress is a function of entrainment, rather like a Reynolds stress, and plots are given of interfacial stress against local Richardson number multiplied by local upper layer velocity.

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The starting point of the study is to determine and trace the water movements resulting from tide and river flows in the Fraser River estuary. These water movements are calculated by determining numerical solutions of the hydrodynamic equations. These solutions are transferred to a convection and

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diffusion computer sub-model as the basis for determining water quality conditions resulting from effluent discharges which enter the system at known points. The method is used to determine the convection and diffusion of conservative pollutants but later work will study non-conservative effects which may be time or flow dependent.

At the present time, the Fraser River is relatively undeveloped and flows are only slightly regulated. Also the river is one of the few remaining major salmon rivers and supports a major portion of both the Canadian and the United States salmon fishing industry. It is considered important to understand fully the behaviour of this estuary before too much development is begun. This study is one step in such a total study which is being actively pursued by a team of scientists and engineers.

The Fraser estuary has certain special features which must be considered and allowed for in the formulation of the computer modeling. The lower estuary divides into a delta with a South Arm which carries some 90% of the flow, a North Arm and a few lesser channels which are later subdivisions of the North and South Arms (Figure 1). There is a large fresh water lake, Pitt Lake, which connects to the estuary through a short channel at about the mid-point of tidal influence. The large flow reversals to and from this lake are frequently of the same order of magnitude as the mean annual fresh water Fraser River flows, and have resulted in a reverse delta extending into the lake. An important feature of the Fraser River is the large seasonal flow variation with winter flows of the order of 30,000 c.f.s. Summer flows have, historically, been as high as 600,000 c.f.s. (Figure 2). The tidal range at the mouth, with a maximum of about 15 feet, and the low winter river flows result in a fairly well stratified salinity intrusion in the seaward reaches. This salinity intrusion has almost certainly been increased by the dredging of the downstream channels. The influence of this flow variation on the tidal variation is indicated in Figure 3.

Pitt Lake is a particularly interesting feature of the estuary system. The large surface area of the lake allows it to accept large volumes of water on each tide so that it acts like a huge hydraulic damper on the tidal propagation. From the water quality viewpoint, Pitt Lake can be a valuable asset because it is continuously sampling the estuary water, so that the lake acts as a long-term integrator of water quality. This is particularly true during the summer months when the lake is stratified, because the cooler, inflowing estuary water plunges below the thermocline while the ebb tide flow is drawn from the surface layers. The lake's biological environment may prove to be a valuable long-term integrator of deteriorating water quality.

The basic hydrodynamic model follows work which is well documented in the literature (Dronkers (2)). This basic model
FIGURE 1. Lower Fraser Estuary
FIGURE 2. Typical Fraser River Discharges

FIGURE 3. Tide Curves at Port Mann Showing Influence of High and Low Freshwater Discharge
is interfaced with more detailed local models which calculate such features as salinity intrusion and its modifications to the flow patterns, and local diffusion of effluents where concentration gradients are high. Finally, the convective model is abstracted from the output of the basic and local models. This convective model traces the movements of individual water masses and keeps account of concentrations.

The low freshwater discharge during winter months produces a density stratified estuary when saline water from the Strait of Georgia intrudes along the river bottom. Any numerical hydraulic modeling must therefore include the capacity to handle the stratified situation. A simple two-layer model for this purpose is presently under development, based on the following hydraulic equations. (Figure 4 shows the notation).

\[
\begin{align*}
    z > h_f & \quad \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + g \frac{\partial h}{\partial x} = -\tau_i / \rho (n+h_f) \\
    \frac{\partial}{\partial x}(n+h_f) + \frac{\partial}{\partial x}(U(n+h_f)) &= 0 \\
    \frac{\partial h_f}{\partial t} + \frac{\partial}{\partial x}(U_s (h-h_f)) &= 0
\end{align*}
\]

where \( \tau_i \) and \( \tau_b \) represent the interfacial and bottom shearing stresses respectively; \( \rho \) and \( \rho_s \) are the fresh and salt water densities. The velocities, \( U \) and \( U_s \), represent mean values integrated over the depth of each layer. The boundary conditions corresponding to these equations are:

(i) the freshwater discharge at the upstream extremity of the salt water; and
(ii) the specifications of \( n_0 \) and \( h_{f0} \) at \( x=0 \) for all time.

Even the simple model involving equations (1) to (4) is computationally difficult and furthermore it contains more unknown quantities than equations. The usual approach is to replace the stress terms by functions of the other dependent variables; traditionally relations in terms of velocity shear. The solutions of these equations are sensitive to the assumption made for the interfacial stress. Therefore, a steady flow model has been extracted from this system of equations and solved to yield information on the interfacial stress term. The approach is similar
\[ R^2 + (n'-2)R^2 = \left(2n' - \frac{(1+n')^2}{\varepsilon} + \frac{2C'}{\varepsilon} \right)R - \left( \frac{2C' n'}{\varepsilon} - \frac{(1+n')^2 n'}{\varepsilon} - \frac{2F^2}{\varepsilon} \right) = 0 \]  

where

\[ \varepsilon = \left( \frac{\rho_s - \rho}{\rho} \right) \]

\[ \frac{2C'}{\varepsilon} = h_{f_0} \left( h_{f_0} - 2 \right) + \frac{(1+n_o')^2}{\varepsilon} + \frac{2F^2}{\varepsilon (n_o + h_{f_0})} \]

\[ F = \frac{u_s (n_s + h)}{\sqrt{gh^3}} = \text{Froude Number} \]

\[ R = h_{f_0} / h \]

\[ n' = n / h, \quad n_o' / h \]

\[ h_{f_0} = h_{f_0} / h \]

FIGURE 4. Stratified Flow Definition Sketch and Solution for Stationary Wedge
to Keulegan's (1) with respect to the starting equations.

The model which represents a simple balance of forces and continuity of mass in each layer, becomes:

$$\frac{d}{dz}(U(n+h_f)) = 0$$  \hspace{1cm} (6)

$$U \frac{dU}{dX} + g \frac{dn}{dX} = -\tau_i / \rho (n+h_f)$$  \hspace{1cm} (5)

$$A \frac{dh_f}{h_f} - \frac{h_f}{z-h_f} \rho \sigma = 0$$  \hspace{1cm} (7)

$$U_s = 0$$  \hspace{1cm} (8)

A rectangular channel of constant width and depth, with no entrainment of salt water from the lower layer have been assumed. Equations (5) to (8) may be solved for the variable $h_f$, by eliminating the interfacial stress, $\tau_i$, and using equation (6) to replace the velocity $U$ by a discharge in the upper layer. This procedure results in an integrable differential equation whose final solution is a cubic in the interfacial depth $h_f$. This solution is an analytic function in terms of the surface elevation and freshwater discharge or Froude number and is shown in Figure 4.

In order to obtain information about $x_s$, the surface profile for a given value of the freshwater discharge must be specified. The one-dimensional tidal equations (2) have been solved explicitly for tidal conditions and rectangular section geometries representative of the lower Fraser River. The results linking discharge and surface elevation have been used to estimate a range of values of these variables applicable to "balanced conditions" in the idealized estuary.

Salt wedge geometries (Figure 5) have been calculated for three discharges typical of velocities just after the turn to ebb tide. These solutions were obtained by imposing a constant surface slope in equation (9), (Figure 4), corresponding in an average sense to the discharge. In Figure 5 the penetration lengths have been normalized using the longest length obtained for the lowest discharge, $Q_s = 30 \text{ ft}^2/\text{sec}$.

Once the salt wedge geometry has been obtained, equation (7) may be solved for the interfacial stress. These stresses are plotted in Figure 6 for the three discharges considered previously. In each case the stress increases to a maximum value near the upstream end and then decreases rapidly to a theoretical value of zero at the wedge toe.
FIGURE 5. Calculated Salt Wedge for Three Different Fresh Water Discharge.

FIGURE 6. Calculated Interfacial Stress As A Function of Distance From Seaward End.
We are now in a position to examine some assumptions about the variation of the interfacial stress term with respect to other flow variables. For example, if the stress is plotted against the freshwater velocity squared -- reflecting the mean velocity shear squared, since the salt water is assumed motionless -- the results shown in Figure 7 are obtained. The interfacial stress decreases in the direction of increasing velocity or velocity shear. If we think of velocity shear as the necessary mechanism for creating the stress, the results are contradictory.

The stress variations in Figure 6 were calculated from the pressure force in the lower layer and involve only the free surface and interfacial slopes. This can be seen if equation (7) is rearranged as:

$$\tau_i = \rho g (h_h \frac{dh}{dx} - \frac{dL}{dx})$$

It is possible that the constant surface slope is an unreal boundary condition, resulting in a stress decrease in the seaward direction that is too rapid. However, equation (10) shows that, even if

$$\frac{dn}{dx} \to 0$$

as we move seaward, the interfacial stress will continue to decrease as

$$\frac{dh_f}{dx}.$$

Evidence for a surface profile modification was obtained by applying the model for higher discharges and slope values. In such cases, the calculated stress reversed in direction when

$$\frac{dn}{dx}$$

exceeded the value of $$\frac{dh_f}{dx}$$. Clearly, such stress reversals are unphysical and suggest that $$\frac{dn}{dx}$$ approaches zero near the seaward end to maintain the correct force balances. The convective accelerations would account for such a modification as they create a retarding force on the flow acting like an adverse pressure force.

If the interfacial stress is principally a product of mixing; that is, a transfer of momentum across the interface like a Reynold's stress, then a seaward decrease in the force term may not be unreasonable. Indeed, preliminary field data indicate a higher degree of stratification near the seaward end, and a better mixed region near the wedge toe. This suggests a greater interaction between the layers near the upstream end and the possibility of higher interfacial stresses in this region.

Assumptions made for the interfacial stress in the unsteady equations (1) and (3), will undoubtedly have a large influence on their solution. On the basis of the "steady-state" model, replacing this stress by functions of velocity or velocity
Figure 7. Interfacial Stress Plotted Against Velocity Squared, Showing Inverse Correlation

Figure 8. Interfacial Stress Plotted Against Velocity Multiplied by Richardson Number
squared will introduce incorrect stress variations with distance. Better representations may be found in terms of the surface slopes or parameters indicating the intensity of mixing. For example, if the interfacial stress is plotted against an "entrainment velocity," the results in Figure 8 are obtained. This entrainment velocity is simply the average local velocity of the upper layer multiplied by the local Richardson number. This approach was suggested from a study of Ellison and Turner's work (3) although the results and formulation are not directly comparable or compatible. A similar, but slightly less good, fit is obtained by plotting stress against just Richardson number. As yet the actual mixing mechanism in the river regime is not well understood and presents perhaps the most challenging part of this problem.

The steady flow model has also shed light on the role of the convective acceleration term in equation (5). Equations (5) to (8) may easily be solved in the absence of the term; in which case the predicted salt wedge penetration averages approximately 40 per cent of the lengths shown in Figure 5. Thus the convective accelerations serve to reduce the transfer of momentum across the layer boundaries, thereby reducing the interfacial stress term. In the simple stationary model, this means the lower can now move further upstream and produce the longer penetration lengths. Thus it is necessary that any models of this nature retain the convective acceleration terms.

CONCLUSIONS

A digital computer model has been written which calculates the tidal flows in the Fraser River estuary. Information is extracted from this basic model to calculate convection and dispersion of conservative pollutants in the estuary system. In the seaward reaches of the estuary salt water intrudes during periods of moderate to low river stages and an additional model of this aspect has yielded some understanding of salt-fresh water interaction.

The solution of the salt water intrusion problem depends upon a correct representation of the interfacial stresses. A study of the arrested wedge reveals that this interfacial stress is not a function of velocity or velocity squared. In fact the interfacial stress is perhaps more nearly inversely proportional to velocity and is governed by entrainment, rather like a Reynolds stress. To confirm this result, a plot is given of local Richardson number multiplied by local upper layer velocity against computed stress. In this case the Richardson number is simply the inverse of the local densimetric Froude number.
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REFERENCES

