CHAPTER 26

KINEMATICS OF BREAKING WAVES

Edward B. Thornton,
James J. Galvin,
Frank L. Bub and
David P. Richardson

Department of Oceanography
Naval Postgraduate School
Monterey, California 93940

ABSTRACT

Measurement of waves, and vertical and horizontal water particle velocities were made of spilling, plunging and surging breakers at sandy beaches in the vicinity of Monterey, California. The measured breaking waves, derived characteristically from swell-type waves, can be described as highly nonlinear. Spectra and cross spectra were calculated between waves and velocities. Secondary waves were noted visually and by the strong harmonics in the spectra. The strength of the harmonics is related to the beach steepness, wave height and period. The phase difference between waves and horizontal velocities indicates the unstable crest of the wave leads the velocities on the average by 5-20 degrees. Phase measurements between wave gauges in a line perpendicular to the shore show breaking waves to be frequency nondispersive indicating phase-coupling of the various wave components. The coherence squared values between the sea surface elevation and the horizontal water particle velocity were high in all runs, ranging above 0.8 at the peak of the spectra. The high coherence suggests that most of the motion in the body of breaking waves is wave-induced and not turbulent.

INTRODUCTION

Wave theories can be applied outside the surf zone with some degree of certainty, and can be tested in laboratory and field situations. The various theories can be used to carry the waves from deep water through the shoaling process up to the point of near breaking. At the breaker point, however, there is a transition from ordered to apparent turbulent motion and the theoretical description of wave kinematics becomes difficult. One of the greatest deficiencies in our understanding of the surf zone is an appropriate description of breaking waves. The experimental study of the kinematics of breaking waves in the surf zone has progressed slowly due to the problems encountered in making direct field measurements and the difficulty in modeling the surf zone in the laboratory.

The most direct approach to the problem of describing the kinematics of surf zone breaking waves is by field measurements. Advances in instrument design have led to simple, sturdy, devices for measuring waves and velocities with rapid response time. Miller and Ziegler (1964) used both acoustic and electromagnetic flow meters to determine the particle motion
in the surf zone. Walker (1969) made studies using propeller-type flow meters. Wood (1973) measured waves and currents in the surf zone using movies of dye movement and capacitance wave gauges. Phürbötter Büsching (1974) utilized a two-component electromagnetic current meter and pressure-type wave meters to measure simultaneous orbital velocities and water levels. Thornton (1968) used an electromagnetic flow meter and pressure transducers to measure waves inside the surf zone. Steer (1972), Thornton and Richardson (1973), Bub (1974) and Galvin (1975) used pressure meters, capacitance wave gauges and electromagnetic current meters to measure surface profiles and particle velocities within the surf zone; the work presented here is a synthesis of these latter studies.

EXPERIMENT

The experiments were designed to measure kinematics of various types of breaking waves including spilling, plunging and surging breakers. The manner in which waves break depends very much on the characteristics of the beach and near-shore bottom slope. The experimental sites were in the vicinity of Monterey, California. The beaches here were some of the first intensively studied to gain an understanding of amphibious warfare techniques and were described and popularized by Bascom (1964).

The Del Monte Beach, within Monterey Bay, was chosen to measure plunging and spilling breakers. The waves at this location are generally topographically sheltered and severely directionally filtered due to refraction by the geometry of the bay often resulting in swell type waves impinging perpendicular to the shore. Hence, a simplification to a two-dimensional narrowbanded wave description is allowed. A second experimental site was Carmel River Beach, five miles to the south, where the beach is very steep and often has surging type breakers. Again here, the beach is within an embayment, Carmel Bay, and the generally narrow-banded waves impinge almost perpendicular to the shore.

The median grain size at Del Monte Beach is approximately 0.2 mm (taken at the water line), and the beach slope varies between 1:14 to 1:40. The median grain size at the Carmel River Beach is approximately 0.6 mm and the beach slope varies between 1:6 to 1:12. A typical beach profile and instrument location for Del Monte Beach is shown in Figure 1.

Instrumentation. The surf zone is a formidable environment in which to make measurements. The instruments must be rugged and reliable but at the same time be accurate and have a good time response. The instrumentation set-up described below evolved over a number of experiments.

Measurements at Del Monte Beach were made using two electromagnetic flow meters and three capacitance wave gauges. One flow meter and two wave gauges were used at Carmel River Beach. The instruments were mounted on the towers within the surf zone. All equipment was calibrated in the laboratory prior to the experiments.

The electronics package for each of the 2.5 m capacitance wave gauges was constructed using the design of McGoldrick (1969). The electronics package was housed in a water-tight brass case which was mounted on the tower. This enabled the connecting leads to be less than 30 cm, thereby minimizing wire-to-wire capacitance. Accuracy is estimated to be ±0.005 m.
The flow meters were Marsh-McBirney Model 721 and 722 Electromagnetic Current meters. The flow meter operation is based on Faraday's principle of electromagnetic induction. Each probe measures water velocity in two orthogonal directions. The flow meters were calibrated with an oscillating platform attached to an eccentric arm driven by a variable speed motor. Measurement accuracy was determined to be ±0.005 m/sec during calibration.

The instrument towers are 6.3 cm outside diameter steel pipes which were 3.6 m high with a 1.0 m baseplate and 0.6 m steel pipe bottom extension. A typical tower and sensor arrangement is shown in Figure 2. The towers were placed on a line perpendicular to the shore and were erected during low tide when the beach was easily accessible. The measurements were then conducted at high tide. The tidal range in the Monterey area is typically two meters. The towers were supported by steel guy wires fastened to the one meter long blade type anchors. Several types of anchors were tried; the blade anchor was chosen because it works quite satisfactorily in the relatively coarse sand and is easily installed. The flow meters were positioned directly under the wave gauges with the axes aligned horizontally and vertically. A carpenter's level was used to establish axis alignment with an estimated error of ±2 degrees.

A number of problems arose in the course of the experiments. The greatest difficulties in the field are encountered during periods of storms and large waves, a time when the measurements are often of most interest. Flotsam and, in particular, the giant kelp, macrocystis, torn loose from its hold-fasts are thrust ashore during storms. The kelp at times becomes entangled in the instrument towers, greatly increasing the drag, and making the towers susceptible to being knocked down.

The beaches are composed of fine to coarse sand. During wave conditions when the beaches are being eroded and cut back, considerable scour can occur around the anchors and instrument towers causing towers to topple on several occasions.

The first capacitance wave gauge wires were manufactured from one cm diameter stripped RG-11 coaxial cable, (Bub, 1974); these wave wires were susceptible to strumming and broke soon after installation. The construction of the wave wire was modified to a three-eighths inch outside diameter stainless steel rod as the center conductor covered with a jacket of polyurethane plastic tubing as the dialectic; these wave gauges can withstand the severe forces in the surf.

Wave gauges penetrating the surface were used to obtain a true measure of the surface elevation. Pressure transducers, although easier to install and maintain, do not represent the water surface in breaking waves. Breaking waves are highly nonlinear and the conventional technique of converting the pressure measurements to water surface elevation using the spectral transfer function derived from linear theory results in substantial error. As will be shown, the velocities, and hence dynamic pressures beneath breaking waves, are much greater than calculated using linear theory. Furthermore, pressure does not support sharp discontinuities beneath the sharp crests of breaking waves, hence, the pressure records are much smoother and rounded off compared with the capacitance wave gauge measurements.
Figure 1. Location of Instruments.

Vertical exaggeration 10:1

Figure 2. Instrumentation Tower.
All signals were cabled ashore and recorded on a Vidar Corporation 32-channel digital data acquisition system. A Sangamo Model 3500 14-channel FM tape recorder was utilized as a secondary recording system. A Brush 8-channel strip chart recorder was used to monitor the instrumentation performance during recording and as a means to select the appropriate data sections to be analyzed.

Analysis. A mean value was calculated for all data sets and the data were linearly detrended to exclude the rise and fall of the tide. The variance, standard deviation and average period were calculated. The average period was determined by calculating the time between zero up-crossings.

Correlation functions were calculated for signals and smoothed with a Parzen window. The smoothed correlation functions were Fourier transformed to obtain the power and cross spectra. The coherence and phase were calculated from the cross spectral estimates.

The maximum lag-time in calculating the correlation functions was taken as five percent of the record giving a spectral bandwidth resolution of 0.0055 Hz and resulting in 40 degrees of freedom for each spectral estimate. The 90 percent confidence limits for 40 degrees of freedom using a chi-square distribution are found to be between 0.72 and 1.51 of the measured power spectral estimates.

Measurements in the surf zone are not only hampered by the difficulties of a physically hostile environment, but can present conceptual analysis difficulties. The wave heights and spectral characteristics continually change as the waves shoal, break and dissipate across the surf zone. Changes in the wave profile occur over short distances compared to the wave length within the surf zone which does not allow spatial averaging within the surf zone. Point measurements are also troublesome because the breaker position is continually changing. Thus, the waves in the surf zone are spatially nonhomogeneous and tend to be temporally nonstationary.

As the wave height stochastically varies, the wave set-up changes resulting in a change in the mean water level at a particular location in the surf zone. The breaker position is approximately related to the wave height and local depth. Hence, the breaker position tends to wander, depending on the wave conditions. The breaker position tends to wander more on flatter beaches as was evident at Del Monte Beach experiment site. The shoaling processes on steeper beaches occur much more rapidly and over shorter distances, resulting in the breaker position being relatively fixed. The waves broke at nearly the same position for a particular tidal stage at the steeper Carmel River Beach site.

The spatial nonhomogeneity and temporal nonstationarity results in a smearing of spectral information of the breaking processes. This difficulty must be kept in mind when interpreting the results, but does not appear to be a severe limitation.
Qualitative Description. A number of universal similarities of wave form can be observed for various types of breakers occurring on different beaches. Figure 3 is a typical analog record of plunging-spilling breaking waves and horizontal velocities beneath the waves obtained from Del Monte Beach. In general, there is a quick drawdown of water just before the breaker arrives, followed by a steep, vertical leading edge, and a sloping profile toward the trailing edge, giving a generally sawtoothed shape. On the trailing edge, one or more secondary waves are often noted. The secondary waves are harmonics of the primary wave frequency and are indicative of strongly nonlinear waves. The authors have had the opportunity to see in the field the secondary waves develop by standing on an instrument tower as the waves break past the tower. As the wave shoals, the secondary waves start to grow and, as the waves steepen rapidly just before breaking, the secondary waves likewise rapidly develop on the back of the primary wave. The rapid transfer of energy from the primary wave frequency to the secondary wave is "mother nature's" means of maintaining the potential energy across the surf zone rather than converting to kinetic energy in the breaking process. The broken primary wave often reforms and continues toward shore closely followed by the secondary wave; this often results in two waves breaking close behind each other as the beach face is approached.

Surging breakers generally occur on steeper beaches in which the shoaling wave becomes unstable and forms a bore-like profile as the water progresses up the beach face. The breaking process on steep beaches occurs rapidly and secondary waves generally do not have time to develop.

On steep beaches, the swash zone is much more important in the surf zone processes, and on very steep beaches can constitute a major extent of the surf zone. A strong interaction of the backwash of the proceeding breaker and the new breaker can occur which complicates the wave processes. Wave reflection from steep beaches is greater which further complicates the processes.

The water particle velocity trace shown in Figure 3 reflects the general characteristics of the sea surface. The water cannot support sharp discontinuities which results in the velocities being considerably more smoothed. It should be emphasized that the smoothing in the velocity records is real and not a result of the frequency response of the flow meters.

In a spilling breaker, the crest becomes unstable and slides down the face of the wave; the turbulent region is generally confined to the area above the trough and does not penetrate into the body of the flow. The crest of the plunging breaker curls over and penetrates deeper into the water column, although the velocities still have an appearance similar to the wave surface. Surging breakers, which are bore-like, are apparently turbulent on the small scale throughout the water column, but the body of the fluid translates with the wave profile. Hence, the primary motion even under the surging breakers can be associated with the surface and is wave-induced. The important point is that the velocities under breaking waves and in the surf zone appear to be primarily associated with the wave surface and not turbulent, disorganized motion.
FIGURE 3. TYPICAL ANALOG RECORD OF WAVES AND HORIZONTAL VELOCITIES BENEATH THE WAVES FROM DEL MONTE BEACH.
Comparing Measured and Calculated Velocity Spectra Using Linear Wave Theory. A velocity spectrum was calculated from the wave spectrum using the transfer function derived from linear wave theory. Previous work in deeper water by Simpson (1969), Bowden and White (1966), and Thornton and Krapohl (1974) has shown that using linear wave theory to derive the spectral transfer function gave very good results in calculating the water particle velocity spectra under waves. It was not expected that as good results would be obtained in the surf zone, but would serve as a basis for comparison with other theories. Further, spectral analysis assumes superposition of the spectral component which only allows using a constant parameter linear theory for the transfer function.

The elevation of the surface \( n(t) \) can be described as the superposition of an infinite number of sinusoids of the form:

\[
\eta(t) = \sum_{n=1}^{\infty} a_n \cos (k \cdot \hat{x} - \sigma_n t + \epsilon_n) = \sum_{n=1}^{\infty} \eta_n
\]

where \( a_n \) is the amplitude, \( x \) is the horizontal Cartesian coordinate, \( t \) is the time, \( \epsilon_n \) is an arbitrary angle, \( k \) is a horizontal vector wave number, and \( \sigma \) is the frequency related in linear theory to \( k \) by

\[
\sigma_n^2 = g k_n \tanh k_n h
\]

where \( g \) is the acceleration of gravity. The \( \eta(t) \) represents the sum total of all component wavelets. Summing in the manner of (1) implies a linear system and restricts the analysis to the use of linear wave theory to describe the wave-induced motion.

Linear wave theory can be used to calculate the wave-induced water particle velocities. The equation for the horizontal velocity is

\[
u(t) = \sum_{n=1}^{\infty} \frac{a_n \cos (k_n (h+z))}{\cosh k_n h} \cos (k_n \cdot \hat{x} - \sigma_n t + \epsilon_n) = \sum_{n=1}^{\infty} u_n
\]

where \( h \) is the total depth and \( z \) is the vertical coordinate measured positively upward from the still water level. The solution says that the amplitude of the velocities is a function of wave amplitude, frequency and depth, and that the water surface and horizontal water particle velocities are in-phase. The term in (3) in braces represents the complex spectral transfer function, \( H_n(\sigma,z) \), and is used to calculate the wave-induced horizontal velocity spectrum, \( S_u(\sigma) \), from the wave spectrum \( S_n(\sigma) \):

\[
S_u(\sigma) = |H(\sigma,z)|^2 S_n(\sigma).
\]
In the formulation of linear wave theory, the boundary conditions are linearized in order to obtain an analytical solution. In the linearization, it is assumed that the amplitude is small in comparison with the wavelength, that is, $ak \ll 1$. Higher-order solutions to the boundary value problem, generally obtained by perturbation analysis, give a better representation of a constant profile wave. However, the nonlinearities introduced in the solution preclude their use where the principle of superposition is invoked.

Figure 4 shows typical spectra of the measured and calculated water particle velocities using Eq. 4 for the case of plunging-spilling breakers in the surf zone taken at Del Monte Beach. The waves are narrow-banded with a primary frequency of 0.06 Hz (16.6 second period).

Strong harmonics of decreasing energy density are evident. The measured horizontal velocities were always greater than the calculated by 20-100 percent. That is to say, linear theory underpredicts the horizontal velocities. This is not surprising because of the steepness of the waves, but the amount of underestimation is surprising. The amount of deviation from linear theory demonstrates the strength of the nonlinearities of shallow water waves and some contribution by turbulence.

The strong harmonics are both real and artifacts. The secondary waves are real harmonics of the primary wave and show up as energy at harmonic frequencies. Due to the very peaked wave form of the primary wave, spectral analysis will also show energy-density at harmonic frequencies as a result of viewing the breaking waves as an infinite sum of sinusoidal wave components.

Also shown in Figure 4 are the coherence squared (hereafter referred to simply as coherence) and the phase difference between waves and horizontal velocity. The coherence between waves and horizontal water particle velocities was high, ranging above 0.75 at the peak of the spectrum and decreasing at higher and lower frequencies. The generally high coherence indicates the water particle motion is primarily wave-induced.

The decrease in coherence can be attributed primarily to the velocities being converted to turbulence during breaking and the nonlinearities associated with finite amplitude wave motion. The decrease in coherence due to turbulence can be demonstrated by considering the horizontal velocity, $u$, as being decomposed into wave-induced, $U$, turbulent, $u'$, and mean, $\bar{u}$, contributions, such that the velocity is given by

$$u = U + u' + \bar{u}. \quad (5)$$

Assuming the wave-induced and turbulent velocity spectral component are statistically independent, the horizontal velocity spectrum is given by

$$S_u(\sigma) = S_{u'}(\sigma) + S_{\bar{u}}(\sigma). \quad (6)$$

The wave-induced velocity spectrum is calculated from the wave spectrum using equation (4). For a constant parameter linear system the coherence is identically equal to unity,
FIGURE 4. MEASURED AND CALCULATED HORIZONTAL VELOCITY SPECTRA UNDER SPILLING-PLUNGING BREAKERS, 4 MARCH 1975.
Since the turbulent and wave-induced velocities are assumed to be statistically independent, then

\[ S_{u_n}(\sigma) = S_{u}(\sigma) . \]  

(8)

The substitution of (6), (7) and (8) into the definition of coherence between the total horizontal velocity and waves results in

\[ \gamma_{u_n}^2(\sigma) = \left[ 1 - \frac{S_{u_n}(\sigma)}{S_u(\sigma)} \right]^{-1} = \frac{S_u(\sigma)}{S_{u}(\sigma)} \leq 1 . \]  

(9)

Increasing lack of coherence is due to an increasingly high ratio of turbulence (noise) to coherent wave-induced velocity fluctuations (signal). Using this interpretation for the coherence, the results of Figure 4 suggest that 94% of the spectral energy at the primary wave frequencies (peak of the spectrum) is wave-induced.

The phase difference between waves and horizontal velocities for all runs varied between 5-30 degrees at the primary frequency. Theory states that the waves and horizontal velocity are in phase, or have a zero phase difference. The measured phase difference is interpreted as showing the breaking wave crest leading the wave-induced velocities beneath.

Wave and velocity spectra characteristic of collapsing breakers taken at Carmel River Beach are shown in Figure 5. Wave No. 1 and horizontal velocity spectra were measured at the breaker line. Wave No. 2 spectrum was measured 3.5 meters shoreward. The spectra do not exhibit the strong harmonics on this steeper beach because the secondary waves do not have time to develop during rapid shoaling. The energy density is greatly attenuated in the breaking process from Wave No. 1 to Wave No. 2. The coherence and phase difference shown are between Wave No. 1 and horizontal velocity. The phase difference shows the waves leading the velocities at the primary frequency by 15 degrees. The coherence is high, again indicating the motion to be primarily wave-induced.

Wave Celerity. The wave speed, or celerity, was measured using two wave gauges separated in a line perpendicular to the propagating wave crests. Consider a spectral wave component propagating perpendicular to the beach in the x direction measured at point, \( x_1 = 0 \), and a point shoreward by an amount, \( x_2 = \Delta x \), given by

\[ n_1(\sigma) = a_1(\sigma) \cos \sigma t , \]

\[ n_2(\sigma) = a_2(\sigma) \cos (k\Delta x - \sigma t) . \]  

(10)

The phase difference of the waves between the two measurement points is given by
FIGURE 5. WAVE AND VELOCITY SPECTRA CHARACTERISTICS OF COLLAPSING BREAKERS, 29 MAY 1975.
\[
\phi(\sigma) = k\Delta x = \frac{\sigma \Delta x}{c}
\] (11)

where the celerity, \( c = \sigma/k \).

Figure 6 shows two wave spectra measured at Del Monte Beach within the surf zone. Wave gauge one was at the approximate breaker line and wave gauge two was 11 meters shoreward. The wave spectra exhibit strong harmonics. The energy density is shown to decrease as the waves break and progress shoreward. The coherence is high at the primary frequency and decays from the peak. The phase difference between the wave gauges continuously increases with frequency.

Expected phase differences were calculated using both the linear wave theory relationship

\[
c = \left(\frac{g}{k} \tanh kh\right)^{1/2}
\] (12)

and the shallow water relation assuming nondispersive waves

\[
c = \sqrt{g(h + \alpha H)}
\] (13)

where \( h \) is the total depth of water, \( H \) is the wave height and \( \alpha \) a constant. It should be noted that even though the water is shallow, the higher frequency wave components can be considered intermediate or even deep water waves.

The phase differences calculated using equation (11) are shown on Figure 6. The dashed line is the phase difference calculated using the nondispersive wave speed, equation (13), for \( \alpha \) equal to 0.5. The comparison of all measured and calculated phase differences shows that linear theory wave speed is not valid, but that the waves are nondispersive. The nondispersiveness of the waves across the frequency band of significant wave energy is because the wave components are phase-coupled to the primary frequency wave, i.e., the higher frequency wave components travel at the phase speed of the primary frequency wave. The phase-coupling of the wave components is another indication of the strong nonlinearities of breaking waves.

CONCLUSIONS

The measured breaking waves, derived characteristically from swell-type waves, can be described as highly nonlinear, although the kinematics are more orderly than intuitively presupposed. The measured wave and velocity spectra show strong harmonics of the peak frequency of the waves. The harmonics are secondary waves of the primary wave frequency and are indicative of strongly nonlinear waves. As the wave shoals, the secondary waves start to grow and, as the waves steepen rapidly just before breaking, the secondary waves likewise rapidly develop on the back of the primary wave. The development of secondary waves indicates a transfer of energy to higher frequencies.
FIGURE 6. WAVE SPECTRA SEPARATED BY 11 METERS IN THE SURF ZONE, 4 MARCH 1976.
Spectra and cross spectra were calculated between waves and velocities measured directly beneath the waves. The wave energy-density spectral components were converted to velocity spectral components using linear wave theory. The measured horizontal velocities were always greater than the calculated by 20-100 percent. The amount of deviation from linear theory demonstrates the strength of the nonlinearities of shallow water waves.

The measured phase difference between waves and horizontal velocities indicates the waves generally led the velocities on the average by 5-20 degrees, implying that the "curling" crest of the wave arrives prior to maximum water particle velocity. The coherence values between the sea surface elevation and the horizontal water particle velocity were high in all runs, ranging above 0.8 at the peak of the spectra and decreasing at higher and lower frequencies. The decrease in coherence can be attributed primarily to the velocities being converted to turbulence during breaking and the nonlinearities associated with finite amplitude wave motion. The high coherence suggests that most of the motion in the body of the breaking waves was wave-induced. Hence, breaking waves may be more amenable to theoretical treatment than previously thought, although still very nonlinear.

The wave speed was measured using two wave gauges separated in a line perpendicular to the propagating wave crests. The measured phase difference was compared with theoretical values calculated using celerity relations from linear wave theory and the shallow water relation assuming nondispersive waves. The measurements show that the breaking waves are frequency nondispersive; this is further evidence of the strength of the nonlinearities of the waves in the surf zone.

ACKNOWLEDGEMENTS

This work was supported by the Office of Naval Research, Geography Branch, under contract NR 388-114.

REFERENCES


Steer, R., Kinematics of Water Particle Motion Within the Surf Zone, M.S. Thesis, Naval Postgraduate School, Monterey, CA, 1972.


