CHAPTER 22

ORIGIN, EFFECT AND SUPPRESSION OF SECONDARY WAVES

by C.H. Hulsbergen *

1 Abstroct

Beoch profile formation moy be severely affected by secondary waves which, together with the bosic wave, alwoys originote fram a sinusoidally-moving wave baard The ascertoinment af this experimental fact is fallowed by an investigation of the behaviaur of the generated waves ond their interactions It appears thot the many choracteristic features, omong which the spatial beat phenomenon and the secondary crest formotion, ore generally in good accordonce with the theories of Fontanet [14], ond Kravtchenko and Santon [20] It is concluded thot on "outer" onalysis, e g by plotting the x-t-lines of visible peoks, is only of limited use to describe the "inner" character of the complex phenamenan. The subsequent study of the effect of secandary woves on o harizontol sond bed reveols thot not only the wave form, but alsa the sand transport varies spotially, resulting in the farmation of bars and troughs This typical behoviour of the onshare-affshore transport is provisionally investigated in a small pulsating water black. Finally, a methad is described which suppresses the secondary waves, by using a low rectangular sill an the atherwise harizantal battom

2 Introduction

Secandory woves, solitons, ar disturbing waves are three different nomes for a peculior kind of wove phenamenon which hos been reparted under various canditians All descriptians mention that a regular progressive wave ar swell is accampanied by one ar more extra wave crests of o smaller height and with a lower prapagotion speed Secondary waves have been reparted in loboratary experiments with nan-breoking waves over o horizontol bed [4,5,11,15,16,18,24,25,31], an o slope or neor an obrupt variation in depth, with or without breaking [13,16,19,24,26,27,30], under notural conditians [6,10], [21], ond in onolyticol or numericol computotions [7,8,12,14,20,23,24,32,35]. The existence of the phenamenan is na longer o matter of dispute, olthough in its outer oppeoronce it hos sometimes been confused with the - real - wove reflection or with the - non real - "crête secondaire" af Miche [18] With respect ta the origin ond the nature of secondary waves, however, na comman apinian or complete theory exists os yet, in which situotion voriaus deviating interpretotions hove been put forward This paper is mainly confined ta the cose of progressive waves over a horizontol bottom, generated by o sinusoidally-moving piston-type wave boord It combines some experimentol results with existing - but portly

* Project engineer, Delft Hydroulics Laboratory, The Netherlonds

forgotten - theories, trying to describe and understand the observed phenomena.
In 1969 the starting point for this study was the experimentally-observed fact that beach profile formation may severely be affected by secondary waves Fig 1 shows three beach profiles which all had developed to entirely different equilibrium positions in identical wave channels under the same wave conditions The profiles appeared to be strongly influenced by the sand bars and troughs, which had developed from the originally horizontal section of the bed This bar system, although being outside the breaker zone, controlled to a great extent the position and the type of breaking, and thus the water movement and sand transport in the surf zone The formation and the geometry of the bar system, which was not caused by wave reflection, seemed to correspond with and to intensify the secondary surface waves which had been present from the beginning, although they were hardly visible then Apparently, these small secondary waves originated at the wave board, and a set of experiments were conducted in order to establish the exact nature of the produced waves

3 Origin and behaviour of secondary waves

3 1 Experimental conditions and measuring procedure

The tests were conducted in two different wave channels, 1 24 m and 0.91 m wide, with smoothly finished sides and bottoms The lengths of the horizontal sections were 9 m and 13 m respectively, and both channels ended with a 1 in 20 sloping beach as a wave absorber No other wave absorbers or filters were used A vertical board, sinusoidally oscillating in a horizontal plane, was used as a wave generator Wave periods T varied from 1 15 s to 1 92 s, the water depth h varied from 0 10 m to 0.55 m, and the wave height H varied between 0 02 m and 0 15 m The water depth to wave length ratio h/L thus varied from 0.05 to 0 20, and the Ursell parameter $Ur = HL^2/h^3$ varied between 2 and 104 In general four different wave board strokes were chosen for each h/L-value The wave form was measured in the centre line of the channel in points 0 2 m apart over a distance of at least 6 m, starting near the wave board A harmonic analysis yielded the local amplitudes of the first, second, third and fourth harmonic components a_1, a_2, a_3 and a_4, averaged over three wave periods The resulting regular spatial variation of a_n, an example of which is presented in fig. 2, forms the essential basis for a further analysis

3 2 Twa different interpretatians

The typical wave farms did nat differ fram thase reparted earlier, e g [5,15,16]. An almast sinusaidal wave farm near the wave generatar defarms, while prapagating, gradually inta a nan-symmetric ane, with a small hump behind the main crest. This hump may develap, depending an the value af Ur, inta a definite secandary crest, lacated in the traugh af the main wave. On that lacatian, the wave farm is symmetric again with respect ta the crests Further dawnwave this pracess is repeated in reverse, until the appr sine farm is reached again, etc. (see fig 3) At any fixed place the wave farm is canstant in time At a first glance, ane cauld take the hump ar secandary crest as an extra wave, having the same periad as the main wave, and prapagating with a lawer speed because af the difference in wave heights Accarding ta this viewpaint, e.g. [15,16,24], bath waves shauld be just in phase at the lacatians where a sine wave results, which has a smaller height than at all ather lacatians The incarrectness af this viewpaint may be demonstrated by the fallawing argument: if the difference in height wauld be the anly reasan far the difference in celerity, there wauld be na reasan at all far the sine wave ta change its farm, because at that lacatian bath waves wauld have the same height, periad, and water depth, and thus the same celerity
Accarding ta a different paint af view [18,23], the symmetrical wave farm with the secandary crest shauld be regarded as a superpasitian af the main wave with periad T and a smaller wave with periad T/2, the sa-called secand harmanic free wave One crest af this smaller wave cantributes ta the visible secandary crest, while its ather crest caincides with the main wave crest and makes it higher than narmal Thus, the second harmanic free wave is here exactly in phase with the secand harmanic campanent af the basic wave The resulting sine wave, an the ather hand, is caused by the fact that the free and the caupled secand harmanic waves are exactly aut af phase and are almast cancelled aut Care must be taken, hawever, nat ta be misled by the auter appearance af the waves In general, the visible peaks da nat carrespand ta the crests af real waves, simply because a recorded wave farm has na awn identity as saan as it must be regarded as the summatian of mare than one participating wave This secand paint af view agrees well with what may be expected an the basis af literature

3 3 Theoretical considerations

Fontanet [14] predicts the amplitude and the phase of the second harmonic free wave, which is always the by-product of a sinusoidally-moving piston-type wave board (fig 4) For low h/L-values the free and the coupled second harmonic waves are virtually 180° out of phase Kravtchenko and Santon [20] predict the generation and the interesting behaviour of a set of two waves, generated by two interacting free waves, with T_1, C_1, L_1, A_1 and T_2, C_2, L_2 and A_2 as period, celerity, length and amplitude, respectively. The new interaction waves have periods defined by the sum and difference frequencies of the interacting waves:

$$1/T_* = \left| 1/T_1 - 1/T_2 \right| \text{ and } 1/T_*' = 1/T_1 + 1/T_2 \qquad (3\text{-}1)$$

The corresponding celerities are

$$C_* = L_1L_2\ (T_1-T_2)/T_1T_2\ (L_1-L_2) \text{ and } C_*' = L_1L_2\ (T_1+T_2)/T_1T_2\ (L_1+L_2) \qquad (3\text{-}2)$$

The corresponding amplitudes are given by

$$A_* = A_1\ A_2\ \gamma_{1,2} \text{ and } A_*' = A_1A_2\ \eta_{1,2} \qquad (3\text{-}3)$$

where $\gamma_{1,2}$ and $\eta_{1,2}$, non-dimensionless coefficients of interaction, are very lengthy functions of L_1, L_2 and h [20]

So the interacting basic wave (period T) and Fontanet wave (period T/2) produce two extra waves with periods $T_* = T$ and $T_*' = T/3$ according to (3-1), whereas their celerities C_*, C_*' and amplitudes A_*, A_*' are given by (3-2) and (3-3) As an example, fig 5 presents the values of $\gamma_{1,2}$, $\eta_{1,2}$ and $\eta_{1,2}/\gamma_{1,2}$ as a function of h/L_1 for T_1 = 1 56s and T_2 = 0 78s The basic wave is regarded as a third-order Stokes wave, i e composed of three harmonic components, with amplitudes A_1, A_2' and A_3', all propogating with the same celerity C_1 of the basic wave

We have, then, six different "waves", viz three pairs of waves with periods T, T/2 and T/3 respectively In each pair both waves thus have equal periods T/n (n = 1,2,3), but different celerities (fig 6) Each pair can of course give only a single value of a_n in the harmonic analysis at a specific location, obtained by adding the constituent amplitudes as vectors Each resulting amplitude a_n must then theoretically display a rhythmic spatial behaviour Its maximum and minimum values are the sum and the difference, respectively, of the amplitudes of the participating "waves" Because of the difference in celerities, the faster wave of the pair will overtake the slower one within a certain distance, the overtake length L_{ov} It follows immediately that

$$L_{ov} = L_{slow}\ \ C_{fast}/(C_{fast}-C_{slow}) \qquad (3\text{-}4)$$

where the subscripts fast and slow refer to the faster and slower wave, respectively

As an example, fig 7 shows the expected behaviour of a_1 as a function of the phase angle $\varphi(x)$ between the constituent amplitudes A_1 and A_* It should be noticed that the celerities of a_1 and A_1 are in general not equal.

3 4 Comparison of theory with experiments

The second harmonic amplitude a_2

Supposing that a_2 is the vector summation of the second harmonic Stokes amplitude A_2' and the Fontanet amplitude A_2, both amplitudes follow from the maximum and minimum a_2 values (see figs 2,7):

$$\text{Stokes 2nd order: } A_2' = (a_{2max} + a_{2min})/2 \quad . \quad .(3\text{-}5)$$

$$\text{Fontanet} \quad : A_2 = (a_{2max} - a_{2min})/2 \quad .(3\text{-}6)$$

provided that $A_2' \geqslant A_2$, which is true according to Fontanet [14]. In fig. 8 the experimentally determined value of A_2', A_2 and A_2/A_2' have been plotted, together with the respective theoretical curves of Miche [28] and Fontanet [14] The fact that a_{2min} is always found near the wave board (fig 2) is in support of Fontanet's phase relationship (fig 4) From (3-4) it follows for the overtake length of a_2:

$$L_{ov2} = L_2C_1/(C_1-C_2) \quad . \ (3\text{-}7)$$

or in dimensionless form:

$$L_{ov2}/L_1 = C_2/2(C_1-C_2) \text{ or } L_{ov2}/L_1 = L_2/(L_1-2L_2) \quad . \ . \ (3\text{-}8)$$

The measured and theoretical values of L_{ov2}/L_1 have been plotted in fig 9, which show a reasonable agreement For relatively deep water, $L_1 = 4L_2$ and $C_1 = 2C_2$, so that from (3-8):

$$L_{ov2} = L_1/2 \quad . \ . \ (3\text{-}9)$$

In this case, the overtake length is apt to be mixed up with the reflection phenomenon For shallow water conditions, $(C_1 - C_2)$ diminishes to very small values (see fig 6) which would give very long overtake lengths according to (3-8) However, in shallow water the celerity increases with the wave height, which affects C_1 more than C_2 Therefore, the denominator in (3-8) is increased remarkably, causing the overtake length to decrease, if C_1 increases only slightly So in order to find the correct value of L_{ov}, one must insert the correct (wave height adapted) values of C and L in (3-8). In fig. 9 the 3rd order Stokes theory [34] was used to find L_{ov2}/L_1 for $H/h = 0.4$, but a cnoidal theory may perhaps work out better In view of the difficulty to measure the exact celerity of a wave, especially when secondary waves are present, the argument may be reversed in that from (3-7) and from the exactly-measurable overtake length the proper wave celerity can be determined:

$$C_1 = L_{ov2}C_2/(L_{ov2} - L_2) \quad . \ .(3\text{-}10)$$

The first harmonic amplitude a_1

From fig 2 it appears that L_{ov1} is equal to L_{ov2} When the theoretical expressions for C_1, C_*, L_1 and L_* are substituted in (3-4), it follows that $L_{ov1}/L_1 = L_2/(L_1-2L_2)$, so that indeed $L_{ov1} = L_{ov2}$ The experiments (fig 2) show that $a_{1\ max}$ is always found near the wave board; the theory [20] is not clear on this point In order to determine A_1 and A_* from the experimental values of a_{1max} and a_{1min}, the wave reflection must be taken into account, because reflection alone also causes a certain variation in a_1 With respect to A_1, reflection does not interfere and the normal formula holds:

$$A_1 = (a_{1max} + a_{1min})/2 \quad . \quad (3\text{-}11)$$

According to [17] the reflection coefficient is about 5 %, and especially for Ur < 28 the reflection appeared to be the dominating feature, causing much scatter. So only for Ur > 28 the values of A_* have been determined as

$$A_* = (a_{1max} - a_{1min})/2 \qquad - 0\ 05\ A_1 \quad .. \quad (3\text{-}12)$$

Fig 10 shows for the larger values of h/L_1 a remarkable discrepancy between $A_{*experim.}$ according to (3-12) and A_{*theor} according to (3-3) The reason may be that the magnitude of A_*, which is only a few mm, is small compared to the disturbing influence of reflection

The third harmonic amplitude a_3

From the experimental results (fig 2) it appears that a_3 has a similar spatial behaviour as a_2 Indeed, when the theoretical values for C_1, C_*', L_1 and L_*' are substituted in (3-4), the result is again that $L_{ov3} = L_{ov2}$, so

$$L_{ov1} = L_{ov2} = L_{ov3} \quad . \quad (3\text{-}13)$$

One complication appears in the behaviour of a_3 For the higher Ur-values (fig 2, run 30), there seems to be an extra, smaller overtake length, apparently caused by yet another wave with period T/3 This might be a third order free wave, generated by the wave board in analogy with Fontanet's wave, but no information exists with respect to such a wave So a certain error must be accepted in determining the amplitudes A_3' and A_*' according to

$$A_3' = (a_{3max} + a_{3min})/2 \quad . \quad (3\text{-}14)$$

and $A_*' = (a_{3max} - a_{3min})/2 \quad . \quad (3\text{-}15)$

For the lower Ur-values, a_3 is so small (order 1 mm) that a further analysis is useless in view of the scatter. Therefore, only those results were used in (3-15) for which $A_3' \geqslant 2$ mm. A reasonable agreement is shown to be present in fig. 11 between $A_{*}'{}_{experim.}$ and $A_{*}'{}_{theor.}$, although there is considerable scatter. The ratio A_{*}'/A_3' has been plotted in fig. 12; it displays a similar trend as A_2/A_2' in fig. 8.

The fourth harmonic amplitude a_4

The harmonic analysis was done for four harmonics, but a_4 was so small that it has not been plotted. As an average value, it may be stated that the magnitude of a_4 was about 50 % of a_3.

3.5 The wave form

The presence of a number of waves with different celerities, predicted by the combined theories and confirmed by the experiments, has as a consequence that the wave form varies from place to place The wave form at any place is predictable if the amplitudes, the celerities and the initial phase-angles of all participating waves are known. From section 3 4 it follows that this is indeed the case to a certain degree of accuracy, especially for the celerity and the initial phase angles An example of a resulting x-t diagram for the participating wave crests is given in fig 13 for $h/L_1 = 0.10$, with the following relative celerities (compare fig 6):

$C_2/C_1 = 0.821$, $C_{*}/C_1 = 0.695$ and $C_{*}'/C_1 = 0.868$

Near the wave board, indicated as place no 1, the six waves are phased as follows:

$$\left.\begin{array}{l} A_1 \text{ and } A_{*} \text{ are in phase} \\ A_2' \text{ and } A_2 \text{ are } 180^\circ \text{ out of phase} \\ A_3' \text{ and } A_{*}' \text{ are } 180^\circ \text{ out of phase} \end{array}\right\} \text{ place no 1, } x = 0 \qquad \ldots\ldots (3\text{-}16)$$

With increasing x the initially symmetric wave form loses its symmetry, because the phase-relation (3-16) changes But as soon as the crest of A_2 and A_{*}' has just been overtaken by the corresponding harmonic components of the basic wave, another symmetric wave results, quite different from (3-16).

$$\left.\begin{array}{l} A_1 \text{ and } A_{*} \text{ are } 180^\circ \text{ out of phase} \\ A_2' \text{ and } A_2 \text{ are in phase} \\ A_3' \text{ and } A_{*}' \text{ are in phase} \end{array}\right\} \text{ place no 2, } x = L_o/2 \qquad \ldots\ldots (3\text{-}17)$$

Because the basic wave is faster than any other wave, firstly some undisturbed basic waves will pass along a certain point far enough from the wave board Then, as the slower waves reach this point one by one, the wave form will be unstationary for some time. Only after the arrival of the slowest wave, a new stationary wave develops, now containing all disturbing waves together with the basic wave [3] So the basic wave of permanent form is always present, but generally not in an explicitly visible form. Starting for example from the conditions $h/L_1 = 0.10$, $H/h = 0.36$ and $T_1 = 1.56s$, the following amplitudes result from section 3.4:

$A_1 = 3.82$ cm		
$A_2' = 1.72$ cm		3rd order Stokes wave, basic wave
$A_3' = 0.79$ cm		
$A_2 = 1.24$ cm	-	2nd harmonic free wave (Fontanet)
$A_* = 1\ 44$ cm		interaction waves (Kravtchenko and Santon)
$A_*' = 0.73$ cm		

Based on the experimental results in figs 10 and 11, A_* and A_*' have been reduced to 0,965 cm and 0,635 cm, respectively With these amplitudes, and their phase relationships from fig 13, various wave forms were reconstructed in fig. 14 for place no. 1, place no. 2 and 9 intermediate locations; for comparison also the undisturbed basic wave has been plotted Obviously, the crest of the composed wave form does in general not coincide with the crest of the basic wave, nor is a secondary crest identical with the crest of one of the participating smaller waves Two more comments may be made on figs. 13 and 14 Firstly, the total wave height varies and has a minimum at place no. 1 and a maximum near place no 2 Secondly, a horizontal section through the x-t diagram results in an instantaneous wave surface which shows in general no regular spatial recurrence system, because the overtake length is in general not a multiple of the various wave lengths involved

Of all locations, places no 1 and 2 display the most characteristic wave forms, which will be analyzed in some detail

At place no 1, $x = k\ L_{av}$, where $k = 0,1,2,3, \ldots$. Here practically a sine wave results, consisting mainly of A_1 and A_*, a_1 being maximum; a_2 and a_3 reach their minimum values. Of all measured values, the following averages result:
$a_{2min}/a_{1max} = 0.10$ and $a_{3min}/a_{1max} = 0\ 027$.

At place no. 2, $x = (k + 1/2).L_{ov}$ where $k = 0,1,2,3,\ldots$. Here the resulting wave is far from sinusoidal, a_1 being minimum and both a_2 and a_3 being maximum. Of all measured values in this place, the ratios a_{2max}/a_{1min} and a_{3max}/a_{1min} have been plotted in fig. 15 as a function of Ur Two lines have been drawn to represent these points and an additional line in agreement with section 3.4 represents the fourth harmonic. On the basis of these lines, and taking into account the relevant phase relationship (3-17), wave forms were constructed for various values of Ur (fig. 16) For Ur $\geqslant 13$ a secondary crest exists in the trough centre, which is in accordance with Madsen's value of $4\pi^2/3$ [23]. These "reconstructed" wave forms may be compared with experimental wave forms recorded in place no. 2, presented in fig. 17. For both figures 16 and 17 the relative height of the secondary crest increases with Ur as shown in fig. 18, where also some of Galvin's results [15] have been plotted. The discrepancy between the reconstructed and the direct experimental values of H'/H_{tot} may partly be due to the fact that, especially for higher Ur-values, 5th and higher harmonic components do participate in fig. 17, but not in fig 16

4 Influence of secondary waves on a horizontal sand bed

Considering sand transport, the behaviour of the orbital motion near the bed is of more direct relevance than the fluid surface [1] Simultaneous measurements of the wave profile and the orbital velocity near the bed carried out in a long wave channel with a fixed horizontal bottom, revealed that their behaviour is virtually the same (fig. 19). This is substantiated by a harmonic analysis of the orbital velocity, plotted as a function of the distance from the wave board (fig. 20) The regular spatial behaviour of the orbital velocity field must have as a consequence that the onshore-offshore sand transport varies spatially, too. In order to check this, tests were run in a 1.20 m wide wave channel, with a smooth horizontal concrete bottom over the first section of 2 m from the wave board. A horizontal flat sand bed extended over the next 9 m, terminating in a 1 on 20 sloping spending beach Fig 21 shows the experimental conditions and the resulting bed forms. Ur varied from 40 to 57 The wave lengths produced in the sand bed are very clearly equal to the respective overtake lengths. In test T73-1, a bar-trough system with a smaller reference length, caused by wave reflection, is superimposed on the large scale bar system. Looking back from these results to fig. 1, the cause of the undulating bed profile is clear now, realising that L_{ov} is appr. 3.20 m for the given conditions. These bars, once formed from an initially flat bed, may on their turn provoke new secondary waves, which all interfere with the breaker type and breaker location, the water circulation, and the morphological development in the surf zone and on the beach. On certain occasions, the undulations have even been seen to develop into large breaker bars. Although the drastic influence of the disturbing waves, via the formation of bars and troughs, on the beach profile development was quite clear in the case under consideration, it is also clear that such an influence may be present without recognizing it as such, so that a general warning seems useful here For instance, from Bagnold's clear description [2, p461 etc] it can safely be concluded that he was confronted with similar phenomena in his tests On the other hand, the interrelation between secondary waves, bar systems and beach behaviour seems not to be restricted to model experiments [22], so that the present study may be of a more general application. Therefore, the bed profiles in fig. 21 have been converted into a rate of transverse sand transport by using the sand balance Fig 22 shows the result for T73-2, where also the wave form is presented for various locations For x = 5 m, one overtake length from the wave board, the sinusoidal motion has no preference for a certain direction, and consequently the sand transport rate is zero In general, the magnitude and the direction of the sand transport appears to be remarkably dependent on slight differences in the wave form.

5 Experiments in a pulsating water block

5 1 Apparatus and procedure

In order to study the effect of higher harmonic components in the orbital motion on the transverse sand transport, some preliminary tests were run in a very simple and small pulsating water block. This apparatus, originally an idea of Silvester [29, 33] consists of a bottomless perspex box, forced by a programmable wave generator to oscillate over a bed (fig. 23). In this bed, a sand bed and two sand traps are installed. The block moves under water in a perspex tank. Before each test, the sandbed is smoothed and made flush with the fixed bed. The tests are divided into periods of 5 minutes, after which the trapped sand is collected in order to define the net sediment transport rate and direction. Ripples are formed in the first 1 or 2 minutes, sometimes staying in fixed positions, sometimes moving, but not necessarily in the same direction as the net sediment transport

5 2 Test results

Throughaut the tests, a basic period of 1 40s was used, with an amplitude $A_1 = 0.05$ m. In the first series of tests, of 15 min. each, the influence of the 2nd harmonic companent A_2 was investigated With a sinusaidal movement, an almost zero net transport is found, as expected (fig. 24) By adding the second harmonic component with $A_2/A_1 = 5$ °/o and 10 °/o, respectively, and with the same phase relationship ($\varphi_2 = 0$) as occurs in a 2nd order Stokes wave, a marked influence appears on the transport pattern. By visual observation, this was caused by the circumstance that vortex formation and behaviour is very sensitive to the form of the orbital velocity. The correspondig velocity is also shown in fig 24

In a second series of tests of 25 min. each, a second harmonic component with $A_2/A_1 = 20$ °/a was added, but naw with a phase relatian varying fram $\varphi_2 = 0^{\circ}$ (like in a 2nd order Stokes wave) to $\varphi_2 = 90^{\circ}$ (fig 25) The resulting sand transports for twa sand diameters are presented in fig. 26 Clearly, these are all only quite preliminary results, both qualitatively and ouantitatively speaking, and further tests with better equipment are planned Nevertheless, it seems that this very close dependence of the direction and the rate of transverse sand transport on slight variations in the wave form and the arbital velocity field, is important for any basic study of beach profile development

6 Suppression of secondary waves

In order to suppress the parasitic waves, Biésel and Suquet [4] suggested already in 1951 to use a more realistic motion of the wave board than a simple harmonic oscillation. Work along this line has recently led to encouraging results [9,23] . Also a different method may be thaught of, which has provisianally been tested In this method, the experimental fact is used that a bar or sill, placed on the horizontal bottom of a wave channel, generates free higher harmonic waves when regular waves proceed aver it. [16,19,27] . Na theory being available on this subject, a trial and error method was used in order to find a sill of such dimensions and on such a location, that it would produce a second harmonic free wave of the same height and exactly 180° out of phase with respect to the Fontanet wave. One af the results is presented in figs. 27 and 28, giving the characteristics of the sill, the wove forms and the hormonic analysis for two runs with the same rectongular sill on two different locations. Without o sill, a secondary wave was clearly visible In bath cases the influence af the sill - with a thickness af anly 0 1 h - was surprisingly great In run D, a virtually permanent wave farm resulted dawnwave fram the sill, while a distinct secandary wave was still visible between the wave flap and the sill. This aptimum result was obtained with the far end af the sill at a distance of 3 m fram the wave flap, carresponding ta L_{av}. When the sill was maved to different lacatians, the resulting wave farm immediately deteriorated. The warst result was abtained far run B, where the sill was shifted aver a distance af $L_{av}/2$. This suggests that the avertake length is an impartant parameter in determining the aptimum sill lacatian, althaugh the physical pracess is nat well understaad. By laaking at it, a certain analagy seems ta exist with the effect af a bulb an the waves generated near a ship's baw.

7 Conclusians

a. Secandary waves may be generated by the wave baard, ar may be pravaked by a sill, bar, slape, ar by breaking waves
b Secandary waves, generated by the wave baard, may adequately be described by cambining the thearies af Fantanet [14], and Kravtchenka and Santan [20].
c. Secondary waves may have a very pranaunced influence on beach profile farmation, althaugh this influence may be quite difficult ta recagnize as such.
d The rate and the direction af the transverse sand transport under waves is very delicately dependent af the wove farm, i.e the form of the orbitol velocity field.
e An adequately designed sill af rectangular crass-sectian may be used ta suppress the Fantanet wave

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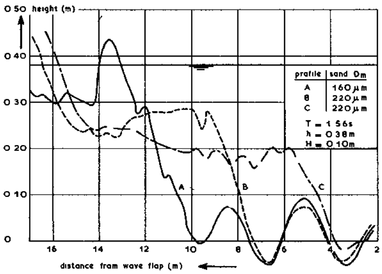


Figure 1: Beach profiles affected by secondary waves

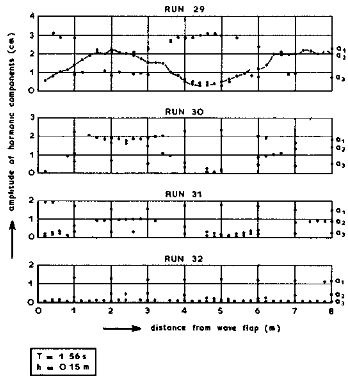


Figure 2: Typical variation of harmonic amplitudes

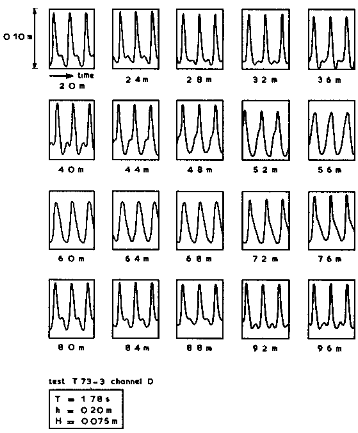


Figure 3: Typical variation of wave forms

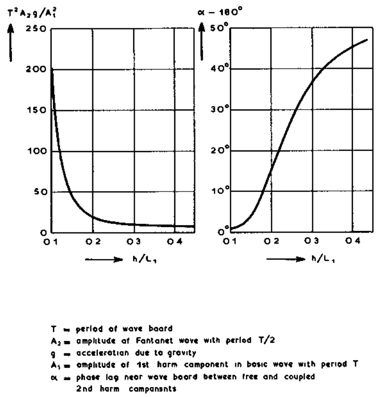


Figure 4. Characteristics of Fontanet wave

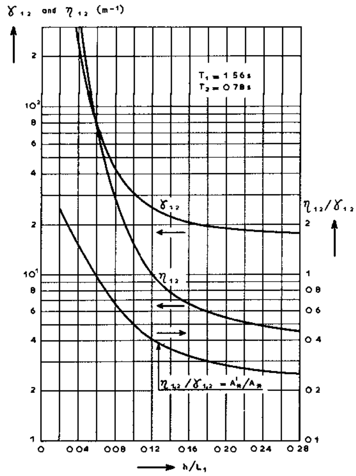


Figure 5: Coefficients of interoction according to Kravtchenko and Sonton

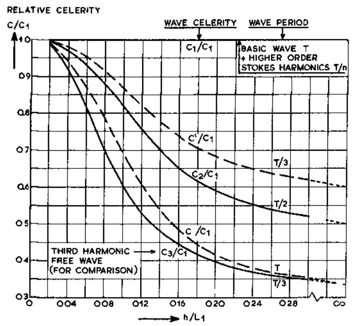


Figure 6: Relative wove celerities; influence of wove height neglected

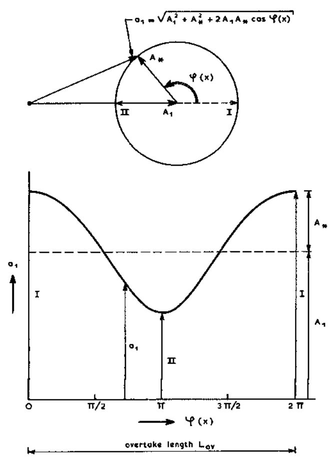


Figure 7: Vector summotion of A_1 ond A_* ond resulting spotiol behaviour af a_1

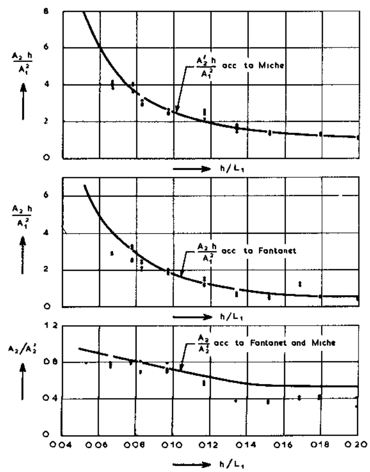


Figure 8: Comporison of theory ond experiments for 2nd hormonic components A_2 and A_2'

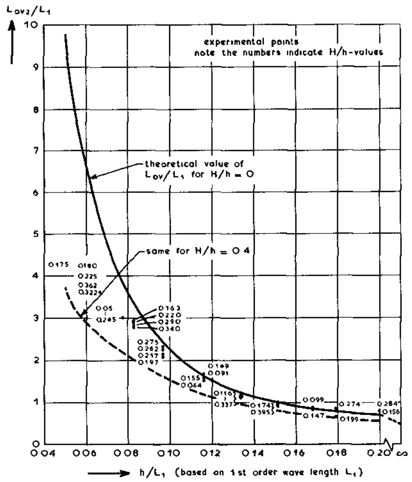


Figure 9: Dimensionless overtake length L_{ov2}/L_1

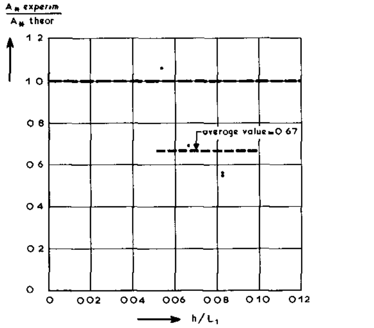


Figure 10: Comparison of theory and experiments for 1st harmonic component A_*

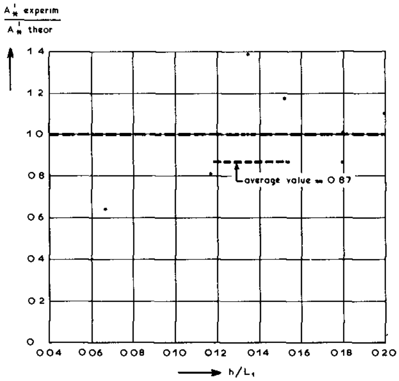


Figure 11: Comparison of theory and experiments for 3rd harmonic component A_*'

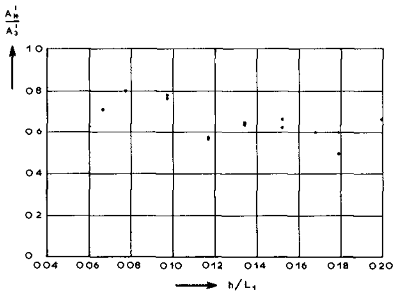


Figure 12: Experimental ratio A_*'/A_3'

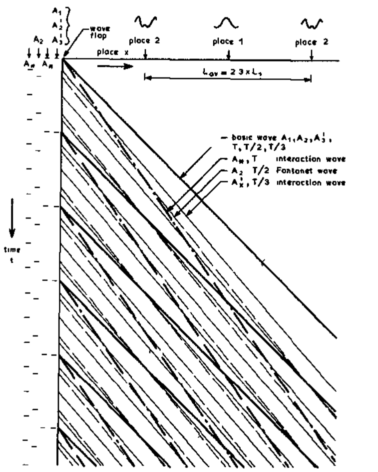


Figure 13: x-t diagram af participating wave crests far $h/L_1 = 0.10$

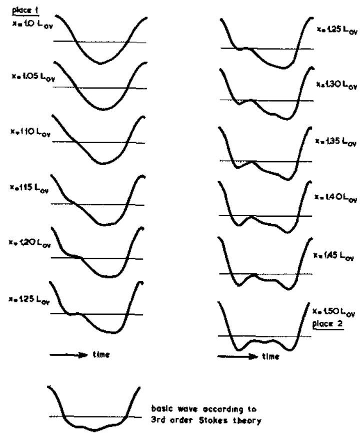


Figure 14: Thearetical wave farms far $h/L_1 = 0.10$ and $A_1/h = 0\ 18$

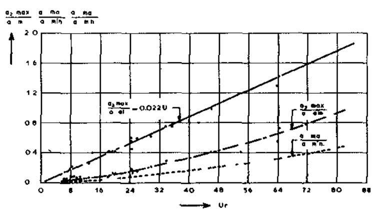


Figure 15: Relative amplitudes at $x = (1/2 + k)\ L_{av}$

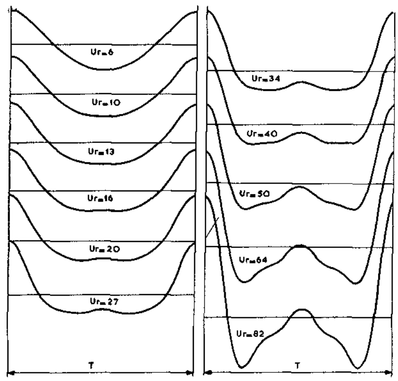


Figure 16: Wave forms at $x = (1/2 + k)\ L_{av}$, recanstructed fram figure 15

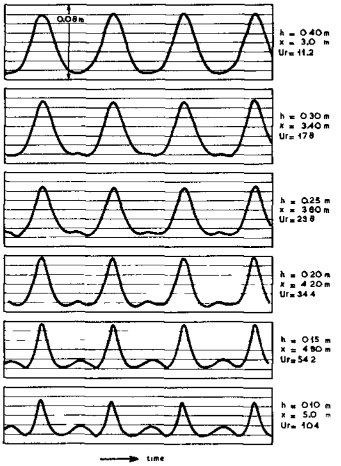


Figure 17: Experimental wave forms for $T = 1\ 92$ s at $x = 1/2\ L_{ov}$

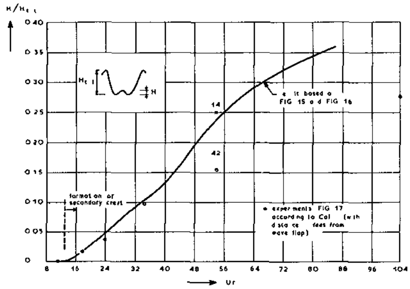


Figure 18: Relotive height of secondory crest at $x = (1/2 + k)L_{ov}$

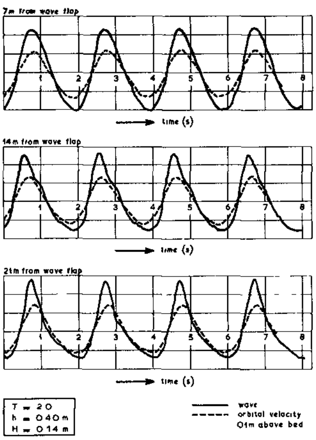


Figure 19: Simultaneous wove and orbitol velocity

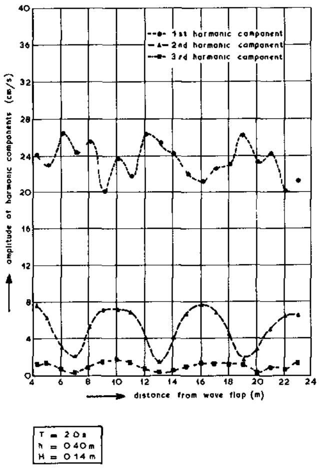
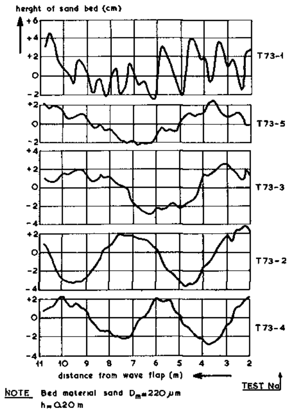


Figure 20: Spatiol behaviour of hormonic omplitudes in orbital velacity



Test No	T	H	Duration of test	L_1	L_{ov}
T 73 - 1	1 98 s	6 4 cm	7 hrs	2 68 m	8.0 m
T 73 - 2	1 60 s	8.0 cm	6 hrs 30min	2 12 m	5 0 m
T 73 - 3	1 78 s	7 6 cm	7 hrs	2 38 m	6.2 m
T 73 - 4	1 48 s	8 4 cm	4 hrs	1 95 m	3 9 m
T 73 - 5	1 88 s	7 0 cm	7 hrs	2 54 m	7 0 m

Figure 21: Influence of secondary waves on an initially flat horizontal bed

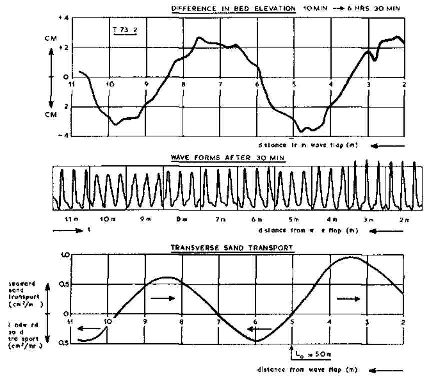


Figure 22: Typical wave forms and resulting sand transport

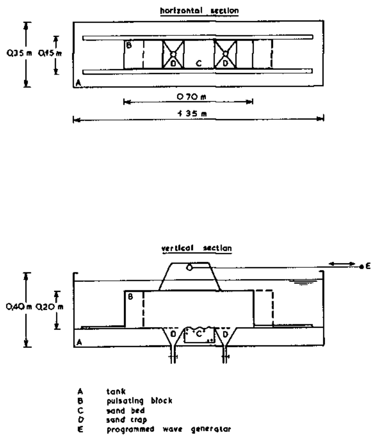


Figure 23: Pulsating water block

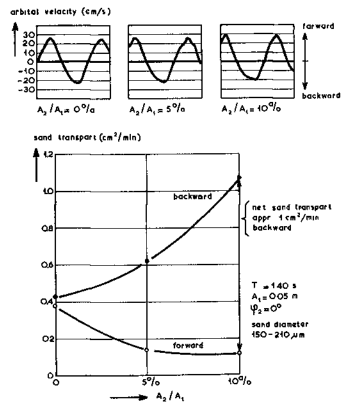


Figure 24: Influence of 2nd harmonic component on orbital velocity and sand transport

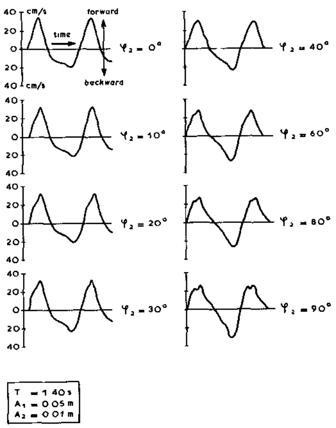


Figure 25. Velocity of pulsoting block with vorioble φ_2

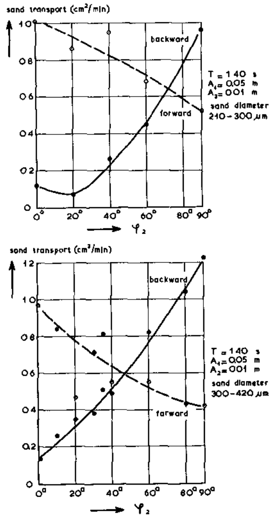


Figure 26· Influence of phase of 2nd harmonic component on sond transport

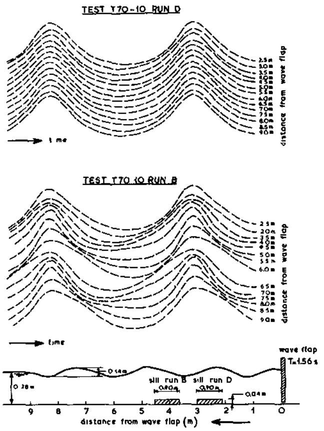


Figure 27: Typicol wove form voriotion os influenced by locotion of low sill

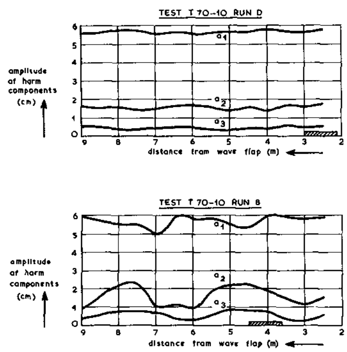


Figure 28: Hormonic omplitude voriation os influenced by locotion of sill