# CHAPTER 127

## GENERATION OF TROUGHS BY DENSITY CURRENTS

Hans Speekenbrink *

### Abstract

Along the northern bank of the so-called "Maasgeul" scour has occurred, causing a trough. It is concluded from field measurements that the trough development is induced by density currents along the slope of the bank. The stratified flow system is dealt with mathematically. The calculated results are compared with the field measurements and agree fairly well. Trough development can be avoided by flattening the slope or by a bottomprotection.

### Introduction

Since early 1970 scour has occurred along the northern boundary of the so called "Maasgeul", the dredged shipping route to the "Europoort" and Rotterdam harbours (see fig. 1), over a distance of about two miles. By this scour a trough has developed (see fig. 2 and 3) with a depth of 5 to 10 m below the bed of the navigation channel (M.S.L. - 24 m) with slopes of about 1 : 3 (see fig. 4). The surrounding sea bed has a depth of M.S.L. -16 m at the eastern- and of M.S.L. -19 m at the western end of the trough.

The material eroded from within the trough sedimentates south of it in the channel, from where it has to be removed right away in order to keep the guaranteed navigational depth at all times.

Widening of the trough in easterly direction possibly can endanger the stability of the end of the northern harbour mole.

*Rijkswaterstaat, Department for Coastal and Sea Research, The Netherlands

Investigations have now been carried out into :

a. The mechanism, that causes the generation of the trough

b. The future equilibrium profile of the trough

c. Possible measures to avoid or reduce trough development.

Trough development

Scouring has occurred along the northern bank of the "Maasgeul" from about 300 m west of the end of the northern harbour mole to about 4 km west of this mole. This section of the "Maasgeul" may be considered as a transition zone between open sea and the mouth of the river Rhine, which discharges its water via the "Rotterdam Waterway".

In January 1970 scouring started about 500 m west of the end of the mole over a distance of about 50 m and widened to the east (150 m) and to the west (200 m) in 4 months time. Some months later scouring was observed about 2 - 3 km west of the first trough over a distance of 1000 m. This trough widened in easterly direction to join the first trough although a small ridge remained between the two trough sections (see fig. 3).

Trough developments were first observed by routine echo soundings made to guard the navigational depth. In August 1974 with scour still continuing it was decided to carry out echo soundings more frequently. Every month, if possible in respect of weather conditions, echo soundings were carried out in lanes perpendicular to the trough axis at distances of 250 m. Until now scouring still continues almost linear in time (see fig,5), although it seems that scouring in summer is slightly greater than in wintertime.

The slopes of the northern bank steepened from about 1:10-15 to 1:3 in the first two years of trough development (see fig. 6). This slope has been rather stable since, although periodically slopes up to 1:1 were observed. The slopes of the southern bank have steepened to a value of about 1:4.

The northern bank remained very stationary but the southern bank moved in a southerly direction over an average distance of about 15 m in two years time.

No significant widening of the trough in easterly nor in westerly direction could be distinguished during the last two years of observations.

The mechanism, causing trough generation

To find an explanation for the generation of the trough, surveys were carried out into the vertical distribution of the current velocities and directions and of the salt concentrations and temperatures. The measurements were related to the vertical tidal movement in Hook of Holland. The situations of the measuring points are indicated on fig. 2.

All measurements showed stratification during low tide. Differences in current direction up to 90° were found in upper and lower layer. In the upper layer the less saline water, discharged from the "Rotterdam Waterway" flows in a westerly direction, parallel to the channel axis (see fig. 7). Velocities reach values of 0.5 - 0.8 m/s at this tidal stage. The flow in the lower region shows a more southerly direction, almost perpendicular to the channel axis, corresponding to the direction of the tidal stream along the Dutch Coast at low tide. The velocities in this layer are about 0.5 - 0.6 m/s at the measuring point north of the trough, 0.8 - 1.0 m/s at the trough and about 0.5 - 0.6 south of it.

From these measurements of current velocities and density distributions it was concluded that density currents occur at low tide. A sketch of the stratified flow system is given in fig.8.

The heavier underflow accelerates beneath the discharged, less saline river water, becoming critical at the upper edge of the bank of the trough and supercritical further down along the slope due to the steepness thereof.

Supercritical flow occurrs at internal Froude-numbers $F_i$ greater than 1.

$$F_i = \sqrt{\frac{u_i}{\varepsilon g a_i}} \qquad i = 1,2 \qquad (1)$$

in which u=velocity; $\varepsilon = (\rho_2 - \rho_1)/\rho_2$ ; $\rho$ =density; g=gravitation acceleration; a=layer depth; and subcripts 1 and 2 = upper and lower layer respectively.

The increasing velocities will cause erosion and thus steepening of the bank. In this bank there is a clay layer with a thickness of about 3m. The top of this layer is found at a depth of about M.S.L. - 18 m at the eastern end of the trough and about M.S.L. - 20 m at the other end. The clay is very stiff and is resistant to scour, thus limiting erosion to the foot of the bank. When the slope steepens more than the natural slope of the bed material, the clay layer is undermined causing a slide down and thus an intermittent shift of the bank in a northerly direction. Although this shift can not be determined from the echo soundings it may be derived from core samples taken at the trough in 1974 that a local slide down of the clay layer occurred.
This clay layer may also be responsible for the fact that scouring did not start before 1970 and only after deepening the navigation channel to a level below this layer.

The foot of the bank may be regarded as a so called plunge-pool from which also sediment is removed.

The current measurements show that the flow is supercritical at the trough in the navigation channel. The conversion will take place via an internal hydraulic jump at the southern bank of the trough. Downstream this jump the current velocities will be too low for erosion and even sedimentation occurs, causing a ridge along the trough.

## Mathematical model for two-layer stratified flow

A mathematical model may be derived for the flow across the trough. The following assumptions were made: (1) the flow is steady; (2) the flow in each layer is one dimensional i.e. there is a constant velocity and density at each cross section in each layer; (3) stratification is stable; there is a sharp interface between the layers and there is no entrainment across the interface; therefore the density is also constant longitudinally in each layer; (4) shear stresses are acting along the bottom and the interface and are proportinal to the velocity differences across the boundaries.

North of the trough the bottom has a gentle slope and the water surface and the interface are assumed to have gentle slopes too. The vertical accelerations may be neglected so the water pressure can be considered hydrostatic. The following equations, as given by Schijf and Schonfeld [1] may be applied.

The equations of motion for the upper and lower layers can be written as :

$$\frac{da_1}{dx}+\frac{da_2}{dx}+\frac{u_1}{g}\frac{du_1}{dx}+\frac{f_i\,|u_1-u_2|\,(u_1-u_2)}{ga_1}+S_b=0 \qquad (2)$$

and

$$(1-\varepsilon)\frac{da_1}{dx}+\frac{da_2}{dx}+\frac{u_2}{g}\frac{du_2}{dx}-\frac{f_i\,|u_1-u_2|\,(u_1-u_2)}{ga_2}+\frac{f_b\,|u_2|\,u_2}{ga_2}+S_b=0 \qquad (3)$$

respectively, in which x = longitudinal coordinate taken as positive from north to south; $f_i$ = friction factor at the interface; $f_b$ = friction factor at the bed; $S_b$ = bottom slope.

The continuity equations read :

$$\frac{dq_i}{dx}=0 \qquad i = 1,2 \qquad (4)$$

in which q = au = discharge per unit width.

Substituting the continuity equations in the equations of motion to eliminate $du_i/dx$ and $da_1/dx$ and neglecting $u_i^2$ in relation to $a_i g$ according to the field measurements, the resulting equation may be written as :

$$\frac{da_2}{dx} = \frac{(u_1^2 - \varepsilon g a_1) a_2 S_b + (1-\varepsilon) f_i (a_1 + a_2) |u_1 - u_2| (u_1 - u_2) - f_b a_1 |u_2| u_2}{u_2^2 a_1 + u_1^2 a_2 - \varepsilon g a_1 a_2} = 0 \quad (5)$$

$$\frac{da_2}{dx} = \frac{q_1^2 - \varepsilon g a_2^3 S_b + (1-\varepsilon) f_i (a_1 + a_2)(a_2 q_1 - a_1 q_2) |a_2 q_1 - a_1 q_2|}{(q_2^2 a_1^3 + q_1^2 a_2^3 - \varepsilon g a_1^3 a_2^3)} +$$

$$+ \frac{-f_b a_1^3 |q_2| q_2}{(q_2^2 a_1^3 + q_1^2 a_2^3 - \varepsilon g a_1^3 a_2^3)}$$

At the upper edge of the northern bank of the trough critical flow will occur if the denominator equals zero :

$$q_2^2 a_1^3 + q_1^2 a_2^3 - \varepsilon g a_1^3 a_2^3 = 0 \quad (6)$$

Deriving the total depth at the edge from echo soundings and vertical tide measurements, the discharges from flow measurements and $\varepsilon$ from salinity and temperature measurements the depth of the interface at the edge may be computed from equation (6). With this depth as boundary condition the flow north of the trough may be calculated from the upper edge of the bank in negative x-direction using equation (5).

The northern bank has a steep slope and the flow in the lower layer will accelerate. It is assumed that the shear stresses in the lower layer are negligeable and the energy equations are applied.

The energy equations for the lower and the upper layer may be written as

$$H = \frac{\rho_1}{\rho_2} a_{1_x} + a_{2_x} + a_{3_x} + \frac{1}{2g} u_{2_x}^2 \quad (7)$$

and

$$H_u = a_{1_x} + a_{2_x} + a_{3_x} + \frac{1}{2g} u_{1_x}^{2} + h_x \tag{8}$$

respectively, in which H = energy head elevation relative to the trough floor; $a_3$ = bed surface elevation relative to the trough floor; index x refers to the distance along the bank; index u refers to the upper edge and $h_x$ = inertia losses.

Assuming that the inertia losses $h_x$ equal the drop in velocity head of the upper layer, thus

$$h_x = \frac{u_{1_u}^{2}}{2g} - \frac{u_{1_x}^{2}}{2g} \tag{9}$$

and substituting equation (9) in equation (8) yields

$$a_1 + a_2 + a_3 = \text{constant} \tag{10}$$

Differentiating equation (7) and using equation (4) and (10) the resulting equation may be written as

$$\varepsilon S_b + \left(\frac{\varepsilon g a_2^{3} - q_2^{2}}{g a_2^{3}}\right) \frac{da_2}{dx} = 0 \tag{11}$$

Now the flow on the northern bank can be calculated from the upper edge of the bank in positive x-direction using equations (6) and (11).

The calculations may be simplified by assuming constant friction factors and constant slopes at both sides of the upper edge of the bank.

Calculations were carried out, taking $f_i$ =0.001 and $f_b$ =0.004. The boundary conditions $q_2$, $q_1$, $\varepsilon$ and the slopes at both sides of the edge were derived from field measurements. As an example a calculated interface depth is given in figure 10 as well as the measured depth. As may be seen there is a fair agreement. The discharges used as boundary conditions in this example were derived from the velocity measurements in figure 7. The density profiles were derived from salinity and temperature measurement at the same time (see fig. 11).

At the southern bank an internal hydraulic jump will occur. The flow system in this section is very complicated and has to be investigated by means of hydraulic model tests, which have not been made so far.

Future equilibrium profile

Erosion of the trough floor probably will continue until the hydraulic jump will be submerged. At what depth of trough this will occur has to be investigated in a hydraulic model.
A further widening of the trough is not expected. The longitudinal stability of the trough may be explained at the eastern end by the presence of a bottom protection around the toe of the northern harbour mole. At the western end the density difference decreases due to longitudinal mixing. Besides the steepness of the bank slope is much less (about 1 : 40) as well as the difference in depth between the channel and the surrounding sea bed. Due to these reasons the velocities along the bank are less too.

Measures to be taken to reduce or avoid trough development

To restrain the erosion a bottom protection on the trough floor and the banks may be considered. On the northern bank drag forces and gravity act in the same direction. The critical velocity for stability of loose grained material, as for instance shingle, on slopes can be compared with the critical velocity on a horizontal bed according to

$$u_{cr,s} = \sqrt{\cos S_b}\ \sqrt{1 - \frac{\tan S_b}{\tan \theta}}\ u_{cr,h} \qquad (12)$$

in which $u_{cr}$ = critical velocity for initiation of movement of bed material; $\theta$ = natural slope of the bed material; and subscripts s and h referring to slope and horizontal bed respectively. Using the Sheilds diagram and taking $\theta = 45^o$ it can be calculated that the minimum required grain diameter for the trough bank protection is about 3 cm. At the trough floor the minimum grain diameter may be even more due to the strongly curved flows here.

Another possible measure to restrain the erosion is reducing the high current velocities and avoiding supercritical flow on the bank by flattening the slope. The flow in the lower layer will not accelerate when $da_2/dx \geq 0$. Using equation (5) and taking $da_2/dx = 0$, the equation for the critical slope, $S_{b,cr}$, may be written as

$$S_{b,cr} = \frac{f_b\, a_1^3 q_2^2 + (1-\varepsilon) f_i (a_1+a_2)(a_2 q_1 - a_1 q_2)^2}{a_2^3 (q_1^2 - \varepsilon g\, a_1^3)} \tag{13}$$

Assuming $q_1=0$ and critical flow on the bank (equation (6) the equation may be rewritten as

$$S_{b,cr} = -f_b - (1-c)\, f_i \left(\frac{a_1+a_2}{a_1}\right) \tag{14}$$

From this equation it may be derived that the critical slope is about 1 : 150. Due to possible variations in the main parameters, the friction factors, the critical slope must be considered of the order of 1 : 100 to 1 : 250. These flat slopes involve more than about 4 $10^6$ $m^3$ to be dredged.

By flattening the slope the upper edge of the bank is shifted in a northerly direction and may become situated outside the surface front of the discharged less saline water. Due to this the flow in the lower layer will stay subcritical.

Because sand is needed in the near future for beach nourishment immediately north of the northern harbour mole, it is considered now to flatten the northern wall of the trough to a slope of about 1 : 50. For this measure a quantity of about 1.600.000 $m^3$ has to be dredged. Plans are being worked out now and dredging activities will probably be carried out in autumn 1976. The outcoming clay (about 600.000 $m^3$) will be dumped in the trough as refillment. The effect of this measure will be closely watched by making echo soundings every month, if possible in respect of weather conditions.

## Summary and conclusions

Along the northern bank of the "Maasgeul" scour has occurred due to density currents induced by density differences between the discharged river water and the heavier sea water. Stratified flow occur at low tide.
The flow accelerations along the slope, causing trough generation, may be dealt with mathematically. The results of the mathematical model agree fairly well with field data.
The trough development, increasing the amount of maintenance dredging and endangering the stability of the northern harbour mole may be reduced by a bottom protection or by flattening the slope. The latter possibility will be carried out in the near future.

## Acknowledgements

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## References

[1]
Schijf, J.B., and Schonfeld, J.C., "Theoretical Considerations on the Motion of Salt and Fresh Water", Proceedings of the Minnesota International Convention, I.A.H.R. and A.S.C.E., pp 321-333, 1953.

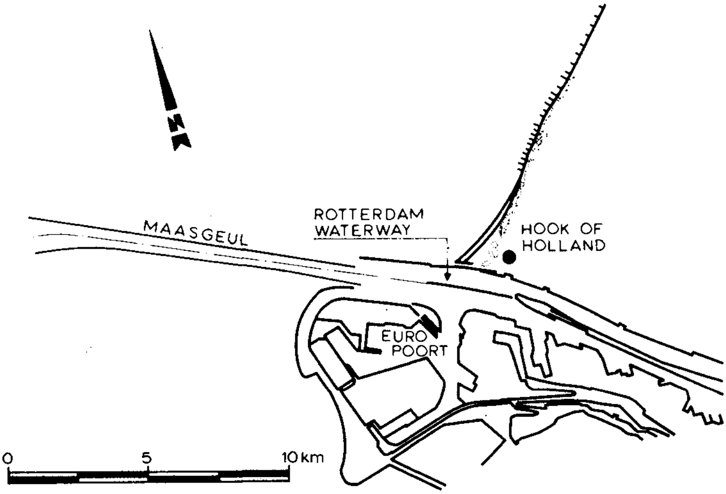


FIG 1 THE MAASGEUL NAVIGATION CHANNEL TO EUROPOORT

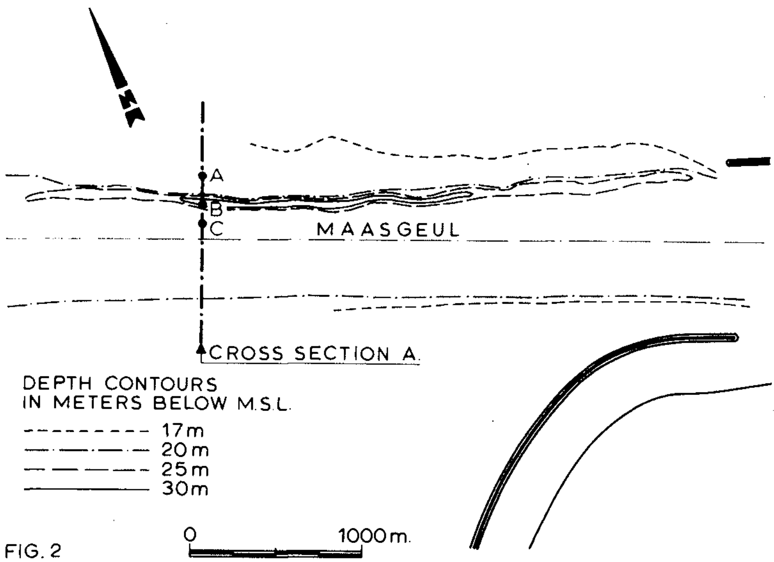


FIG. 2

TROUGH IN THE MAASGEUL ALONG NORTHERN BOUNDERY

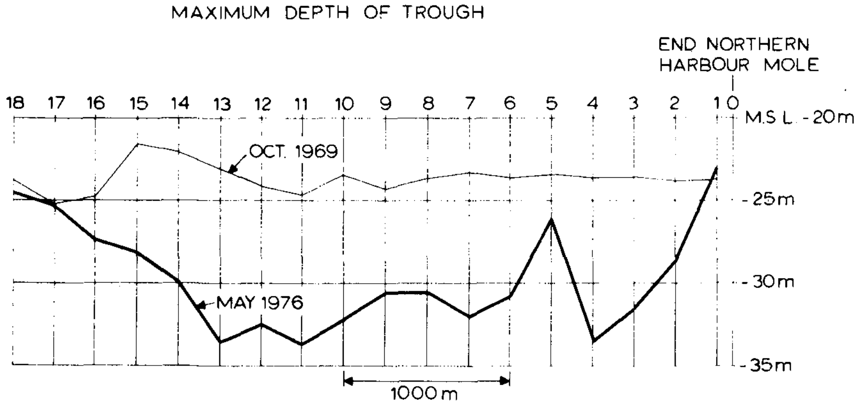


FIG. 3

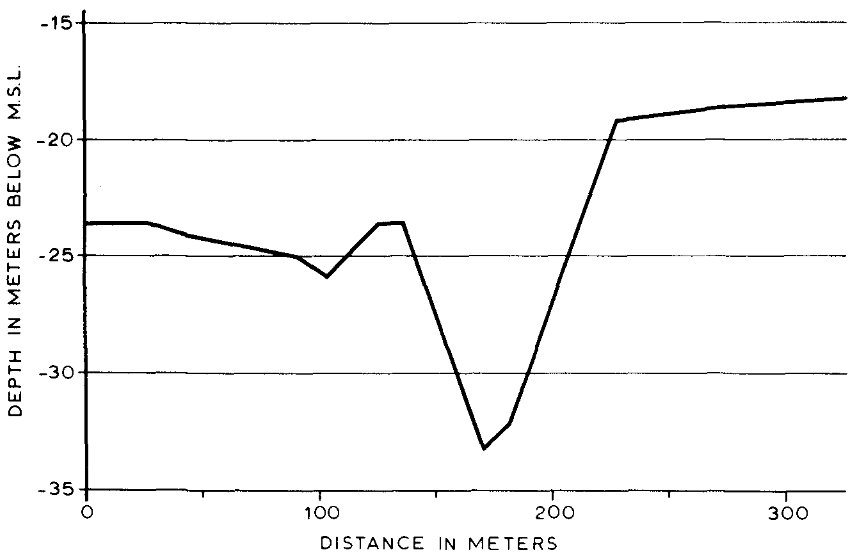


FIG. 4 CROSSECTION A. OF TROUGH

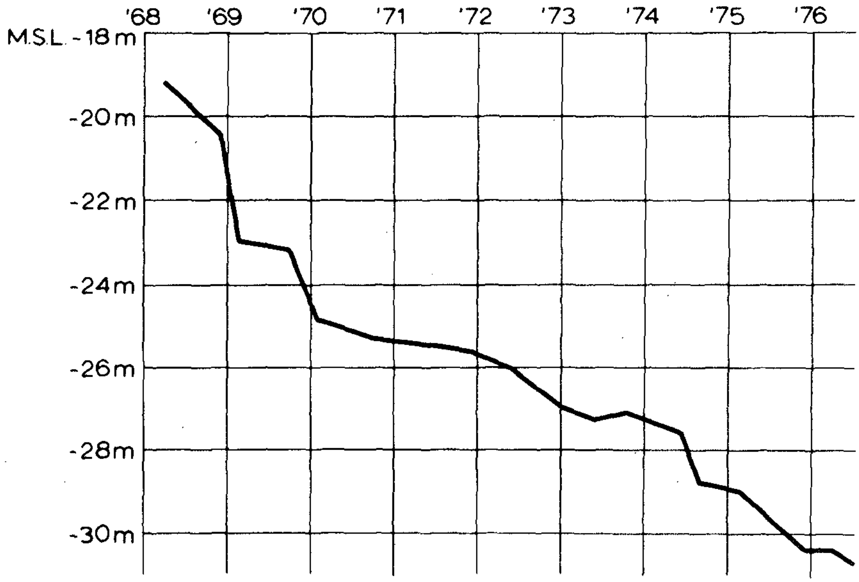


FIG. 5 INCREMENT OF MEAN DEPTH OF TROUGH AS FUNCTION OF TIME

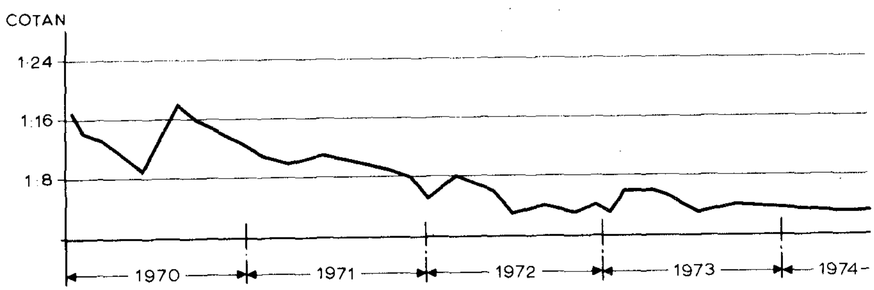


FIG.6 CHANGE OF SLOPE OF NORTHERN TROUGH BANK IN TIME

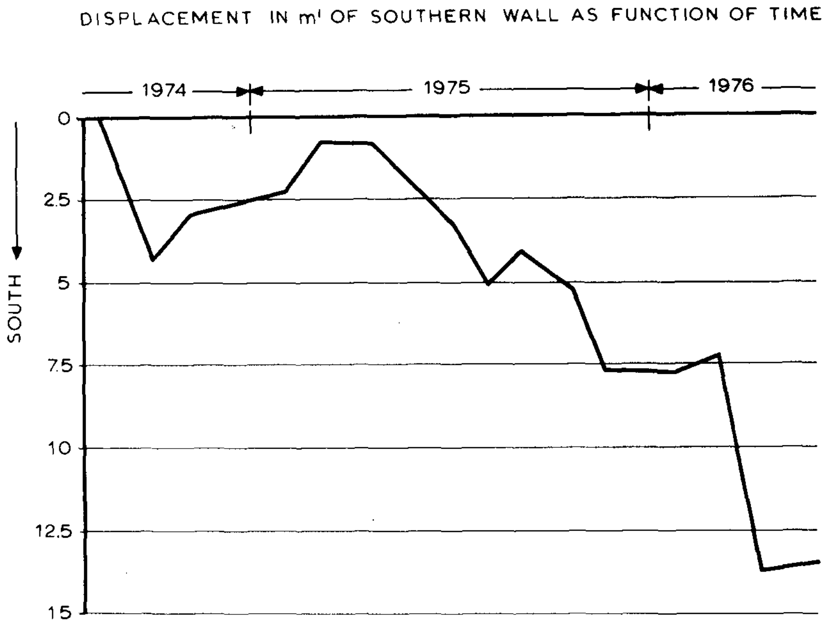
DISPLACEMENT IN m' OF SOUTHERN WALL AS FUNCTION OF TIME
1974
1975
1976
0
2.5
5
7.5
10
12.5
15
SOUTH

FIG. 7a

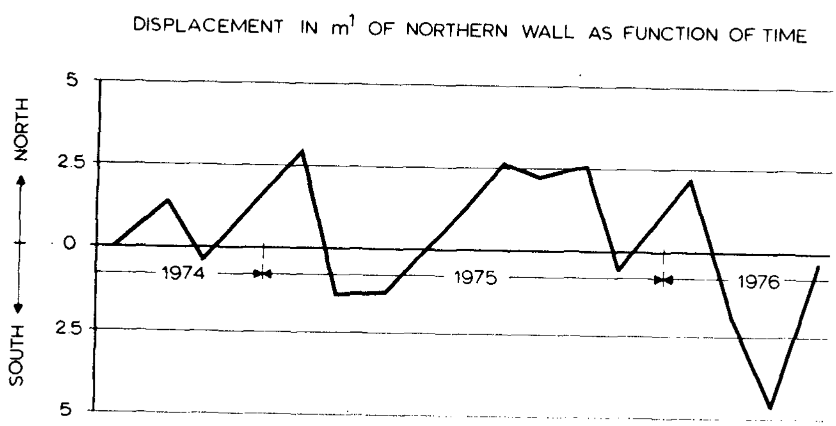
DISPLACEMENT IN m' OF NORTHERN WALL AS FUNCTION OF TIME
5
2.5
0
2.5
5
NORTH
SOUTH
1974
1975
1976

FIG. 7b

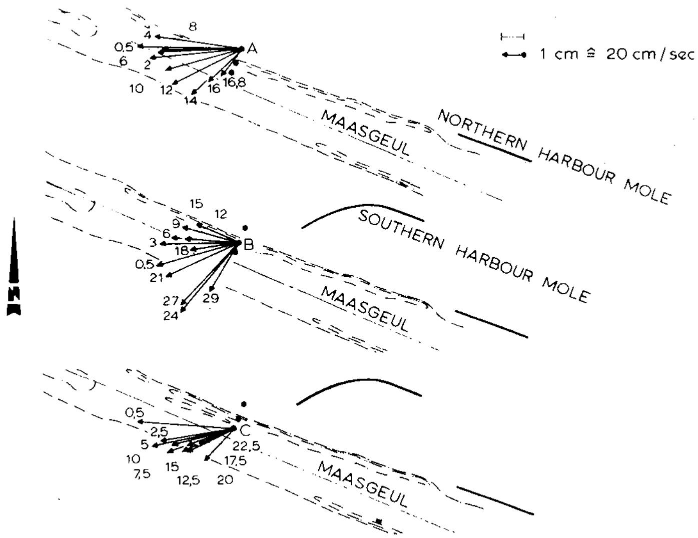


FIG 8 CURRENTVELOCITIES AND DIRECTIONS ON DIFFERENT DEPTHS AT LOW WATER AT POSITIONS IN THE TROUGH, NORTH AND SOUTH OF THE TROUGH

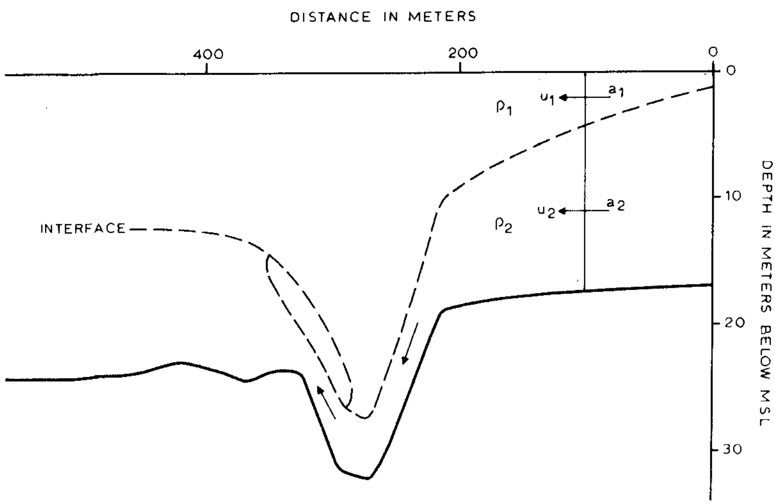


FIG 9 DEFINITION SKETCH STRATIFIED FLOW ACROSS TROUGH

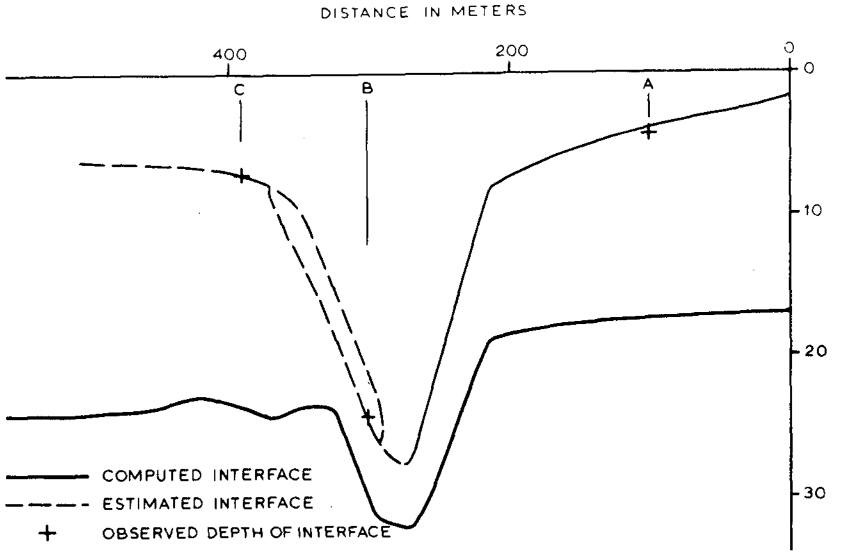


FIG. 10 DEPTH OF INTERFACE

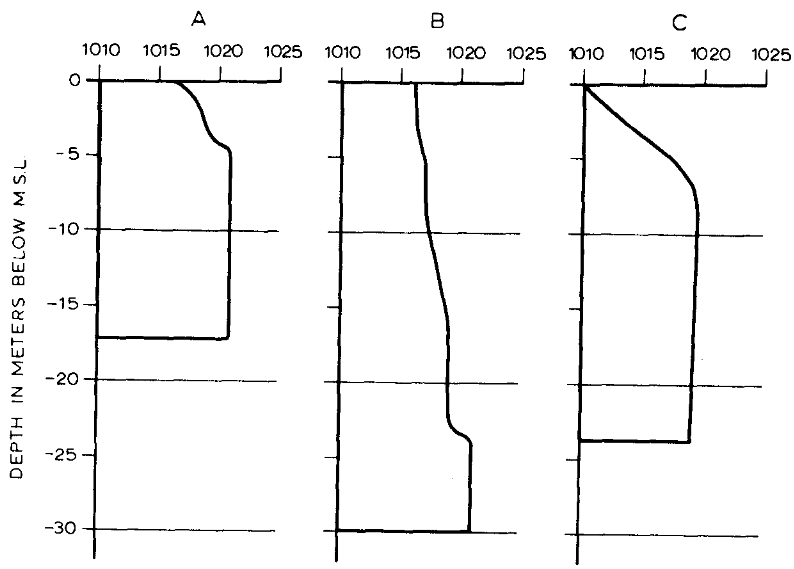


FIG. 11 DENSITYPROFILES AT POSITIONS A, B AND C.