CHAPTER 3

WIND WAVE FREQUENCIES IN A TROPICAL CYCLONE REGION

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INTRODUCTION

Australia's Coral Sea coast from Bundaberg north to Cape York has a wind wave climate that is almost unique. The coastline is afforded unparalleled protection from the 1900 km Great Barrier Reef, yet it lies in a tropical cyclone region and must expect recurrent intense wind and wave conditions.

The Great Barrier Reef is a continuous chain of quite separate coral reef clusters located near the edge of the continental shelf. The separate reefs are often exposed at low tide, the inner fringe of the clusters ranges from 10 km offshore north of Cairns to 200 km offshore south of Rockhampton and the outer fringe is typically some 50 km further offshore, beyond which the ocean bed drops rapidly away. Incident wave energy from the Coral Sea is invariably dissipated on the outer edge of the Reef and wave conditions on the continental shelf can reasonably be considered due to local wind conditions. The Reef imposes an effective fetch limitations on wave generation over the continental shelf and there is, as a consequence, a moderately rapid response of wave conditions to changes in local wind conditions. A pronounced diurnal variation in the wind climate is reflected also in the wave climate and the stability of the region's tropical climate leads to frequent calm to slight sea conditions. This stability however is occasionally exploded by the generation and passage of a tropical cyclone in mid to late summer. Large waves can be generated by the intense winds of the tropical cyclone (hurricane or typhoon), often an order of magnitude greater than those in response to non-cyclonic events.

The rational design of coastal structures and the rational pursuit of coastal zone management requires appropriate estimates of the frequency of occurrence of waves of various heights. Ideally such information is obtained from an extreme value analysis of long term wave records at the particular site in question. Permanent wave recording programs unfortunately have only become common practice in the present decade and wave records, if they exist at all for a particular site, are rarely long enough to allow a satisfactory extreme value analysis. It is clear, in the Australian context at least, that historical wave data alone is not yet sufficient to derive satisfactory estimates of long term wave frequencies. The alternative is system modelling.

Wind is a major meteorological variable and its long term recording has been a standard meteorological practice now for over half a century.

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Suitable data is normally available to estimate the statistical characteristics of a local wind climate. There is also now a developing confidence in wind wave forecasting techniques. Used together, it is possible to simulate several thousand years of statistically likely wind wave observations. A summary of these simulated observations will provide a reasonable estimate of long term wind wave frequencies for a particular site.

METHODOLOGY

Statistical simulation is now established as a basic technique in the study of a wide range of geophysical phenomena; earthquakes, precipitation, floods, wind, storm tides and wind waves have all been considered in this manner. The need for a stochastic approach stems from the recognised inadequacy of current deterministic models to make predictions beyond a few days. The present approach can be described as stochastic - deterministic; long term prediction of winds is essentially stochastic whereas the short term forecasting of the waves generated by a predicted wind field is deterministic. The approach involves a largely stochastic input (wind) to a deterministic system (wind-wave generation) yielding of course a stochastic output (wind waves). As pointed out by Nolte (7), the determination of wind wave statistics involves three important considerations: (i) desired end use, (ii) existing data base, and (iii) mathematical system model. All three should be defined and their compatibility established.

Desired End Use. - Essentially the objective is the estimation of the significant wave height - frequency relationship for a typical coastal site behind the Great Barrier Reef along Australia's Coral Sea coast. Reliable estimates of waves with average recurrence intervals in excess of 100 years are sought. It is recognised however that it is not always the maximum wave in any one year that causes the most damage. It may be that sustained attack from a more modest wave can be just as significant. The technique adopted should potentially allow the estimation of a wave height - duration - frequency relationship as well as the more common wave height - frequency relationship.

Existing Data Base. - Available data held by the (Australian) Bureau of Meteorology is far from perfect and the final result can be of course no better than the basic data. The adopted data base was drawn from Bureau records held in computer compatible form (1), specifically Card 7 "Hourly Surface Observations" and Card 13 "Tropical Cyclones and Tropical Disturbances". The Card 7 data includes three-hourly observations of mean wind speed and direction at coastal city airports. These sites are removed somewhat from the continental shelf but the quality of the data is good and the records are comprehensive. The wind speed observations are taken as wind speeds at the standard height of 10 m in the boundary layer. Card 13 data, while clearly the more important data source, is disappointing in terms of data quality. Notwithstanding some reservations they constitute the only historical data available for the estimation of long term wind fields, cyclonic and non-cyclonic, in the region.

SYSTEM MODEL

Even a brief consideration of the wind wave climate behind the Great Barrier Reef establishes the need to consider non-cyclonic as well as cyclonic conditions. While extreme wave conditions are invariably associated with a tropical cyclone, years pass without such an event in the immediate region. Even when a cyclone does influence the region it may not pass sufficiently close to affect the long term frequency of wind waves at the site. More often than not non-cyclonic winds will be responsible for the maximum wave conditions during any year and this fact must be included in the mathematical model.

Geophysical and especially hydrological time series $Z(t)$ are frequently postulated (5,8) as the summation of three separate components (see Fig. 1), such that

$$Z(t) = P(t) + C(t) + R(t). \tag{1}$$

$P(t)$ is a periodic component that is often possible to reconcile with known physical effects. Astronomical tide and, in the present context, diurnal and annual weather cycles are obvious examples. $C(t)$ is an infrequent catastrophic event such as an earthquake, a landslide and, in the present context again, a tropical cyclone. The residue $R(t)$ is a non-pure random series with a clear element of persistence. The component P is represented as a deterministic function of time whereas C and R are stochastic functions of time. Eq. 1 has been adopted as the basis of the mathematical system model.

The simulation methodology adopted is illustrated in Fig. 2. The cycle is initiated by determining the existence or otherwise of a tropical cyclone. Should one exist it must then be decided if it passes sufficiently close to the nominated site to influence the local wave climate. If it does not, this fact is recorded and the cycle is repeated. Should it influence the site the storm intensity and track are chosen and an estimate made of the maximum significant wave height at the site during the passage of the storm. This information is also recorded and the cycle repeated. When a tropical cyclone does not exist, and this is most of the time, non-cyclonic wind conditions are appropriate and the significant wave height at the site is predicted. Again this information is recorded and the cycle repeated. Details of this simulation cycle follow, firstly for non-cyclonic wind waves and then tropical cyclone wind waves.

Wave conditions throughout are represented solely in terms of significant wave height, with no consideration of directional spectra or even one-dimensional spectra. Further the Sverdrup-Munk-Bretschneider forecasting technique (13), deterministic and empirical, is adopted in the estimation of wind-generated waves under both cyclonic and non-cyclonic conditions.

CHARACTERISTICS OF NON-CYCLONIC WIND RECORDS

Following the adopted simulation methodology for non-cyclonic winds, the first step is the generation of a statistically equivalent wind record (magnitude and direction). A necessary preliminary step is an evaluation of the statistical characteristics of the wind climate, as represented by the Card 7 data, to assist in the selection of an appropriate synthetic

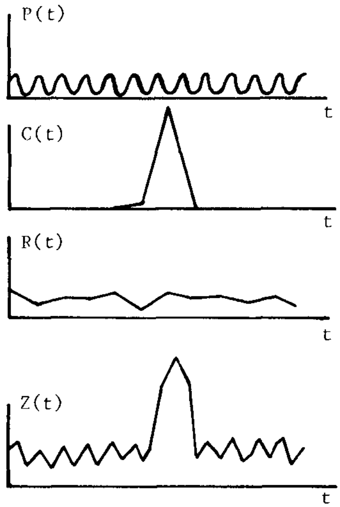

Fig. 1 GEOPHYSICAL TIME SERIES DECOMPOSITION

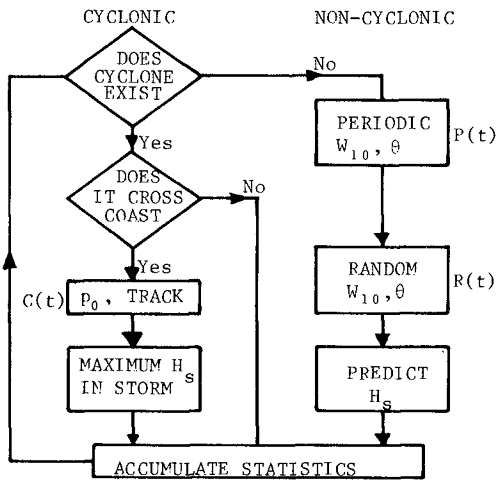

Fig. 2 WIND WAVE SIMULATION METHODOLOGY

generation technique. The available three-hourly wind records for two typical sites, Townsville and Rockhampton airports, were subjected to standard time series analysis techniques, namely spectral, correlation and probability analyses.

Spectral Analyses. - It was anticipated that the wind spectrum would cover a wide range of frequencies from seasonal effects around 0.003 cpd (cycles per day) to diurnal variations at 1 cpd to turbulence and gusts above 1000 cpd. Spectral resolution is bounded at the lower end by the time step Δt of the data series and at the upper end by the length of the data series, which is of course limited by computer storage. Using the FFT technique and a second data series formed from daily averages of the three-hourly records, a reasonable estimate of the wind magnitude and wind direction spectra can be obtained over the frequency range 0.000122 cpd to 4 cpd. The maximum frequency available is well below the range for gusts and turbulence which can not be discerned from the data available. This is no handicap however as such information is not required for the SMB wave forecasting technique.

Composite wind speed and wind direction power spectra for Townsville and Rockhampton airports are given elsewhere by Sobey, Rossow and McMonagle (11). All four spectra show the same trend, namely clear spectral peaks at frequencies of 1 cpd and around 0.0027 cpd, representing observable weather periodicities at periods of 1 day (diurnal) and 1 year (seasonal), together with a significant residual of essentially random noise throughout the spectrum.

Correlation Analyses. - Time domain analyses of the wind data in the form of auto and cross-correlation computations were undertaken in addition to the frequency domain analyses above. The base data only (Δt = 3 hr) was subjected to this analysis and a typical result (wind magnitude at Rockhampton airport) is shown in Fig. 3.

These results confirm the indications of the spectra. The diurnal periodicity is particularly clear and extension of the time-lag scale would also show the seasonal periodicity. The randomness of the residue is represented by the decay of the mean position to zero as the time-lag increases. The gradual exponential-like decay exhibits the characteristics of a Markov process in that substantial "memory" of preceding conditions is retained.

Probability Analyses. - Finally consideration was given to the amplitude domain characteristics. Figs. 4(a) and (b) show frequency histograms of wind magnitude and direction for Rockhampton airport. Davenport (3) considered the statistics of the total population of mean wind speeds and, assuming initially that the wind is isotropic and that there is no "prevailing" wind, showed that the mean wind followed the Rayleigh distribution. However prevailing wind conditions are normally experienced and Davenport has indicated that directional characteristics can reasonably be accommodated by the Weibull distribution, whose cumulative distribution function (CDF) is

$$F(W_{10}) = 1-\exp\left[-\left(\frac{W_{10}}{c}\right)^k\right] \qquad (2)$$

Additional flexibility is introduced by this two parameter distribution

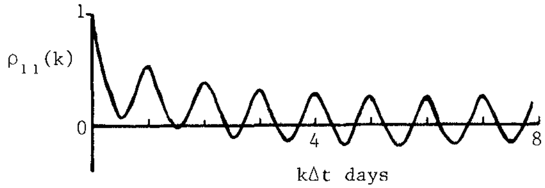

Fig. 3 WIND SPEED CORRELOGRAM AT ROCKHAMPTON

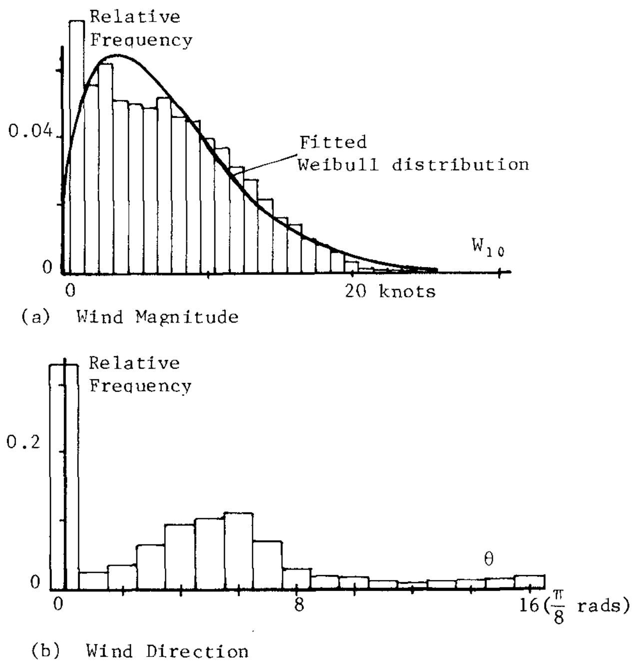

Fig. 4 WIND FREQUENCY HISTOGRAMS AT ROCKHAMPTON

compared with the single parameter Rayleigh distribution (The Weibull distribution reduces to the Rayleigh distribution when $c = (2)^{\frac{1}{2}}\sigma$ and $k = 2$). It is apparent from the histogram in Fig. 4(a) that a Weibull distribution would be appropriate.

Calm Observations. - The wind records for both Townsville and Rockhampton contain a significant number of calm observations (approximately 30% of the record); this is a common characteristic of normally stable tropical conditions and appropriate consideration must be given to this situation. For analysis purposes it may be considered in terms of conditional probability - a Bernoulli distribution for calm or non-calm conditions and appropriate distributions for magnitude (Weibull) and direction (discrete), given that a wind exists.

This aim was implicitly achieved in the following manner. Calm observations were retained only in the wind direction records where they were entered as zeros, whereas the sixteen compass points from NNE through S to N were entered as integers 1 to 16. The left hand block of the histogram in Fig. 4(b) represents the calm observations. If zero is retained as a "direction", a simulated direction of zero represents a calm day.

WEATHER PERIODICITIES

A major feature of the non-cyclonic wind records is the diurnal weather cycle. An annual weather cycle is also apparent but less pronounced. On this evidence the periodic component P(t) of Eq. 1 has been assumed to comprise only two Fourier components having angular speeds of $\omega = 2\pi$ rad/day and $\Omega = 2\pi/365.25$ rad/day respectively. The periodic component can be written as

$$P(t) = a \sin(\omega t + \phi) + A \sin(\Omega t + \Phi). \qquad (3)$$

It is necessary however to consider both magnitude and direction of the wind; magnitude can be represented at $P_1(t)$, direction as $P_2(t)$ and Eq. 3 rewritten as

$$P_{1,i} = a_1 \sin(\omega t_i + \phi_1) + A_1 \sin(\Omega t_i + \Phi_1) \qquad (4a)$$

$$P_{2,i} = a_2 \sin(\omega t_i + \phi_2) + A_2 \sin(\Omega t_i + \Phi_2). \qquad (4b)$$

Separation of the periodicities (i.e. determination of a_1, ϕ_1 ... A_2, Φ_2) is somewhat subjective. The initial approach was classical harmonic analysis, fitting Eq. 4 to the data by the method of least squares. Typically however the random component R(t) is at least the same order of magnitude as the periodic component and, while harmonic analysis was numerically successful, the computed amplitudes did not match in magnitude the measured spectral peaks. As an alternative the spectral peaks themselves can be used in the estimation of a_1, ϕ_1 ... etc.

From Parseval's theorem the amplitude of a single periodic component is related to the single-sided power spectrum P(f) as

$$a^2 = 2 \int_0^{f_N} P(f)\, df, \qquad (5)$$

from which it is possible to directly evaluate the amplitudes a_1, a_2, A_1, A_2 in Eq. 4 from the measured spectra. Estimation of the phase angles ϕ_1, ϕ_2, Φ_1, Φ_2 was much more subjective; it was based on visual estimates from a number of sample records, three-hourly observations for ϕ_1 and ϕ_2 and monthly averaged observations for Φ_1 and Φ_2. No doubt it is possible to improve this estimation but time did not permit pursuance of this point. The time datum throughout was taken as 00^{00} hours 1 July 1950.

With the periodic signal P(t) thus defined deterministically it may be subtracted from the complete record Z(t) leaving theoretically (see Eq. 1) the components C(t) and R(t). Strictly it would be further possible to separate tropical cyclone winds C(t) from this record. However the deletions from the record would be small in number and their retention would not unduly influence the population statistics of R(t). They would of course have a significant influence on the extreme value statistics of R(t) but the wind data has not been used for such a purpose.

RANDOM WIND COMPONENT

The auto-spectra for Townsville and Rockhampton indicate a substantial random-like residue that would remain after extraction of the ω and Ω periodicities; this is the component R(t) of Eq. 1. The associated correlograms (Fig. 3 is typical) support this observation to some extent but also show a high level of persistence or memory. Physically this represents the observable tendency of high winds from a particular direction to follow high winds from much the same direction and alternatively for calm conditions to follow calm conditions. In time series analysis this persistence is measured by the auto-correlation function ρ_k which is a measure of the degree of linear dependence between observations separated by time $k\Delta t$, Δt being the time interval (3 hrs) of the observations.

Again both components of the residue must be considered, $R_1(t)$ representing wind speed magnitude and $R_2(t)$ representing wind direction, the two series being of course related. A multivariate Markovian generating process proposed by Matalas (6) for the synthetic generation of streamflow at two sites in the same river basin would appear to be an appropriate and relatively straightforward generating process in the present context. For two related series $x_1(t)$ and $x_2(t)$ that are Normally distributed, the Matalas algorithm preserves the sample estimates of:

μ_1, μ_2	- the means of x_1 and x_2
σ_1, σ_2	- the variances of x_1 and x_2
$\rho_{11}(1)$, $\rho_{22}(1)$	- the lag-one auto-correlations
$\rho_{12}(0)$, $\rho_{21}(0)$	- the lag-zero cross-correlations, and
$\rho_{12}(1)$, $\rho_{21}(1)$	- the lag-one cross-correlations.

The multivariate generating process is

$$X_{i+1} = A\,X_i + B\,\varepsilon_{i+1} \tag{6}$$

where

$$X_i = \begin{bmatrix} x_{1,i} - \hat{\mu}_1 \\ x_{2,i} - \hat{\mu}_2 \end{bmatrix}$$

ε_{i+1} is a vector of independent N(0,1) distributed sampling deviates and the matrices A and B are defined as

$$A = M_1 M_0^{-1} \tag{7}$$

and $BB^T = M_0 - M_1 M_0^{-1} M_1^T$

where $$M_1 = \begin{bmatrix} \hat{\rho}_{11}(1)\hat{\sigma}_1^2 & \hat{\rho}_{12}(1)\hat{\sigma}_1\hat{\sigma}_2 \\ \hat{\rho}_{21}(1)\hat{\sigma}_1\hat{\sigma}_2 & \hat{\rho}_{22}(1)\hat{\sigma}_2^2 \end{bmatrix}$$

and $$M_0 = \begin{bmatrix} \hat{\sigma}_1^2 & \hat{\rho}_{12}(0)\hat{\sigma}_1\hat{\sigma}_2 \\ \hat{\rho}_{21}(0)\hat{\sigma}_1\hat{\sigma}_2 & \hat{\sigma}_2^2 \end{bmatrix}$$

Young and Pisano (15) have shown that B is not unique and, with $C = BB^T$, suggest using the lower triangular form

$$B = \begin{bmatrix} \sqrt{C_{11}} & 0 \\ \dfrac{C_{12}}{\sqrt{C_{11}}} & \sqrt{C_{22} - \dfrac{C_{12}^2}{C_{11}}} \end{bmatrix} . \tag{8}$$

There remains a problem in that the $x_1(t)$ and $x_2(t)$ series must be Normally distributed. As seen in Fig. 4 the distributions of the R_1 and R_2 series are clearly not Normal.

A preliminary step is the adoption of suitable distributions to describe the R_1 and R_2 series respectively. R_1 has been fitted to a Weibull distribution by the method of maximum likelihood. No attempt was made to fit any distribution to the R_2 series and it was retained in a discrete histogram form.

Young and Pisano discuss "minimum skewness" transformations of the base data series to yield series that more closely follow a Normal distribution, the minimum skewness referring to the fact that the Normal distribution has zero skewness. They mention logarithmic and square-root transformations in the context of streamflow simulation and adopt the former. In the present context the two related series are not physically the same, as for example in the dual site streamflow case, and it would seem unlikely that the same transformation would be equally satisfactory for both wind magnitude R_1 and wind direction R_2.

The adopted approach is rather direct; each residual is transformed to another having the same mean and variance but nominally zero skewness by matching the cumulative probabilities of the residual and normal distributions. The residual distribution was chosen as Weibull for the R_1 series and discrete for R_2. The $R_2 \rightarrow x_2$ transformation contains an additional step, the rotation of the direction datum so that the peak region is central to the record and the relocation of the calm observations (direction zero) to harmonise with this distribution. In both cases it was necessary to adopt truncated Normal distributions, omitting wind speeds less than zero and directions less than zero and greater than sixteen. This CDF transformation has proved moderately successful. The reverse transformation in both cases is equally straightforward.

The steps involved in preparation for the synthetic generation are -

1. Separate R(t) from data files Z(t)
2. Calculate $\hat{\mu}_1$, $\hat{\sigma}_1$ from $R_{1,i}$ and $\hat{\mu}_2$, $\hat{\sigma}_2$ from $R_{2,i}$
3. Fit Weibull distribution to $R_{1,i}$
4. CDF transformations on residual series $R_{1,i} \rightarrow x_{1,i}$ and $R_{2,i} \rightarrow x_{2,i}$
5. Calculate $\hat{\rho}_{11}(1)$, $\hat{\rho}_{22}(1)$, $\hat{\rho}_{12}(0)$, $\hat{\rho}_{21}(0)$, $\hat{\rho}_{12}(1)$ and $\hat{\rho}_{21}(1)$ from $x_{1,i}$ and $x_{2,i}$

The steps involved in the actual synthetic generation are -

1. Generate synthetic $x_{1,i}$ and $x_{2,i}$ series from Matalas algorithm
2. Inverse CDF transformation to synthetic $R_{1,i}$ and $R_{2,i}$ series: $x_{1,i} \rightarrow R_{1,i}$ and $x_{2,i} \rightarrow R_{2,i}$

Questions remain concerning the adequacy of the first-order Markov generating process in modelling long term persistence, for example the Hurst phenomenon. According to O'Connell (8), the retention of only lag-zero and lag-one correlations leads to the reproduction of only high frequency behaviour and cannot capture long term persistence. Several alternate generating processes have been proposed in the hydrology literature but there does not seem as yet to be any general agreement even in the case of streamflow. The Matalas algorithm certainly reproduces many of the observable features of geophysical time series and is just as certainly not the weak link in the present methodology.

NON-CYCLONIC WIND WAVE PREDICTION

At time t_i during non-cyclonic wind simulation the wind speed is

$$W_{10} = P_{1,i} + R_{1,i} \tag{9}$$

and the wind direction is

$$\theta = P_{2,i} + R_{2,i} \,. \tag{10}$$

Wind conditions are simulated every 3 hours. This wind is assumed active in direction θ for sufficient time that the generated sea condition is fetch-limited. This would certainly not always be true but becomes an increasingly better assumption for higher wind speeds where interest is of course centred; it will always be conservative. Adopted fetches along radiating lines, corresponding to the sixteen compass points, from the nominated site to the coast or the inner edge of the Great Barrier Reef are taken direct from appropriate navigation charts.

Assuming deep water conditions wind waves have been predicted directly from the empirical equations (13) to the Sverdrup-Munk-Bretschneider forecasting method:

$$\frac{gH}{W_{10}^2} = 0.283 \tanh\left[0.0125\left(\frac{gF}{W_{10}^2}\right)^{0.42}\right] \tag{11}$$

$$\frac{C}{W_{10}} = 1.20 \tanh\left[0.077 \left(\frac{gF}{W_{10}^2}\right)^{0.25}\right] \tag{12}$$

$$\frac{gt_D}{W_{10}} = K \exp\sqrt{A\ \ell n^2 \frac{gF}{W_{10}^2} - B\ \ell n \frac{gF}{W_{10}^2} + C_* + D\ell n \frac{gF}{W_{10}^2}} \quad (13)$$

where K,A,B,C_* and D are constants, H is the significant wave height, C the wave celerity, F the fetch length and t_D the wind duration. Again the assumption of deep water conditions is conservative. Eq. 11 has been used directly to estimate wave height at time t_i corresponding to the wind magnitude and direction indicated by Eqs. 9 and 10.

TROPICAL CYCLONE OCCURRENCES

The advent of a tropical cyclone is an infrequent event and has been considered as a completely separate influence on the local wind and wave climates. The basic steps of the non-cyclonic wind wave simulation described above, namely simulate the wind field, then predict the resulting wave field, are repeated in principle for tropical cyclone wind waves with the details of course much altered. These steps however must be preceeded by the simulation of the existence of a tropical cyclone, a step that has no parallel in the previous section. Given that a cyclone occurs, the methodology proceeds implicitly to the simulation of a moving wind field system and thence to the prediction of wind waves that would be generated at the nominated site by the moving wind field.

The stochastic modelling of tropical cyclone or hurricane occurrences in a particular region has been considered in detail by Russell (9), the approach adopted herein differing in detail but not in spirit. A suitable stochastic model should permit predicting multiple occurrence or none at all during the cyclone season. Within each southern hemisphere season, taken as mid-December to mid-April (a total of 121 days or 122 in a leap year), it should permit prediction of the time of occurrence of any events. Storm occurrences are assumed to follow a uniform Poisson process, which describes independent incidents occurring along a continuous time axis with a constant average rate of occurrence of λ per season. The distribution of storm occurrence time at particular time t_i then follows a Poisson distribution with parameter λt_i. The distribution of inter-arrival time τ between separate storms is more relevant in the present context however and, for a Poisson process, τ has an exponential distribution with parameter λ:

$$f(\tau) = \lambda e^{-\lambda\tau} \text{ for } \tau > 0. \quad (14)$$

No detailed analysis of the appropriateness of the Poisson process assumption has been undertaken, there being no reason to doubt its applicability in these circumstances.

TROPICAL CYCLONE WIND FIELD

The meteorological characteristics of a tropical cyclone and Australian data on the statistics of the various cyclone parameters are discussed in detail in Ref. 10. The tropical cyclone is essentially an atmospheric phenomenon, developing over tropical seas in mid to late summer. Extremely low central pressures (< 960 mb at M.S.L.), high vortex winds (> 40 m/s) and the presence of an eye are the dominant features of the atmospheric flow structure. The aerodynamics of the tropical cyclone and the hydrodynamics of the underlying water body are coupled at the water surface by the atmospheric pressure and wind shear stress. The M.S.L. pressure and sustained wind both change significantly with distance from the eye, which itself moves forward, typically in a south-westerly direction in the Coral Sea. The hydrodynamic response is complex and transient and comprises both long waves (surge) and the short waves (wind waves), the latter being of interest in the present context.

According to current wind wave generation theories it is the near surface spectrum of the turbulent pressure fluctuations that is directly responsible for wind wave generation. Information on this aspect of tropical cyclones is just not available although the mean flow pressure structure can be predicted with some confidence. Existing tropical cyclone wind wave theories tend to relate wave generation to the near surface mean flow wind structure, specifically in the SMB approach to W_{10}. The spatial distribution of W_{10} within a moving storm is still the matter of some debate and the parameterised wind field adopted in Ref. 10 will also be adopted here. This is basically the N.H.R.P. tropical cyclone model (4); in Ref. 10 this model has been mathematically formalised and suitably modified to describe a Coral Sea tropical cyclone. The basic storm parameters number four:
(1) Central pressure p_0 at Mean Sea Level
(2) Maximum sustained wind W_{10} at a height of 10 m above M.S.L.
(3) Radius to maximum wind R
(4) Speed V_E and direction of eye (i.e. track or path)
and moving wind fields are built up in terms of these parameters. A sample wind field is given in Ref. 11.

Following Russell, the next step is to assign, from the historical data or otherwise, suitable probability distributions to these four parameters. The adopted procedure to some extent follows Sobey, Harper and Stark (10) and Stark (12) and is closely related to the available historical data.

Central Pressure. - The Card 13 meteorological data was searched by computer to extract all tropical cyclones passing within 200 n miles of the nominated site. As the Card 13 data does not distinguish between tropical cyclones and tropical disturbances only those "hits" less than or equal to 990 mb have been retained and termed tropical cyclones for the purpose of this study, the less intense disturbances being implicitly considered as non-cyclonic events. It was assumed that the partial duration series so formed followed a Gumbel or Extreme Value Type I distribution, using smallest value criteria: the probability density function (PDF) is

$$f(p_0) = \alpha \exp[\alpha(p_0 - u) - e^{\alpha(p_0 - u)}], \qquad (15)$$

where α is the dispersion and u the mode of the Cumbel distribution. These distribution parameters are estimated from the partial duration series by the method of maximum likelihood. Also calculated was the Kolmogorov-Smirnov goodness-of-fit statistic, which measures the suitability of the assumed CDF in describing the observed cumulative frequency histogram. The K-S statistics indicate that the Cumbel distribution is by no means a prefect representation of the historical data. This however is considered more as a measure of the unsatisfactory nature of the data than a condemnation of the Gumbel distribution. Certainly other extreme value distributions could have been adopted but this would not improve the quality of the data.

Maximum Sustained Wind. - The modified N.H.R.P. model relates V_{10} directly to the central pressure as:

$$V_{10} = K\sqrt{\frac{p_\infty - p_0}{\rho_a e}} \quad . \qquad (16)$$

No probabilistic interpretation is introduced into this choice. The M.S.L. ambient pressure p_∞ has been taken as 1013 mb, the wind coefficient K has been set at unity (Ref. 10) and ρ_a is the mass density of air.

Radius to Maximum Wind. - Russell adopts a log-normal distribution for R from U.S. data for the Gulf of Mexico but equivalent Australian data is not available. Card 13 data has an entry for distance from storm centre to position of maximum *reported* mean wind in the area influenced by the tropical cyclone. This is of course unlikely to be the radius of maximum winds, as the data recording network together with additional ship reports provide only a very sparse coverage in northern Australia. In many cases also this entry has been left blank, in implicit recognition of this problem. The feasibility of relating R to central pressure and latitude is discussed in Ref. 10, but not adopted. It was concluded that a subjective decision on R must be made, which was to set it at 30 km for all Coral Sea tropical cyclones with no regional variation. This approach was also adopted herein.

Storm Track. - The Card 13 data is somewhat more helpful on this aspect although it is still not entirely adequate. Russell, for the Gulf of Mexico, adopted a Normal distribution for storm speed V_E, a uniform distribution for coast crossing position and a linear relationship between coast crossing heading and position; the Australian data is insufficient to allow such a detailed description of storm movements but it is possible to make some reasonable assumptions.

The data is entered as position co-ordinates (latitude and longitude) at various times throughout the life of the tropical cyclone, one such entry per card. The time spacing between cards is a minimum of 3 hrs but is often much longer. In the case of storm speed the data that can be extracted is rather scattered but an average value of 28 km/hr was established in Ref. 10 and is also adopted here. The data was considered too scattered to attempt the adoption of a distribution about this mean.

Data on coast crossings was also too sparse and too scattered to draw any conclusions. Russell's data was apparently not too much better and he found it necessary to assume a uniform distribution for coast crossing position for landfalling storms. This assumption is quite reasonable and is adopted for landfalling storms herein.

In formal terms the probability of a landfalling tropical cyclone crossing the coast a distance y from a nominated coastal site with equal probability over a length of coast extending a distance L both sides of the site is described by the PDF

$$f(y) = \begin{cases} \frac{1}{2L} & \text{for } -L \leqslant y \leqslant L \\ 0 & \text{elsewhere.} \end{cases} \tag{17}$$

The remaining decision concerns storm direction and here some very subjective assumptions are necessary as the Card 13 data is very inconclusive (10). It has been assumed that only one-half of all tropical cyclones passing within 200 n miles of the site actually cross the coast in this region and that these storms cross essentially at right angles to the general line of coastline; the remaining storms are assumed to move away from the site and not to unduly influence the coastal wave climate. In formal terms again the probability of a landfalling tropical cyclone, given that the storm exists, is described by the Bernoulli distribution: the probability mass function (PMF) is

$$p = \begin{cases} 0.5 & \text{for a landfalling storm} \\ 0.5 & \text{for a non-landfalling storm,} \end{cases} \tag{18}$$

TROPICAL CYCLONE WIND WAVES

Existing wind wave generation theories attempt to describe a process of gradual energy transfer to the sea across the air-sea interface, in which both wave height and wave celerity gradually increase under a steady wind towards some plateau level determined by the physical characteristics of the atmospheric boundary layer (represented by W_{10} alone in the SMB approach). The time scale of the energy transfer process is measured in hours and the atmospheric forcing must be maintained for the wave to continue to grow as it propagates across the sea. Should the wave impinge on a shoreline (a fetch limitation) or should the atmospheric forcing not be sustained (a duration limitation), then the wave may cease to grow before it reaches the plateau level, termed a fully arisen sea. Sustained forcing beyond the stage where there is sufficient time and sufficient fetch to attain the plateau level will not further increase the height or celerity of the wave. In physical terms the waves have been built up to a limiting steepness beyond which further energy transfer is dissipated in wave breaking.

Wave decay in deep water however is an even more gradual process. Swell waves behave almost as an ideal fluid and may propagate over hundreds of kilometres of ocean without appreciable decay. An opposing wind would of course hasten decay.

Wave generation within a tropical cyclone is clearly a very complicated process. Field observations are very sparse and rarely amount to more than water level versus time traces at a handful of sites. The spatial structure of the storm largely determines the distribution of wind waves and although peak waves do not necessarily coincide with peak wind velocities, it is implicit that there is a gradual decay away from the region of maximum waves. In deep water well beyond the edge of the continental shelf the spatial distribution of wind waves about the storm eye is thought to be broadly axi-symmetric, except for the forward motion of the storm, in a similar manner to the spatial distribution of sustained wind within the tropical cyclone. Fetch limitations imposed by the coastline as the storm crosses the continental shelf would have a substantial influence on this structure; the Great Barrier Reef introduces further fetch limitations and further complications. The spatial distribution of wind waves over the continental shelf will certainly be different from the offshore situation. A numerical model developed by Cardone, Pierson and Ward (2) to predict the directional spectra of hurricane-generated waves largely confirms this general description.

MODIFIED WILSON MODEL

The Cardone et al model is based on the numerical integration of the wave energy balance or radiative transfer equation. A much less time consuming though certainly less satisfactory approach is to represent the developing sea conditions in terms of just significant wave height rather than directional spectra. Such an approach can be based on the SMB empirical forecasting equations, rather than wind wave generation theories. This was in essence the approach adopted for non-cyclonic conditions.

Wilson (14) has proposed a numerical technique, based on the SMB equations, for estimating deep water wave generation by a wind system that varies in both space and time. Waves are implicitly assumed to propagate along a *straight line* path in co-ordinate direction s, along which the wind W_{10} varies with both position s and time t. The method also assumes that local wave conditions are uniquely described by the significant wave height H. The method adopted herein is essentially Wilson's except that due consideration has been given to the mathematical formulation of the problem and the consequent numerical integration.

In mathematical terms the wave generation process is assumed to be described in deep water by two simultaneous functional relationship of the form

$$H = H(t, W_{10}) \qquad (19a)$$

and

$$C = C(t, W_{10}) \qquad (19b)$$

where $W_{10} = W_{10}(s, t)$. When W_{10} is a constant, Eqs. 19 are identical to the SMB Eqs. 11 to 13. When W_{10} is not a constant the rate of change of H and C with time is

$$\frac{dH}{dt} = \frac{\partial H}{\partial t} + \frac{\partial H}{\partial W_{10}} \cdot \frac{dW_{10}}{dt} \qquad (20)$$

and

$$\frac{dC}{dt} = \frac{\partial C}{\partial t} + \frac{\partial C}{\partial W_{10}} \cdot \frac{dW_{10}}{dt} \qquad (21)$$

At a particular position s, W_{10} is a function of t only:

$$\frac{dW_{10}}{dt} = f(s,t) \qquad (22)$$

where the function f(s,t) is specified uniquely by the adopted moving wind system. The wave being generated travels at the instantaneous group velocity C_g ($=\frac{1}{2}$ C in deep water):

$$\frac{ds}{dt} = C_g. \qquad (23)$$

Eqs. 20 to 23 form a system of four simultaneous ordinary differential equations that can be integrated numerically in time by the Runge-Kutta method. Following Wilson in spirit but not in detail the partial derivatives on the right hand sides of Eqs. 20 and 21 can be estimated from the deep water SMB equations. The details can be found in Ref. 11.

There is some difficulty with the SMB equations when the wind opposes the direction of wave propagation for a period of time. The SMB equations themselves are strictly inapplicable but numerical integration remains continuous - $\frac{dH}{dt}$ and $\frac{dC}{dt}$ become negative, H and C both decrease and eventually a negative wave height is predicted. Physically of course this would not happen but there would be some attenuation. A conservative approach to this difficulty is adopted by setting the right hand sides of Eqs. 20 and 21 to zero whenever a negative value is computed.

The remaining problem in the mathematical formulation is the specification of suitable initial conditions at t = 0. It is assumed in general that the wave is initiated in still water, i.e.

$$H = C = s = 0 \text{ and } W_{10}(0,0) \text{ at } t = 0.$$

MAXIMUM SIGNIFICANT WAVE AT NOMINATED SITE IN A TROPICAL CYCLONE

The application of the above mathematical model to the estimation of tropical cyclone wind waves on the continental shelf is relatively straight-forward. Given the storm central pressure and track, a moving wind system is predicted. According to the Wilson model any straight line drawn across the continental shelf will represent a fetch along which a wave will propagate. It is a simple exercise in trigonometry to determine at any time from the moving wind field the wind speed component directed along the adopted fetch line; this becomes the local $W_{10}(s,t)$. Interest is largely centred on coastal sites and all waves arriving at such sites are assumed to have been initiated at the inner fringe of the Great Barrier Reef; no wave energy is assumed to penetrate beyond the Reef from the Coral Sea. To avoid the necessity to consider shoaling and refraction in the wave model the nominated coastal site is assumed to be a near shore deep water site typically 10 km offshore such that shallow water effects need not be considered; wave routing shoreward from this point would need to consider shoaling, refraction and wave breaking to arrive at a site design wave.

For a particular near shore site a number of radiating straight-line "fetches" can be drawn towards the Reef and waves can be computed along each of these fetches in turn. Under the complicated forcing resulting from the passage of a tropical cyclone, wave energy can reach a site from potentially all points of the compass, contributing to the local directional spectrum at that time. The significant wave height at that particular time is an integral measure of the directional spectrum. The wave model predicts significant waves arriving at the coastal site at a number of different times from a number of different directions; what to take as *the* maximum significant wave at the site is not particularly clear. The adopted procedure outlined below attempts to take some account of the development of the directional spectrum in the adoption of the maximum significant wave:

1. Radiating fetch lines are drawn from the coastal site towards the Great Barrier Reef at 15^0 spacings.
2. Waves are initiated at the Reef end of these fetch lines every half-hour during the passage of a tropical cyclone.
3. The two waves reaching the coastal site respectively before and after each integral hour along each fetch are interpolated to estimate the wave height for that fetch at that hour. Interpreting the radiating fetches as the contributions to the directional spectrum at the site at that time, an appropriate estimate of the wave height at the site at that integral hour is the root mean square value of the respective fetch contributions.
4. The maximum significant wave height for the particular site and storm track is taken as the highest value computed in 3 above; it typically occurs at one hour before storm landfall.

A computer program was developed for this purpose and could be used in the wave simulation procedure. However it was rather time consuming and it was decided instead to develop a somewhat simplified model of the situation in the form of a "universal" maximum significant wave profile that would obviate the need for such an extreme computational exercise.

COASTAL PROFILE OF MAXIMUM SIGNIFICANT WAVE

The steps adopted in the construction of a coastal profile of maximum significant wave height during the passage of a landfalling tropical cyclone were as follows:

1. The appropriate region of continental shelf between the Queensland coast and the Great Barrier Reef is idealised to a straightline coast and a straightline Reef perhaps angled to the coast and at a distance offshore from the site. The water over the continental shelf is assumed deep in the sense of the SMB forecasting equations.
2. Passage of a landfalling tropical cyclone across the continental shelf is assumed to be satisfactorily represented by storm movement at right angles to the straightline coast from sea to land.
3. A number of nearshore deep water coastal sites 10 km offshore are considered at 25 km spacings for distances N and S of landfall.
4. The maximum significant wave height was estimated at each of these coastal sites for storms with central pressures of 990 mb, 960 mb and 930 mb respectively. A typical result, for Cleveland Bay at Townsville, is shown in Fig. 5. The shape of these profiles was physically expected and is somewhat analogous to the storm surge profile that is forced by the same tropical cyclone. The waves peak approximately a radius of maximum winds to the south of the landfall position. In the range of central pressure considered there appeared to be an approximate linear relationship between increase in wave height at a particular position and decrease in central pressure.

TROPICAL CYCLONE WAVE SIMULATION

The steps involved in the tropical cyclone wind wave simulation can be summarised as follows:

1. Simulate the occurrence of the next tropical cyclone from the exponential distribution (Eq. 14) by the Monte Carlo procedure.
2. Simulate storm central pressure from the Cumbel distribution (Eq. 15) below 990 mb by the Monte Carlo procedure.
3. Simulate the storm track from the Bernoulli distribution (Eq. 18) by the Monte Carlo procedure. If not a landfalling storm return to the simulation of non-cyclonic conditions. If a landfalling storm simulate the coast crossing position from the uniform distribution (Eq. 17) by the Monte Carlo procedure.
4. For a landfalling storm predict the maximum significant wave height at the site from Fig. 5 or its equivalent for another site.

TYPICAL RESULTS

Some 5000 years of statistically likely wind wave records, nominally from 2001 AD to 7000 AD, can be simulated following the methodology described above. Typical results can be found in Ref. 11. While tropical cyclones are clearly responsible for the majority of the very high waves, non-cyclonic conditions make a dominant contribution to the long term statistics in the moderate wave height range.

The resulting significant wave height - average recurrence interval curve for Cleveland Bay at Townsville is shown in Fig. 6. The tendency towards a bi-modal character again illustrates the separate contributions to the long term statistics from cyclonic and non-cyclonic conditions. Such a curve would be appropriate for design purposes.

The procedures adopted for the wind wave simulation methodology will nominally estimate also the wave period associated with the predicted wave height at any given time. This would considerably increase the computational effort and was not attempted. However if the assumption is made that a fully arisen sea condition exists then a wave period appropriate to a nominated wave height may be estimated from the universal one-dimensional wave spectrum of Pierson and Moskowitz (13). Assuming the distribution of wave heights within the particular storm follows the accepted Rayleigh distribution, the significant wave height can be related to the total spectral energy and hence to the peak frequency or wave period of the Pierson-Moskowitz spectrum as

$$T_p = \pi\left(\frac{5}{\alpha}\right)^{\frac{1}{4}}\left(\frac{H}{g}\right)^{\frac{1}{2}} \qquad (24a)$$

where the Phillips constant α is 0.0081. Evaluating the constant term, Eq. 24a becomes

$$T_p = 15.66 \left(\frac{H}{g}\right)^{\frac{1}{2}} \qquad (24b)$$

The assumption of a fully arisen sea will not always be valid, in which case Eq. 24 will overestimate the wave period.

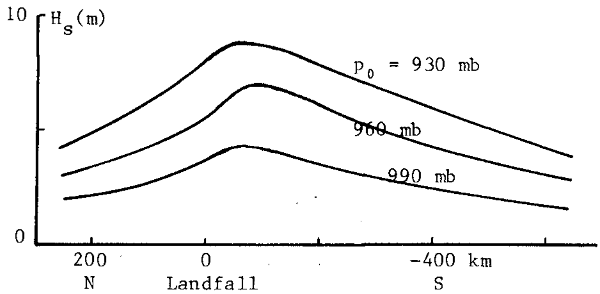

Fig. 5 COASTAL PROFILE OF TROPICAL CYCLONE WIND WAVES NEAR TOWNSVILLE

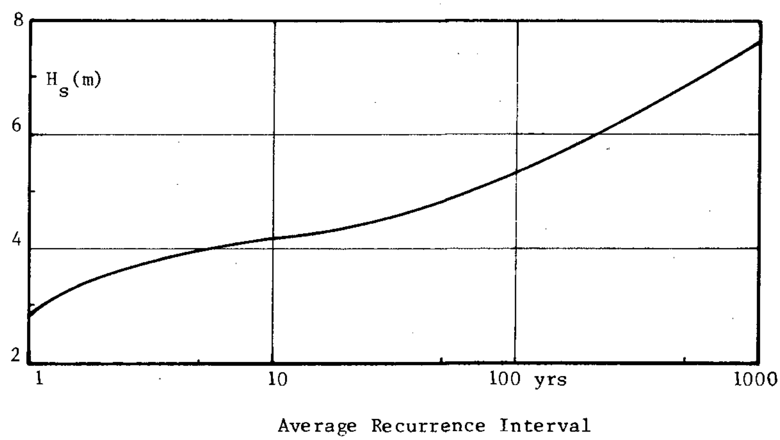

Fig. 6 WIND WAVE RETURN PERIODS FOR CLEVELAND BAY, TOWNSVILLE

CONCLUSIONS

In a general sense mathematical system modelling has been confirmed as a realistic alternative to historical wave data in the prediction of long term wind wave frequencies in a tropical cyclone region. In the vast majority of cases where historical wave data is either not available or covers only a few years, system modelling is clearly a superior approach when maximum advantage is taken of historical wind and tropical cyclone records, as in the present model.

In a tropical cyclone region the importance of non-cyclonic conditions for short and medium term recurrence intervals has been established. The resulting frequency distribution is clearly bi-modal representing the separate influences of cyclonic and non-cyclonic conditions.

More specifically for Australia's Coral Sea coast, the Great Barrier Reef effectively prevents wave penetration on to the continental shelf from further offshore and provides a significant fetch limitations on locally generated wind waves.

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