CHAPTER 122

DESIGN OF DISTORTED HARBOR WAVE MODELS

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ABSTRACT

An investigation of errors incurred by scale distortion for fixed-bed harbor wave models has been performed and results of this study applied to the design, construction, and operation of a model of Los Angeles and Long Beach Harbors. Objectives of the model investigation must be clearly prescribed in order that each phenomenon affected by scale distortion can be analyzed. In some cases, if study objectives are too broad, a distorted model may be incompatible with these objectives; however, it will be found that in many circumstances a distorted model will produce valid data with considerable time and cost savings in construction and operation.

INTRODUCTION

The design of scale model harbors for the investigation of wave phenomena can be divided into two basic categories consisting of the type problem under study. These two areas are (1) harbor oscillations which are of major importance for large commercial harbors due to the potential for moored ship surging and (2) excessive wave heights from the normal wind waves which are of paramount importance for small-craft harbors and of some importance for commercial harbors. There are certainly other types of harbor problems such as tidal flushing, transport of cold and warm water discharges, and tsunami effects that warrant investigation in distorted wave models; however, the scale model design for this class of problem is considerably more elementary because of the validity of the long-wave assumption for tides and tsunamis. The two classes of problems under investigation in this paper encompass the intermediate wave period range for which neither the long nor short wave assumption is valid.

Quantitative data can be obtained from distorted harbor wave models provided the model design is conducted on a comprehensive basis. Some of the more important wave phenomena which may be effected either directly or indirectly by model scale distortion are refraction, diffraction, energy transmission through breakwaters and other structures, reflection from the shoreline and coastal structures, viscous friction at the bottom, harbor resonance, reflections within the harbor, breaking height, breaking depth, wave steepness, longshore currents, runup, and mass transport. It becomes immediately obvious that study objectives must be well defined if one has any hope of intelligently analyzing those phenomena which are important. An example of the application of the principles enumerated in this paper are applied to a model of Los Angeles and Long Beach Harbors.

DESIGN CONSIDERATIONS

This section discusses some of the more important phenomena in design considerations for harbor wave models. It always is desirable for wave action models to be constructed to an undistorted scale if time, cost, and space allow.

Viscous Friction Effects

The viscous dissipation of energy at the model bottom can become an extremely important consideration in models involving large prototype areas of relatively shallow water. In the United States this can be a problem in models of areas on the East and Gulf Coasts and in some areas in the Great Lakes. In the prototype, viscous dissipation of energy at the bottom is practically nil (almost always being less than one percent); however, if the model scale is too small and propagation distances are large, then this can result in a considerable scale effect in the model. Since scale distortion acts to improve this particular scale effect (use of a horizontal scale ratio smaller than the vertical scale ratio will reduce the relative amount of excessive energy dissipation in the model), the greater the distortion the less the scale effect. Computations on the viscous dissipation of energy at the bottom can be based on the original work of Keulegan (1950). The effect of viscous dissipation of energy at the bottom on the wave height is given by

$$H_2 = H_1 \, e^{-\int_0^X \delta dX} \tag{1}$$

over a propagation distance of X where H_1 is the wave height at X = 0 and H_2 is the height after the wave has traveled a distance, X, in water of depth, d, and

$$\delta = \frac{5\Pi \sqrt{\Pi \nu T}}{L^2 \left\{\sinh \frac{4\Pi d}{L} + \frac{4\Pi d}{L}\right\}} \tag{2}$$

Actually the calculation of viscous energy dissipation must be performed along wave orthogonals. It is possible that complicated offshore topographies could result in different amounts of viscous dissipation of energy near the shoreline. This must be analyzed in detail for each specific case.

Wave Refraction

Refraction effects are a function of d/L only; thus, if it is required that $(d/L)_m = (d/L)_p$, then linear wave refraction is correctly reproduced in the model and the result is that the ratio of the model to prototype wave period is proportional to one divided by the square root of the vertical length scale. Any other scaling method will result in errors in the wave refraction pattern and coefficients. Therefore, the choice of any other scaling method will necessitate the detailed investigation of errors introduced into the refraction pattern. This is accomplished by computing wave refraction patterns for the undistorted case and repeating the computation for the distorted model scales and the scaled wave periods. A comparison of the wave front positions and the refraction coefficients will yield the desired information.

Wave Diffraction

Similitude of linear wave diffraction effects from model to prototype requires that $(x/L)_m = (x/L)_p$ where x is a horizontal distance and L is the wavelength. The scale effect present in the diffraction pattern for a distorted scale model depends upon the similitude relation chosen to determine the wave period. If diffraction is the major phenomena of interest then the diffraction effects can be preserved when the time scale is proportional to the square root of the horizontal length scale. Of course, the longer the wavelength the less important are diffraction scale effects.

Wave Reflections

Two types of wave reflections should be considered in analyzing the effect of model distortion. Reflections from breakwaters or other structures and from the shoreline comprise one type and the other is the cumulative reflection from bottom slopes as the wave propagates from deep water to the shoreline. The scale effect arising from increased reflections from distorted bottom slopes is a function of the existing bottom slope. If the bottom slope is very gradual, there will probably be an extremely small scale effect; however, if the bottom slope is relatively steep or the distortion too large, quite significant scale effects can result. The latter reflection problem mentioned above occurs from the position of the wave generator to the point of breaking nearshore.

The magnitude of waves reflecting from slopes is a function of the slope, the wave steepness, and the type of material comprising the slope. An estimate of the reflection coefficient from the breakwater or the shoreline can be obtained, to a first order of approximation, by the classicial theory of Miche. Keeping in mind that the theory is applicable only for small reflections and small amplitude waves (linear theory) and realizing that the theory over-estimates the reflection coefficient (at least when it is large), application of the theory can determine if scale effects relative to reflections must be contended with. The reflection coefficient R is given by

$$R = \frac{H_r}{H_i} = \rho R' = \frac{H_o/L_{o\,max}\ \text{theory}}{H_o/L_o} = \rho\,\frac{2\alpha}{\Pi}\,\frac{\sin^2\alpha}{\Pi}\,\frac{1}{H_o/L_o} \qquad (3)$$

where α is the average beach slope in radians and ρ is an empirical coefficient estimated to be 0.8 for smooth impermeable slopes (such as a fixed-bed wave action model). Analysis of the cumulative wave reflection during propagation over the variable bottom topography can be estimated from the theory of Rosseau (1952). Usually there will be a negligible reflection scale effect due to this phenomenon.

Wave Breaking Location

If the time scale for a distorted scale model is determined on the basis of obtaining the correct refraction pattern (requiring that $d_m/L_m = d_p/L_p$), the ratio of a vertical to a horizontal length will be correct presuming each is indeed to scale. The implication is significant; that is, the wave steepness ratio ($H_m/L_m = H_p/L_p$) also is correct provided that the wave height is modeled correctly. This means that when all factors affecting the wave height (refraction, shoaling, bottom friction, reflections, diffraction, and bottom percolation) are either modeled correctly, or made to be compensatory, the wave steepness will be modeled correctly. The effects of these factors, briefly stated, are as follows:

a. Refraction. Refraction effects are modeled correctly by requiring that $d_m/L_m = d_p/L_p$.

b. Shoaling. Wave shoaling is a function of d/L only; thus, shoaling is modeled correctly if refraction is correctly modeled.

c. Bottom friction. The greater the distortion factor the nearer one comes to modeling the correct prototype bottom friction. It can be demonstrated that an increase in the incident wave height can compensate for the excessive loss in wave energy due to bottom friction. Thus, by the time the model waves reach the shoreline their height can be correctly reproduced.

d. Reflections. The problem of wave reflection was discussed and it was ascertained that an estimate of the increased wave reflection from the shoreline and from offshore structures could be made. These scale effects, if significant can be compensated for by performing some wave flume tests to design the model breakwaters and shoreline from some material which will yield the proper reflection coefficient. It also was ascertained that reflection scale effects from the underwater topography were probably negligible but could be calculated and if significant could be compensated for by an increase in the incident wave height at the generator.

e. Diffraction. Since, to the first order of approximation, diffraction effects are a function of the ratio of a horizontal distance to the wavelength, the requirement that $d_m/L_m = d_p/L_p$ to produce the correct refraction and shoaling effects, introduces an error into the diffraction pattern. Each case must be analyzed in detail to determine the magnitude of this error and to ascertain if it is acceptable. Should diffraction effects be the phenomenon of major importance, then the time scale can be based on similitude of linear wave diffraction.

f. Bottom percolation. Bottom percolation acts to reduce the wave height in the same manner as bottom friction; however, this effect is less than the attenuation due to bottom friction in the prototype and is negligible in most instances.

In view of the above considerations, one realizes that the wave steepness at the breaking location will be correct provided the incident wave is increased at the wave generator to compensate for any excessive model wave height attenuation due to bottom friction and bottom reflection scale effects. Thus, one comes directly to the conclusion that wave breaking characteristics will be correct (both depth of breaking and angle of breaking) regardless of whether or not the breaking is depth dependent or steepness dependent. The only factor which might cause an error is that breaking is somewhat dependent on the bottom slope (Street, 1966); however, for the slopes and distortions usually considered in harbor wave models there will be a small error due to the slope dependence of the breaking location.

Longshore Velocity

In order to analyze the possible scale effect due to model distortion on the longshore velocity, it will be assumed that for any scale model in which longshore velocity is of paramount importance, the time scale must be predicated on the relation $(d/L)_m = (d/L)_p$ which has as the consequence of producing the correct breaking location as previously discussed. We will resort to the following analytically derived equation given by Johnson and Eagleson, in Ippen (1966):

$$V_L^{\,2} = \frac{3}{8} \; \frac{g\, H_b^{\,2}\, n_b}{d_b} \; \frac{\sin\alpha \, \sin\Theta_b \, \sin 2\Theta_b}{f} \tag{4}$$

where

V_L = longshore velocity component

H_b = breaking wave height

n_b = ratio of group to phase velocity at the breaker

d_b = depth at breaking

Θ_b = angle of breaking

α = average beach slope

f = Darcy-Weisbach coefficient

f is computed using the Kármán-Prandtl resistance equation for steady uniform flow in a rough conduit

$$f = \left[2 \log_{10} \frac{d_b}{k_e} + 1.74\right]^{-2} \tag{5}$$

where k_e is the absolute bottom roughness. k_e for natural sand is assumed to be 0.0033 ft, 0.0010 ft for sheet metal or smooth cement, and 0.0208 ft for pea gravel. For the model computation we will assume k_e = 0.0010 ft since it is constructed of smooth cement.

Thus the longshore velocity can be written as

$$V_L = (\tfrac{3}{8})\ [g\ (\frac{H_b}{d_b})\ H_b\ n_b]^{\frac{1}{2}}\ \frac{[\sin\alpha\ \sin\Theta_b\ \sin 2\Theta_b]^{\frac{1}{2}}}{[2\log_{10}\frac{d_b}{k_e} + 1.74]^{-1}} \tag{6}$$

or

$$V_L = (\tfrac{3}{8})^{\frac{1}{2}}\ [g\ \frac{H_b}{d_b}\ H_b\ n_b]^{\frac{1}{2}}\ [2\log_{10}\frac{d_b}{k_e} + 1.74]\ [\sin\alpha\ \sin\Theta_b\ \sin 2\Theta_b]^{\frac{1}{2}}$$

$$\frac{V_{L_m}}{V_{L_p}} = \frac{[H_b\ n_b\ \sin\alpha]_m^{\frac{1}{2}}}{[H_b\ n_b\ \sin\alpha]_p^{\frac{1}{2}}}\ \frac{[2\log_{10}\frac{d_b}{k_e} + 1.74]_m}{[2\log_{10}\frac{d_b}{k_e} + 1.74]_p} \tag{7}$$

since $(H_b/d_b)_m = (H_b/d_b)_p$ and $(\Theta_b)_m = (\Theta_b)_p$ assuming that the angle of breaking and breaking height and depth are modeled correctly as discussed in previous section. Thus, the scale effects can be analyzed on the basis of the variation of the second term below from 1

$$\frac{(V_L)_m}{(V_L)_p} = \frac{(H_b)_m}{(H_b)_p}^{\frac{1}{2}}\ \frac{(n_b\ \sin\alpha)_m^{\frac{1}{2}}}{(n_b\ \sin\alpha)_p^{\frac{1}{2}}}\ \frac{(2\log_{10}\frac{d_b}{k_e} + 1.74)_m}{(2\log_{10}\frac{d_b}{k_e} + 1.74)_p} \tag{8}$$

since by the scaling requirement that $d_m/L_m = d_p/L_p$ one obtains that the velocity ratio is determined by the square root of the vertical scale ratio (i.e. $(H_{b_m}/H_{b_p})^{\frac{1}{2}}$). Thus, if we compute the variation of the second term above from 1 as a function of model and prototype wave characteristics, we can ascertain the model scale effects due to distortion.

It can be shown that

$$n_b = \frac{1}{2}\ [1 + \frac{2k_b\ d_b}{\sinh\ 2k_b\ d_b}] = \frac{1}{2}\ [1 + \frac{\frac{4\Pi d_b}{L_b}}{\sinh\ \frac{4\Pi d_b}{L_b}}] \tag{9}$$

Thus, since our scaling relation requires that $d_m/L_m = d_p/L_p$ for reproduction of wave refraction effects,

$$\frac{(n_b)_m}{(n_b)_p} = 1 \tag{10}$$

and, if we let $V_{L_{se}}$ represent the scale effect in the longshore velocity,

$$V_{L_{se}} = \frac{(\sin\ \alpha)_m^{\frac{1}{2}}}{(\sin\ \alpha)_p^{\frac{1}{2}}}\ \frac{(2\ \log_{10}\frac{d_b}{k_e} + 1.74)_m}{(2\ \log_{10}\frac{d_b}{k_e} + 1.74)_p} - 1 \tag{11}$$

The first term represents that portion of the scale effect due to the distortion of the beach slopes, which tends to increase the model longshore velocity. The second term representes that portion of the scale effect due to increased bottom friction, which tends to decrease the model longshore velocity.

Wave Transmission through Structures

One problem of extreme importance for distorted harbor wave models is energy transmission through the protective structures. The structure must be built to the geometrically distorted scale of the model. The only reliable manner in which to insure that no adverse scale effects are present is to perform wave transmission tests through a model test section of the prototype breakwater in a wave flume at an undistorted scale sufficiently large so that no Reynolds number scale effects occur. Usually a scale of 1:20 to 1:30 is sufficient for this purpose. Subsequently, two-dimensional flume tests must be conducted for a cross section of the distorted scale model breakwater in order to insure that approximately the correct transmission coefficient is obtained.

DESIGN OF LOS ANGELES AND LONG BEACH HARBORS MODEL

In the case (Los Angeles and Long Beach Harbors, see fig. 1) under detailed study, the objective was to determine the effect of planned major revisions and improvements to the harbors (channel deepening and construction of additional basins and piers, see fig. 2) in order to insure that satisfactory mooring conditions will obtain with respect to wave and current conditions and their effects on ship surge. Also, it is desired to determine whether the proposed construction plans will increase wave and surge action conditions in the existing harbor areas and whether tidal flushing of the harbor areas will be adversely affected by the proposed expansion. While primary interest is focused on wave periods in the range from 20 seconds to 2 minutes, it is suspected that long-period swell could be a problem for some of the proposed harbor revisions; thus, it is desired to obtain accurate model data for prototype wave periods at least as small as 15 seconds. If possible, it also is desirable to investigate tsunami effects. Since harbor and basin resonance is undoubtedly the major problem, it must be prescribed that wavelengths within the harbor are correct (i.e. relative to the horizontal scale of the model). It should be noted that this requirement will produce some scale effect errors in both the model refraction and diffraction. Wave periods will be based on the following relationship for intermediate water depths:

$$T_m = T_p \left[\frac{\ell_{hm}}{\ell_{hp}}\right]^{1/2} \left[\frac{\tanh \frac{2\Pi}{\Omega} \frac{d_m}{L_m}}{\tanh 2\Pi \frac{d_m}{L_m}}\right]^{1/2} \qquad (12)$$

where, T_m = model wave period

T_p = prototype wave period

ℓ_{hm} = horizontal length scale in the model

ℓ_{hp} = horizontal length scale in the prototype

Ω = distortion

d_m = model depth of the inner harbor

L_m = model wavelength

Refraction diagrams constructed from open ocean directions between west and southeast indicated that significant long-period wave energy can penetrate to the harbor site only from the south. A typical refraction diagram for 60-second prototype waves is shown in fig. 3. The refraction diagram showed a strong convergence zone (caustic) due to the shape of the underwater contours seaward of the harbor site. Since presently available refraction theory is

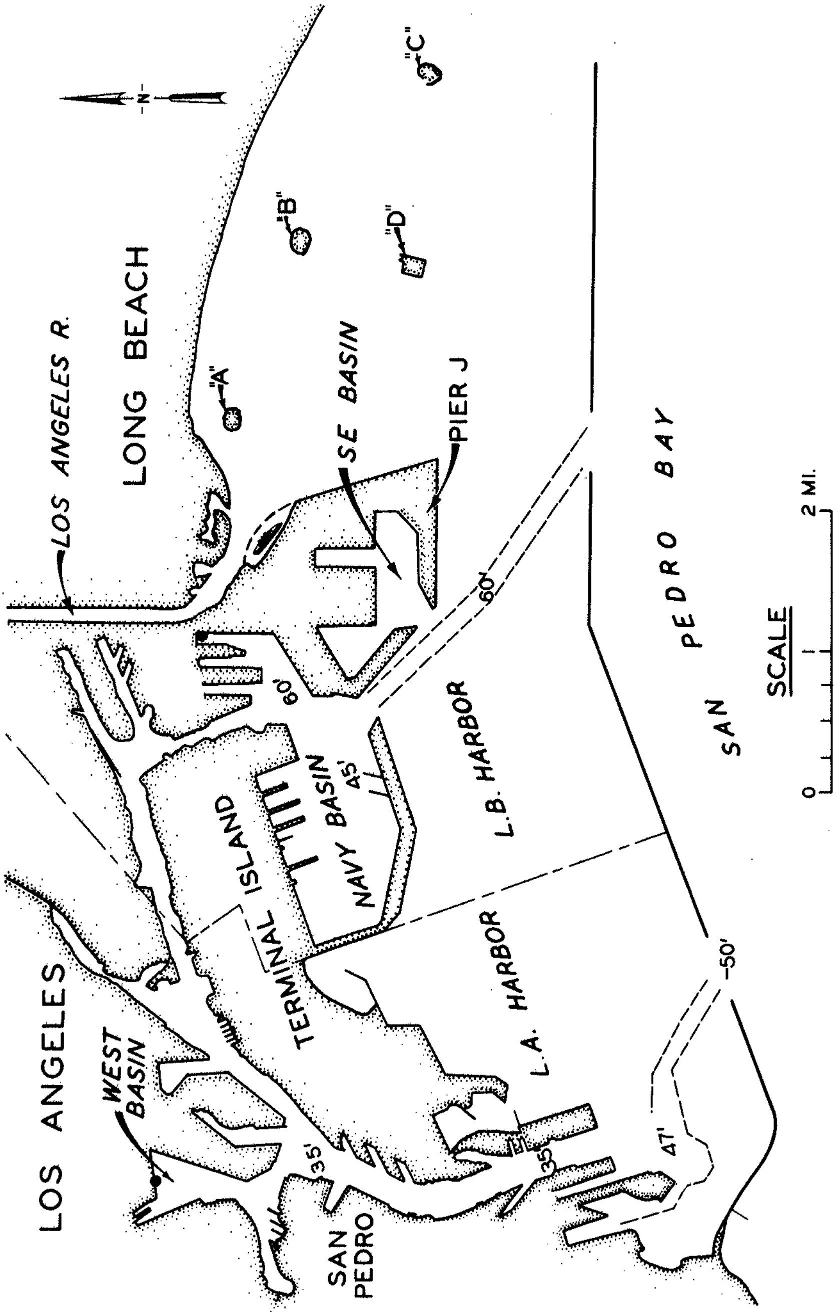

Figure 1. Los Angeles-Long Beach Harbors; Existing Conditions.

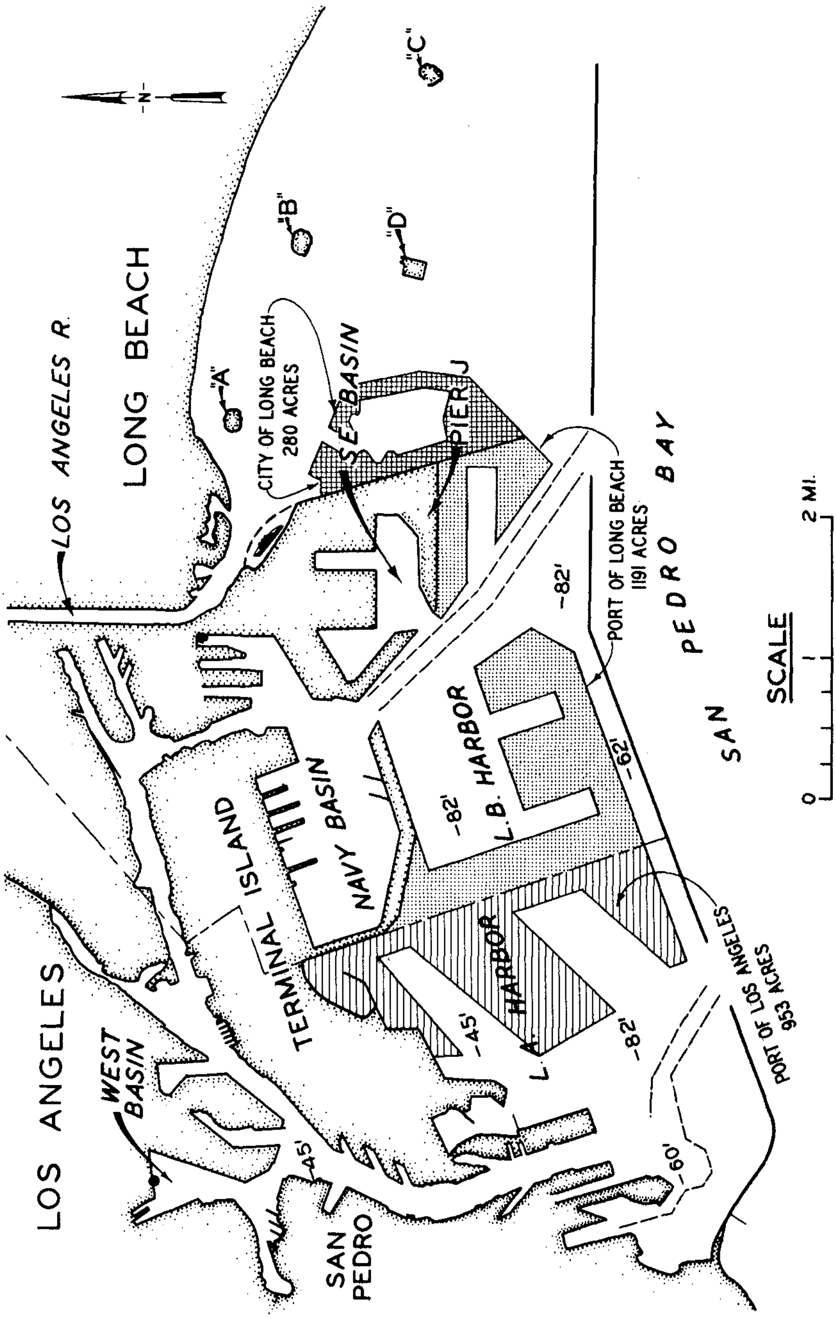

Figure 2. Los Angeles-Long Beach Harbors; Proposed Revisions.

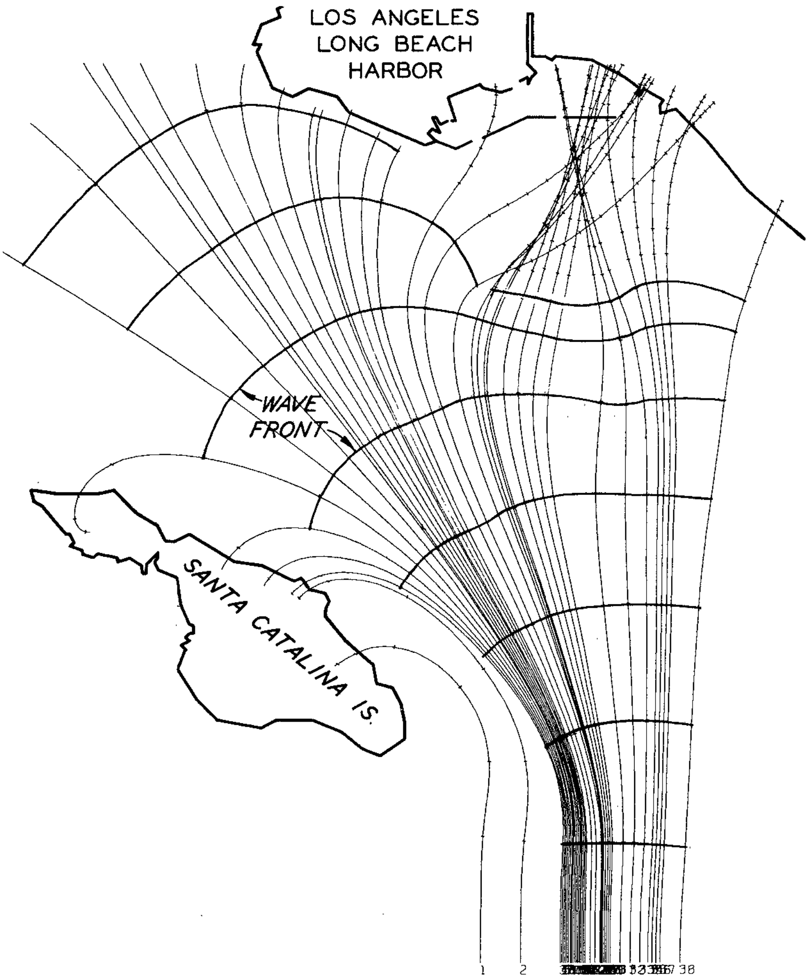

Figure 3. Refraction Diagram for 60-Second Wave Period.

not able to accurately predict what happens in strong convergence zones, it was necessary to reproduce this area in the model so that wave fronts approaching the harbor would be correct. Consequently, underwater contours were reproduced in the model to a prototype depth of 300 feet.

The wave front at the -300 contour (ranging from a straight line for 15-second waves to an "S" shaped curve for 360-second waves) will be reproduced by a 200-foot-long hydraulic piston-type wave generator with time synchronized sections. By varying the individual machine settings, the wave height can be varied along the wave front (to allow for convergence or divergence which has taken place due to refraction seaward of the -300-foot contour).

To evaluate the effects of distortion, refraction diagrams were constructed for representative wave periods for the following cases:

Vertical scale	Horizontal scale	Distortion
1:1	1:1	None
1:100	1:200	2
1:100	1:300	3
1:100	1:400	4
1:64	1:256	4

A comparison of these diagrams indicated no significant differences between the distorted and undistorted cases for wave periods of one minute (prototype) or above. For the shorter wave periods (15-30 seconds prototype) there were some differences in direction and energy content, but adjustments will be made in the initial wave generator position and stroke setting to compensate for these. Model scales of 1:100 (vertical) and 1:400 (horizontal) were selected based on the refraction analysis (distortion = 4).

The model viscous friction effect was evaluated using Keulegan's equation for wave-height attenuation. The computations made are included in the following table:

Horizontal Scale	Vertical Scale	Distortion	H_x for T = 15 sec	H_x for T = 360 sec
1:200	1:100	2	.8925	.9438
1:300	1:100	3	.9642	.9545
1:400	1:100	4	.9833	.9595
1:256	1:64	4	.9816	.9705

These computations indicate that for a given vertical scale, viscous friction decreases as distortion increases because the travel distance is less. For the selected model scales (1:100 vertical and 1:400 horizontal) and the range of wave periods to be tested (15-360 seconds prototype), attenuation due to viscous friction would be from 2 to 4% in the extreme reaches of the model. A small correction for this scale effect can easily be applied to the wave data.

The rubble-mound breakwaters protecting the harbor are relatively porous and considerable long period wave energy passes through the voids of these structures. Two-dimensional tests of typical breakwater sections were conducted at undistorted scales of 1:20 (large enough to have negligible scale effects) to determine the wave transmission and wave reflection characteristics. A typical breakwater section is shown in fig. 4. Two-dimensional tests were then conducted using different sizes of rock in the distorted breakwater section until the correct wave transmission and reflection were duplicated. Figure 5 shows a typical plot of wave transmission versus wave period. It was found that using the vertical scale (1:100) to geometrically scale the rock sizes would produce the correct wave transmission through the breakwater structures. A typical section of the model breakwater is shown in fig. 6.

Reflections from underwater topography seaward of the breakwaters for the natural and distorted cases were calculated using the theory of Rosseau (1952). The practical application of this theory requires a fitting to the actual bottom of the parametric representation of the Rosseau bottom contour. That is, given depths d_1 and d_2, one must determine the best γ which produces a good fit to the bottom. As $\gamma \to \Pi/2$ the bottom contour approaches a step and as $\gamma \to 0$ the bottom approaches a constant depth. The reflection coefficient is given by:

$$C_R = \frac{\tanh \Pi S_1 - \tanh \Pi S_2}{\tanh \Pi S_1 + \tanh \Pi S_2} \tag{13}$$

where S_1 and S_2 are solutions of

$$\begin{aligned} &S_1 \tanh \frac{\Pi S_1}{N} - \frac{NF_1}{\Pi} = 0, \\ &S_2 \tanh \frac{\Pi S_2}{N} - \frac{NF_1}{\Pi\rho} = 0, \\ &F_1 = \frac{\omega\, d_1}{g}, \text{ and } \gamma = \Pi/N \end{aligned} \tag{14}$$

where ω = wave frequency

ρ = water density

For the contours seaward of the Los Angeles-Long Beach Harbors breakwater and for the range of wave periods (15-360 seconds, prototype) under consideration, reflections would be one percent or less for the undistorted case. For the distorted model contours, reflections would range from 0 to about 28 percent for the steeper slopes. Fortunately the steeper slopes are oriented so that the waves will be reflected toward the model side wall and not back toward the wave generator. A rubberized fiber wave absorber will be installed around the

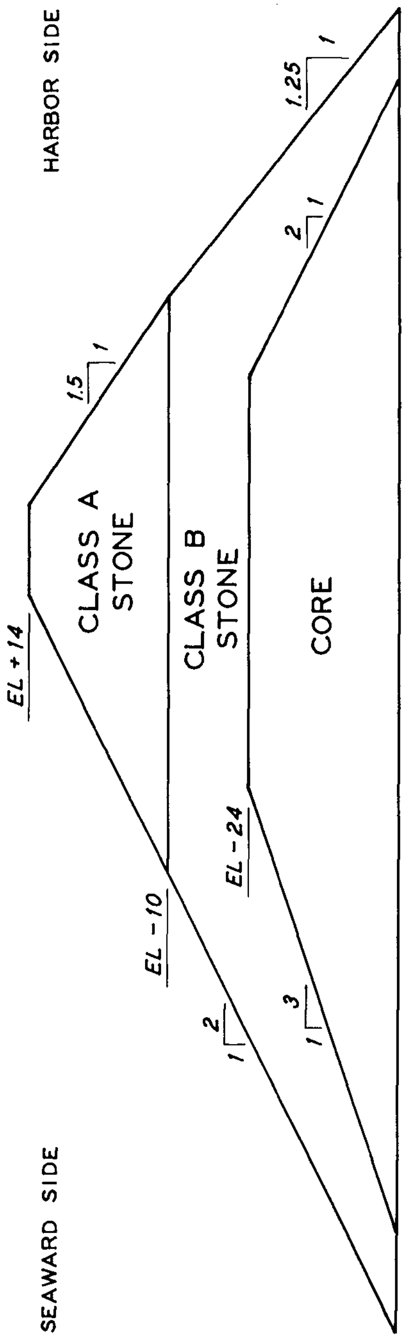

Figure 4. Typical Section of Los Angeles-Long Beach Harbors Breakwater.

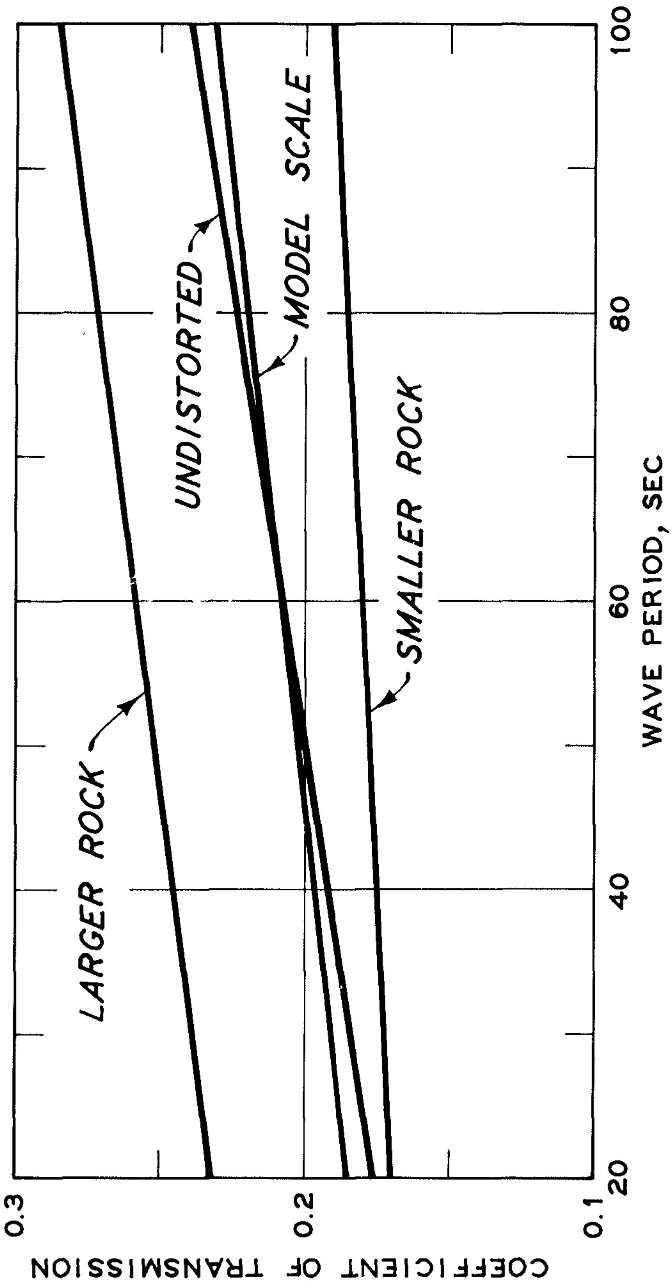

Figure 5. Comparison of Breakwater Transmission for Distorted and Undistorted Sections.

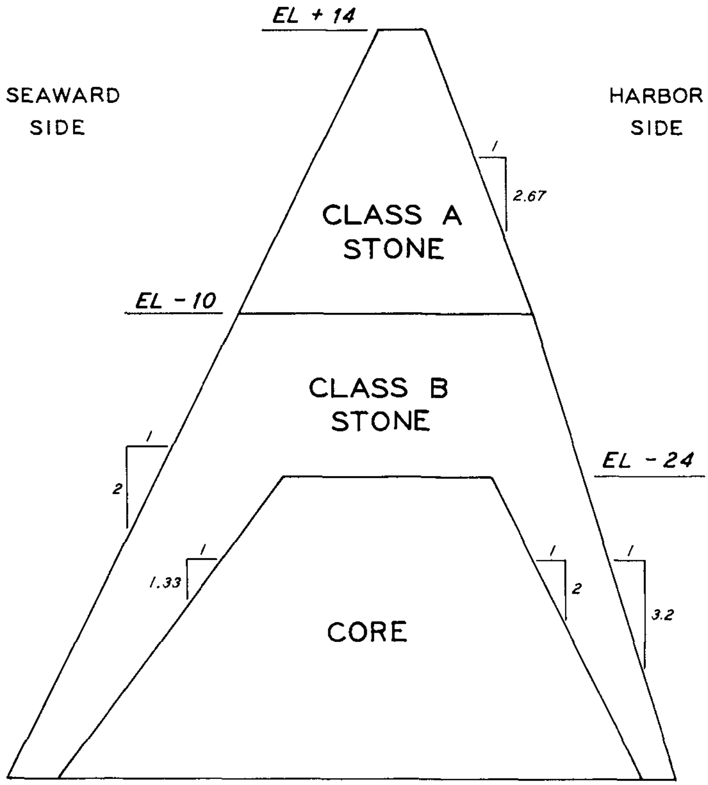

Figure 6. Typical Section of Model Breakwater; Distortion = 4; Rock Sizes Scaled Using Vertical Scale (1:100).

model perimeter walls and a wave filter of the same material will be installed in front of the wave generator to dampen waves which would be re-reflected from the wave generator.

Reflections from the sides of the harbor basins were studied two-dimensionally for the distorted and undistorted cases. These sides have an average slope of 1v on 1.5H and reflection coefficients for the undistorted case ranged from about 0.3 to 0.6. The distorted slopes increased the reflection coefficients about 10 to 20 percent (ranged from about 0.4 to 0.7). This increase was not considered significant enough to warrant addition of artificial roughness or wave absorbers around the basin sides.

CONCLUSIONS

After consideration of the various items described above, it was concluded that valid data can be obtained from the Los Angeles and Long Beach Harbors model for a vertical length scale of 1:100 and a distortion of 4.0. However, every time model distortion is considered, an analysis of all phenomena relevant to the study objectives must be carefully conducted. Since no two study objectives or prototype conditions will be identical, it is impossible to state specific conclusions of universal applicability. Construction of the Los Angeles and Long Beach Harbors model has been completed and model operation is underway. The model area is 44,000 square feet which represents the largest wave model constructed in the United States (see figs. 7 and 8).

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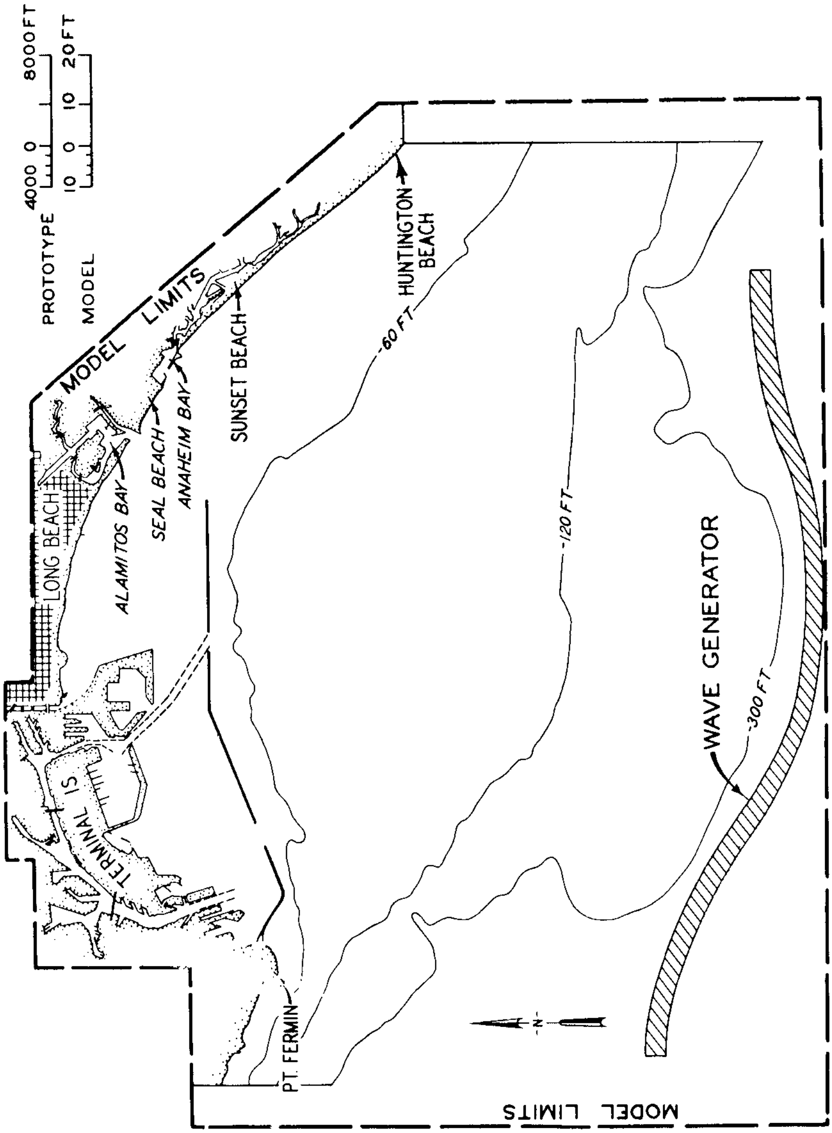

Figure 7. Los Angeles-Long Beach Harbors Model Layout.

Figure 8. General View of Los Angeles-Long Beach Harbors Model.

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