CHAPTER 4

COMPARISON OF SHIPBORNE WAVE RECORDER AND WAVERIDER BUOY DATA USED TO GENERATE DESIGN AND OPERATIONAL PLANNING CRITERIA

by

C.G. Graham¹, G.Verboom² and C.J.Shaw²

ABSTRACT

This paper presents the results of recent investigations at three sites where waves have been monitored simultaneously by two commonly used deep-water wave recorders, over a total period of 16 sensor-years. The study confirms earlier statements that there are relative differences between the wave parameters and statistical values calculated from the measurements of the two instruments. However, the large amount of data has enabled the authors to quantify the results in engineering terms and to assess the implications for extreme value analysis, spectral analysis and wave climate operational planning.

INTRODUCTION

A considerable proportion of the instrumental wave recordings collected in Northwest European Waters has been obtained using either shipborne wave recorders (SBWR) or wave-rider buoys (WRB). The offshore engineer, who needs to establish operational and design wave criteria for a particular site, will draw upon the nearest reliable set of wave data, which may have been collected by either or both instruments.

Where a relative accuracy (in terms of gain and offset) of ±10% is acceptable, then the sensor used to collect the measurements is probably immaterial (provided of course it has been properly maintained and calibrated). However, in the situation where a higher degree of accuracy is required then the 'absolute' accuracy of the instrument(s) used becomes a prime consideration.

A number of workers, for example Draper (ref. 9) and Van Aken (ref. 18) have reported the results of field and laboratory investigations into both the 'absolute' and relative accuracies of the SBWR and WRB. This paper is intended to complement this work, not by comparing a small sample of data, but by presenting the results of an overall examination of simultaneous measurements collected over a total period of 16 sensor-years. It is hoped that these results will provide a useful guide to interpreting SBWR and WRB data, especially where they both appear in the same data set.

PURPOSE OF THE INVESTIGATION

The last 10 years have seen a rapid expansion in hydrocarbon exploration and production activities in the waters of the Northwest European Continental Shelf. In order to carry out these activities as safely, efficiently and economically as possible, a considerable amount of information about the environment is needed. Ten years ago very little measured data were available for the Continental Shelf areas away from the coast (water depths 50m to 200m).

Today, with five to eight years of data from particular sites (though the data return has often only been in the order of 60 to 70%), our knowledge has increased significantly, but it must be realised that the amount of data is, statistically speaking, still limited. To provide the environmental operational and design criteria statistics required by the offshore oil and gas industries to the accuracy and confidence levels often quoted would, from a mathematical point of view, need many more years of data. Since it is totally unrealistic to wait such a length of time, theoretical methods (such as hindcasting and correcting for relative severity) have been developed which compensate for the limited amount of data. It is essential that the accuracy of the measurements themselves should be considered if these methods are to have any significant meaning.

What accuracy levels are required? For most offshore operations carried out by the oil industry a wave height accuracy, in the field and at the planning stage, in the order of ±4% is probably acceptable. However, what is unlikely to be acceptable is an offset or difference in gain between two measuring instruments — particularly at low wave heights — which would create an unwelcome bias to the results of any analysis work. For this situation, it is important to quantify the relative difference between the two instruments.

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The ‘absolute’ accuracy of wave heights is important for structural design, however, in particular for extreme value analysis. In the formulation of extreme wave height criteria, an accuracy of ±2% is sometimes quoted (e.g. Draper, ref. 9). While this degree of accuracy is certainly attainable with individual components in the measurement system, it is doubtful if the system itself is capable of such ‘absolute’ accuracy. Indeed in high sea states the problem is compounded since the sea surface itself becomes difficult to define.

This apparent need for such high degrees of accuracy is understandable when it is realised that an extra 0.3 metre on the design wave height for an offshore structure (approximately +1% in the Northern North Sea) could increase the cost of the project by as much as $2 million. While the industry certainly has no wish to under-design, neither does it wish to over-build since this is a waste of both money and scarce resources.

In some respects, all design work is empirical, so that the ‘absolute’ accuracy of an instrument might be regarded as academic if the data measured by one particular instrument (when fully calibrated) are generally accepted as the standard and are used as a basis for all design work. Measurements from the WRB (manufactured by Datawell) tend to be viewed in this light by some engineers.

When a relative difference appears consistently between two instruments — such as seen between the SBWR and WRB — then there is a real need to quantify the difference and this is the purpose of this paper.

BASIC DESCRIPTION OF THE SBWR AND WRB

The shipborne wave recorder was developed by the UK Institute of Oceanographic Sciences and has been described in detail by Tucker (ref. 16). It comprises two pairs of accelerometer and pressure units, one each side of the ship, approximately on the pitch axis. The accelerometers sense the heave of the ship which is itself a measure of the ship’s response to waves of wavelength rather longer than the vessel’s length, whilst the pressure units sense the short wavelength waves within the length of the ship. The output from the four sensors is combined to produce a wave signal which needs to be corrected for depth attenuation and instrumental response. As described by Van Aken (ref. 18), the SBWR has an advantage over the other wave measuring systems in that it is relatively easy to install (although dry-docking is usually necessary). It does have a drawback in terms of accurate calibration, originating from the fact that the wave signal is obtained synthetically from the ship’s movement and the water pressure on the vessel’s hull.

Details on calibrating and establishing the frequency response of the SBWR may be found in papers by Cartwright (ref. 1), Ewing (ref. 10), Darbyshire (ref. 2) and Van Aken (ref. 18).

A new integrated circuit version of the SBWR has recently been designed by I.O.S. and has been tested in service.

In 1965, the Datawell accelerometer Waverider buoy became available. This buoy is spherical, has a diameter of 0.7m (standard size) and produces a wave displacement signal by integrating twice the vertical accelerations of the buoy under the influence of wave action. This signal is then used to modulate a radio transmitter whose transmission can be received up to about 25 km away. The buoy accelerometer is kept in a vertical position by means of a cardanic suspension in a glass sphere filled with fluid.

The buoy itself can be moored in a number of different ways but the most common utilises 15 metres of rubber cord beneath the buoy, connected to a weighted mooring line leading to an anchor weight on the sea bed.

The response characteristics of the WRB are almost constant for wave frequencies between 0.065 Hz (15.4 sec) and 0.50 Hz (2 sec) being within 3% amplitude error, but for higher frequencies this response first increases due to resonance then tails off rapidly (ref. 3). The WRB is not truly ‘wave following’ since it is not a fixed point reference. It tends to rotate in the horizontal plane with the passing of each wave and in short-crested seas may have a tendency to ‘roll off’ the wave crests.
DATA SOURCES

The input wave data for this comparison study were collected by the UKOOA Oceano-
graphic Committee over a total period of nearly 16 wave sensor-years using weatherships at
two stations on the UK Continental Shelf, as shown in Figure 1.

<table>
<thead>
<tr>
<th>Position</th>
<th>Water Depth</th>
<th>Data Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stevenson Station</td>
<td>61°20'N., 0°00'E</td>
<td>159 from Feb. '73 to Feb. '76</td>
</tr>
<tr>
<td>Fitzroy Station</td>
<td>60°00'N., 4°00'W</td>
<td>122 from Dec. '73 to May '76</td>
</tr>
<tr>
<td>Boyle Station</td>
<td>50°40'N., 7°30'W</td>
<td>107 from May '74 to May '77</td>
</tr>
</tbody>
</table>

The following weatherships were used:

<table>
<thead>
<tr>
<th>Vessel</th>
<th>Length Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>m/v Edelstein (later renamed Silver Pit)</td>
<td>41.9 m</td>
</tr>
<tr>
<td>m/v Famita</td>
<td>41.9 m</td>
</tr>
<tr>
<td>m/v Ami</td>
<td>41.8 m</td>
</tr>
<tr>
<td>m/v Jomi</td>
<td>40.9 m</td>
</tr>
<tr>
<td>m/v Skagerak</td>
<td>40.9 m</td>
</tr>
</tbody>
</table>

Each ship used was fitted with a SBWR and maintained station within 2 miles of a moored
WRB. The output from both instruments was recorded simultaneously on paper chart rolls
as 15 minute samples every three hours. As an additional data recording method, the wave
traces were also recorded continuously on FM analogue magnetic tape.

Both SBWR and WRB instrument sets were subject to regular maintenance and calibration
checks. The SBWR transducer depths on the five ships ranged from 1.0 to 1.9 metres below
the water line.

DATA PROCESSING

The SBWR and WRB chart roll records were processed on-shore according to the standard
Tucker-Draper $H_1, H_2$ method of hand chart analysis.

A full description of this method is described in Tucker (ref. 17) and Draper (ref. 6).

From each 15-minute sample, the following wave parameters were computed:

- $H_s$: Significant Wave Height, the average height of the highest one third of the waves in
the sample. $H_s$ is related to the root mean square wave height and the square root of the sea
surface energy. In more simple terms, it is approximately the same as the average wave
height reported by an experienced observer through visual estimation.

- $H_{\text{max}}$ (3 hrs): The highest wave height computed to occur within the 3-hour recording interval by the
Tucker-Draper method.

- $T_z$: The Mean Zero Up-crossing Period. This is obtained by dividing the duration of the record
(in seconds) by the number of times the wave trace passes through the mean water level in an
upward direction.

For the SBWR, a correction is required in the calculations to compensate for depth attenu-
ation and instrumental response. This correction is defined by the equation (ref. 14):

$$K_{\text{SBWR}} = 0.83 \left[ 1 + (8.8\mu)^{-2} \right]^{3/2} \exp (\beta\mu^2 d/g) \ldots \ldots \ldots \ldots \ldots (1)$$

where

- $\mu = \frac{2\pi}{T_z}$ radians/sec
- $\beta = 2.5$ (a dimensionless constant)
- $g = 9.81$ m/sec$^2$
- $d = \text{depth of pressure unit below waterline in metres}$
Figure 1 - U.K.O.O.A. Weather Ship Stations
For the WRB, K usually equals 1.0 depending upon the instrument calibration, since the
instrument response is generally regarded as constant for the majority of wave frequencies
(ref. 3).

The following points are worth noting about the data processing method:

1. If an $H_s$ value is computed using the Tucker-Draper method from each of a series of
wave records taken under the same stationary wave conditions, then the $H_s$ values
obtained will be found to vary from record to record in a random manner. The value
required is the average of the large number of records measured at the same time in the
same sea state but since only one reading is usually available in practice, it is important
to realise that there is random error associated with each data value computed. The
error has been quoted by Draper (ref. 6) to be in the order of ±10% (one standard
error). The overall effect of this random error, when dealing with many data samples,
is smoothed out, so that the long-term wave height distribution of $H_s$ is unlikely to
be affected except at the tail of the distribution containing values for the single event
samples at the peak of the largest storm.

The implication of this for comparing WRB and SBWR $H_s$ values, computed by the
Tucker-Draper method, is that the comparison is 'clouded' by the random error effects
which produce appreciable scatter in the $H_s$ data points derived from the two instru-
ments. However, it is expected that a realistic smoothed out relationship between the
two instruments can be derived.

2. In the Tucker-Draper method, the computation of $H_s$ is related to the $T_z$ value. Thus
any differences in the $T_z$ values computed from the records of the two instruments will
also be reflected in the $H_s$ values computed. However, by examining Table 1 of Draper
(ref. 6) it will be seen that differences in $T_z$ would need to be quite large to affect the
value of $H_s$, so this factor will have only minor influence.

3. In equation (1) above a single coefficient 'K' is derived to correct the SBWR records
for the effects of depth attenuation and instrumental response. It can be seen that K
is dependent upon values input for $\beta$ (a constant), d (the transducer depth) and $T_z$.
Their relative influence on 'K' — and hence on the SBWR $H_s$ values computed — is
clearly demonstrated in Figures 2 and 3.

Figure 2 is for a transducer depth of 1.0 metre below the water line, while Figure 3 is
for 1.75 metre depth. Obviously at the greater depth a larger correction is needed and
therefore the value of 'K' used is greater. Curves are also presented in the figures for
the range of $\beta$ values sometimes applied (for these data $\beta = 2.5$ was used). Finally on
each graph has been presented a curve for $\beta = 2.5$ and $T_z$ multiplied by 1.25. This
curve has been presented to demonstrate that for mean wave periods below about 9.0
seconds, the value of 'K' is very dependent upon any changes in $T_z$. For example, for
d = 1.0m, $\beta = 2.5$ and a $T_z$ of 5 seconds, 'K' = 1.26. However, if a period of 6.25
seconds had been used, then 'K' would only equal 1.09.

4. Since both the Tucker-Draper $H_s$ and SBWR K values are related to $T_z$, it should be
emphasised that the SBWR/WRB comparison study results presented here are generally
only true for the particular size of weathership used, the depths of SBWR pressure
sensors and the particular wave climatic conditions experienced at the three weather-
ship station locations.
Figure 2 — SBWR Correction Curve — Transducer Depth 1.00m

Figure 3 — SBWR Correction Curve — Transducer Depth 1.75m
STATISTICAL METHODS USED TO COMPARE THE SBWR AND WRB PROCESSED DATA

The following statistical procedures were employed to compare the SBWR and WRB simultaneous Hs and Tz values:

1. **STANDARD LINEAR REGRESSION ANALYSIS**
   
   This analysis establishes how well a straight line defines the relationship between the two variables 'Y' and 'X'. The 'Y' on 'X' regression analysis computes the equation for a straight line by a least-squares fit parallel to the 'Y' axis, through the data points. The 'X' on 'Y' regression analysis, on the other hand, computes a similar straight line but by a least-squares fit parallel to the 'X' axis.

   The degree of scatter and 'goodness of fit' of a linear relationship between 'Y' and 'X' can be defined in terms of the dimensionless linear correlation coefficient, r. When r = 1, this indicates a perfect correlation, while r = 0 signifies no correlation at all.

   Also computed in this analysis is the 'standard error of estimate' which has properties analogous to the standard deviation. For example, in the 'Y' on 'X' regression analysis, the standard error of estimate, Syx, is computed. If we were to construct lines parallel to the regression line of 'Y' on 'X' at respective vertical distances Syx, 2Syx and 3Syx from it, we should find with sufficient data samples, that there would be included between these lines about 68%, 95% and 99.7% of the sample points.

   In the 'X' on 'Y' regression, a similar standard error of estimate is computed parallel to the horizontal axis.

2. **MAJOR AXIS LEAST-SQUARES REGRESSION ANALYSIS**
   
   In the standard linear regression analysis, two 'best lines' are fitted to the data, the first by minimising the error parallel to the vertical axis and the second by minimising parallel to the horizontal axis. However, in most cases, and in particular for this study, the co-ordinates of the sample data points are in error parallel to both axes together. Therefore, neither of the standard regression analysis 'best lines' is appropriate by itself and even if an arithmetic mean line is computed between them, the result is rather unsatisfactory.

   An improved method is to find a line such that the sum of the squares of the perpendicular distances of the points from this line is a minimum. This line is termed the Major Axis line and is described by York (ref. 20).

3. **MAJOR AXIS ANALYSIS, LINE CONSTRAINED TO PASS THROUGH ORIGIN**
   
   For this, the major axis computations are adapted to produce a line constrained to pass through the origin. In this way the relationship between the 'Y' and 'X' data points can be simply defined in terms of the slope of this line. Such an analysis becomes particularly appropriate when more than two variables are compared together, as illustrated by the comparison of three wave sensors in Pitt et al. (ref. 15).

4. **GROUPED CLASS INTERVAL ANALYSIS**
   
   The fourth method employed is described as 'Grouped Class Interval Analysis' and was applied in an attempt to see how any differences between 'Y' and 'X' varied from class interval to class interval.

   The 'X'-axis is split up into equal intervals. For each of these intervals the mean 'X' and mean 'Y' are calculated together with the standard deviation of 'Y'. Unfortunately this analysis method suffers from the same limitations as the standard 'Y' on 'X' linear regression analysis method in that only scatter parallel to the vertical axis is examined, while the scatter parallel to the horizontal axis is ignored.
5. **EQUI-PROBABILITY ANALYSIS**

If there are a series of $N$ values of $X$ and $N$ values of $Y$, then these two data sets may be arranged in ascending order:

\[
\begin{align*}
X_1, X_2 & \ldots \ldots X_m, \ldots \ldots X_n \\
Y_1, Y_2 & \ldots \ldots Y_m, \ldots \ldots Y_n
\end{align*}
\]

such that $Q(X_m)$ and $Q(Y_m)$ are the respective cumulative probabilities of $X$ and $Y$ exceeding $X_m$ and $Y_m$.

If $Q(X)$ is set equal to $Q(Y)$ then an equi-probability analysis can be carried out to determine the relationship between $Y$ and $X$ over the whole of the cumulative probability distribution.

This form of analysis is particularly appropriate since the $H_s$ wave height cumulative probability distribution is often used directly by the offshore engineer in both the planning and design function.

6. **EXTREME VALUE ANALYSIS**

One method of extreme value analysis is to extrapolate cumulative probability distributions (all data points included) in order to estimate the values for extreme events with return periods of 1, 10, 50 or 100 years. (N.B. by themselves, 5 years or so of wave data are inadequate for this process without some form of mathematical correction for the relative severity of the particular 5 years covered by the measurements.)

For a comparison of SBWR and WRB data it is therefore important to assess the effect of any differences on the results of the extreme value analysis.

The extreme value analysis itself is based on the best fit of the cumulative probability distribution to a particular mathematical equation. In graphical terms, this best fit is reflected in how well the data points making up the distribution plot out as a straight line on the relevant cumulative probability graph paper.

For wave height data a number of different distribution equations are commonly applied. These include the Weibull, log-normal, Gumbel's Third Asymptote and Gumbel's First Asymptote. For this study, the Weibull scale distribution (ref. 17) was chosen on the basis of Jenkinson (ref. 14).

The SBWR and WRB $H_s$ cumulative probability distribution data points were plotted out on the Weibull scale and a least-squares criterion was used to establish the two ‘best fit’ lines. These lines were then extrapolated up to the probability levels associated with return periods of 1, 10, 50 and 100 years, making it possible to compare the extreme event $H_s$ values predicted by the SBWR and WRB distributions.

Out of these six comparison methods it was found that the Major Axis Line Through the Origin and the Equi-probability Analyses produced the most meaningful results.

**PRESENTATION AND DISCUSSION OF STUDY RESULTS**

The statistical methods described were applied in turn to the SBWR and WRB processed $H_s$ and $T_z$ values. Initially each monthly set of data was examined, before carrying out an overall analysis for each of three weathership stations and all three stations combined. The characteristics displayed in the overall analyses were reflected in the monthly data. Though fluctuations were noted from month to month, no significant trend was evident — except of course for a natural bias in the summer months towards lower wave heights compared with the winter months. A sample month of simultaneous $H_s$ and $T_z$ data is presented in Figures 4 and 5.

Co-ordinates:  

\[
\begin{align*}
Y & \text{ axis } - \text{SBWR (H}_s\text{ or T}_z\text{ values)} \\
X & \text{ axis } - \text{WRB (H}_s\text{ or T}_z\text{ values)}
\end{align*}
\]
Figure 4 — Comparison of SBWR and WRB — Significant Wave Height

Figure 5 — Comparison of SBWR and WRB — Mean Zero Up-crossing Wave Period
COMPARISON OF SBWR AND WRB SIGNIFICANT WAVE HEIGHTS

Table 1 below presents the results of the linear regression and major axis analyses:

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Station A</th>
<th>Station B</th>
<th>Station C</th>
<th>All Stations</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of data point pairs</td>
<td>4273</td>
<td>3350</td>
<td>3860</td>
<td>11,483</td>
</tr>
<tr>
<td>Linear correlation coefficient 'r'</td>
<td>0.952</td>
<td>0.942</td>
<td>0.943</td>
<td>0.949</td>
</tr>
<tr>
<td>Y on X Linear Regression Analysis</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope ($M_{YX}$)</td>
<td>0.975</td>
<td>1.016</td>
<td>1.013</td>
<td>1.002</td>
</tr>
<tr>
<td>Intercept ($C_{YX}$)</td>
<td>+0.334m</td>
<td>+0.206m</td>
<td>+0.145m</td>
<td>+0.225m</td>
</tr>
<tr>
<td>Standard Error of Estimate ($S_{YX}$)</td>
<td>0.507m</td>
<td>0.539m</td>
<td>0.403m</td>
<td>0.487m</td>
</tr>
<tr>
<td>X on Y Linear Regression Analysis</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope ($M_{XY}$)</td>
<td>0.929</td>
<td>0.874</td>
<td>0.878</td>
<td>0.899</td>
</tr>
<tr>
<td>Intercept ($C_{XY}$)</td>
<td>-0.042m</td>
<td>+0.146m</td>
<td>+0.123m</td>
<td>+0.063m</td>
</tr>
<tr>
<td>Standard Error of Estimate ($S_{XY}$)</td>
<td>0.495m</td>
<td>0.500m</td>
<td>0.375m</td>
<td>0.461m</td>
</tr>
<tr>
<td>Major Axis Line (Y on X)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope (b)</td>
<td>1.025</td>
<td>1.084</td>
<td>1.079</td>
<td>1.059</td>
</tr>
<tr>
<td>Intercept (a)</td>
<td>+0.190m</td>
<td>+0.010m</td>
<td>-0.004m</td>
<td>+0.073m</td>
</tr>
<tr>
<td>Standard Deviation of Slope $\sigma_b$</td>
<td>0.005</td>
<td>0.007</td>
<td>0.006</td>
<td>0.003</td>
</tr>
<tr>
<td>Standard Deviation of Intercept $\sigma_a$</td>
<td>0.758m</td>
<td>0.856m</td>
<td>0.703m</td>
<td>0.759m</td>
</tr>
<tr>
<td>Major Axis constrained to pass through origin</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope (b)</td>
<td>1.077</td>
<td>1.086</td>
<td>1.078</td>
<td>1.080</td>
</tr>
<tr>
<td>Standard Error of Estimate</td>
<td>0.364m</td>
<td>0.372m</td>
<td>0.279m</td>
<td>0.340m</td>
</tr>
</tbody>
</table>

Table 1 - Results of Linear Regression and Major Axis Analysis - SBWR/WRB Hs Comparison

The linear regression analysis results presented above reflect the scatter associated with the Tucker-Draper method, quoted earlier at ±10% for the computation of $H_s$. Some variations in the results from the three stations are noted, in particular with the 'Station A' ordinary major axis analysis. However, the final analysis of the major axis line constrained to pass through the origin produced virtually identical results for all three weathership locations in that on average:

$$H_s^{SBWR} = 1.08 H_s^{WRB}$$
Further analyses were applied to see if this relationship varied with wave height; the most meaningful results were obtained from the equi-probability and extreme value analyses. The results for each threshold wave height are set out for each station in Table 2 below:

<table>
<thead>
<tr>
<th>WRB H_s Threshold Wave Height</th>
<th>Percentage Increase H_s SBWR over WRB</th>
<th>Station A</th>
<th>Station B</th>
<th>Station C</th>
<th>All Stations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>+18.0</td>
<td>+17.0</td>
<td>+10.0</td>
<td>+14.0</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>+17.3</td>
<td>+11.3</td>
<td>+4.7</td>
<td>+10.0</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>+11.0</td>
<td>+10.5</td>
<td>+7.5</td>
<td>+10.0</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>+11.2</td>
<td>+8.0</td>
<td>+6.8</td>
<td>+8.0</td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td>+9.3</td>
<td>+8.3</td>
<td>+9.0</td>
<td>+7.3</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>+8.6</td>
<td>+7.1</td>
<td>+8.6</td>
<td>+7.1</td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td>+7.5</td>
<td>+8.0</td>
<td>+7.5</td>
<td>+8.3</td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td>+6.9</td>
<td>+7.1</td>
<td>+11.3</td>
<td>+8.9</td>
<td></td>
</tr>
<tr>
<td>5.0</td>
<td>+6.2</td>
<td>+6.0</td>
<td>+10.0</td>
<td>+6.0</td>
<td></td>
</tr>
<tr>
<td>5.5</td>
<td>+5.6</td>
<td>+7.3</td>
<td>+9.1</td>
<td>+5.5</td>
<td></td>
</tr>
<tr>
<td>6.0</td>
<td>+3.5</td>
<td>+8.3</td>
<td>+8.3</td>
<td>+8.3</td>
<td></td>
</tr>
<tr>
<td>6.5</td>
<td>+4.6</td>
<td>+9.2</td>
<td>+4.6</td>
<td>+6.2</td>
<td></td>
</tr>
<tr>
<td>7.0</td>
<td>+6.3</td>
<td>+7.1</td>
<td>+1.4</td>
<td>+7.1</td>
<td></td>
</tr>
<tr>
<td>7.5</td>
<td>+3.7</td>
<td>+5.3</td>
<td>-1.3</td>
<td>+5.3</td>
<td></td>
</tr>
<tr>
<td>8.0</td>
<td>+6.3</td>
<td>+6.3</td>
<td>-3.8</td>
<td>+5.0</td>
<td></td>
</tr>
<tr>
<td>8.5</td>
<td>+7.9</td>
<td>+2.9</td>
<td>0.0</td>
<td>+5.9</td>
<td></td>
</tr>
<tr>
<td>9.0</td>
<td>+8.9</td>
<td>+7.8</td>
<td>0.0</td>
<td>+5.6</td>
<td></td>
</tr>
<tr>
<td>9.5</td>
<td>+7.9</td>
<td>+6.3</td>
<td>*</td>
<td>+5.3</td>
<td></td>
</tr>
<tr>
<td>10.0</td>
<td>*</td>
<td>+5.0</td>
<td>*</td>
<td>+3.0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Return Period</th>
<th>Percentage Increase H_s SBWR over WRB</th>
<th>Station A</th>
<th>Station B</th>
<th>Station C</th>
<th>All Stations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Year</td>
<td>+3.4</td>
<td>+4.4</td>
<td>+1.7</td>
<td>+5.1</td>
<td></td>
</tr>
<tr>
<td>10 Years</td>
<td>+3.2</td>
<td>+3.7</td>
<td>+0.8</td>
<td>+4.6</td>
<td></td>
</tr>
<tr>
<td>50 Years</td>
<td>+3.6</td>
<td>+3.2</td>
<td>+0.2</td>
<td>+4.3</td>
<td></td>
</tr>
<tr>
<td>100 Years</td>
<td>+2.8</td>
<td>+2.9</td>
<td>-0.1</td>
<td>+4.2</td>
<td></td>
</tr>
</tbody>
</table>

*Insufficient data points for computation*

Table 2 — Results of Equi-probability and Extreme Value Analyses

The results for all stations combined are presented in Figure 6, the left-hand ordinate is scaled in terms of the SBWR threshold wave heights, while the right-hand ordinate is scaled in terms of the percentage increase of the SBWR H_s over the WRB.

The equi-probability analyses do exhibit a fairly constant percentage increase of the SBWR H_s over the WRB for the central band of wave heights. However, at higher wave heights this percentage appears to reduce, particularly at Station C, though the effect is less at Station A and B. The extreme value analysis goes on to confirm the tendency but it must be borne in mind that this latter exercise is bound by two limitations:
Figure 6 — Results of $H_s$ Equi-probability and Extreme Value Analyses — All Stations

Figure 7 — Results of $T_z$ Equi-probability Analysis — All Stations
1. The data have been constrained to follow the equation of the Weibull cumulative probability distribution.

2. The extrapolation process is very dependent upon the scatter in the data points and the 'best fit' line computed.

It would therefore be dangerous to conclude too much from these results but they do, nevertheless, demonstrate the practical implications when interpreting or extrapolating data sets from the two sensors.

The apparent convergence between the SBWR and WRB $H_s$ values as the wave height increases is probably explained by the operation of the SBWR instrument itself. At wavelengths longer than the length of the weathership (which are usually associated with high wave heights), the SBWR accelerometer components predominate (the ship effectively becomes a waverider) while the pressure transducers contribute very little. For seas shorter than the length of the ship (and therefore lower wave heights) the situation is reversed. In other words, from the analysis viewpoint, at high wave heights, the value of 'K' in equation (1), used to correct the SBWR data for instrument response, is approximately equal to 1.0. At lower wave heights, when the wave periods tend to be shorter, the value of 'K' needs to be greater than 1.0 to correct for the now influential pressure transducers.

Further analyses such as comparing wave heights in particular wave period bands were considered but are outside the scope of this particular study.
Comparison of SBWR and WRB Mean Zero Up-Crossing Periods

Table 3 below presents the results of the linear regression and major axis analyses:

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Station A</th>
<th>Station B</th>
<th>Station C</th>
<th>All Stations</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of data point pairs</td>
<td>4273</td>
<td>3350</td>
<td>3860</td>
<td>11,483</td>
</tr>
<tr>
<td>Linear Correlation Coefficient – ‘r’</td>
<td>0.753</td>
<td>0.755</td>
<td>0.814</td>
<td>0.780</td>
</tr>
<tr>
<td>Y on X Regression Analysis</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope (M_{yx})</td>
<td>0.713</td>
<td>0.706</td>
<td>0.691</td>
<td>0.710</td>
</tr>
<tr>
<td>Intercept (C_{yx})</td>
<td>+2.324 sec</td>
<td>+2.630 sec</td>
<td>+2.656 sec</td>
<td>+2.471 sec</td>
</tr>
<tr>
<td>Standard Error of Estimate (S_{yx})</td>
<td>0.926 sec</td>
<td>0.889 sec</td>
<td>0.794 sec</td>
<td>0.879 sec</td>
</tr>
<tr>
<td>X on Y Regression Analysis</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope (M_{xy})</td>
<td>1.611</td>
<td>0.807</td>
<td>0.959</td>
<td>0.856</td>
</tr>
<tr>
<td>Intercept (C_{xy})</td>
<td>+0.796 sec</td>
<td>+1.530 sec</td>
<td>+0.112 sec</td>
<td>+1.063 sec</td>
</tr>
<tr>
<td>Standard Error of Estimate (S_{xy})</td>
<td>0.978 sec</td>
<td>0.951 sec</td>
<td>0.936 sec</td>
<td>0.965 sec</td>
</tr>
<tr>
<td>Major Axis Line (Y on x)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope (b)</td>
<td>0.930</td>
<td>0.915</td>
<td>0.818</td>
<td>0.888</td>
</tr>
<tr>
<td>Intercept (a)</td>
<td>+0.591</td>
<td>+0.853</td>
<td>+1.656</td>
<td>+1.035</td>
</tr>
<tr>
<td>Standard Deviation of Slope (σ b)</td>
<td>0.012 sec</td>
<td>0.014 sec</td>
<td>0.009 sec</td>
<td>0.007 sec</td>
</tr>
<tr>
<td>Standard Deviation of Intercept (σ a)</td>
<td>2.496 sec</td>
<td>2.500 sec</td>
<td>1.832 sec</td>
<td>2.228 sec</td>
</tr>
<tr>
<td>Major Axis constrained to pass through origin</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td>1.002</td>
<td>1.014</td>
<td>1.022</td>
<td>1.012</td>
</tr>
<tr>
<td>Standard Error of Estimate</td>
<td>0.721 sec</td>
<td>0.699 sec</td>
<td>0.670 sec</td>
<td>0.699 sec</td>
</tr>
</tbody>
</table>

Table 3 — Results of Linear Regression and Major Axis Analysis — SBWR/WRB Tz Comparison

The linear regression analysis results — in particular the correlation coefficients in the order of only 0.75 — reflect a considerable scatter in the Tz calculated from the records of the two instruments. Significantly, for all three stations, each major axis line constrained to pass through the origin has a slope of approximately 1.0, thus indicating that on average the mean zero up-crossing periods from the two instruments are equal. This apparent equality should be judged only in the context of the associated scatter.
The relatively larger intercept values and slopes less than 1.0 in the equations of the ordinary major axis lines might suggest that the SBWR tends to give longer wave periods. This aspect was investigated further and the most realistic results were produced by the equi-probability analysis. It was found that for all three weatherships, the SBWR and WRB Tz cumulative probability distributions were virtually the same. However, a slight tendency was noted for the SBWR wave periods to be longer than the WRB at low periods, while at high periods the opposite was found. This tendency can be seen in the Tz equi-probability results presented in Figure 7.

In effect, all the analyses showed that, on average, the SBWR and WRB Tz's are nearly the same, while for individual wave samples, considerable scatter can be expected. This scatter is probably caused by one or more of the following factors acting together:—

1. The SBWR and WRB are not measuring the same waves. Though sample times are simultaneous, the two instruments are separated by anything up to 2 miles.
2. The response characteristics of the weathership in different sea states and combinations of wind-waves and swell will vary and thus might affect the recordings.
3. Ship's heading relative to the sea.
4. The inherently different noise characteristics of the two instruments.

SUPPLEMENTARY ANALYSIS — WAVE SPECTRA

A comparison of simultaneous SBWR and WRB data in the frequency domain is presently under study and has yet to be completed.

It is expected that this work will produce different comparison results from those presented above. It is felt that the prime reason for this will be that in the frequency domain, it is possible to correct for the SBWR instrument response at each frequency component, while in the time domain analysis, the correction is dictated solely by the Tz count.

Figure 8(a) shows an example of simultaneous SBWR and WRB spectra which happen to compare rather favourably while Figure 8(b) is an example of a not so favourable comparison.

SUMMARY OF CONCLUSIONS

1. On average, the SBWR was found to record significant wave heights 8% higher than the WRB. This percentage difference was greater at low wave heights and less at high wave heights.
2. For most planning and design work it is recommended that this relative difference should be taken into account, especially in the situation where a data set contains a mixture of Hs values from the two instruments.
3. The convergence of the SBWR and WRB data as the wave height increases has important implications for extreme value analysis. The convergence is carried through into the extrapolations and into any extreme values thereby derived. This effect should be considered in the evaluation process to establish design wave height values.
4. On average, it was found that mean zero up-crossing periods from the two instruments were the same. However, appreciable scatter about the average was noted in the results.
5. Care should be exercised in applying the results of this study to significantly different SBWR systems (in terms of vessel length and transducer depth).
6. This study has highlighted the obvious benefit of having two sensors measuring the same parameter. Equipment faults and sensor drifts which did not show up initially often become very apparent when data from the SBWR and WRB were processed and then compared.

ACKNOWLEDGEMENTS

The authors wish to thank the Oceanographic Committee of the United Kingdom Offshore Operators Association for permission to utilize their proprietary wave records in the preparation of this study.
Figures 8(a) and 8(b) — Comparison of Simultaneous Shipborne Wave Recorder and Waverider Buoy Spectra.
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